MICRO-523: Optical Detectors

Week Two: Detector Formalism and Noise

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Outline

Exercise 2.1: Quantum Efficiency and Detectivity

Consider a semiconductor photodiode with a band gap E_g and an ideal quantum efficiency.

Sketch:

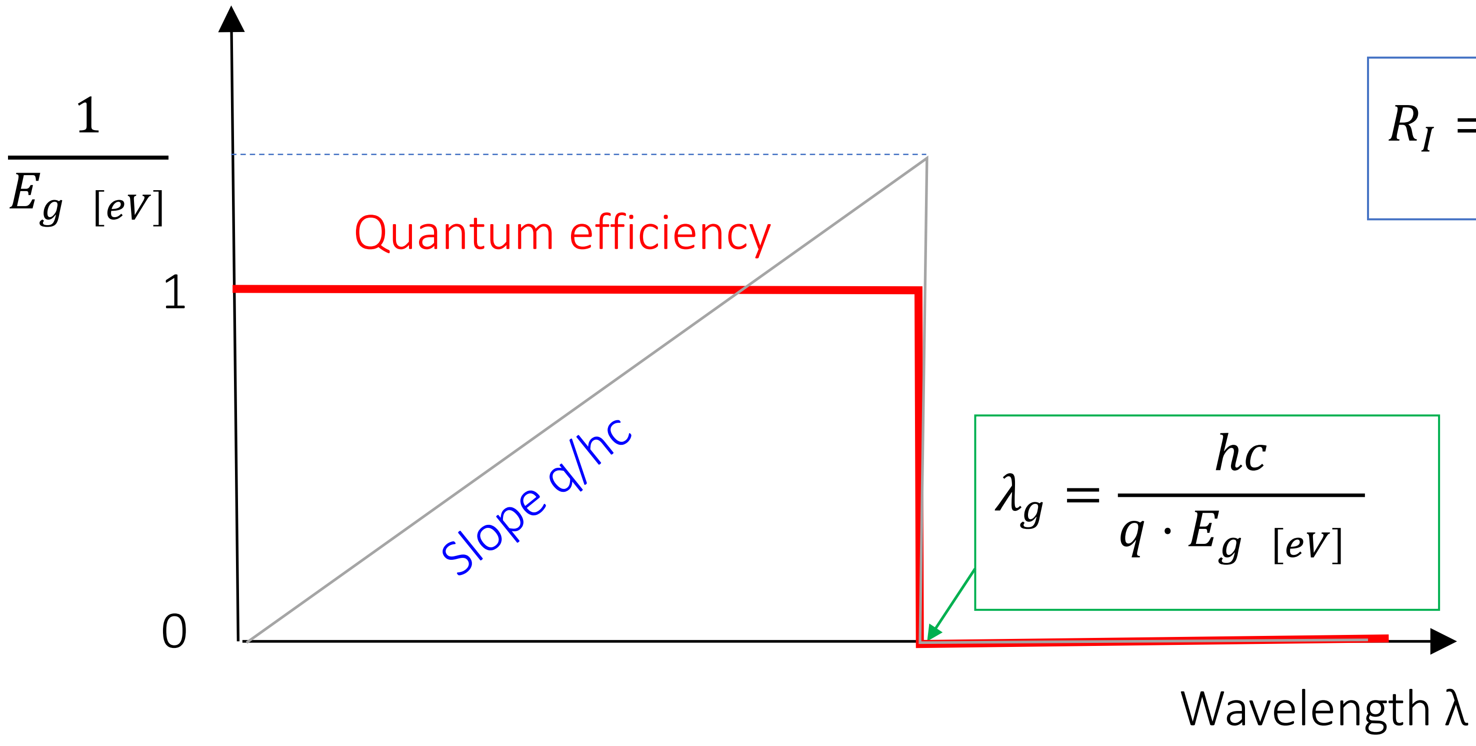
- its quantum efficiency η and
- its responsivity R_I

as a function of the wavelength of the incident photons.

Consider the noise N to be independent of wavelength.

Sketch its detectivity as a function of its wavelength.

Exercise 2.1: Quantum Efficiency

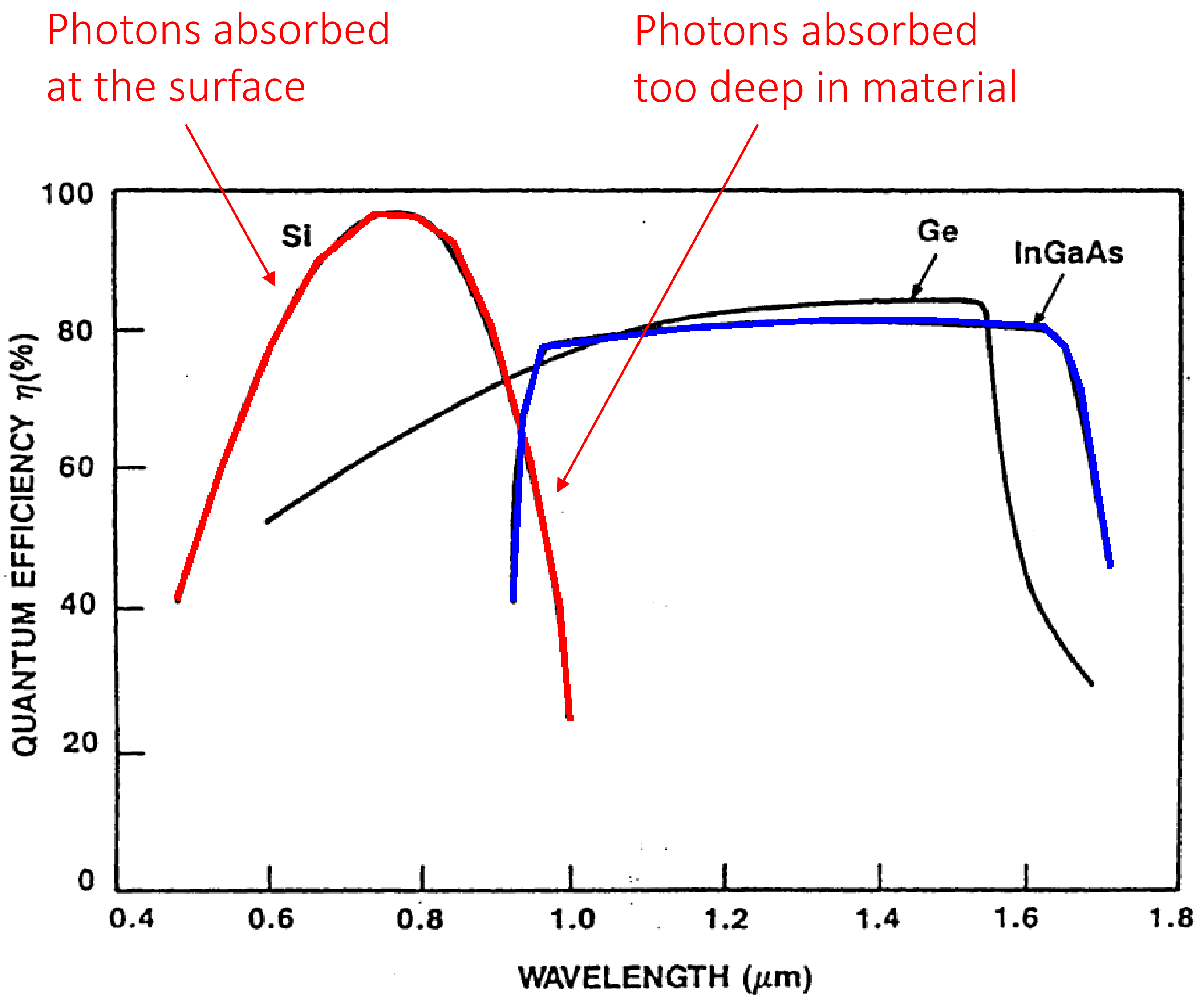


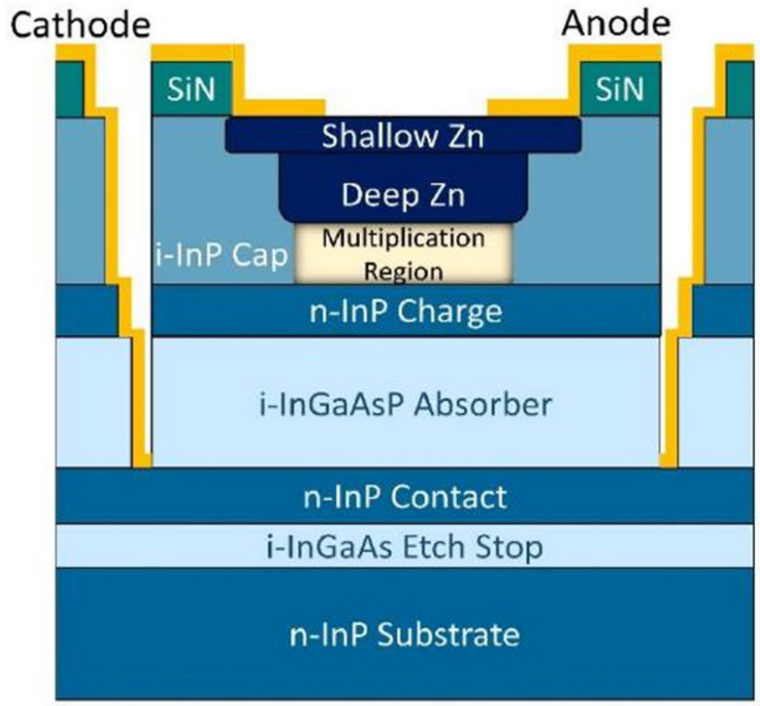
$$R_I = \frac{\eta q}{hc}\lambda$$

$$\frac{S}{N} = 1 \quad \Rightarrow \quad \frac{R_I \cdot NEP}{N} = 1 \Longrightarrow \quad D \equiv \frac{1}{NEP} = \frac{R_I}{N} \quad \Longrightarrow$$

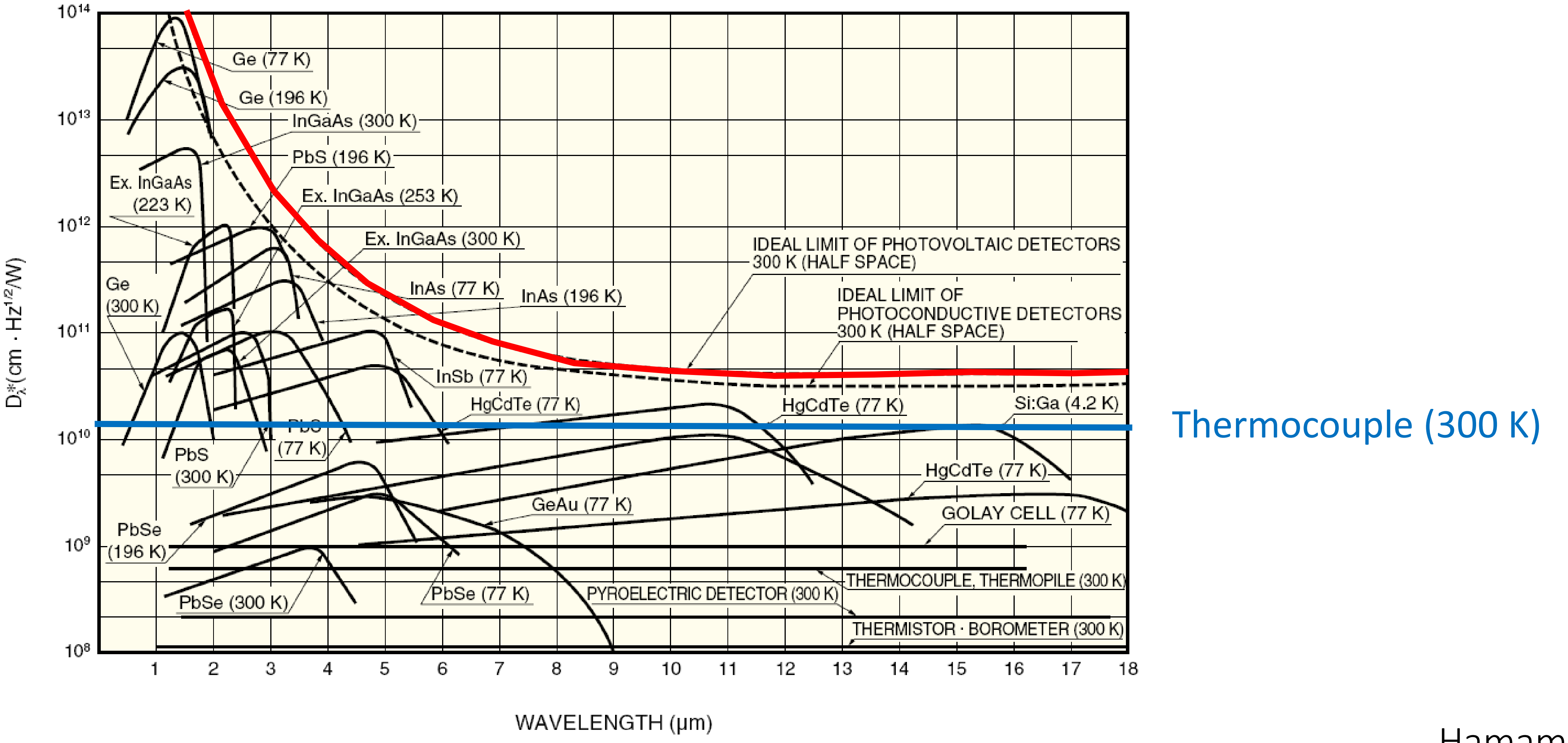
The spectrum of D is similar to that of the responsivity.

Exercise 2.1: Quantum Efficiency: Example





Exercise 2.1: Spectral Dependence of the Responsivity



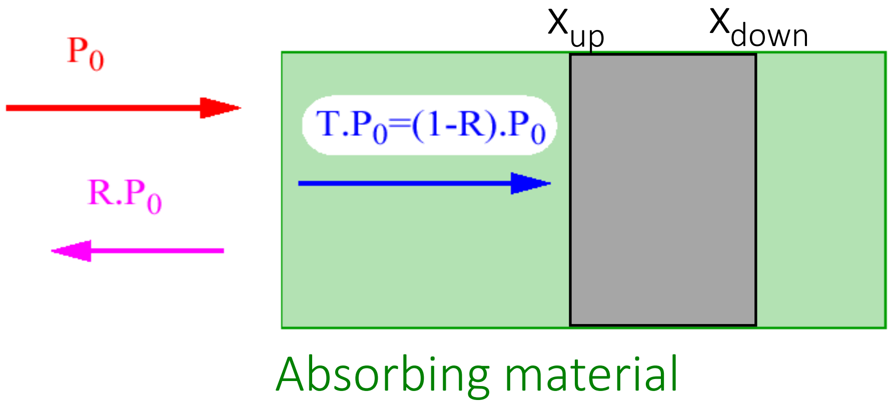
Hamamatsu Catalog

Exercise 2.2: Responsivity and Depth

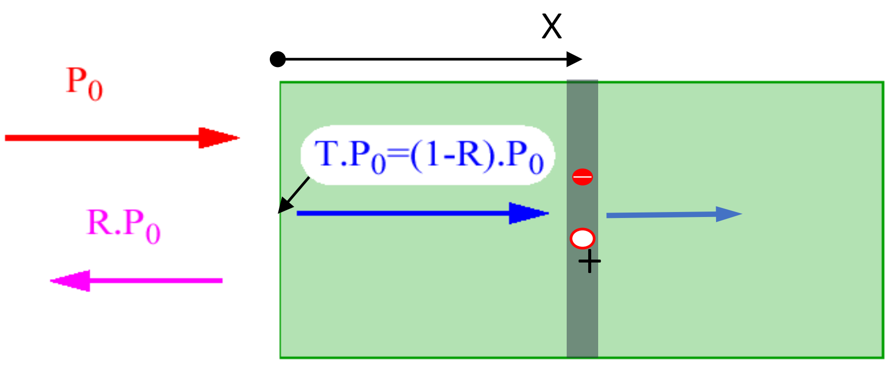
The absorption coefficient of silicon can be approximated as $\alpha_{cm^{-1}}(\lambda_{\mu m}) \cong 10^{7.2-5.5\lambda}$

Consider two detectors and two wavelengths (λ=450nm and λ =600nm).
The first detector is sensitive between x_{1up} = 0.05µm and x_{1down} = 0.3µm.
The second detector is sensitive between x_{2up} = 0.9µm and x_{2down} = 4µm.
The reflection coefficient is 10%.

Calculate the responsivity R_I and the quantum efficiency of both detectors at the abovementioned wavelenghts.



Exercise 2.2: Absorption and Generation Rate



R= reflection coefficient

T= transmission coefficient

g(x)= generation rate of carriers

$$g(x) = \frac{P(x)}{h\nu} \cdot \alpha = \frac{P_0}{h\nu} \cdot (1-R) \cdot e^{-\alpha x} \cdot \alpha \qquad \left[\frac{1}{cm \cdot s}\right]$$

Exercise 2.2: Quantum Efficiency

For photodetectors the quantum efficiency is defined as follows:

$$\eta = \frac{\text{number of optically generated electrons}}{\text{number of incident photons}}$$

It takes into account:

- reflection
- absorption
- recombination
- and electron scattering

It does not consider:

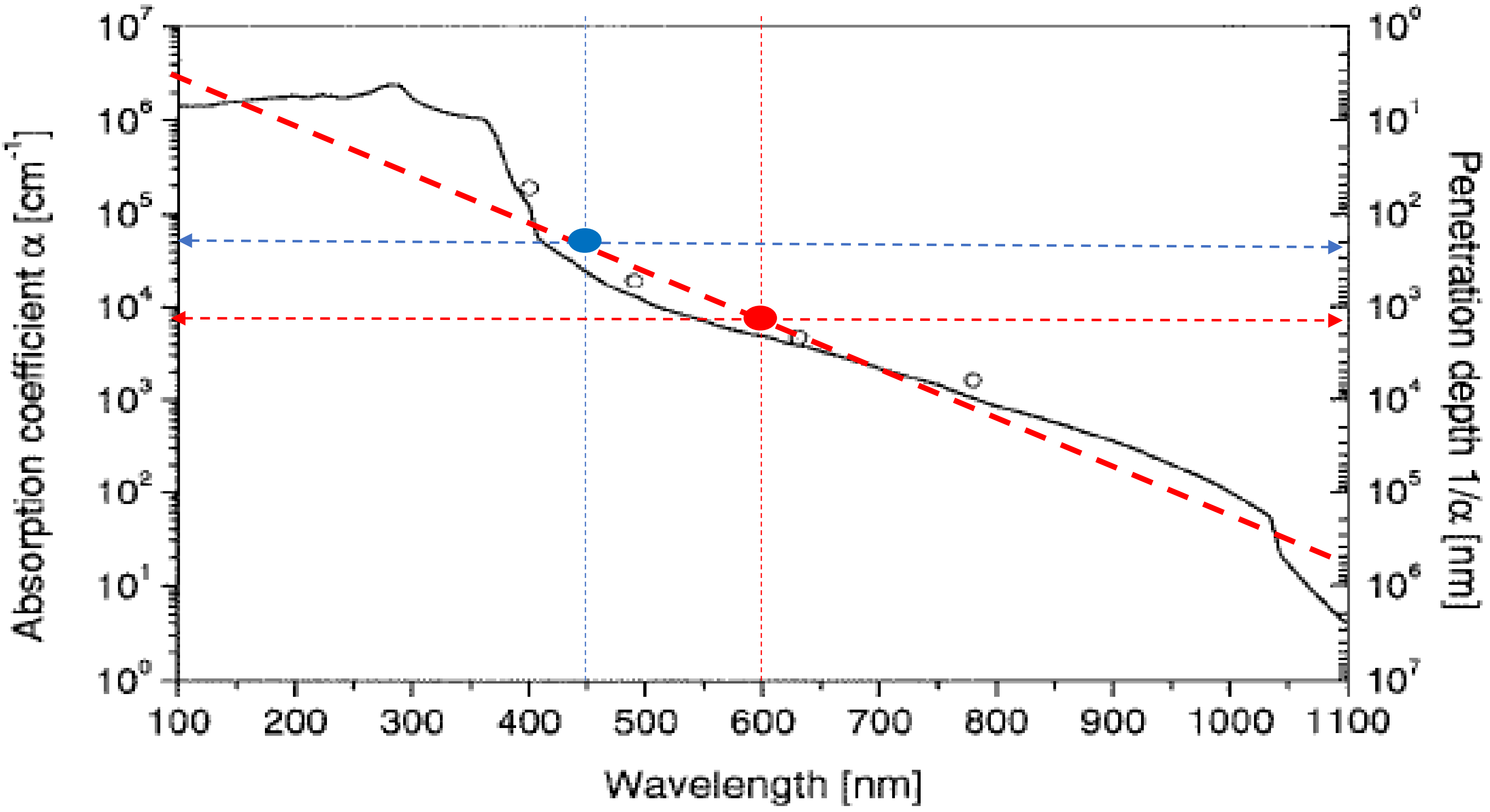
- internal gain
- avalanche phenomena, …

Relationship with responsivity

$$\eta = \frac{I_s/q}{P_s/h\nu}$$

$$\rightarrow \quad R_I = \frac{I_s}{P_s} = \frac{\eta\, q}{h\, \nu}$$

Exercise 2.2: Absorption by silicon



200 nm

1300 nm

Approximation: $\alpha_{cm^{-1}}(\lambda_{\mu m}) \cong 10^{7.2-5.5\lambda}$

Exercise 2.2: Absorption, Responsivity and Quantum Efficiency

Absorption:

$$\alpha_{cm^{-1}}(\lambda_{\mu m}) \cong 10^{7.2-5.5\lambda}$$

Generation rate:

$$g(x) = \frac{P(x)}{h\,\nu} \cdot \alpha = \frac{P_0}{h\,\nu} \cdot (1-R) \cdot e^{-\alpha\,x} \cdot \alpha \qquad \left[\frac{1}{cm \cdot s}\right]$$

Photocurrent:

$$I = q \cdot \int_{x_{up}}^{x_{down}} g(x) \cdot dx = \frac{q}{h\nu} \cdot (1-R) \cdot (e^{-\alpha x_{up}} - e^{-\alpha x_{down}}) \cdot P_0$$

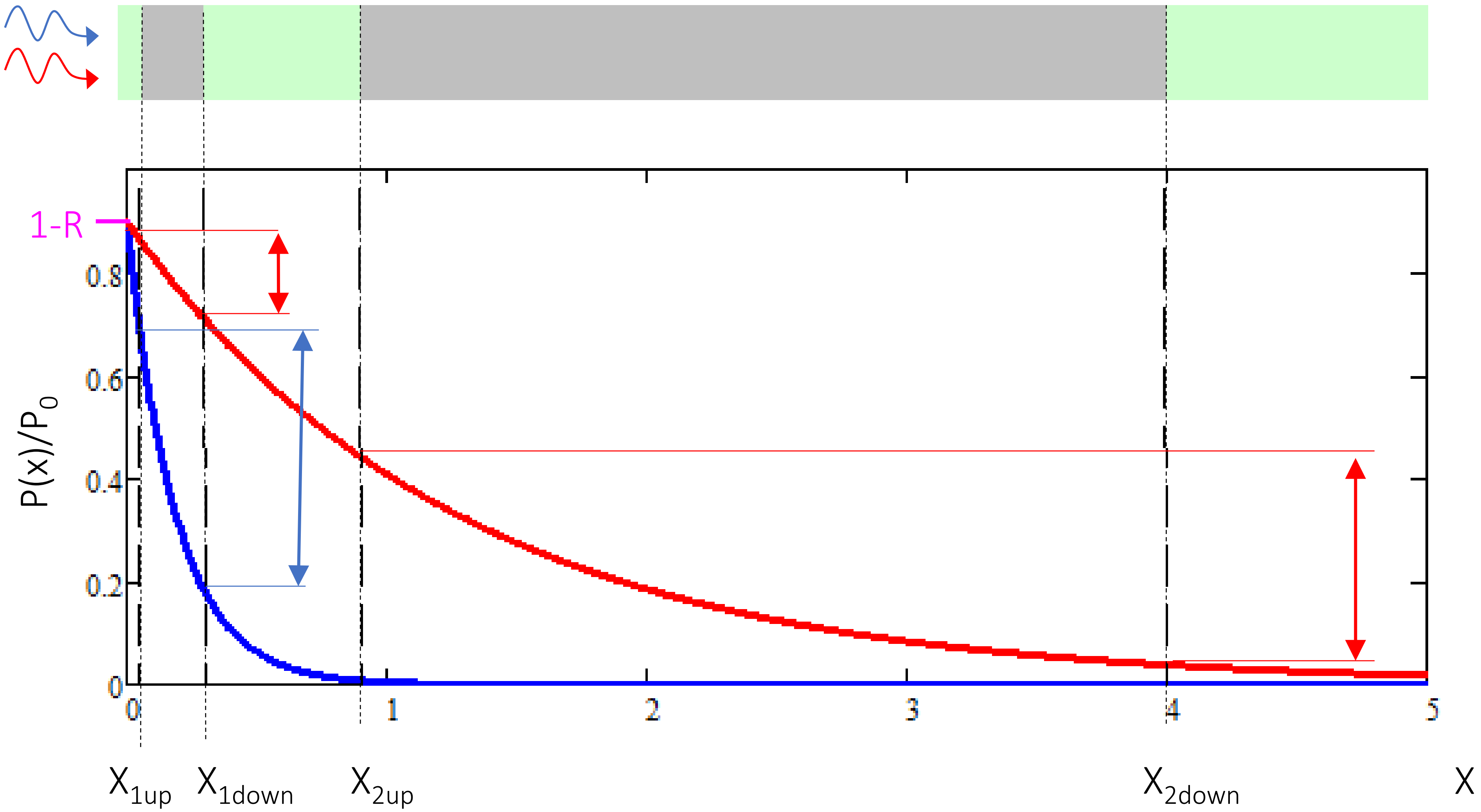
Responsivity:

$$R_I = \frac{I}{P_0} = \frac{q}{h\nu} \cdot (1-R) \cdot (e^{-\alpha x_{up}} - e^{-\alpha x_{down}})$$

Quantum efficiency

$$\eta = R_I \cdot \frac{h\nu}{q} = (1-R) \cdot (e^{-\alpha x_{up}} - e^{-\alpha x_{down}})$$

Exercise 2.2: Interpretation: Quantum Efficiency

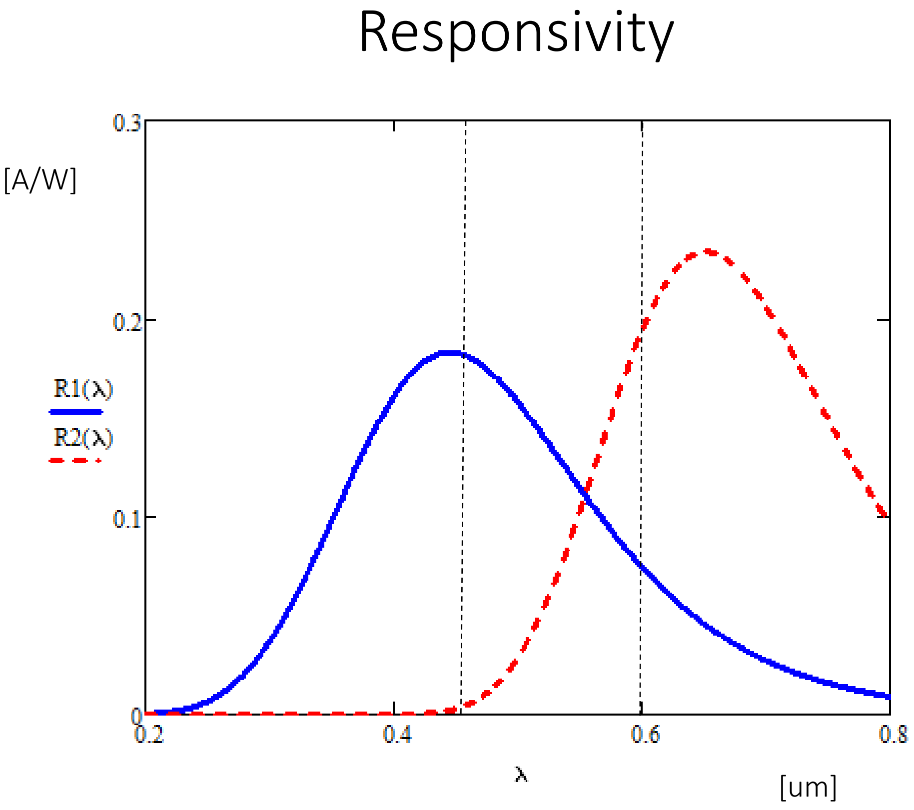


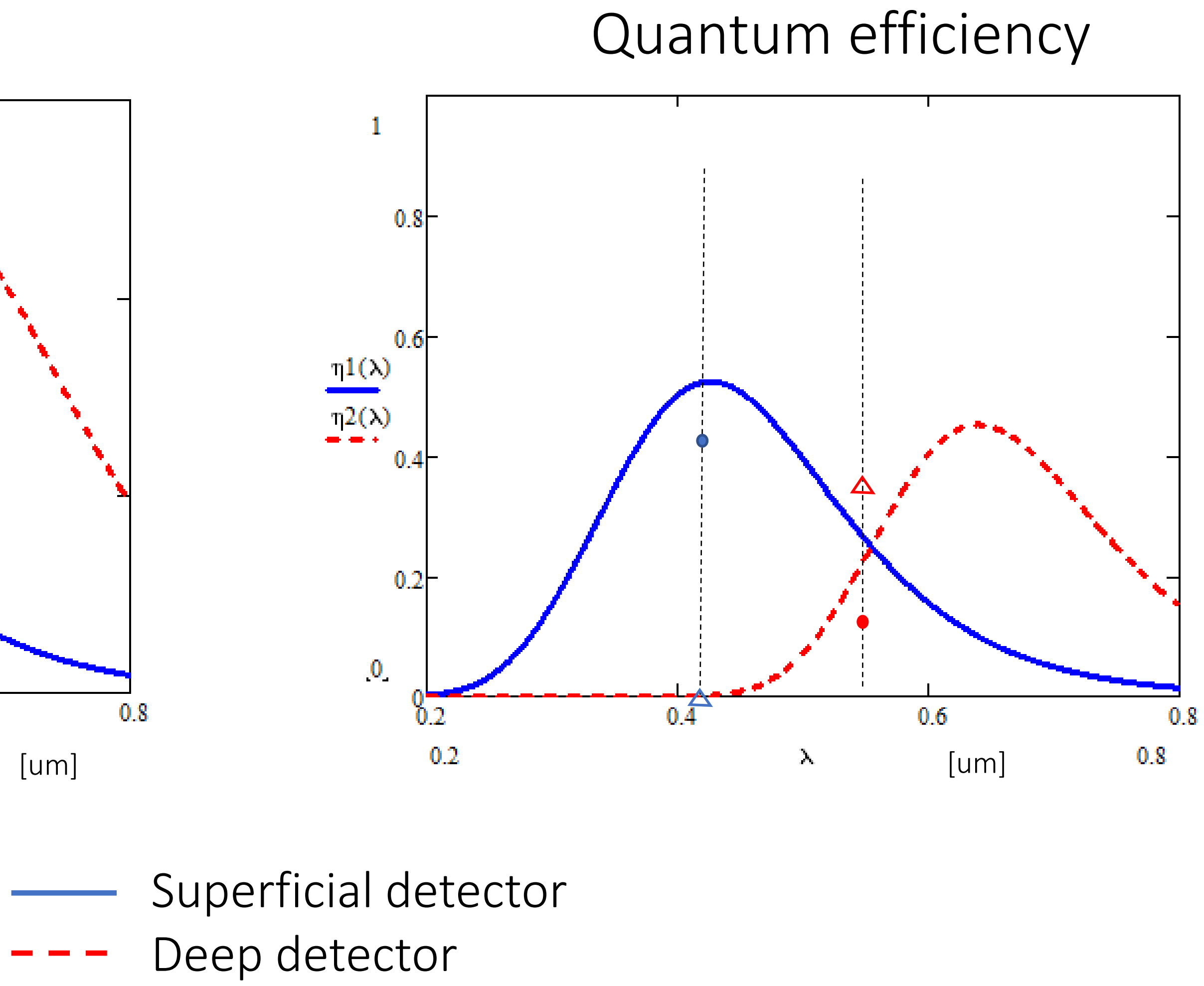
Exercise 2.2: Numerical Values

$$\alpha(450) = 5.3 \cdot 10^4 \ cm^{-1} \qquad \alpha(600) = 7.9 \cdot 10^3 \ cm^{-1}$$

	"Superficial" detector		"Deep" detector	
	R_I [A/W]	η %	R_I [A/W]	η %
λ=450nm	0.183	51 %	0.003	0.8 %
λ=600nm	0.074	15.5 %	0.194	40 %
R_{450}/R_{600} η_{450}/η_{600}	2.5	3.3	0.014	0.02

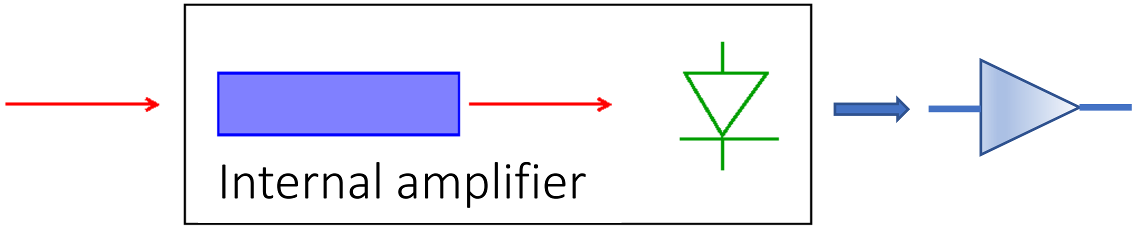
Exercise 2.2: Responsivity and Quantum Efficiency





Exercise 2.3: Optimal Internal Gain

We would like to detect an optical signal with P_0=25 pW using a detector with a variable internal gain G (for example an avalanche photodiode).
Its responsivity is R_I=0.4 A/W,
its bandwidth is Δf=1 MHz,
and its excess noise factor is $F=G^{0.3}$.



Determine as a function of its gain G:

a) The amplified photocurrent $I_{sig}(G)$.
b) The shot noise of this current.

The electronics generate an rms noise of ΔI_{el}=100 pA.

c) Calculate the total noise Δi_{tot} as a function of the gain.
d) Determine the signal to noise ratio as a function of gain.
e) What is the optimal gain G_{opt}?

Exercise 2.3: Internal Gain (1)

Primary photocurrent: $I_0 = R_I \cdot P_0 = 10\ pA$

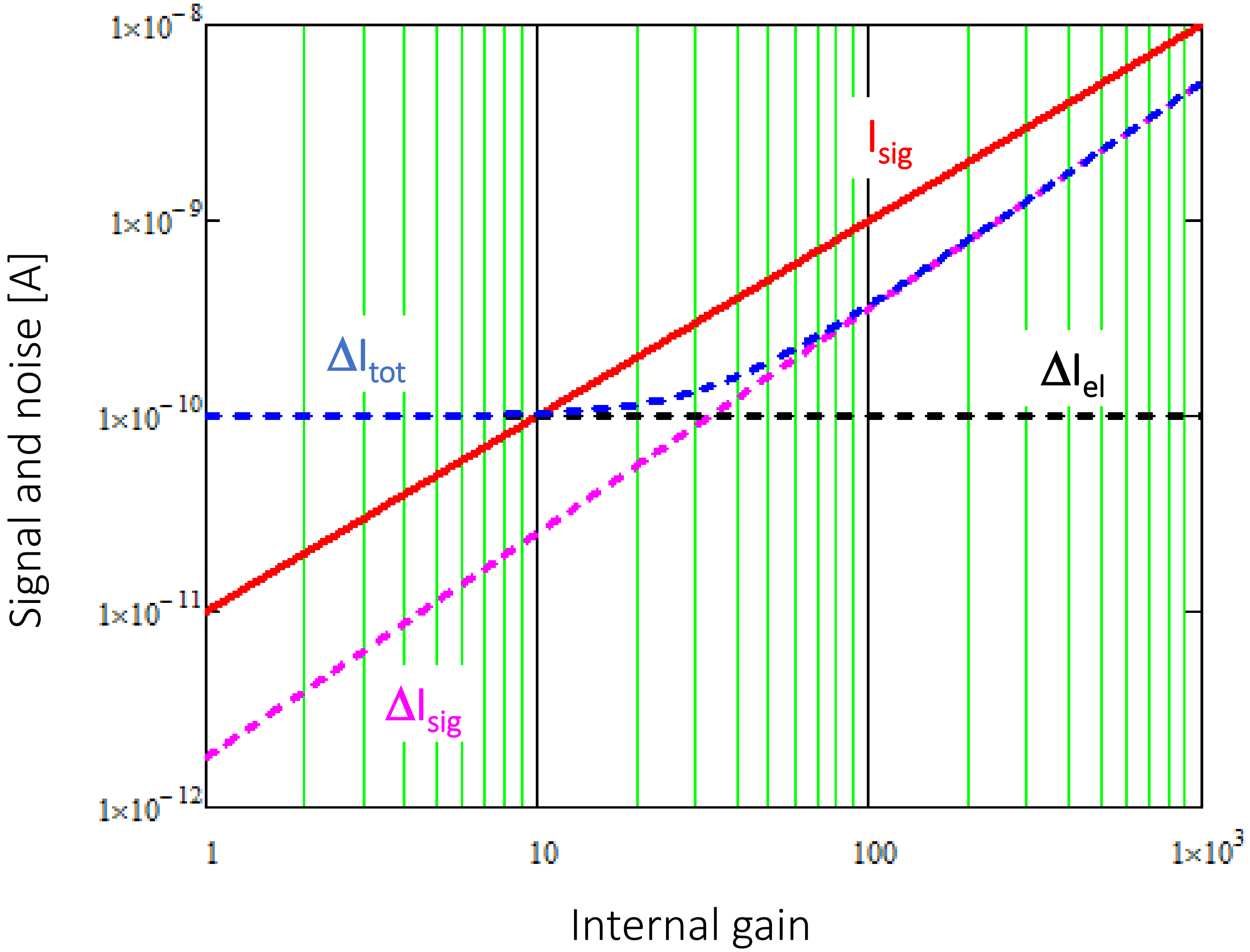
Amplified photocurrent: $I_{sig}(G) = G \cdot I_0$

Shot noise of the amplified signal: $\Delta I_{sig}(G) = \sqrt{2qI_0\Delta f \cdot G^2 \cdot F(G)}$

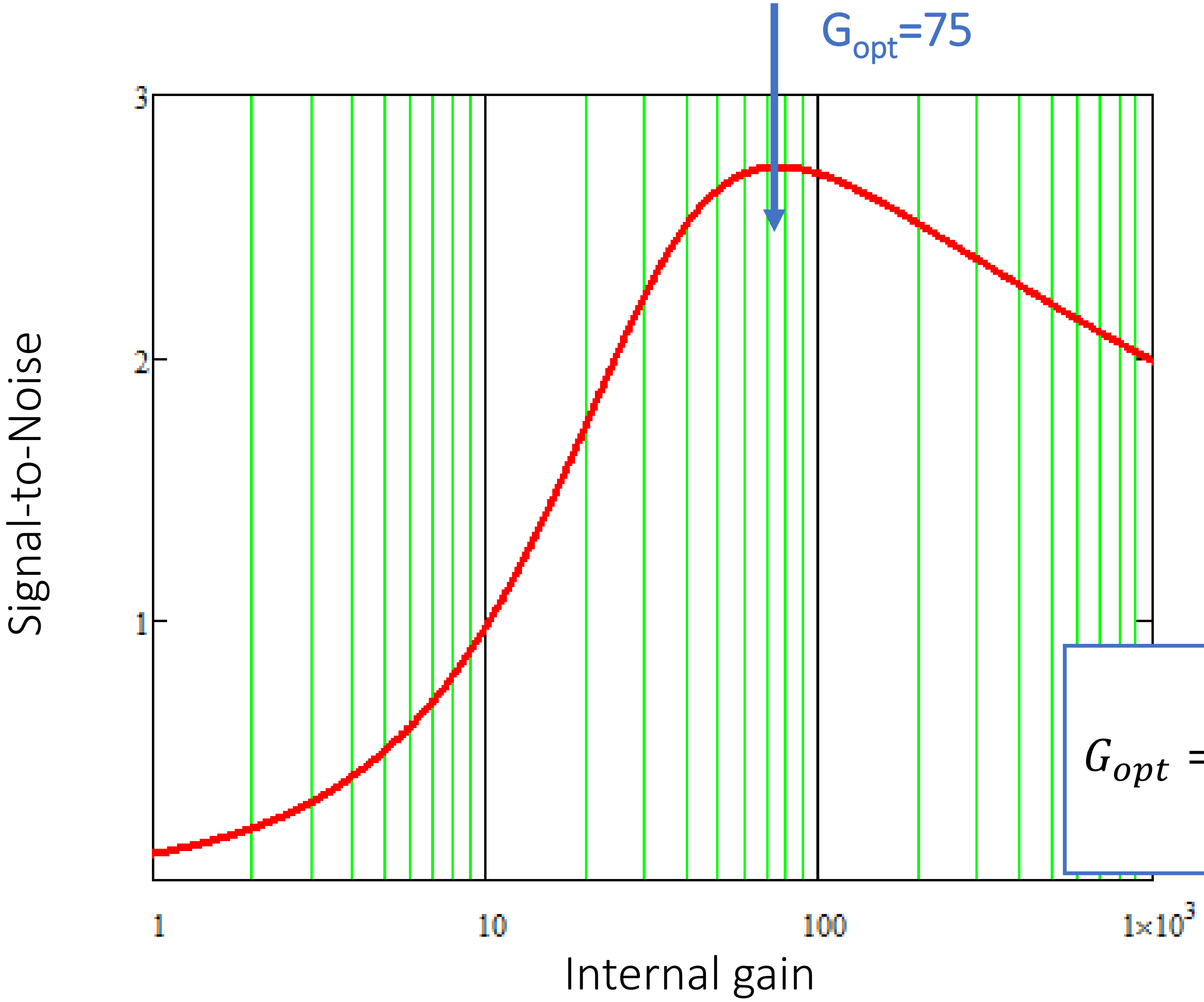
Total noise: $\Delta I_{tot} = \sqrt{\Delta I_{el}^2 + \Delta I_{sig}^2(G)}$

Signal-to-Noise ratio: $S/N = \dfrac{G \cdot I_0}{\sqrt{\Delta I_{el}^2 + 2qI_0\Delta f \cdot G^2 \cdot G^{0.3}}}$

Exercise 2.3: Internal Gain (2)



Exercise 2.3: Internal Gain (3)



$$\frac{\partial(S/N)}{\partial G} = 0$$

$$G_{opt} = \left(\frac{\Delta I_{el}^2}{0.3 \cdot q \cdot I_0 \cdot \Delta f}\right)^{\frac{1}{2.3}} \cong 75$$

Exercise 2.4: Light sources statistics

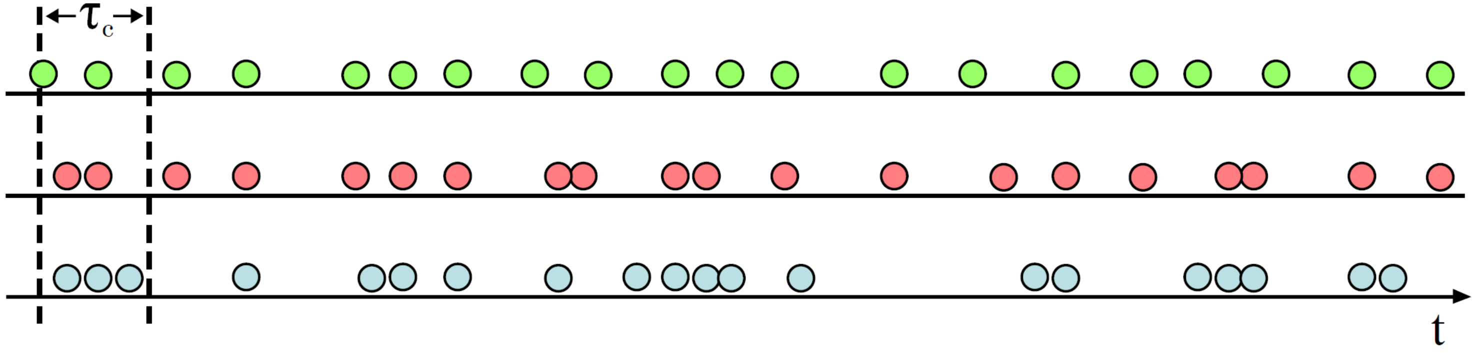
Questions

- Which kind of light sources exist?
- How are their statistical emission properties?

- *Recap: shot noise*

$$N^2(f) = \frac{\langle \Delta I_{shot}^2 \rangle}{\Delta f} = 2q \cdot |\langle I \rangle|$$

$$N^2(f) = \frac{\langle \Delta P_{shot}^2 \rangle}{\Delta f} = 2h\nu \cdot \langle P \rangle$$



Photon detections as function of time for a) antibunched, b) random, and c) bunched light

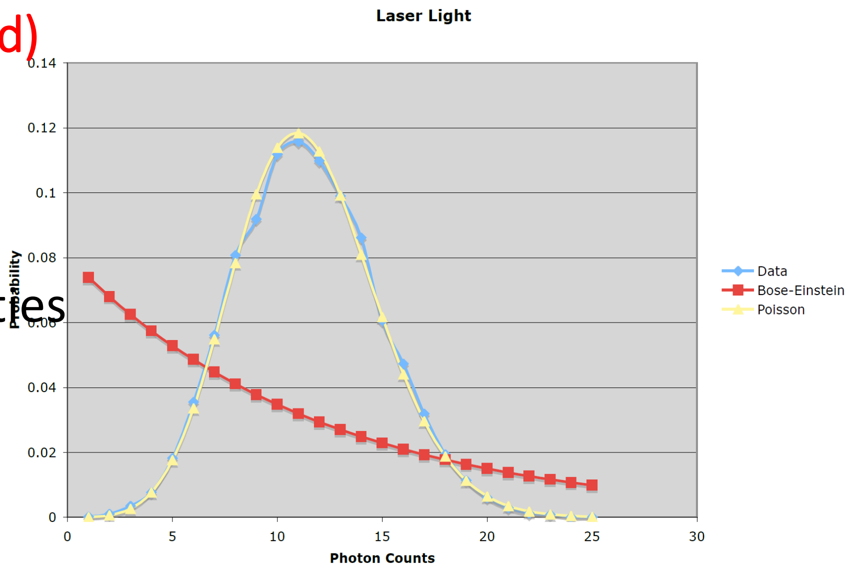
Exercise 2.4: Poisson Distribution vs. Light Sources

- Non-classical light: Sub-Poissonian -> antibunched (anticorrelated)
- Coherent light source (Laser): Poissonian, random spacing (uncorrelated)
- Thermal Light: Super-Poissonian, Bose-Einstein distribution with zero counts as most probable count (bunched, positively correlated)

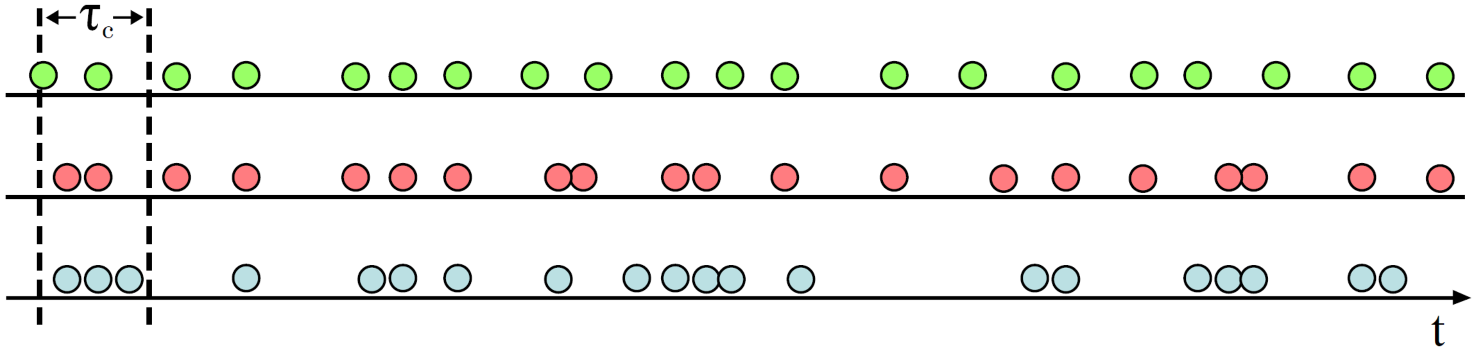
However, in practice it defaults to Gaussian due to the very low coherence time, O(ps), and the corresponding experimental difficultes

Experimentally one can use pseudothermal light*.

https://demonstrations.wolfram.com/PhotonNumberDistributions/



By Ajbura - Vectorised version of File:Photon bunching.png, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=73299604



Photon detections as function of time for a) antibunched, b) random, and c) bunched light

E.g. scattering of a laser beam on a rotating ground glass disc

📖 http://physics.gu.se/~tfkhj/lecture_X_differential_transmission-2.pdf

📖 https://www.stmarys-ca.edu/sites/default/files/attachments/files/GriderJordanFinalReport_0.pdf

Exercise 2.4: Light sources statistics

$\bar{n}$ = average photon number

Non-classical light: Sub-Poissonian $< \sqrt{\bar{n}}$

Coherent light source (Laser): Poissonian

$$P(n) = \frac{\bar{n}^n}{n!} e^{-\bar{n}}, \sigma = \sqrt{\bar{n}}$$

For large photon numbers, the relative fluctuations $\sigma/\bar{n}$ tend to 0

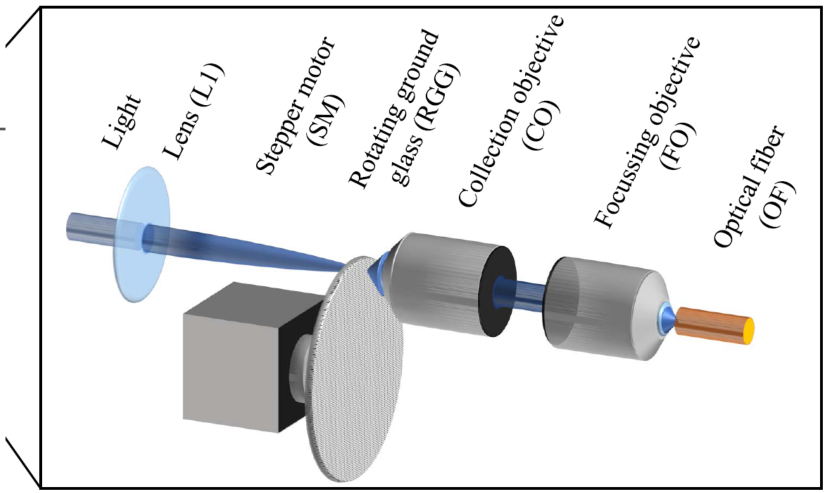
Thermal Light: Super-Poissonian, Bose-Einstein distribution

$$P(n) = \left(1 - e^{-\hbar\omega/k_B T}\right) e^{-n\hbar\omega/k_B T} = \frac{\bar{n}^n}{(\bar{n}+1)^{n+1}}, \bar{n} = \left(e^{\hbar\omega/k_B T} - 1\right)^{-1},$$

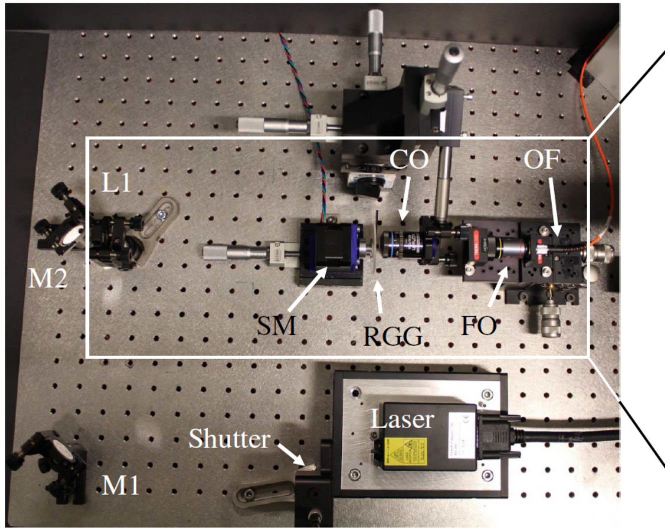
$$\sigma = \sqrt{\bar{n}^2 + \bar{n}} \ \ (for\ T \ll \tau_c) > \sqrt{\bar{n}}$$

For large photon numbers, the relative fluctuations $\sigma/\bar{n}$ tend to 1

(b)



(a)



Pseudothermal light source

Advanced Lab Course (F-Praktikum), Exp. 45, *Photon Statistics*, v. Aug. 21 2017

http://physics.gu.se/~tfkhj/lecture_X_differential_transmission-2.pdf

https://www.stmarys-ca.edu/sites/default/files/attachments/files/GriderJordanFinalReport_0.pdf

T. Stagner *et al.*, *Step-by-step guide to reduce spatial Coherence of laser light using a rotating ground glass diffuser*, OSA Applied Optics 56 (2017).