Image Processing 2, Exercise 2

1 Structure tensor

[intermediate] An example of the power of the structure tensor to encode local gradient information about an image.

(a) Consider computing over an image f the delocalized structure tensor $\mathbf{J}$ characterized by the constant-valued observation window $w = 1$. Use Parseval to give an expression of $\mathbf{J}$ where $\hat{f}$ appears instead of f.

(b) Let an image be $f(\mathbf{x}) = \mathrm{sinc}(2\,x_1 + 3\,x_2)\,\frac{1}{\sqrt{2\,\pi}}\,\mathrm{e}^{-\frac{1}{2}\,(x_1^2 + 4\,x_1\,x_2 + 4\,x_2^2)}$. Determine its gradient $\boldsymbol{\nabla} f$. Hint: To avoid direct calculation of $\boldsymbol{\nabla} f$ which is complex, notice that image f can be written as applying an affine transformation $\mathbf{A}$ to another image g that has a much simpler form, try to find out $\mathbf{A}$ and g.

(c) Determine the Fourier transform $\hat{f}$ of the image f.

(d) Determine the value of the delocalized structure tensor associated to f. You may want to take advantage of $\int_{\mathbb{R}} \mathrm{e}^{-x^2}\,\mathrm{d}x = \sqrt{\pi}$ and $\int_{\mathbb{R}} x^2\,\mathrm{e}^{-x^2}\,\mathrm{d}x = \frac{1}{2}\,\sqrt{\pi}$.

(e) Give the delocalized gradient energy of f.

(f) Give the delocalized coherency of f.

2 Spline Interpolation 1D

[basic] Interpolating the samples of a function using B-splines. Interpolation is fundamental in image processing, because we often want to move back and forth between continuous-domain signals and their discrete-domain representations.

Assume that $f(x) = (10 - |6x + 3|)\,\mathrm{rect}(\frac{x}{4} + \frac{1}{8})$. Find the quadratic spline coefficients $\{c[m]\}_{m\in\mathbb{Z}}$ such that $s(x) = \sum_{m\in\mathbb{Z}} c[m]\beta^2(x - m)$ satisfies the interpolation condition $s[k] = f[k]$ for all $k \in \mathbb{Z}$.