Lecture 11

Impedance Control with Dynamical Systems – Passive DS

Force Control with Dynamical Systems

Today's Lecture
How to perform force control with dynamical systems?

Recap of previous lecture:

Why compliant control?

- Compliant control is crucial to enable robots to interact safely with their environment and with humans.

How to program robots to become compliant?

- Control the robot through impedance control.

Today's lecture:

How to combine impedance control with dynamical systems?

- Impedance control wrapped around dynamical system motion planner
- Definition of passivity and design control parameters to ensure system remains passive
- Shape the control to generate desired force?

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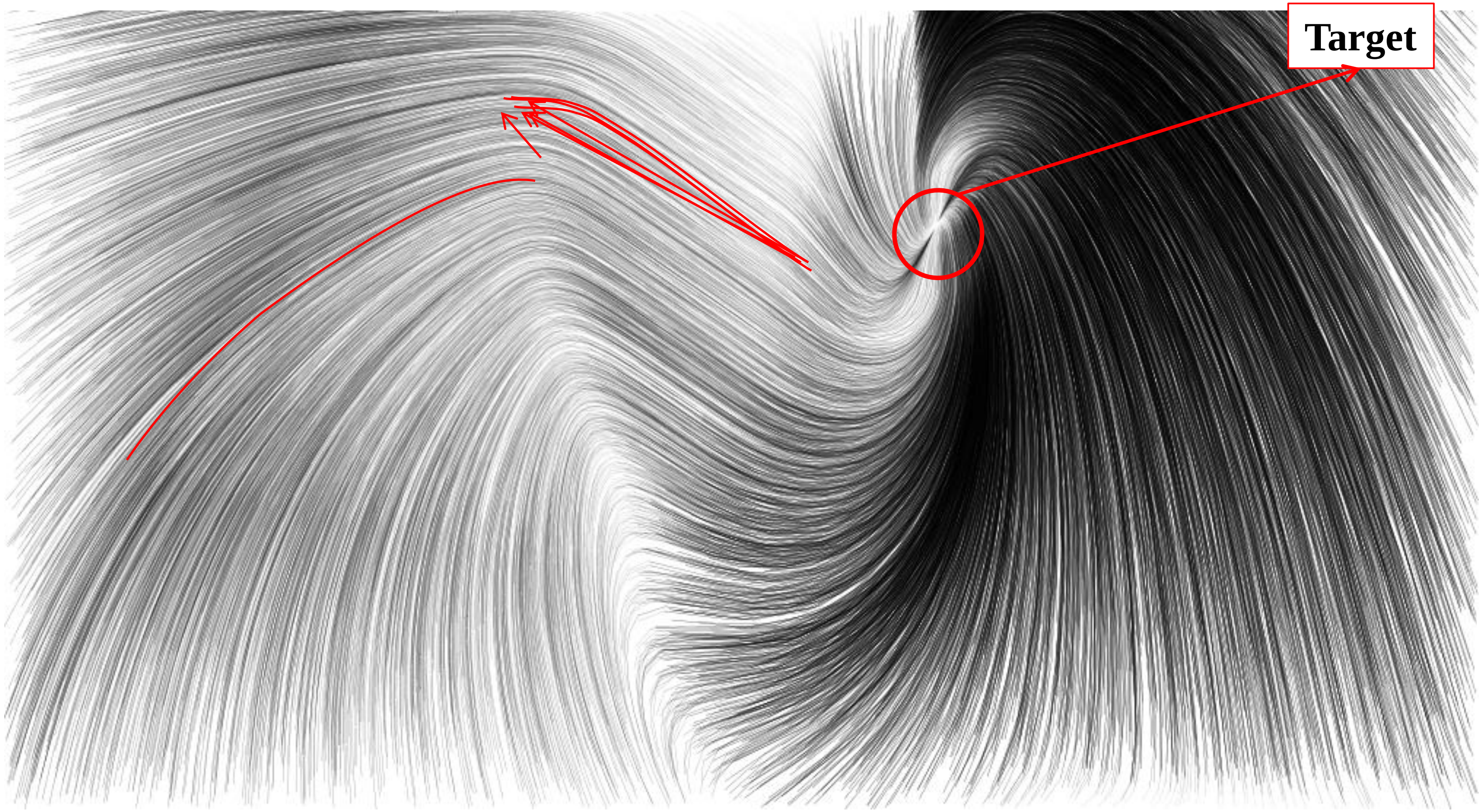
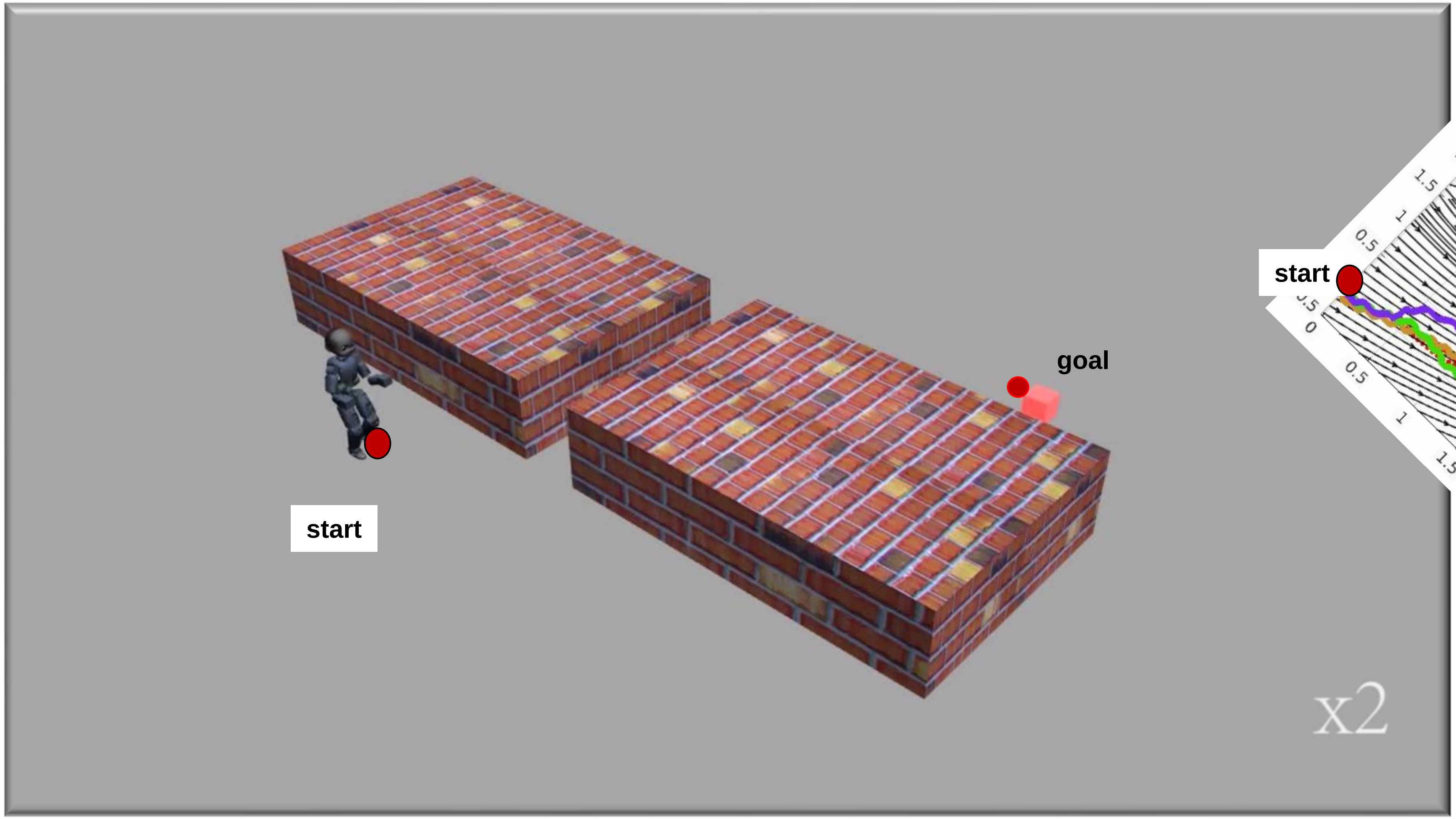
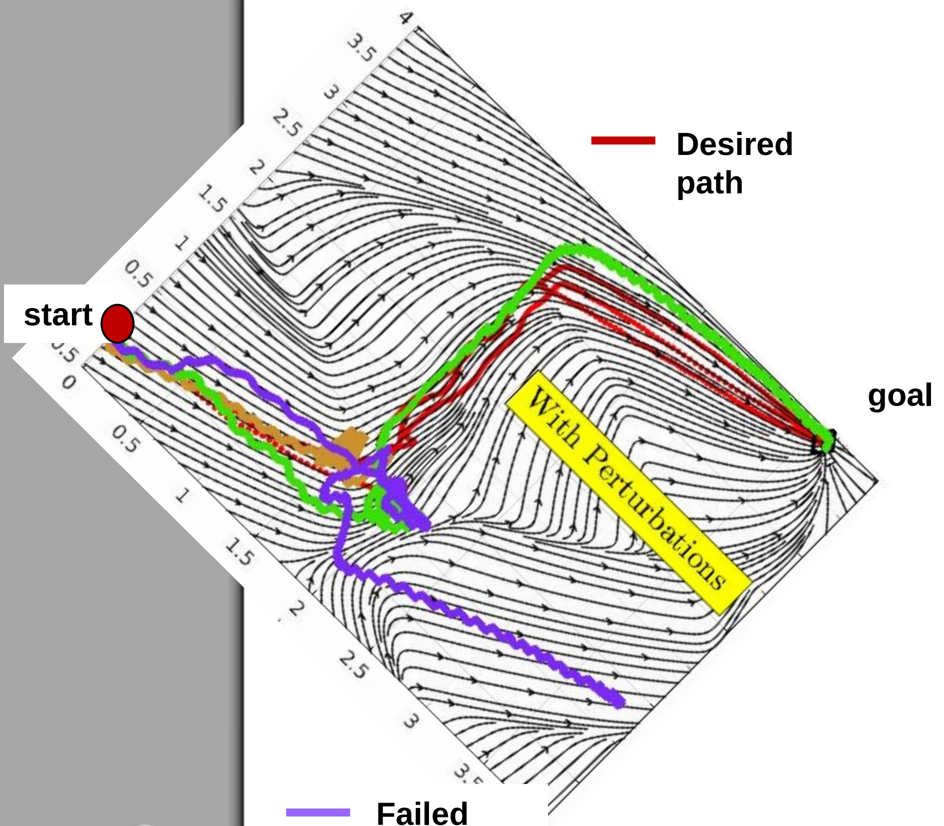
DS control makes the system infinitely compliant!

The trajectory planner given by: $\dot{x} = f(x)$ is infinitely compliant by definition.



DS control makes the system infinitely compliant!





Figueroa, N. and Billard, A., 2022. Locally active globally stable dynamical systems: Theory, learning, and experiments. *The International Journal of Robotics Research*, *41*(3), pp.312-347.

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Robot's Dynamics, Assumptions and Requirements

Dynamics equation of the robot

$$\underbrace{M(x)\ddot{x}}_{\text{Inertia}} + \underbrace{C(x,\dot{x})\dot{x}}_{\text{Coriolis}} + \underbrace{g(x)}_{\text{Gravity}} = \underbrace{\tau_c}_{\text{Control Input}} + \underbrace{\tau_e}_{\text{External Forces}}$$

Design a control law for generating control torques τ_c.

For the system to remain stable under external disturbances, we need to show that it remains ***passive.*** *(see next slides and Annexes A.6)*

Control torques τ_c must be modulated to ensure that the system remains passive.

Stability of the System through Passivity Analysis

$\dot{x} = f(x, u), \quad u \in \mathbb{R}^p$: input

We must verify that the energy injected by the input **u** does not destabilize the system.

→ We must verify that the system is *closed-loop passive.*

Recall*:* To study stability of *f(x)*, we used *Lyapunov stability.* Lyapunov stability uses a measure of the energy of the system and verifies that it decreases over time and eventually vanishes at the attractor.

Passivity extends Lyapunov stability to study stability of systems **subjected to an external input *u*.**

To determine the evolution of the energy of the system, we define a variable:

$y = h(x), \; y \in \mathbb{R}^m$

Passivity: Definition

Definition A.8 *(Passivity): A system with the form*

$$\dot{x} = f(x, u)$$
$$y = h(x) \qquad \text{(A.9)}$$

is passive if there is a lower-bounded storage function $V: \mathbb{R}^N \rightarrow \mathbb{R}_{0\leq}$ *such that*

$$\underbrace{V(x(t)) - V(x(0))}_{\text{Stored energy}} \leq \underbrace{\int_0^t u(s)^T y(s) ds}_{\text{Supplied energy}} \qquad \text{(A.10)}$$

For strict "<", the system is "dissipative".

is satisfied for all $0 \leq t$*, all input functions* u*, and all initial conditions* $x(0) \in \mathbb{R}^N$.

Passivity: Definition

Definition A.10 *(Passivity—definition 3): A system with the form*

$$\dot{x} = f(x, u)$$
$$y = h(x) \qquad (A.12)$$

is passive if there is a continuously differentiable, lower-bounded storage function $V: \mathbb{R}^N \rightarrow \mathbb{R}_{0\leq}$ such that along the trajectories generated by (A.12)

$$\dot{V}(t) \leq u(t)^T y(t) \qquad (A.13)$$

is satisfied for all $0 \leq t$, all input functions u and all initial conditions $x(0) \in \mathbb{R}^N$.

To analyze passivity, we must find a storage function. The storage function keeps track of violation of passivity. If the system violates passivity constraints, we can try to modify the control input in such a way that passivity is restored.

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Goals for the design of the control torques

Dynamics equation of the robot

$$\underbrace{M(x)\ddot{x}}_{\text{Inertia}} + \underbrace{C(x,\dot{x})\dot{x}}_{\text{Coriolis}} + \underbrace{g(x)}_{\text{Gravity}} = \underbrace{\tau_c}_{\text{Control Input}} + \underbrace{\tau_e}_{\text{External Forces}}$$

Design a control law for generating control torques τ_c

Goals for the control system:

- The robot should move according to a desired dynamics, set by $\dot{x} = f(x)$.
- The system should remain *passive*.

Format of Control Torques

Dynamics equation of the robot

$$\underbrace{M(x)\ddot{x}}_{\text{Inertia}}+\underbrace{C(x,\dot{x})\dot{x}}_{\text{Coriolis}}+\underbrace{g(x)}_{\text{Gravity}}=\underbrace{\tau_c}_{\text{Control Input}}+\underbrace{\tau_e}_{\text{External Forces}}$$

Feedback term:

$$\tau_c = \underbrace{g(x)}_{\text{Gravity compensation}} - \underbrace{D(x)\dot{x}}_{\text{Damping}}$$

Control torques τ_c must be modulated to ensure that the system remains passive.
→ Modulate $D(x)$.

Gravity compensation mode available on standard robots

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Constraints on Control Torques for Passivity

Dynamics equation of the robot

$$\underbrace{M(x)\ddot{x}}_{\text{Inertia}} + \underbrace{C(x,\dot{x})\dot{x}}_{\text{Coriolis}} + \underbrace{g(x)}_{\text{Gravity}} = \underbrace{\tau_c}_{\text{Control Input}} + \underbrace{\tau_e}_{\text{External Forces}}$$

Feedback term:

$$\tau_c = \underbrace{g(x)}_{\text{Gravity compensation}} \underbrace{-D(x)\dot{x}}_{\text{Damping}}$$

Passivity analysis

What is the first constraint we must set for $D(x)$?

$$D(x) \succ 0, \quad \forall x$$

We verify that the system remains passive under external disturbances τ_e.

We set: $\begin{cases} u = \tau_e \\ y = \dot{x} \end{cases}$

We define the storage function as W.

We verify that : $\dot{W} \le {\tau_e}^T \dot{x}$

W : kinetic energy $W = \frac{1}{2}\dot{x}^T M(x)\dot{x}$

Passivity Verification I

We verify that : $\dot{W} \leq \tau_e{}^T \dot{x}$ $\qquad W = \frac{1}{2}\dot{x}^T M(x)\dot{x}$

$$\dot{W} = \dot{x}^T M(x)\ddot{x} + \frac{1}{2}\dot{x}^T \dot{M}(x)\dot{x}$$

Replacing using $M(x)\ddot{x} + C(x,\dot{x})\dot{x} + g(x) = \tau_c + \tau_e$ and $\tau_c = g(x) - D(x)\dot{x}$

$$\dot{W} = \underbrace{\frac{1}{2}\dot{x}^T\left(\dot{M}(x) - 2C(x,\dot{x})\right)\dot{x}}_{=0} \underbrace{- \dot{x}^T D(x)\dot{x}}_{<0} + \dot{x}^T \tau_e \leq \tau_e{}^T \dot{x}$$

The system is passive.

$\dot{M}(x) - 2C(x,\dot{x})$

Skew-symmetric

since $D(x) \succ 0$

Tracking Feedback Control Loop

Feedback term:

$$\tau_c = \underbrace{g(x)}_{\text{Gravity compensation}} \underbrace{-D(x)(\dot{x} - f(x))}_{\text{Tracking term}}$$

The system must follow a desired dynamics $\dot{x} = f(x)$

Traditional Tracking Feedback Control Loop

$$g(x) - D(\dot{x} - \dot{x}^d) - \cancel{K(x - x^d)} = \tau_c$$

Tracking in position given by the DS

$$g(x) - D(x)(\dot{x} - f(x)) = \tau_c$$

State-dependent impedance modulation

If *f(x) is Lyapunov stable*, the control should dissipate energy solely in directions perpendicular to *f(x)*.

Shaping the Impedance

$$g(x) - D(x)\left(\dot{x} - f(x)\right) = \tau_c$$

"Eigencomposition" of $D(x)$

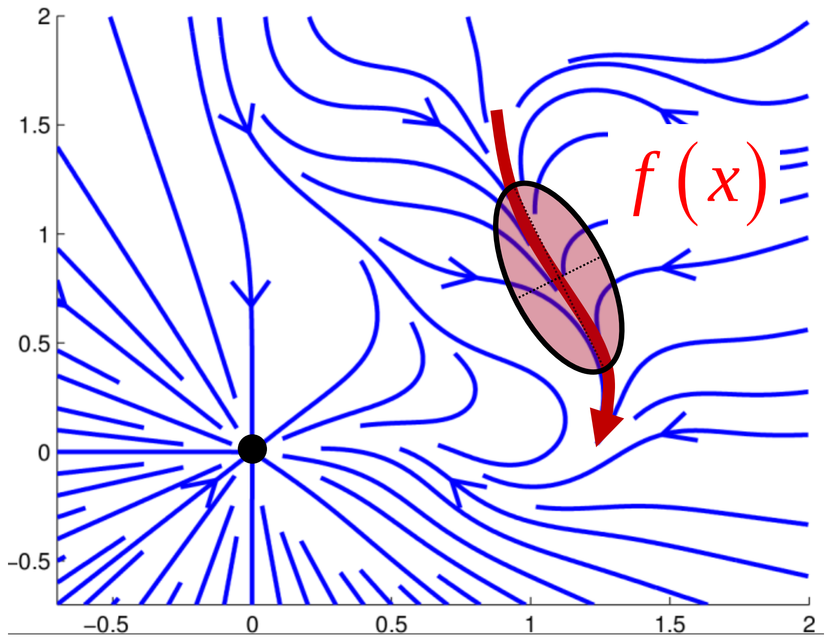
$$D(x) = Q(x)\Lambda(x)Q(x)^T$$

We set *f(x)* to be aligned with an eigenvector of *D(x)*

$$Q(x) = \left[e_1(x)\ e_2(x)\right], \quad e_1(x) = \frac{f(x)}{\|f(x)\|}, \quad e_1(x)^T e_2(x) = 0.$$

The eigenvalues will set the impedance

$$\Lambda(x) = \begin{bmatrix} \lambda_1(x) & 0 \\ 0 & \lambda_2(x) \end{bmatrix}$$



Shaping the Impedance

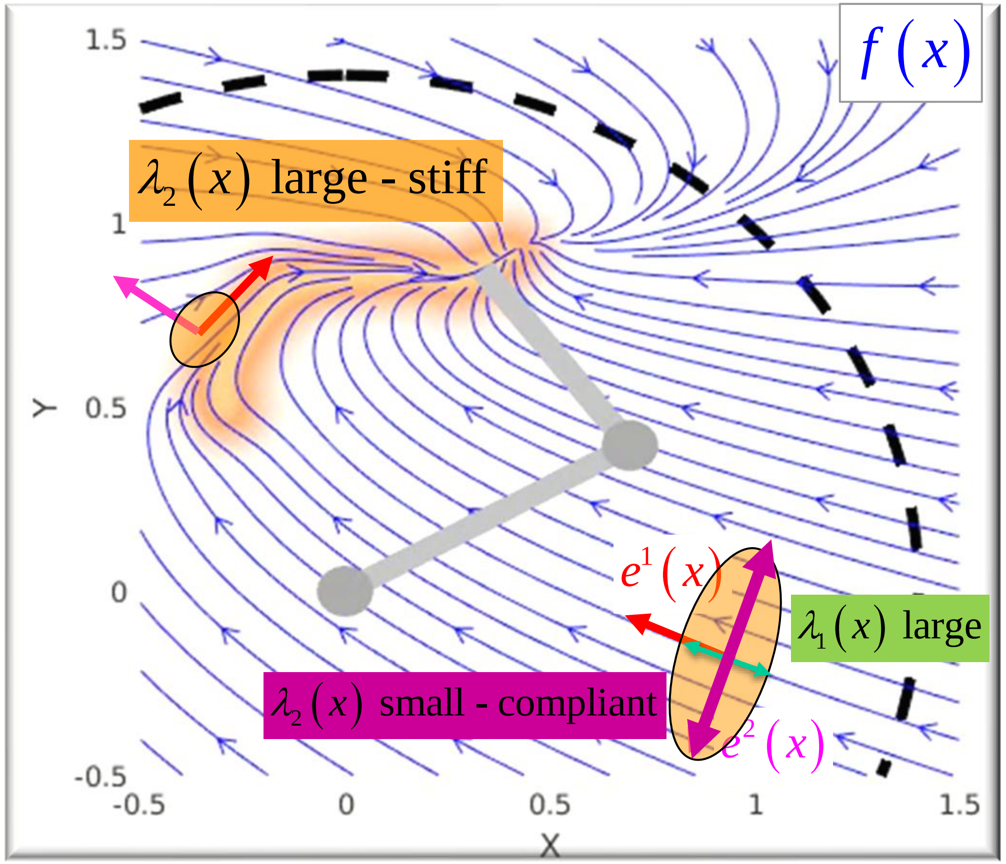
$$g(x) - \color{red}{D(x)}\left(\dot{x} - f(x)\right) = \tau_c$$

The eigenvalues will set the impedance

$$\Lambda(x) = \begin{bmatrix} \lambda_1(x) & 0 \\ 0 & \lambda_2(x) \end{bmatrix}$$

Set $\lambda_1(x)$ to be very stiff for accurate tracking.

Modulate $\lambda_2(x)$ to comply with orthogonal disturbances.



Passivity Analysis

$$g(x) - D(x)\left(\dot{x} - f(x)\right) = \tau_c$$

$\dot{x} = f(x)$ is the first eigenvector of D: $e_1(x) = \frac{f(x)}{\|f(x)\|}$ with associate eigenvalue $\lambda_1(x) = \lambda_1 = cst.$

$$\Rightarrow \quad g(x) + \lambda_1 f(x) - D(x)\dot{x} = \tau_c$$

$\dot{x} = f(x)$ is a Lyapunov stable function

with an associated Lyapunov function $\mathrm{V}_f(x)$.

$\mathrm{V}_f(x)$ is a potential function and we can write:

$f(x) = -\nabla V_f(x)$

Passivity Verification II

We set the storage function: $W = \underbrace{\frac{1}{2}\dot{x}^T M(x)\dot{x}}_{\text{Kinetic Energy}} + \underbrace{\lambda_1 V_f(x)}_{\text{Potential Energy of } f(x)}$ We verify that : $\dot{W} \le \tau_e^T \dot{x}$

$$\dot{W} = \dot{x}^T M(x)\ddot{x} + \frac{1}{2}\dot{x}^T \dot{M}(x)\dot{x} + \lambda_1 \dot{V}_f(x)$$

Replacing with $M(x)\ddot{x} + C(x,\dot{x})\dot{x} + g(x) = \tau_c + \tau_e$, $\dot{V}_f(x) = \nabla V_f(x)^T \dot{x}$

and $g(x) + \lambda_1 f(x) - D(x)\dot{x} = \tau_c$.

$$\dot{W} = \underbrace{\frac{1}{2}\dot{x}^T\left(\dot{M}(x) - 2C(x,\dot{x})\right)\dot{x}}_{=0} \underbrace{- \dot{x}^T D(x)\dot{x}}_{<0} + \lambda_1 \dot{x}^T f(x) + \lambda_1 \dot{V}_f(x) + \dot{x}\tau_e \overset{?}{\le} \tau_e^T \dot{x}$$

Skew-symmetric

since $D(x) \succ 0$

$= 0$

$f(x) = -\nabla V_f(x)$

The system is passive.

Non-conservative DS – Energy Tank

$\dot{x} = f(x)$ is not conservative.

Decompose f into conservative and non-conservative terms:

$$f(x) = f_c(x) + f_r(x)$$

Conservative part follows:

$$f_c(x) = -\nabla V_c(x)$$

The energy injection must now be actively controlled as we are left with an uncontrolled term:

$$\dot{W} = \underbrace{\dots}_{\text{Same as before}} + f_r(x)^T \dot{x}$$

When $f_r(x)^T \dot{x} < 0,$ the system dissipates energy.

Idea: use this "lost" energy to allow the control torque to become non-passive.

Introduce a new variable s to "store" energy dissipated:

$$W(x, \dot{x}, s) = \underbrace{\frac{1}{2}\dot{x}^T \dot{M}(x)\dot{x}}_{\text{Kinetic Energy}} + \underbrace{\lambda_1 V_c(x)}_{\text{Potential Energy of conservative flow } f_c(x)} + \underbrace{s}_{\text{Energy Tank}}$$

Set a tank limit $\bar{s}$ beyond which we do not allow the system to absorb energy anymore.

Non-conservative DS – Energy Tank

The energy injection must now be actively controlled as we are left with an uncontrolled term:

$$\dot{W} = \underbrace{\ldots}_{\text{Same as before}} + f_r(x)^T \dot{x}$$

Introduce a new variable s to "store" energy dissipated:

$$W(x, \dot{x}, s) = \underbrace{\frac{1}{2}\dot{x}^T \dot{M}(x)\dot{x}}_{\text{Kinetic Energy}} + \underbrace{\lambda_1 V_c(x)}_{\text{Potential Energy of conservative flow } f_c(x)} + \underbrace{s}_{\text{Energy Tank}}$$

Variable to account for energy injection at each time step:

$$z = f_r(x)^T \dot{x}$$

Define dynamics of s to reflect changes in the robot's state variable $\dot{x}$

$$\dot{s} = \alpha(s)\dot{x}^T D(x)\dot{x} - \beta_s(z, s)\lambda_1 z$$

$\alpha(s) : \mathbb{R} \rightarrow \mathbb{R}$

$\beta(s, z) : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$

Must be designed to control flow of energy, so as to not exceed a maximum set by the user: $\bar{s}$.

Non-conservative DS – Energy Tank

Define dynamics of s to reflect changes in the robot's state variable $\dot{x}$

$$\dot{s} = \alpha(s)\dot{x}^T D(x)\dot{x} - \beta_s(z,s)\lambda_1 z$$

Adds to virtual storage as long as $s < \bar{s}$.

Variable to account for energy injection at each time step:

$$z = f_r(x)^T \dot{x}$$

$$\begin{cases} 0 \le \alpha(s) \le 1 & s < \bar{s} \\ \alpha(s) = 0 & s \ge \bar{s} \end{cases}$$

$$\begin{cases} \beta_s(z,s) = 0 & s \le 0 \text{ and } z \ge 0 \\ \beta_s(z,s) = 0 & s \ge \bar{s} \text{ and } z \le 0 \\ 0 \le \beta(z,s) \le 1 & \text{elsewhere .} \end{cases}$$

Control torque can add to virtual storage $(z < 0)$ only if $s < \bar{s}$.

Extraction of energy from storage $(z > 0)$ possible only if $s > 0$.

Set the control torques to allow energy injection only when tank is not full:

$$\tau_c = g(x) + \lambda_1 f_c(x) - D(x)\dot{x} + \underbrace{\beta_R(z,s)\lambda_1 f_R(x)}.$$

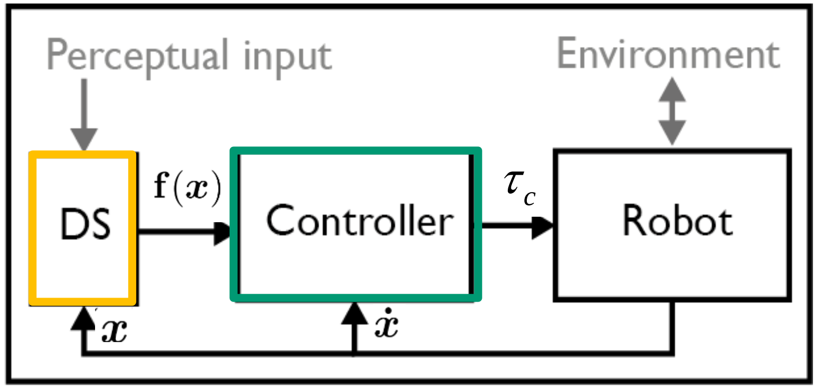
Non passive term activated as long as $\beta_R(z,s)>0$

$$\begin{cases} \beta_R(z,s) = \beta_s(z,s) & z \ge 0 \\ \beta_R(z,s) \ge \beta_s(z,s) & z < 0. \end{cases}$$

The system is passive, see exercise session.

Closed-Loop Control with DS

DS in feedback configuration



DS-based constant replanning of trajectory based on position feedback.

Impedance control based on velocity tracking.

Damping structure constantly aligns with desired direction of motion

You need an autonomous DS:

$$\dot{\boldsymbol{x}} = \mathbf{f}(\boldsymbol{x}) \rightarrow \left\{ \lim_{t\to\infty} \|\boldsymbol{x} - \boldsymbol{x}^*\| = 0 \right.$$

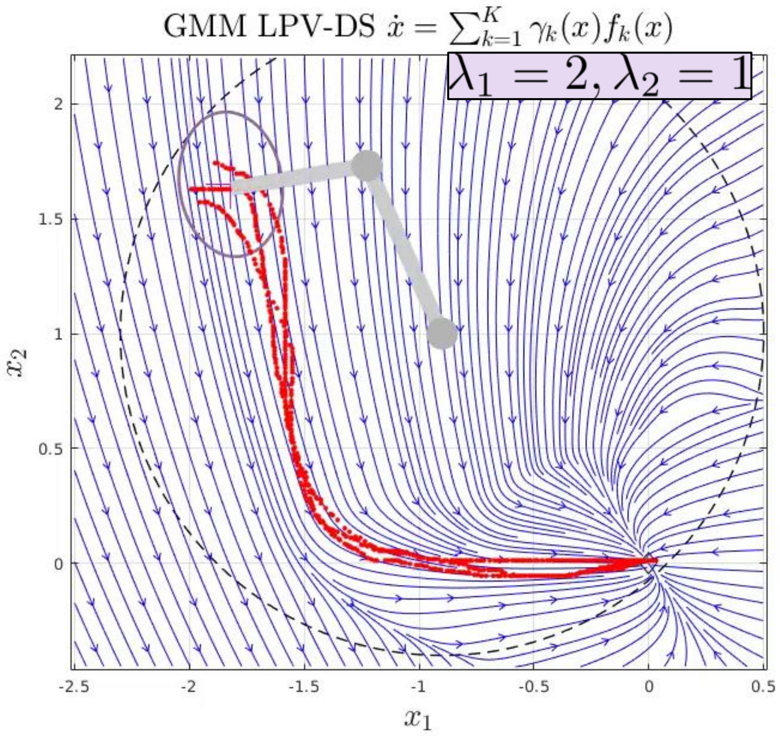
$\boldsymbol{x}\ \dot{\boldsymbol{x}}$

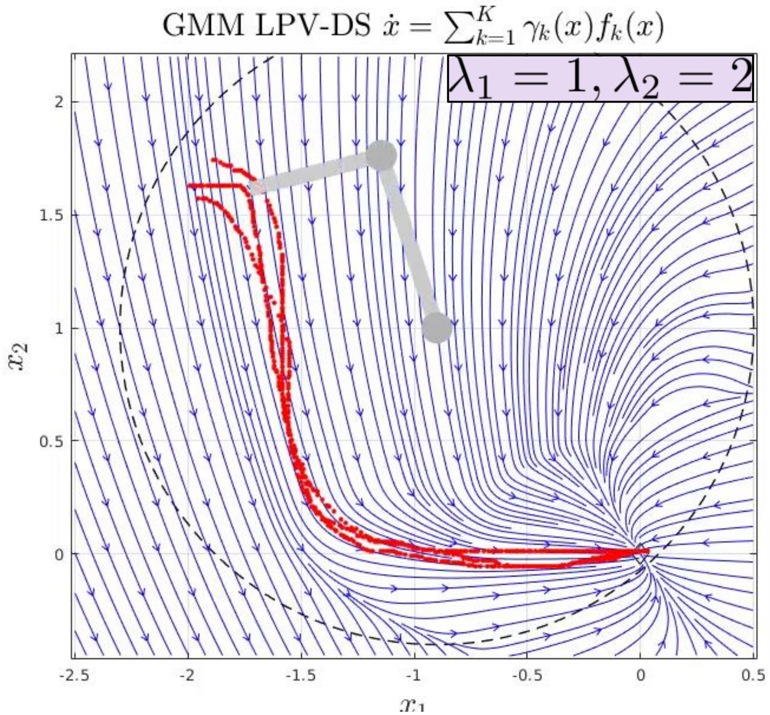
DS can be:

- Linear DS
- Nonlinear DS via LPV-DS or SEDS (Lecture 3)
- Modulated DS (Lecture 6)

that can be conservative or non-conservative

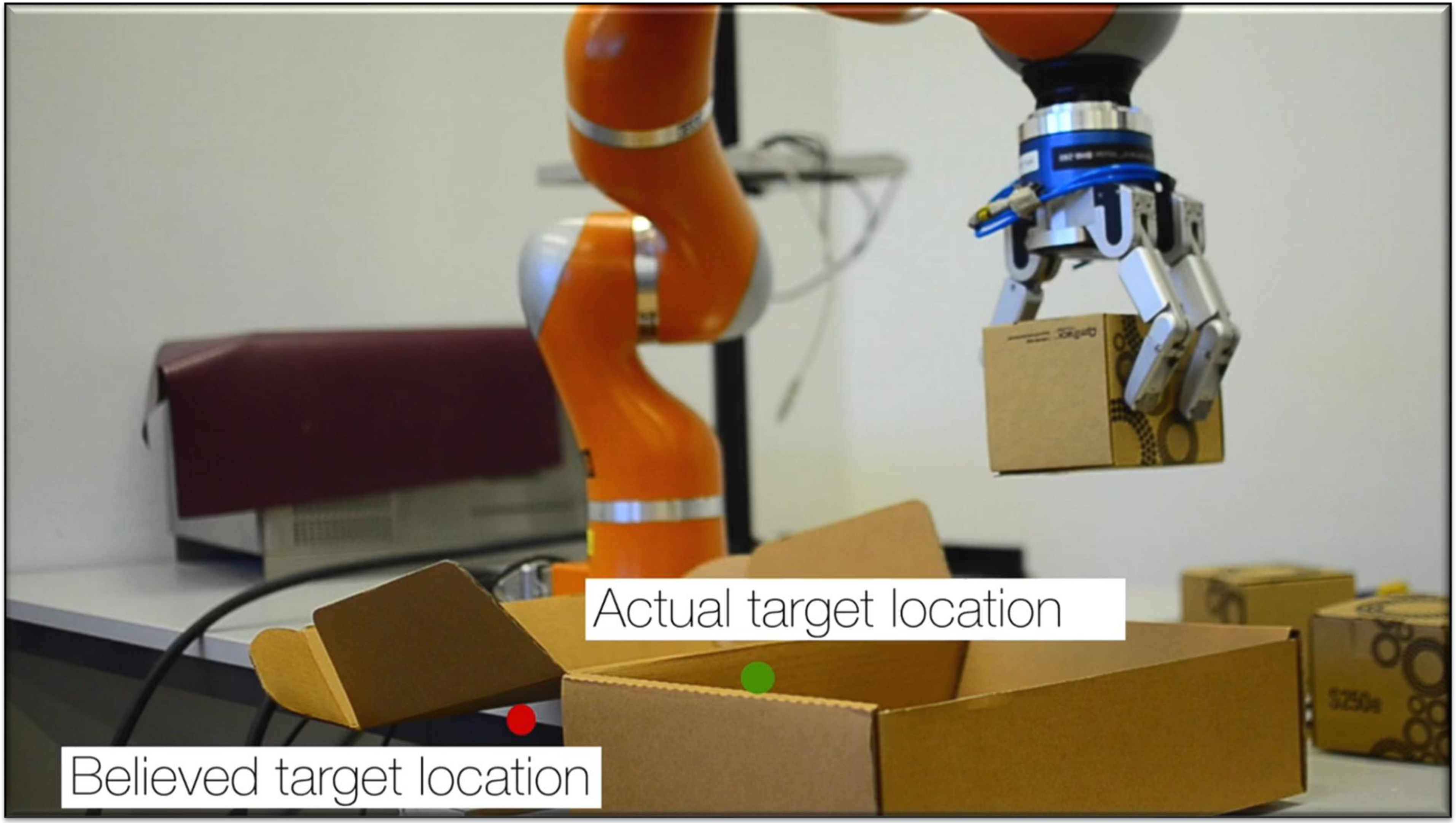
Impedance Modulation on LPV-DS



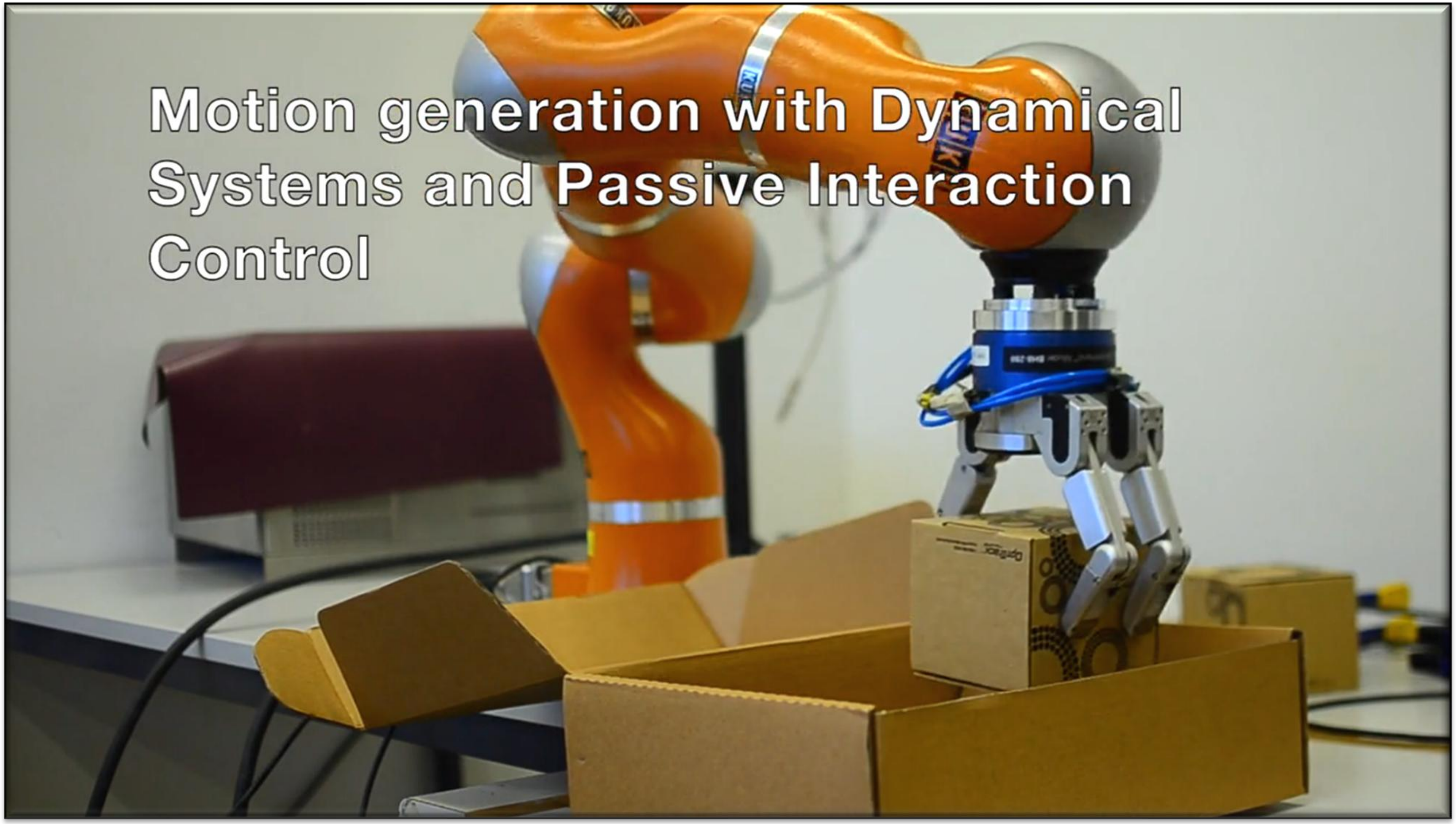


Desired Behavior is determined by choice of damping eigenvalues.

Robot Control: Traditional Time-Dependent Planner in place of DS



Passive-DS for Robot Control



Passive-DS for Robot Control

Summary

- Introduced a means to combine impedance control with DS control.
  - The DS acts as a trajectory generator.
  - Impedance control generates torques to track the output of the DS.

- Introduced the notion of passivity to characterize the energy of a system subjected to disturbances, such as external forces
  - Showed that when the nominal DS is conservative (Lyapunov stable), the system is passive.
  - When the DS is not conservative, one must introduce the notion of a tank to track the energy injected into the system and use this to modulate the controller (see exercises).

- The impedance gains (damping matrix eigenvalues) modulate the response of the system when subjected to external disturbances (external forces).
  - Impedance is directional – aligned with the flow of the DS.
  - High impedance in the direction of the DS will force the system to track accurately the DS.
  - Low impedance in orthogonal directions allows to dissipate energy.