Compliant Control with Dynamical Systems

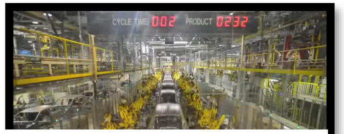
When and why should a robot be compliant?

Automation in a well-structed environment.

Kia Sportage factory production line. 2012

Automation in an unstructured environment.

CNB, 2016



Kia Sportage factory production line.
2012

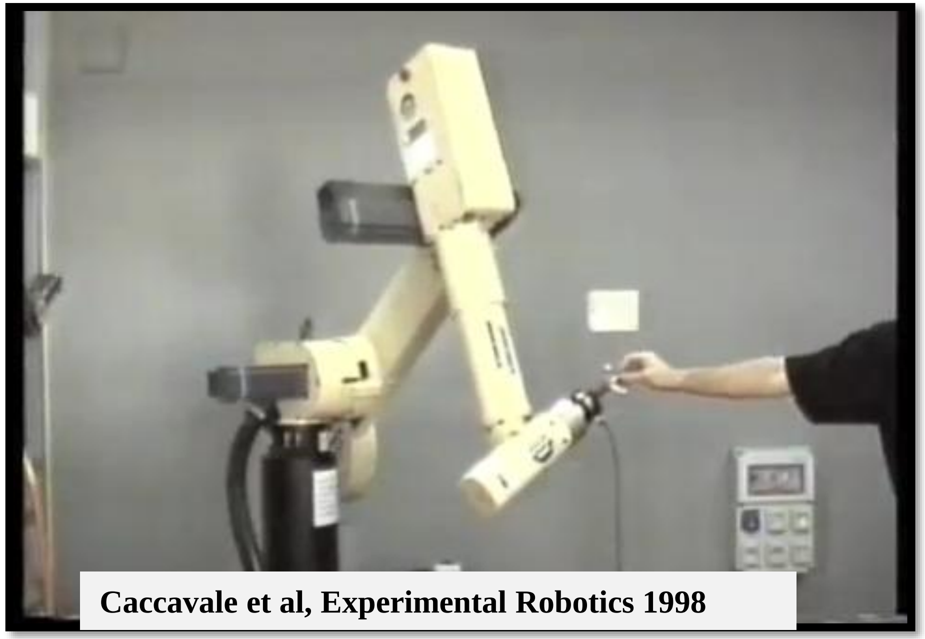
Haddadin *et al*, ICRA 2009

Safety

Haddadin *et al*, ICRA 2009

Robotics Lab at DIAG 2012

Actively backdrivable

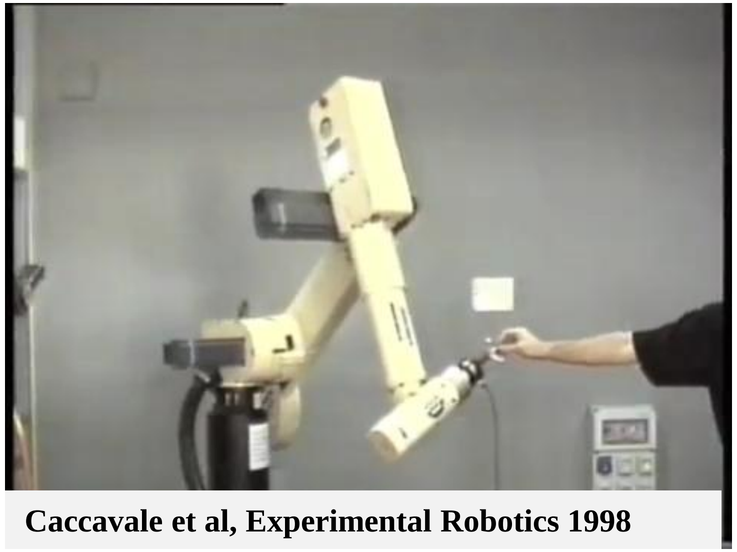


Caccavale et al, Experimental Robotics 1998

EPFL

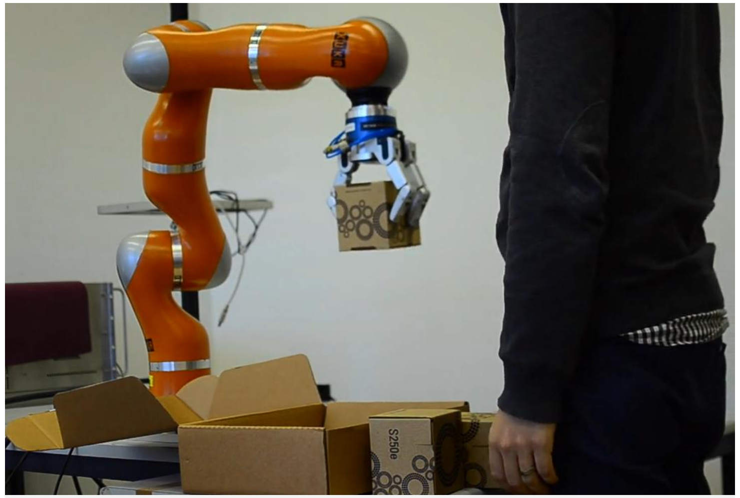
Actively backdrivable

Robotics Lab at DIAG 2012

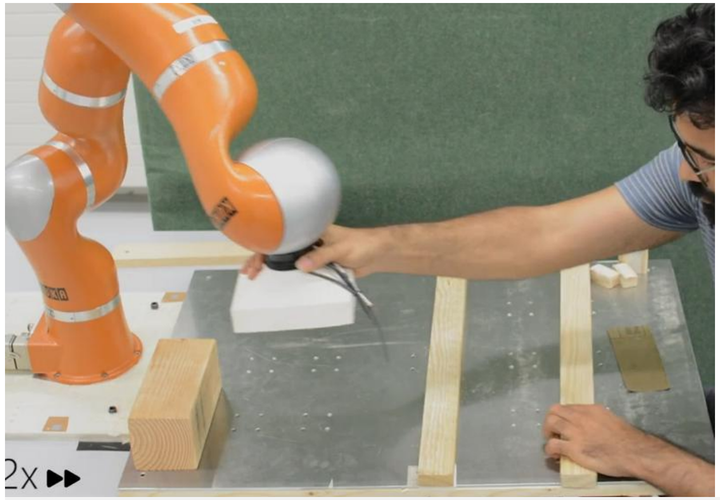


Caccavale et al, Experimental Robotics 1998

Allows for live interactions with humans during task execution



Kronander, Billard, RAL 2016



Khoramshahi, Billard, Autonomous Robots 2018

Compliance with impedance control
Principle

What does Impedance control do?

- It imposes a desired dynamic behavior to the interaction between an object (in our case a robot) and environment.

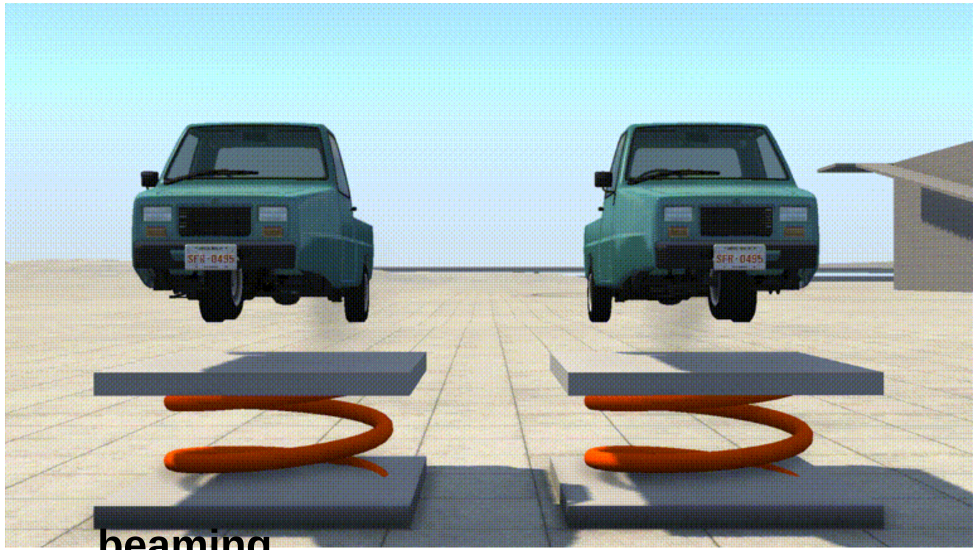
How does Impedance control work?

- The desired performance is specified through a set of *mass-spring-damper equations*:

$$m\ddot{x} + d\dot{x} + k(x - x^*) = F_{ext},$$ *m: mass, d: damping, k: stiffness, F_{ext}: External forces*

This model describes how the system reacts to the external forces with environment deformation.

Very soft



Very Stiff

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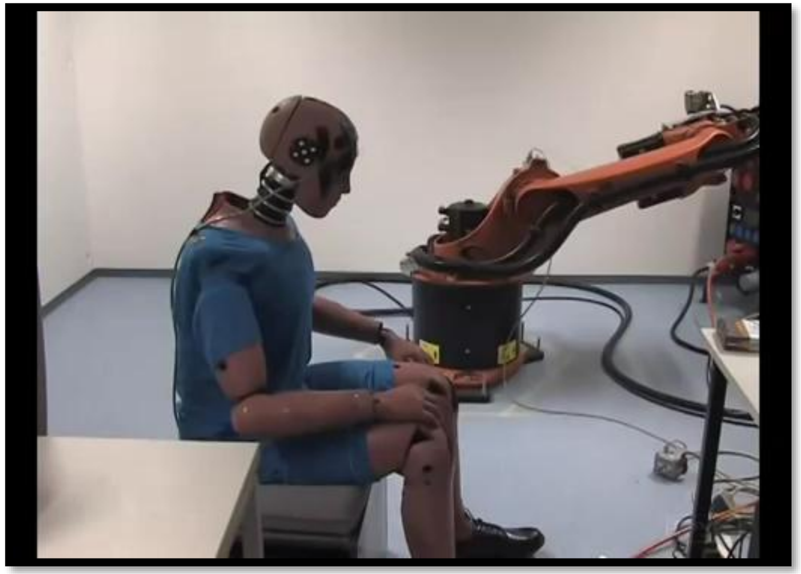
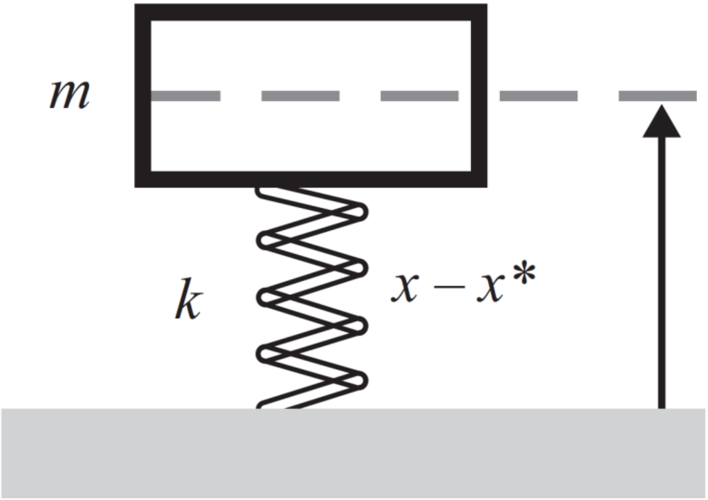
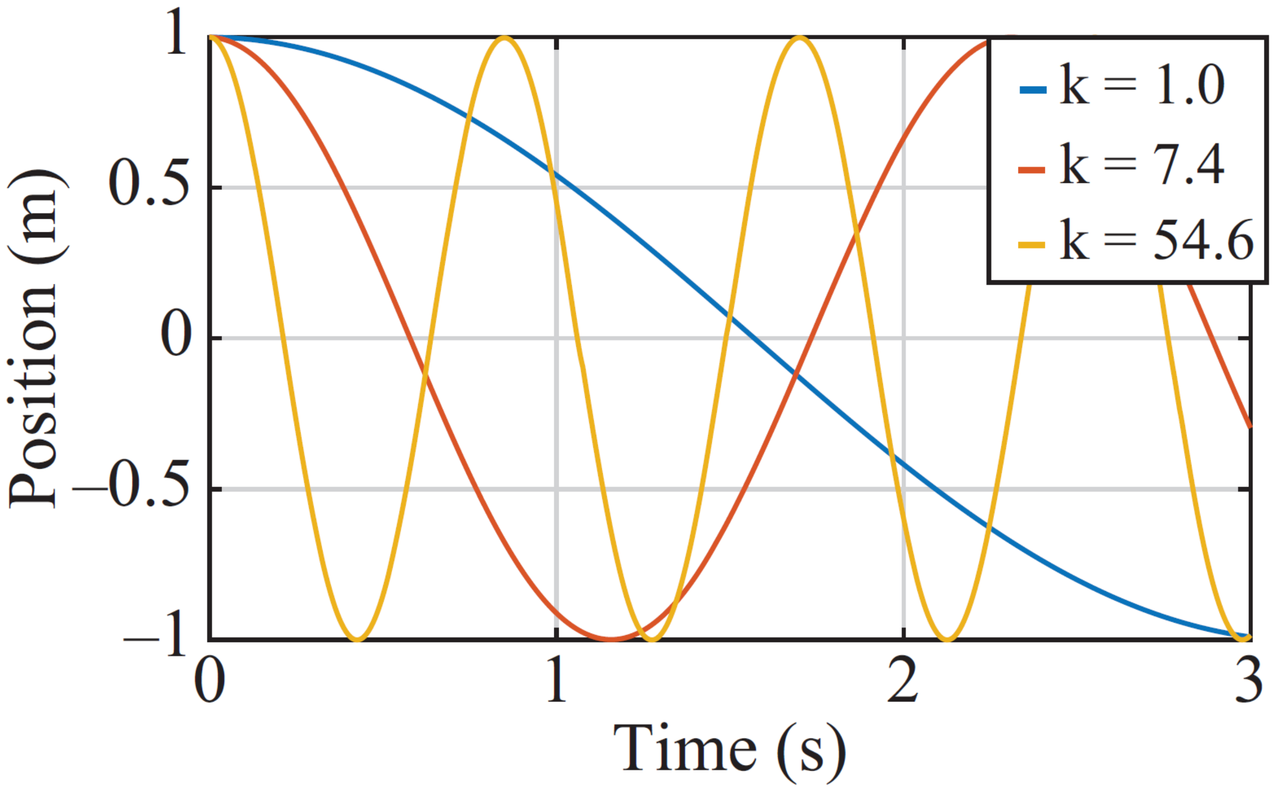


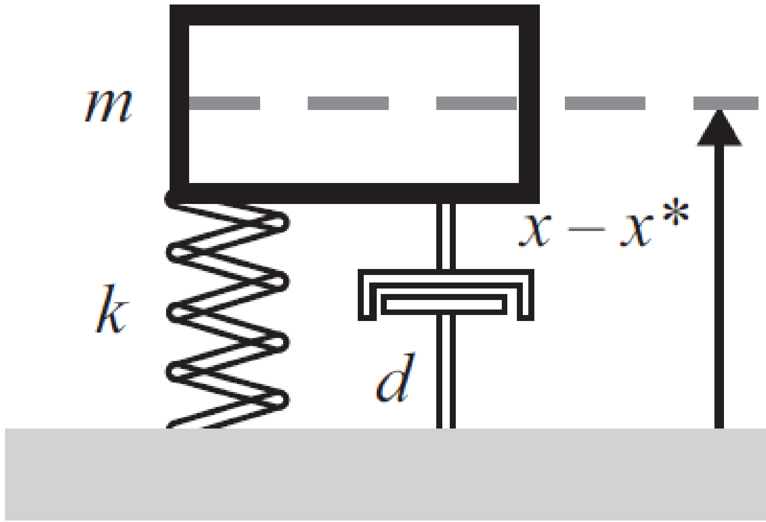
Very Stiff



What will the behavior of the system be for different values of the spring constant *k*?

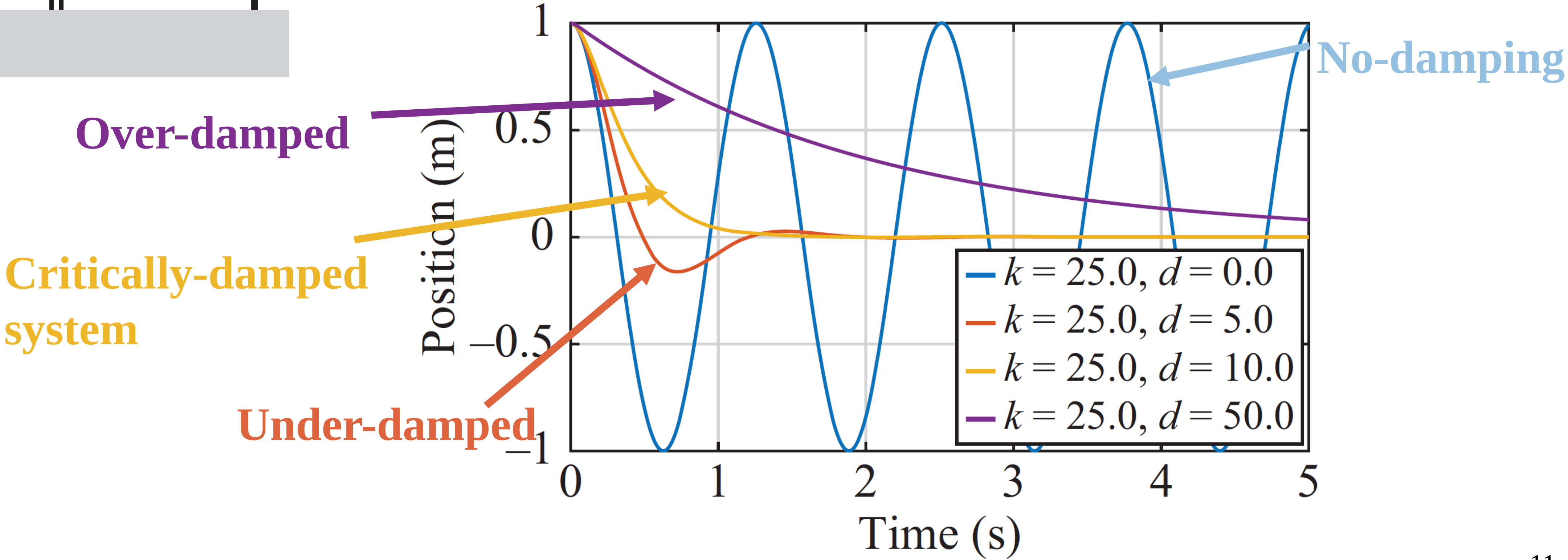
$$m\ddot{x} + k(x - x^*) = F_{ext}$$

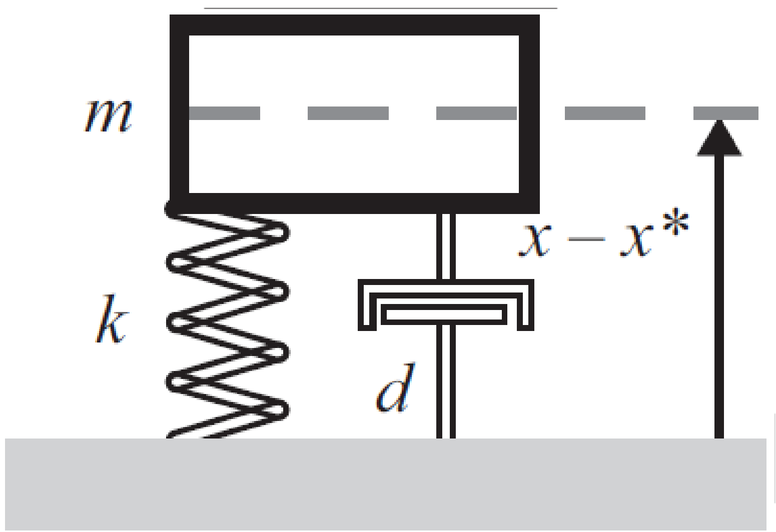




What will the behavior of the system be for different values of the damping constant *d*?

$$m\ddot{x} + d\dot{x} + k(x - x^*) = F_{ext}$$

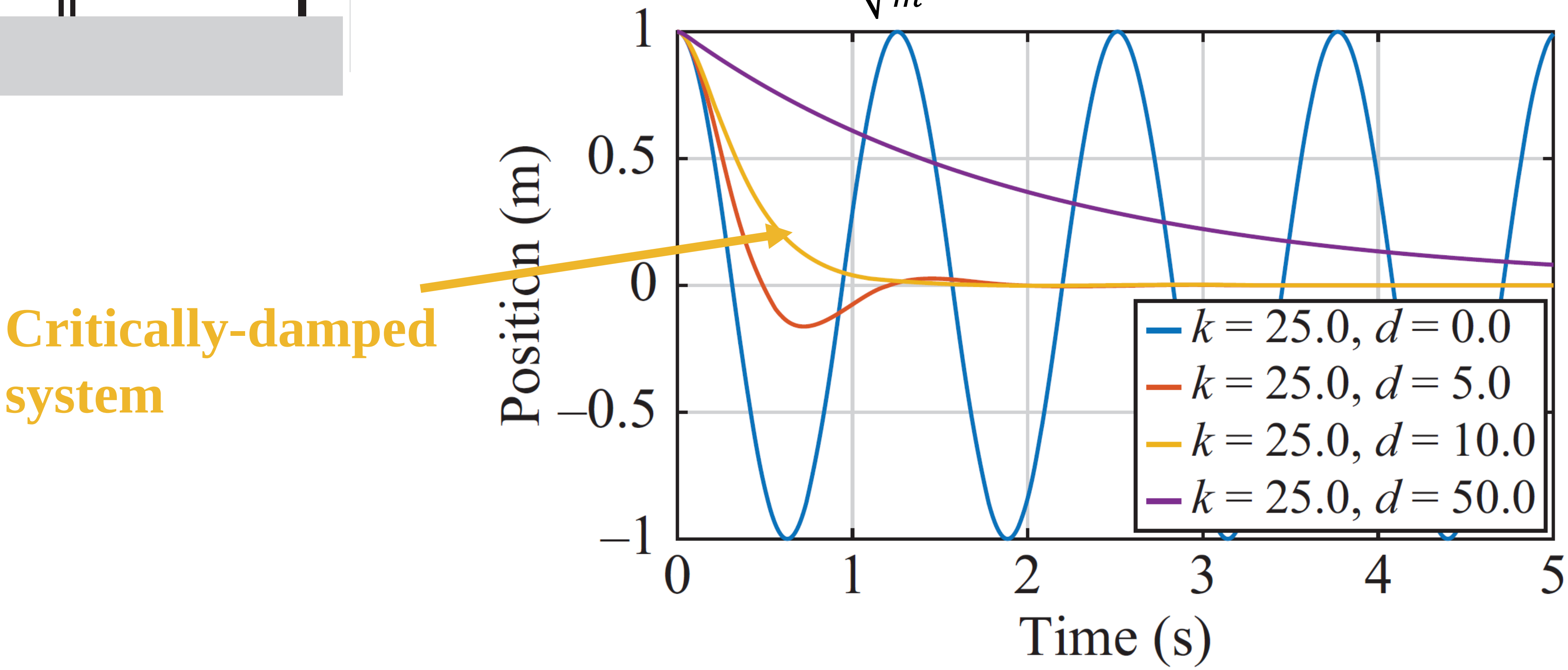




What will the behavior of the system be for different values of the damping constant *d*?

$$\ddot{x} + 2\omega\dot{x} + \omega^2(x - x^*) = F_{ext}$$

$\omega = \sqrt{\frac{k}{m}}$ is the natural frequency of the system.



What does impedance mean in robotics?

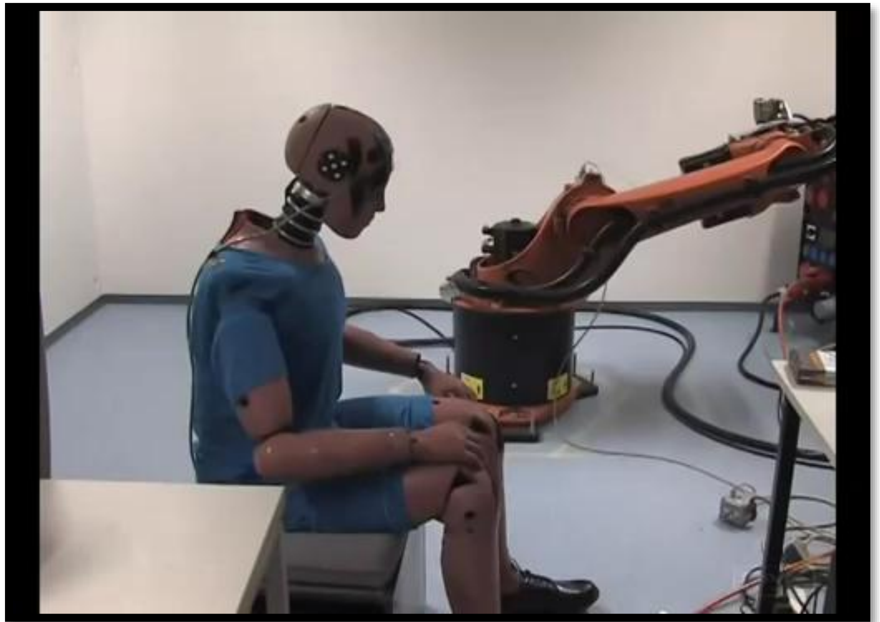
- In control, impedance indicates how much a system resists a harmonic force (i.e., the ratio of the force to the resulting velocity)

$$m\ddot{x} + d\dot{x} + k(x - x^*) = F_{ext}$$

Impedance of a mass-spring-damper is (solution of diff. equation through Laplace transform):

$$\frac{F_{ext}}{\dot{x}} = \frac{s^2 m + sd + k}{s}$$

Low Impedance



High Impedance

Robot dynamics

- Dynamic of a robot (in the joint space):

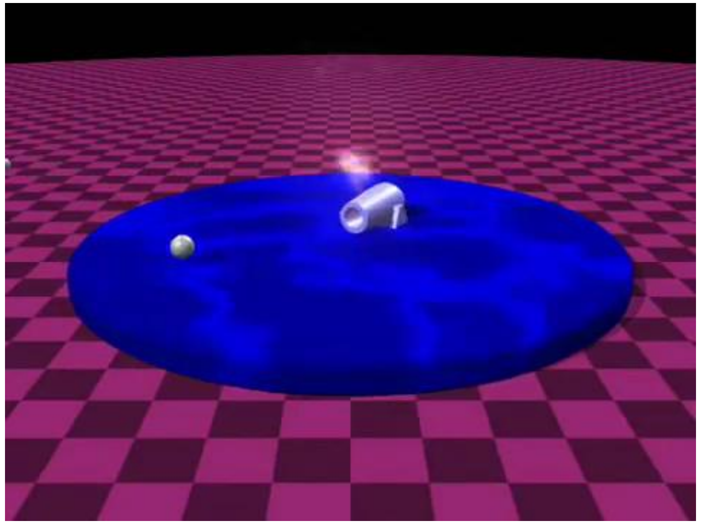
$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = J(q)^T F_{ext} + \tau$$

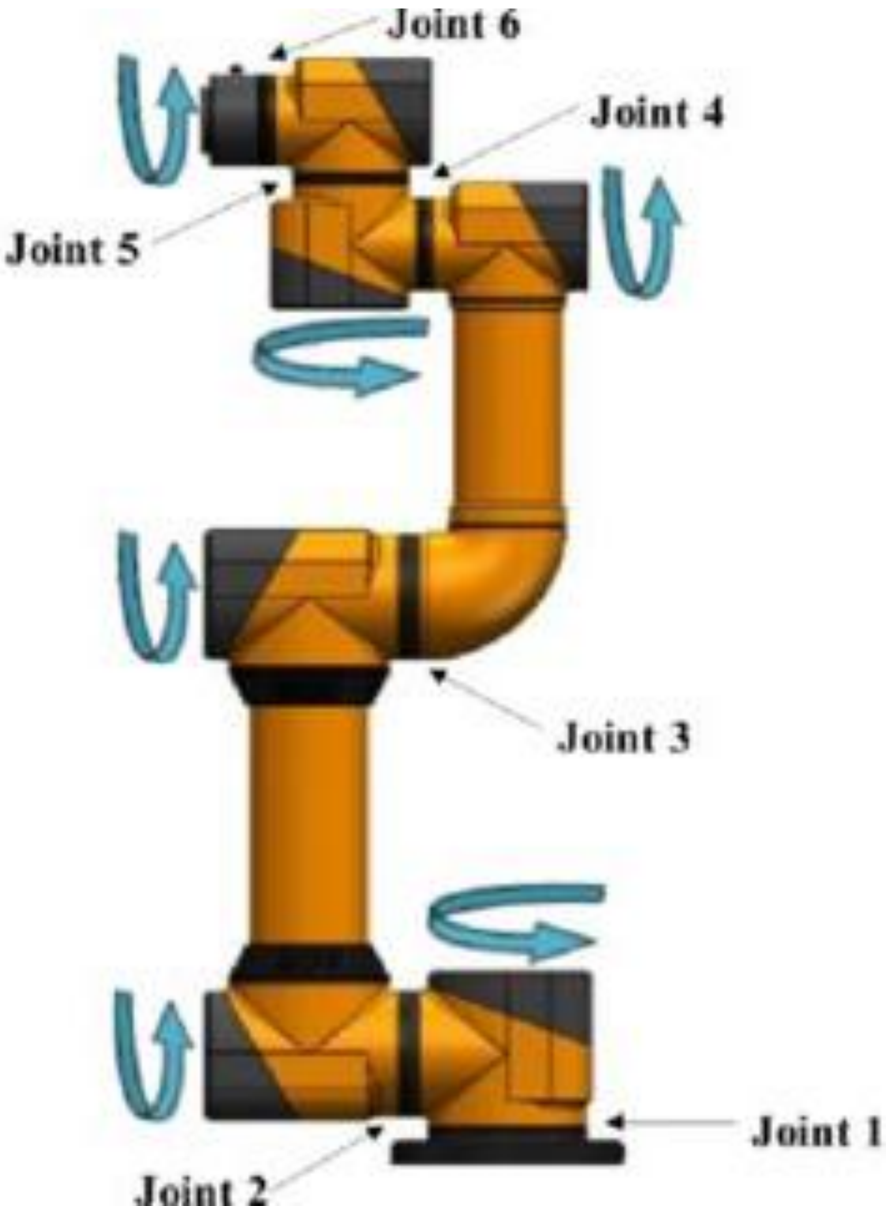
$q \in R^6$ (Joint position)

$M(q) \in R^{6\times6}$ (Mass matrix)

➢ Symmetric, positive definite

$C(q,\dot{q}) \in R^{6\times6}$ (Coriolis and centrifugal forces)





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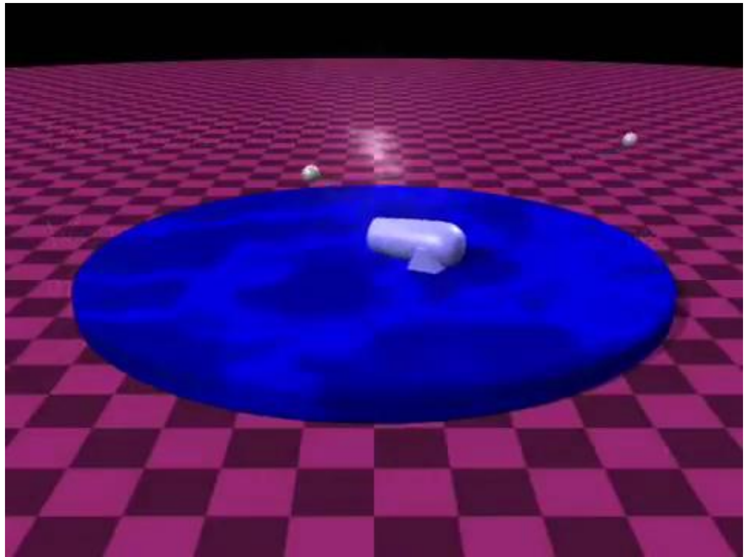

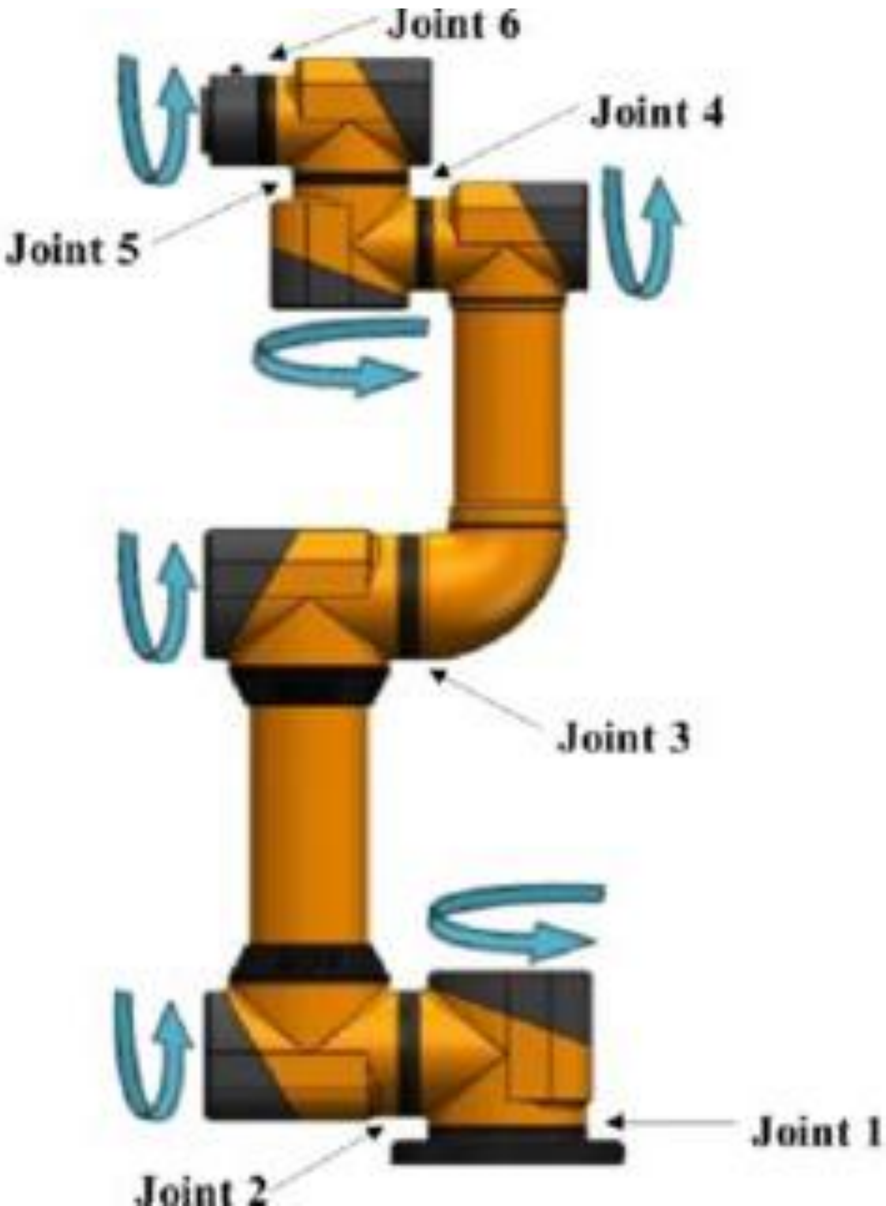

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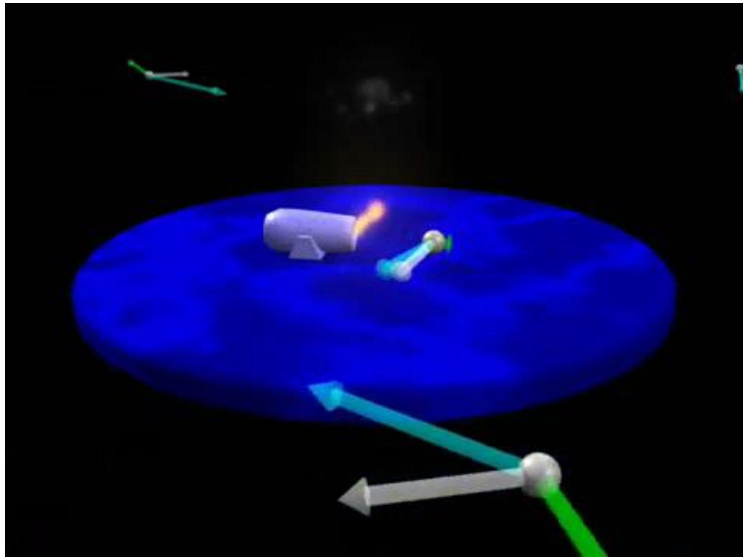

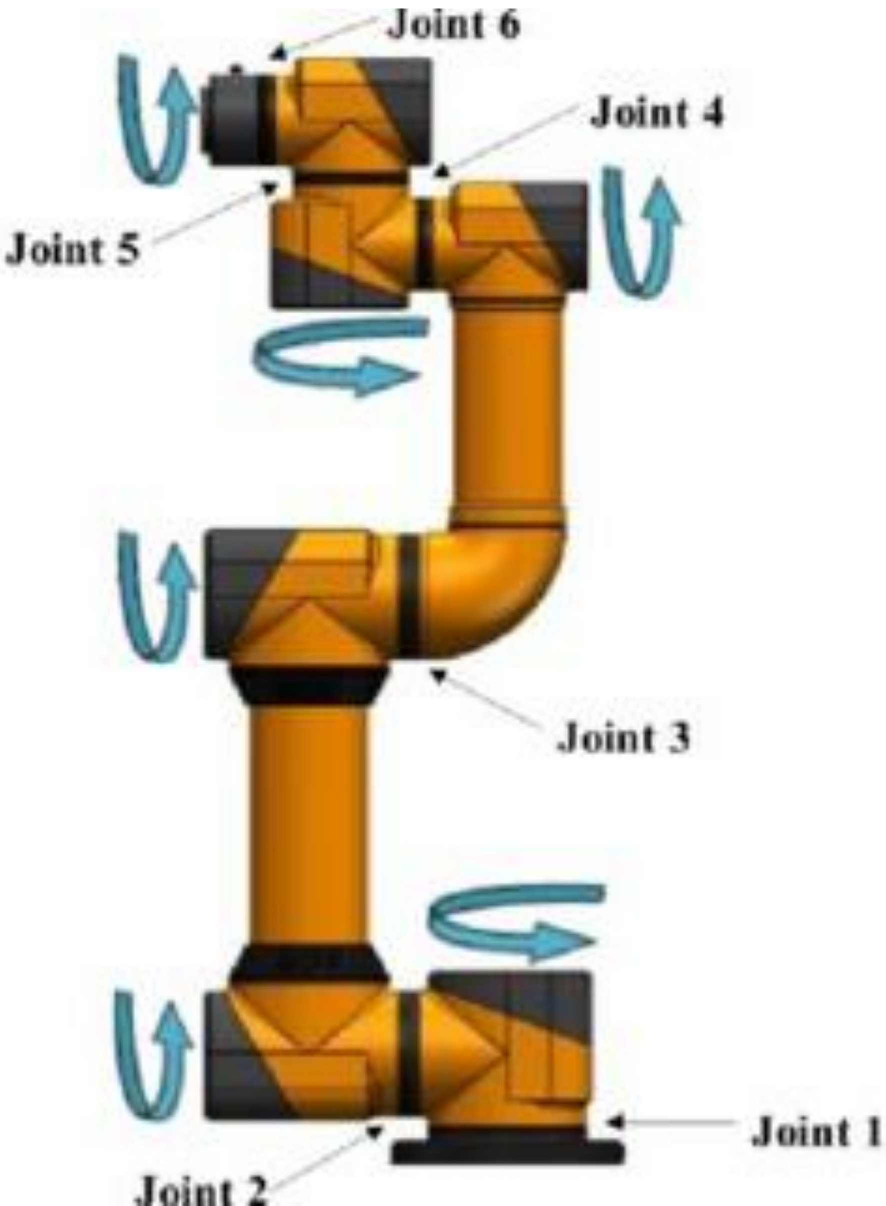

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$C(q,\dot{q}) \in R^{6\times6}$ (Coriolis and centrifugal forces)

➢ $\dot{M}(q) - 2C(q,\dot{q})$ is skew symmetric.

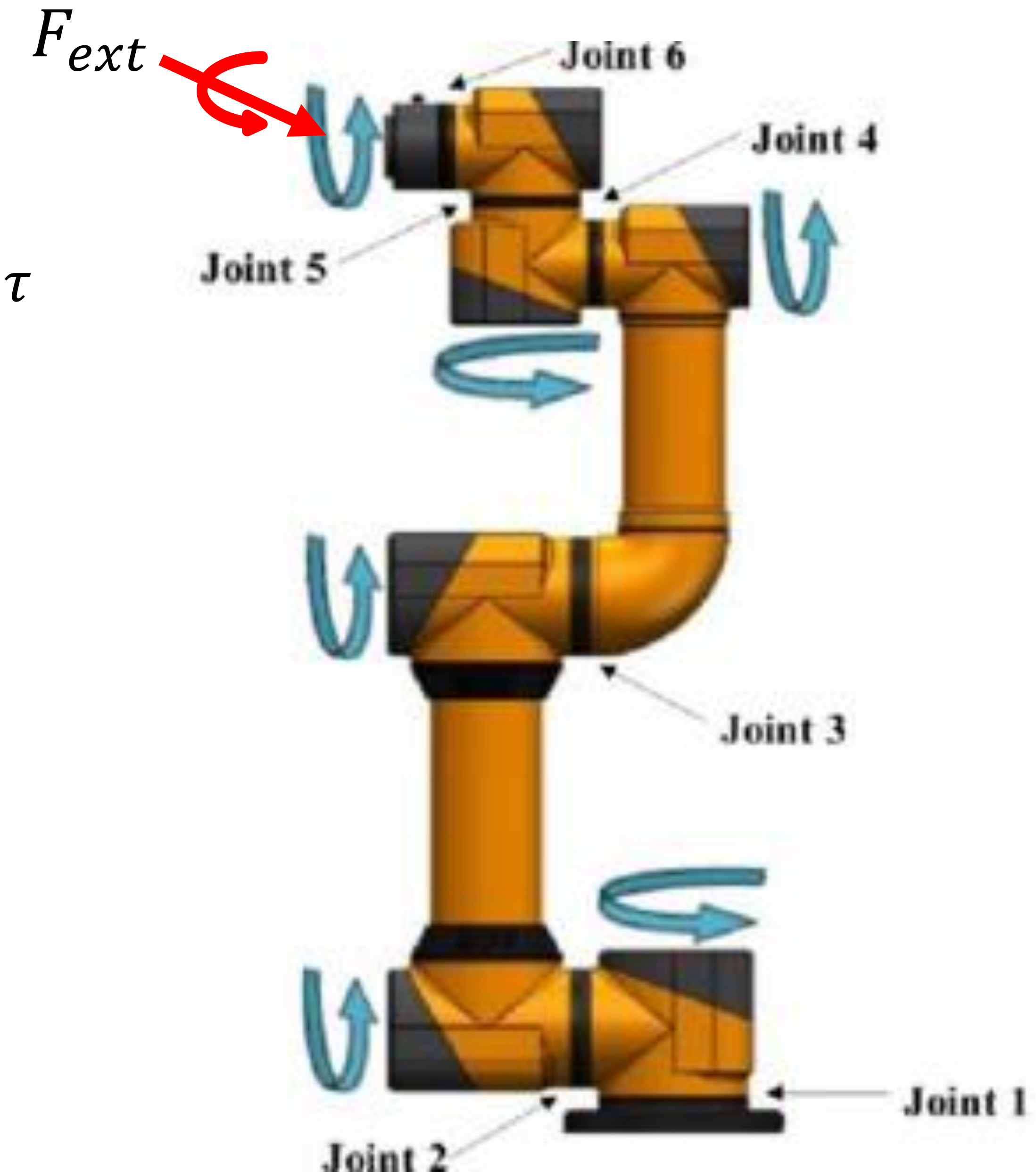
$G(q) \in R^{6\times1}$ (Gravity)

$J(q) \in R^{6\times6}$ (Jacobian matrix)

➢ $v = J(q)\dot{q}$, v: speed of end-effector

$F_{ext} \in R^{6\times1}$ (External force and torque)

$\tau \in R^{6\times1}$ (Control input)



Robot dynamics

- Dynamic of a robot (in the joint space):

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = J(q)^T F_{ext} + \tau$$

$q \in R^{7}$ (Joint position)

$M(q) \in R^{7\times 7}$ (Mass matrix)

➢ Symmetric, positive definite

$C(q,\dot{q}) \in R^{7\times 7}$ (Coriolis and centrifugal forces)

➢ $\dot{M}(q) - 2C(q,\dot{q})$ is skew symmetric.

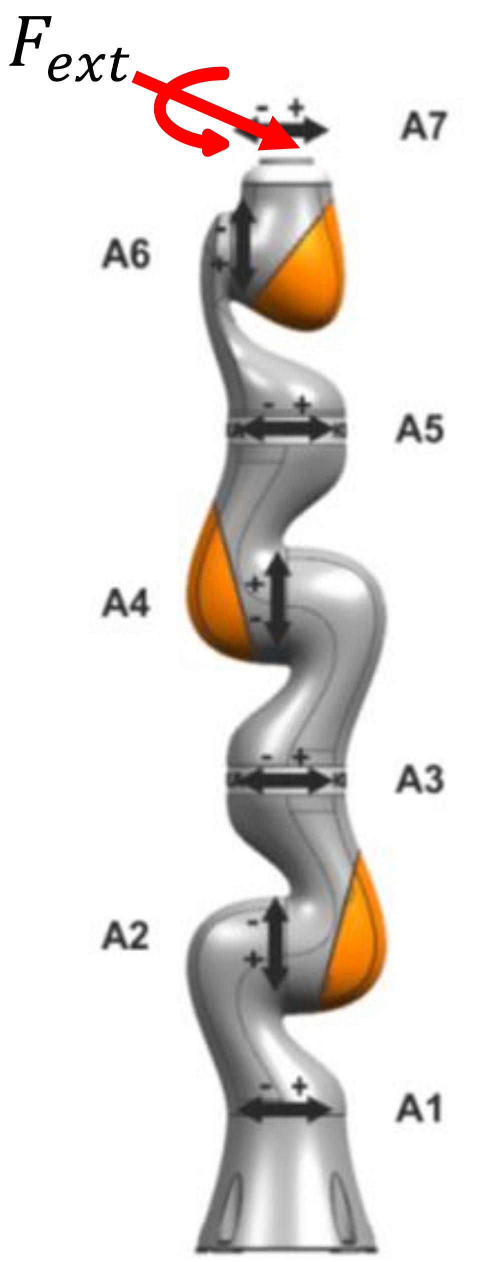
$G(q) \in R^{7\times 1}$ (Gravity)

J(q) $\in R^{7\times 6}$ (Jacobian matrix)

➢ $v=J(q)\dot{q}$

$F_{ext} \in R^{6\times 1}$ (External force and torque)

$\tau \in R^{7\times 1}$ (Control input)



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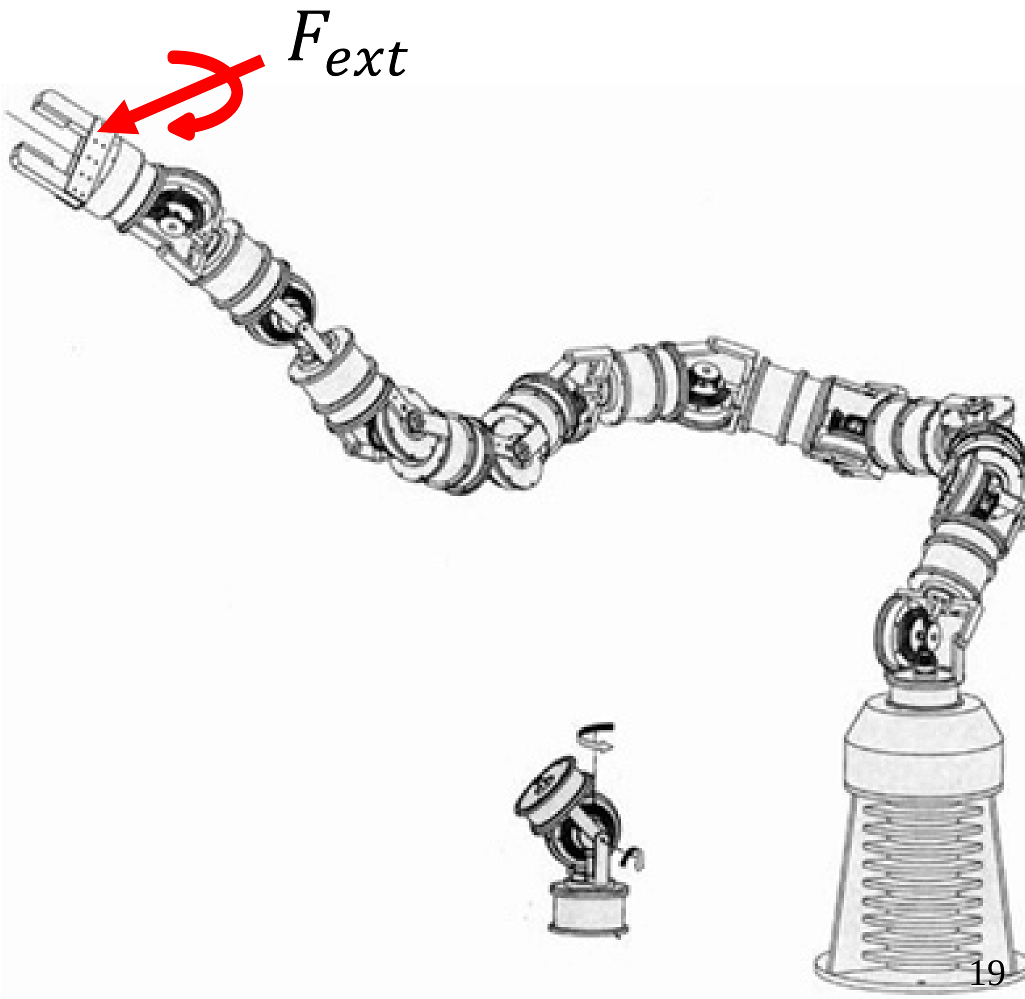

$q \in R^{N}$ (Joint position)

$M(q) \in R^{N\times N}$ (Mass matrix)

➢ Symmetric, positive definite

$C(q,\dot{q}) \in R^{N\times N}$ (Coriolis and centrifugal forces)

➢ $\dot{M}(q) - 2C(q,\dot{q})$ is skew symmetric.

$G(q) \in R^{N\times 1}$ (Gravity)

$J(q) \in R^{N\times 6}$ (Jacobian matrix)

➢ $v=J(q)\dot{q}$

$F_{ext} \in R^{6\times 1}$ (External force and torque)

$\tau \in R^{N\times 1}$ (Control input)

What does impedance mean in robotics?

- Dynamic of a robot (in the joint space):

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = J(q)^T F_{ext} + \tau$$

How the control input should be designed such that the above system will look like a desired mass-spring-damper: $\Lambda\ddot{q} + \mathrm{D}\dot{\mathrm{q}} + \mathrm{Kq} = J(q)^T F_{ext}$?

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Hint:

Feed-back linearization!

- $\tau = C(q,\dot{q})\dot{q} + G(q) + \cdots$

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- $M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = J(q)^T F_{ext} + C(q,\dot{q})\dot{q} + G(q) + \cdots$

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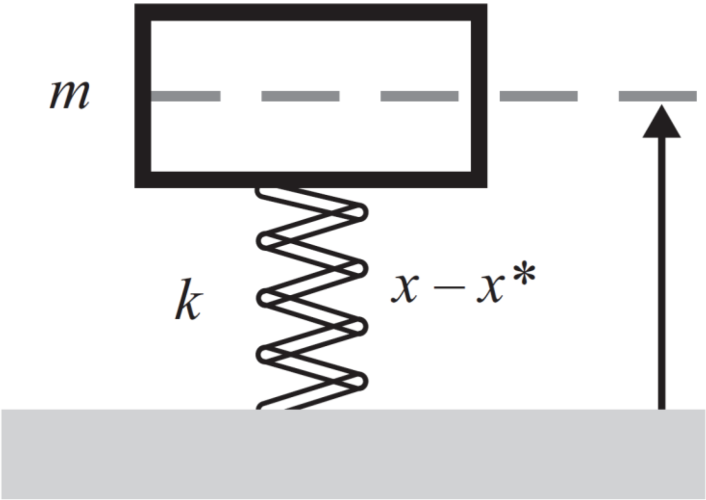
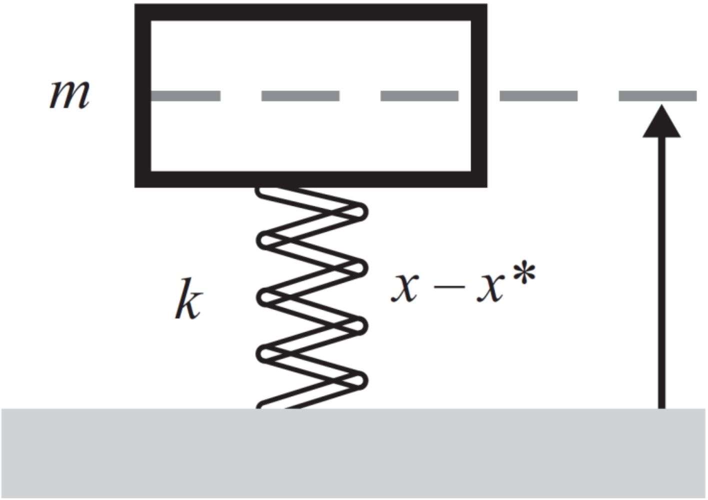
- $\tau = C(q,\dot{q})\dot{q} + G(q) + \cdots$
- $M(q)\ddot{q} = J(q)^T F_{ext} + \cdots$

Why we can't define $\tau = C(q,\dot{q})\dot{q} + G(q) + M(q)\ddot{q} - \Lambda\ddot{q} - \mathrm{D}\dot{q} - \mathrm{K}q$



What are the inputs and the outputs in this system?

$$m\ddot{x} + k(x - x^*) = F_{ext}$$



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How the control input should be designed such that the above system will look like a desired mass-spring-damper: $\Lambda\ddot{q} + D\dot{q} + Kq = J(q)^T F_{ext}$?

Hint:

Feed-back linearization!

- $\tau = C(q,\dot{q})\dot{q} + G(q) + \cdots$
- $M(q)\ddot{q} = J(q)^T F_{ext} + \cdots$

We can't define $\tau = C(q,\dot{q})\dot{q} + G(q) + M(q)\ddot{q} - \Lambda\ddot{q} - D\dot{q} - Kq$ as we can't have $\ddot{q}$ in both sides!

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$M(q)$ and Λ are positive definite

- $\tau = C(q,\dot{q})\dot{q} + G(q) - M(q)\Lambda^{-1}(D\dot{q} + K\,q)$

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- $\tau = C(q,\dot{q})\dot{q} + G(q) - M(q)\Lambda^{-1}(D\dot{q} + K\,q) + (M(q)\Lambda^{-1} - I)J(q)^T\ F_{ext}$

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How the control input should be designed such that the above system will look like a desired mass-spring-damper while following a desired path:

$$\Lambda\ddot{\tilde{q}} + \mathrm{D}\dot{\tilde{q}} + \mathrm{K}\tilde{q} = J(q)^T F_{ext}?$$

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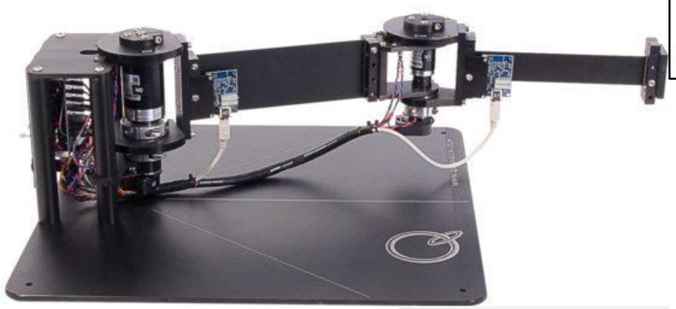
Example: 2Dof planner robot

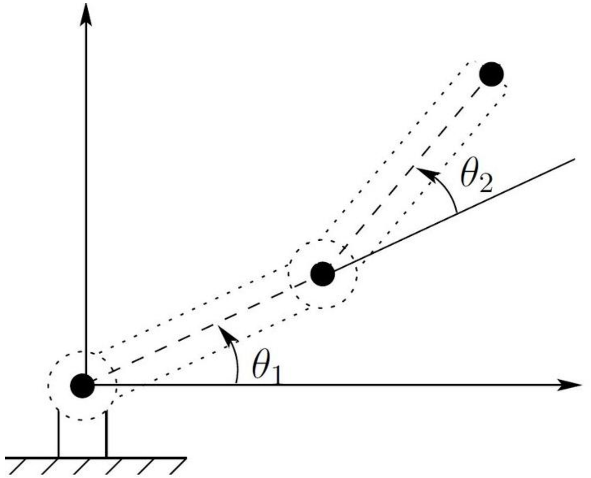
$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = J(q)^T F_{ext} + \tau$$

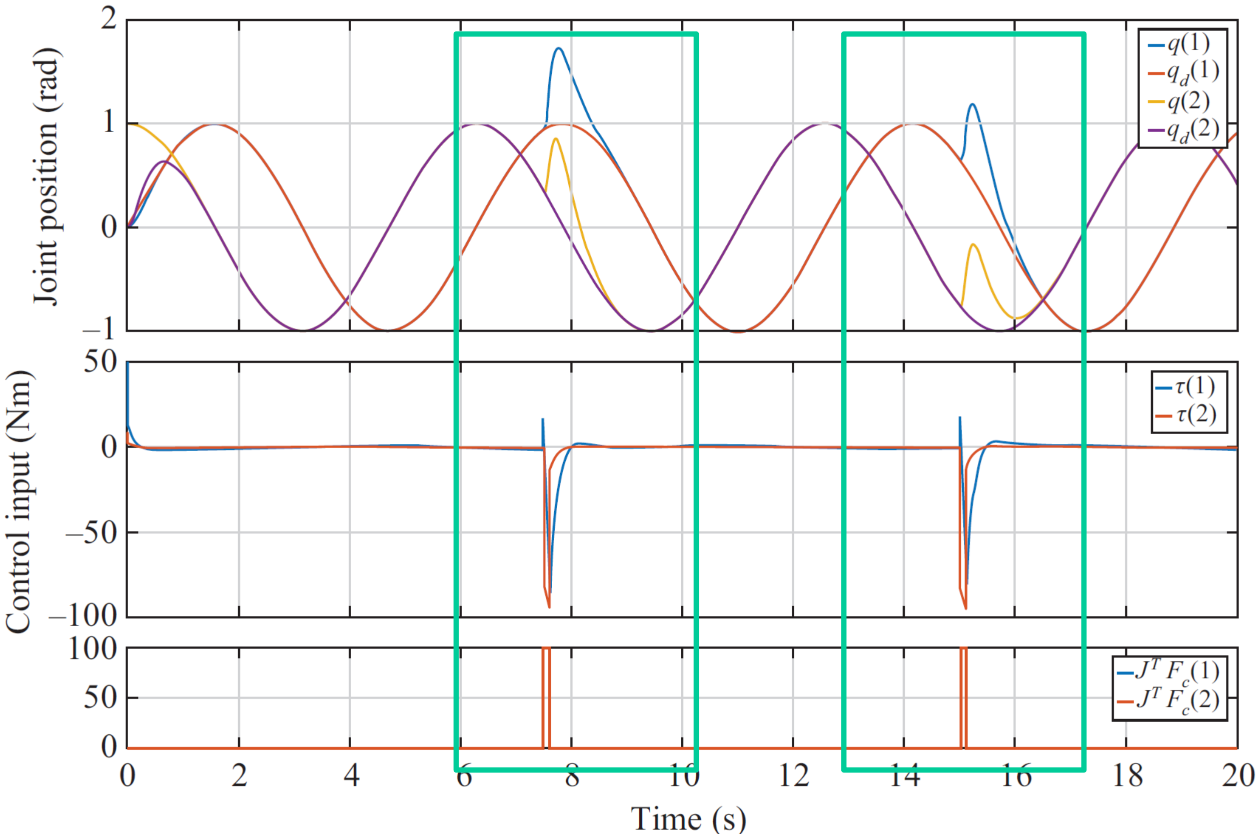
$$\tau = C(q,\dot{q})\dot{q} + G(q) - M(q)\Lambda^{-1}(D\dot{\tilde{q}} + K\tilde{q}) + (M(q)\Lambda^{-1} - I)J(q)^T F_{ext} + \Lambda\ddot{q}^d$$



Quanser

$m_1 = 1.0$ m_2=0.5 *Mass of the first and second links*
l_1=1.0 l_2=0.5 *Length of the first and second links*
$\Lambda = I_{2\times2}, D = 10I_{2\times2}, K = 25I_{2\times2}$





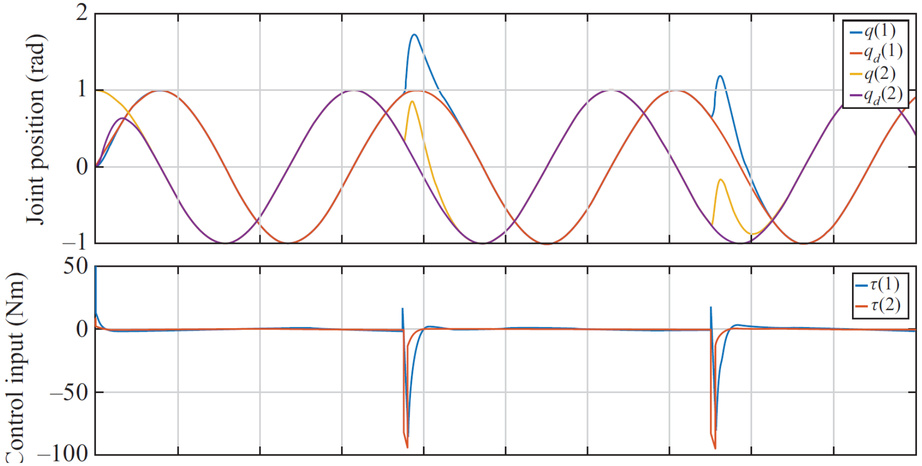
$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = J(q)^T F_{ext} + \tau$$

$m_1 = 1.0$ $m_2 = 0.5$ *Mass of the first and second links*
$l_1 = 1.0$ $l_2 = 0.5$ *Length of the first and second links*
$\Lambda = I_{2\times2}, D = 10I_{2\times2}, K = 25I_{2\times2}$

Example: 2Dof planner robot

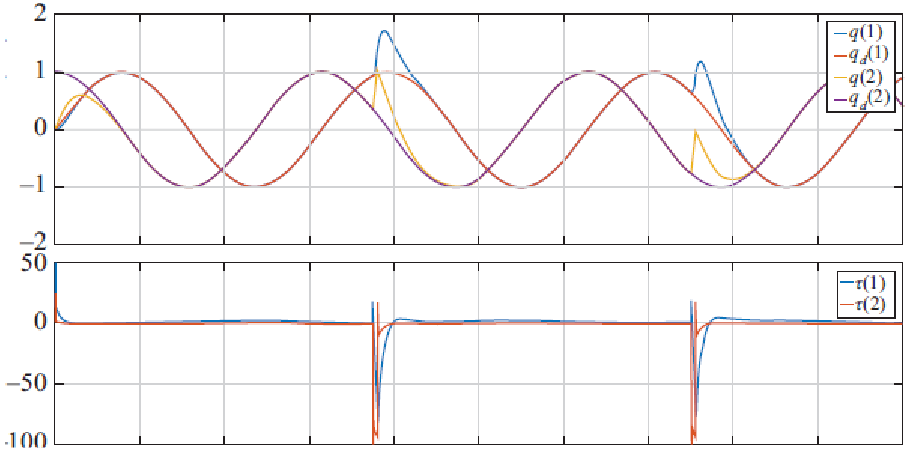
Eq 1

$$\tau = C(q,\dot{q})\dot{q} + G(q) - M(q)\Lambda^{-1}(D\dot{\tilde{q}} + K\tilde{q}) + (M(q)\Lambda^{-1} - I)J(q)^T F_{ext} + \Lambda\ddot{q}^d$$



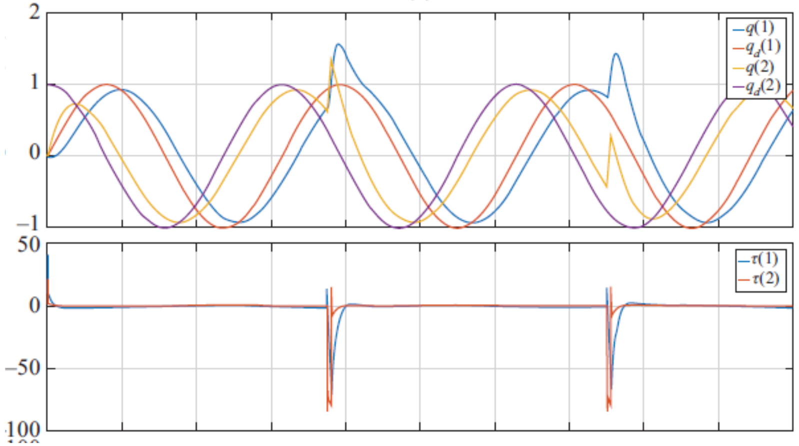
Eq 2

$$\tau = C(q,\dot{q})\dot{q} + G(q) - (D\dot{\tilde{q}} + K\tilde{q}) + M(q)\ddot{q}^d$$



Eq 3

$$\tau = G(q) - (D\dot{q} + K\tilde{q})$$



Why do you think we can't always implement Eq 1 or Eq 2?
(5 minutes open discussion)

Learning the impedance parameters

Setting the impedance parameters

The mass-spring-damper depends on choosing well the impedance parameters: matrices Λ, D and K.

$$\boldsymbol{\Lambda}\ddot{\tilde{q}} + \mathbf{D}\dot{\tilde{q}} + \mathbf{K}\tilde{q} = J(q)^T F_{ext}$$

How to determine the best impedance value?

Setting the impedance parameters

The mass-spring-damper depends on choosing well the impedance parameters: matrices Λ, D and K.

$$\boldsymbol{\Lambda}\ddot{\tilde{q}} + \mathbf{D}\dot{\tilde{\mathrm{q}}} + \mathbf{K}\tilde{\mathrm{q}} = \mathrm{J(q)}^T F_{ext}$$

One must set both the absolute value of impedance and its direction! *Impedance can be directional.*

Why do you think setting the direction of the impedance can be important?
(5 minutes open discussion)


CAR
4028

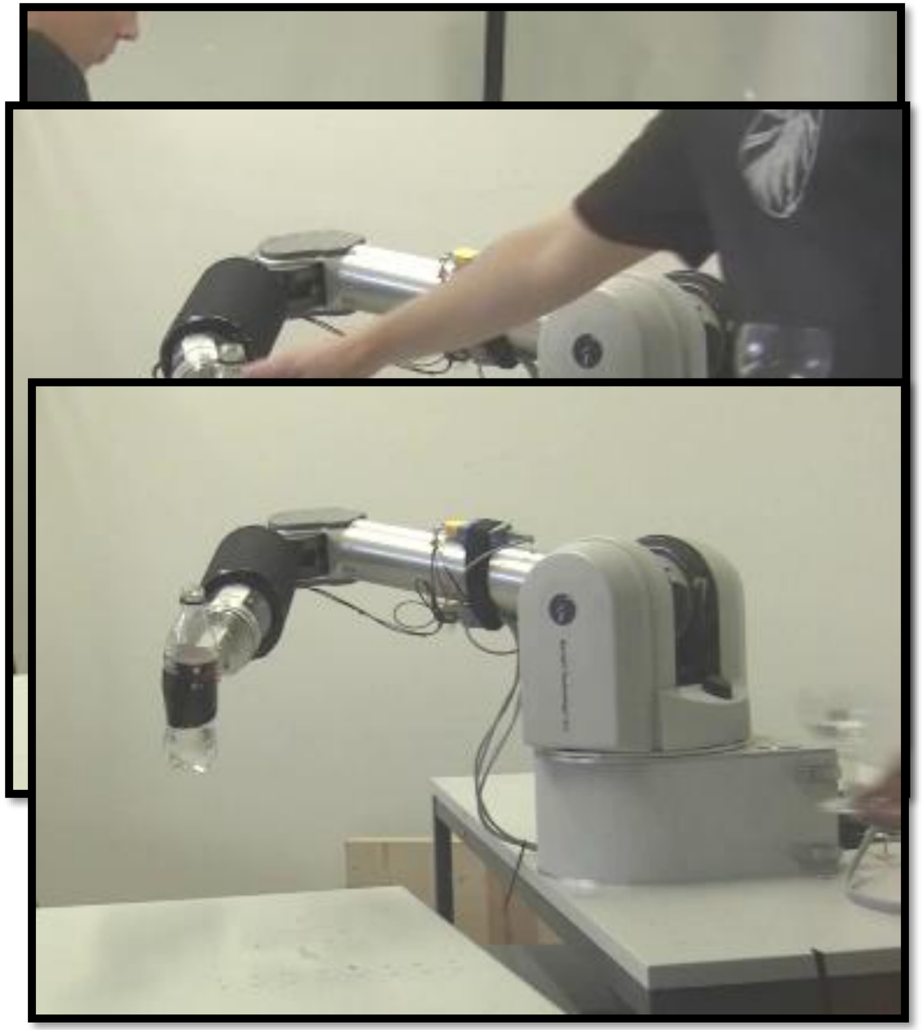

CAR
4028

Example: Required Impedance for pouring a drink

Constant, high stiffness:
Aggressive response to perturbations, spills the coke!

Constant, low stiffness:
Good when reaching

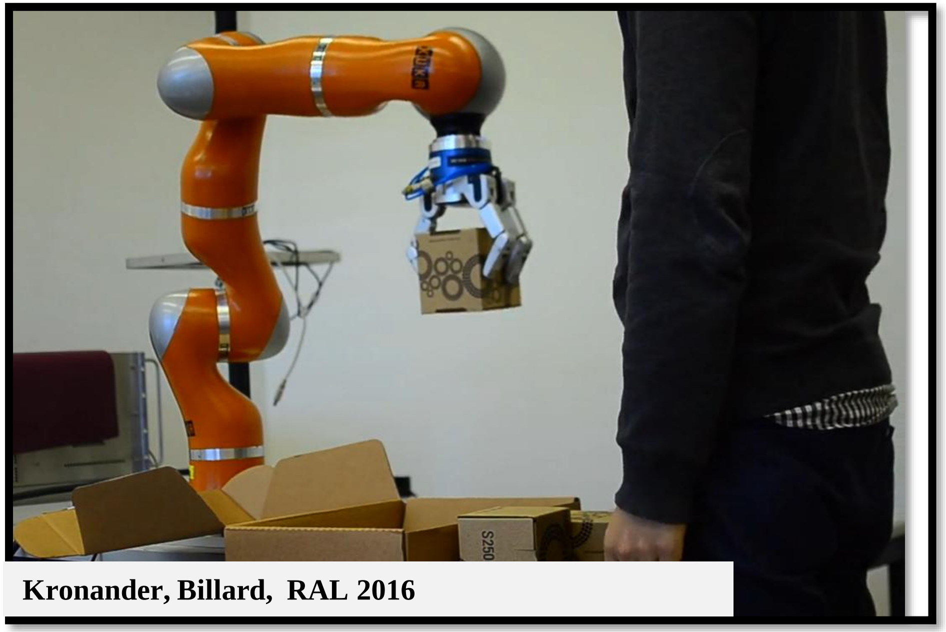
Not good when pouring!



K. Kronander and A. Billard., IEEE Trans. In Haptics, 2014

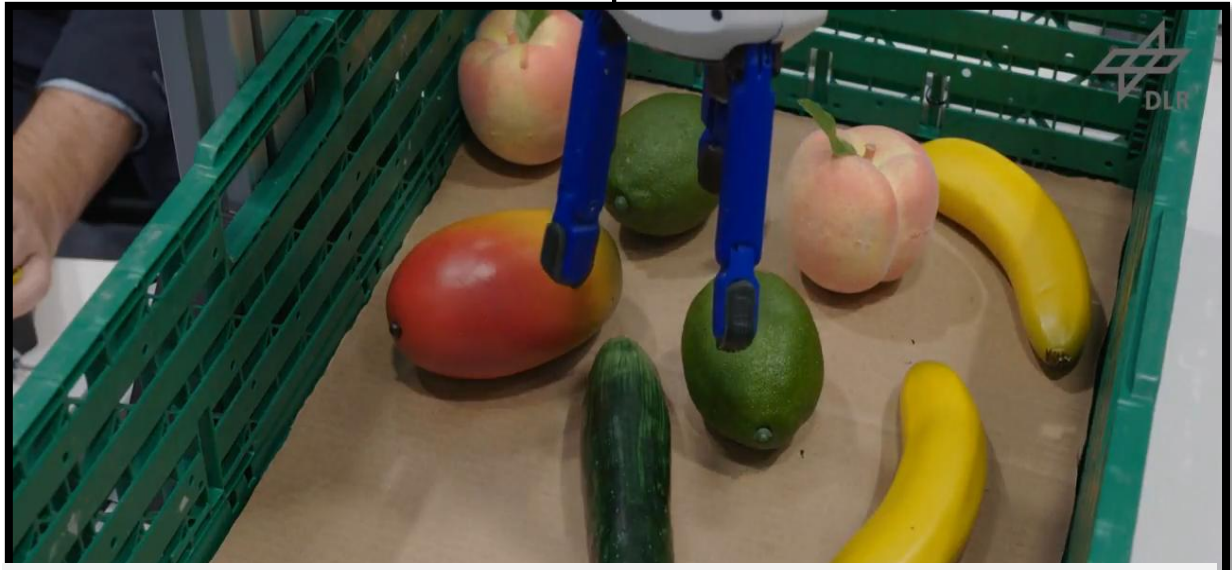
Variable impedance control

Pick and place



Kronander, Billard, RAL 2016

Grasp



DLR

There is no universal impedance value!

Learning the impedance parameters
In variable impedance control

Setting the impedance parameters

The impedance parameters matrices D and K. may vary during the task.

This is expressed by setting an explicit dependency on the state of the system $\tilde{q}, \dot{\tilde{q}}$.

$$\boldsymbol{\Lambda}\ddot{\tilde{q}} + \mathbf{D}(\tilde{q}, \dot{\tilde{q}})\dot{\tilde{q}} + \mathbf{K}(\tilde{q})\tilde{q} = J(q)^T F_{ext}$$

Modeling variable impedance

- We start with our original dynamic of a robot (in the joint space):

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = J(q)^T F_{ext} + \tau$$

How to model the variations of the impedance parameters?

$$\Lambda\ddot{\tilde{q}} + \mathbf{D}(\tilde{q},\dot{\tilde{q}})\dot{\tilde{q}} + \mathbf{K}(\tilde{q})\tilde{q} = J(q)^T F_{ext}?$$

Hint:

Feed-back linearization!

$\mathbf{D}(\tilde{q},\dot{\tilde{q}})$ and $\mathbf{K}(\tilde{q})$ can be modelled by using LPV system:

$$\boldsymbol{K}(\tilde{q}) = \sum_{i=1}^{K_n} \gamma_i^{K_n}(\tilde{q}) K_i \qquad K_i \in R^{N\times N} \qquad \gamma_k^{K_n} \in R_{(0,1)}$$

$$\boldsymbol{D}(\tilde{q},\dot{\tilde{q}}) = \sum_{i=1}^{D_n} \gamma_i^{D_n}(\tilde{q},\dot{\tilde{q}}) D_i \qquad D_i \in R^{N\times N} \qquad \gamma_k^{D_n} \in R_{(0,1)}$$

$$\tau = C(q,\dot{q})\dot{q} + G(q) - M(q)\Lambda^{-1}(\mathbf{D}(\tilde{q},\dot{\tilde{q}})\dot{\tilde{q}} + \mathbf{K}(\tilde{q})\tilde{q}) + (M(q)\Lambda^{-1} - I)J(q)^T F_{ext} + \Lambda\ddot{q}^d$$

Modeling variable impedance

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Example: 2Dof planner robot

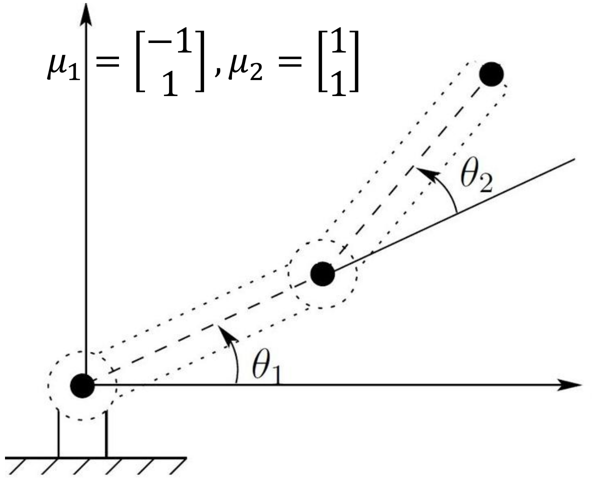
$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = J(q)^T F_{ext} + \tau$$

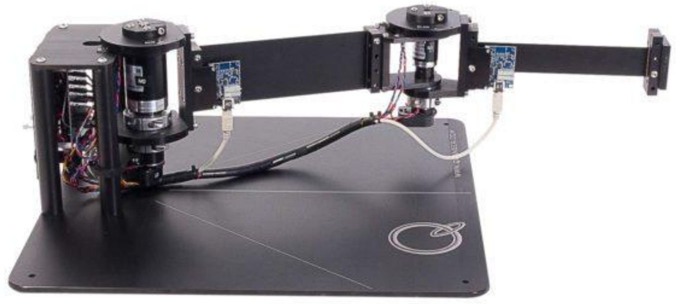
$$\tau = C(q,\dot{q})\dot{q} + G(q) - M(q)\Lambda^{-1}(\mathbf{D}(\tilde{q})\dot{\tilde{q}} + \mathbf{K}(\tilde{q})\tilde{q}) + (M(q)\Lambda^{-1} - I)J(q)^T F_{ext} + M(q)\ddot{q}^d$$

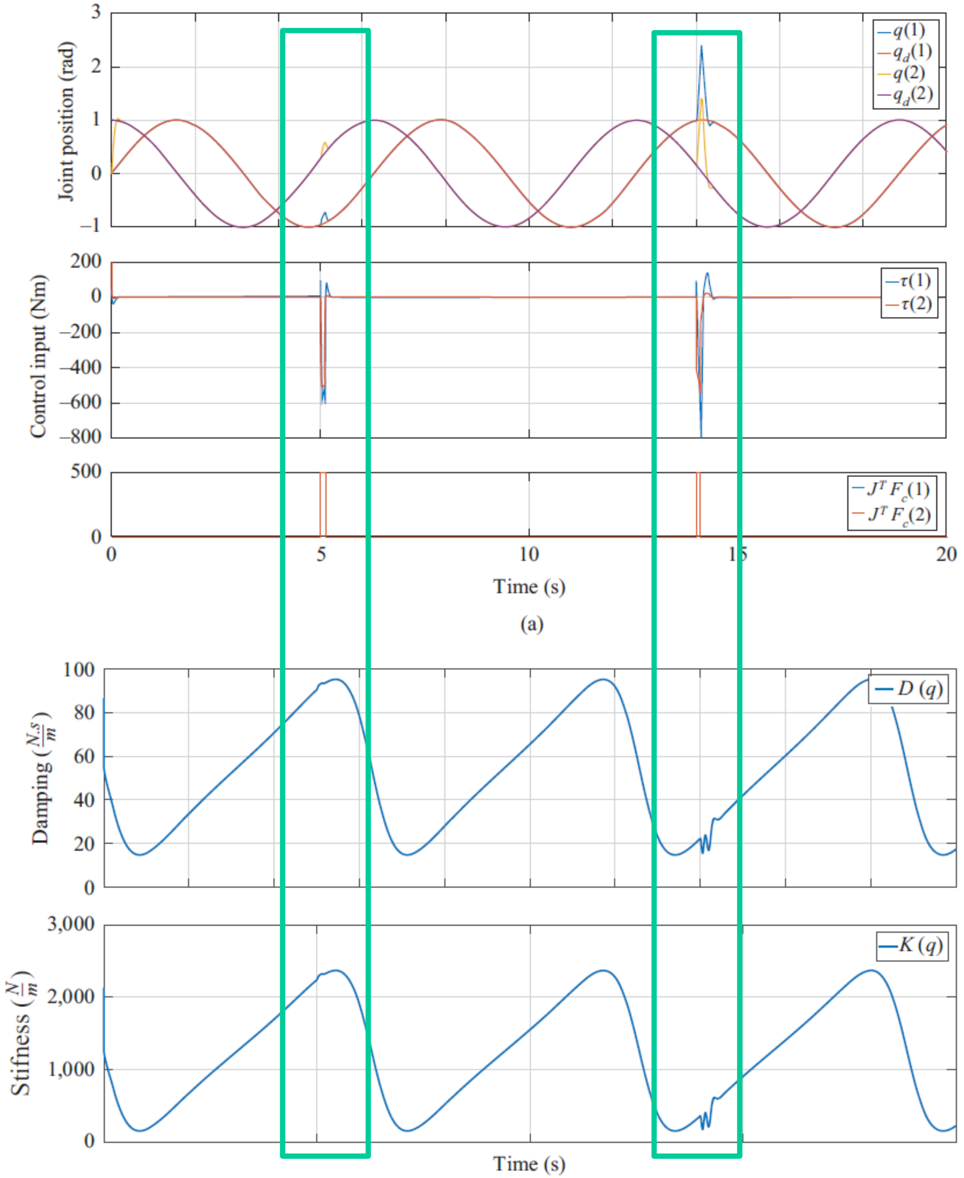
$$\mathbf{D}(\tilde{q}) = \frac{10(q-\mu_1)^T(q-\mu_1) + 100(q-\mu_2)^T(q-\mu_2)}{(q-\mu_1)^T(q-\mu_1) + (q-\mu_2)^T(q-\mu_2)}$$

$$\boldsymbol{K}(\tilde{q}) = \frac{25(q-\mu_1)^T(q-\mu_1) + 2500(q-\mu_2)^T(q-\mu_2)}{(q-\mu_1)^T(q-\mu_1) + (q-\mu_2)^T(q-\mu_2)}$$

$$\mu_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \mu_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$







How to teach a robot to stiffen or unstiffen

Example: Required Impedance for pouring a drink

Constant, high stiffness:
Aggressive response to perturbations,
spills the coke!

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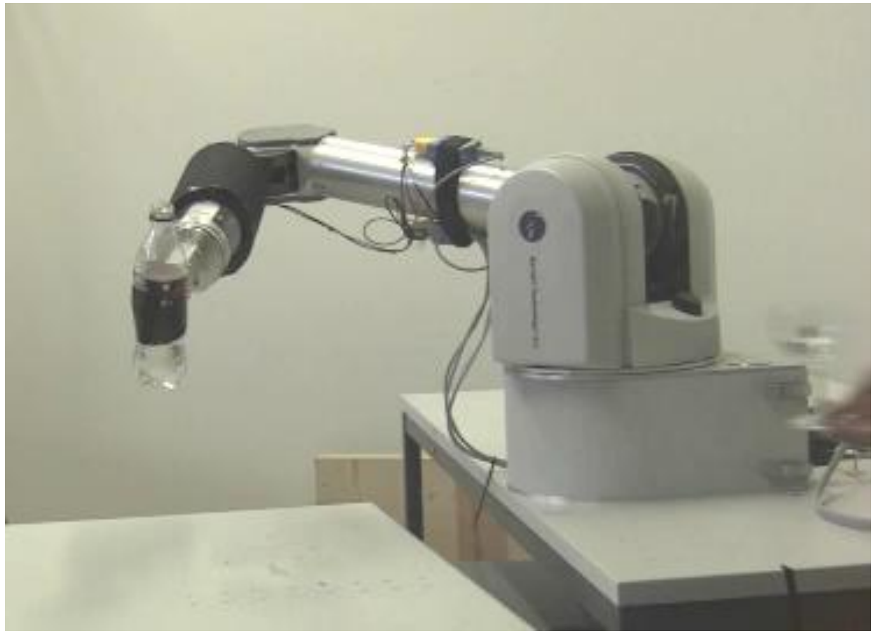
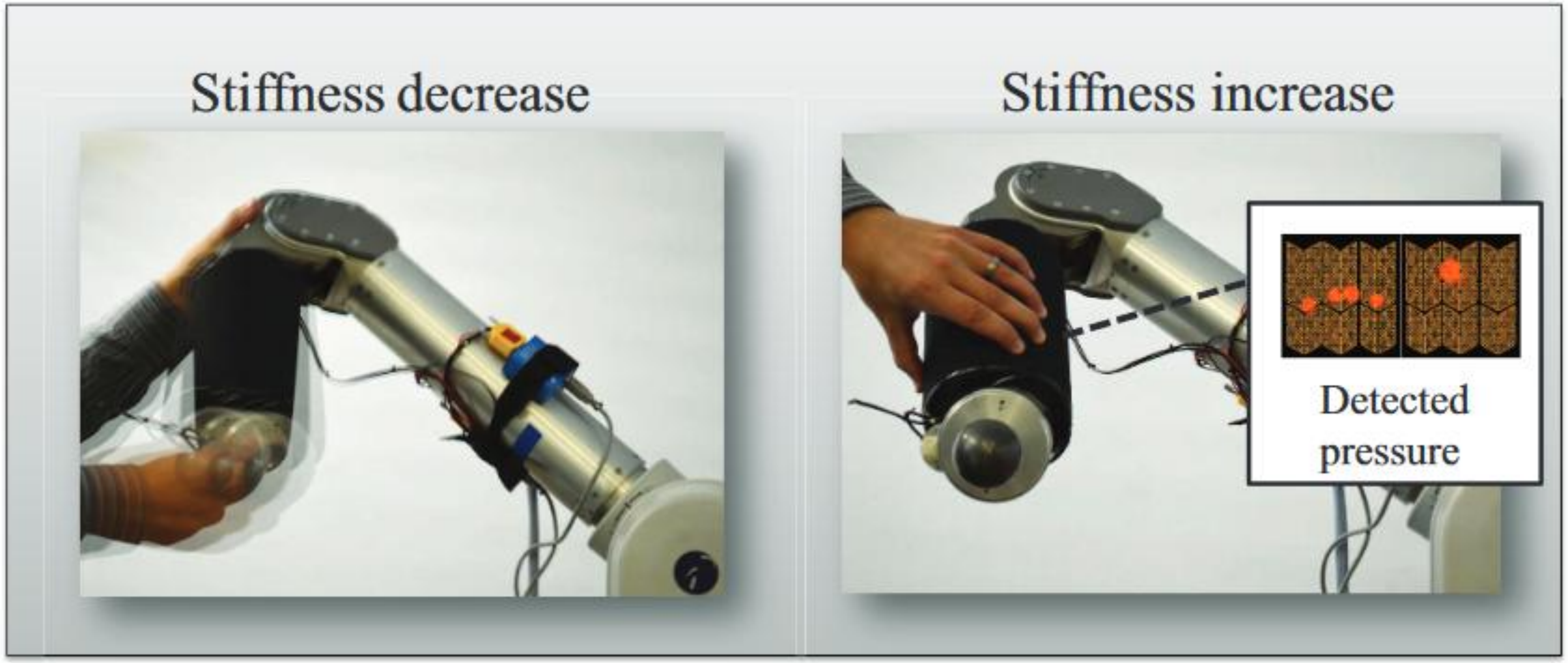


K. Kronander and A. Billard., IEEE Trans. In Haptics, 2014

Learning the Desired Impedance Profiles

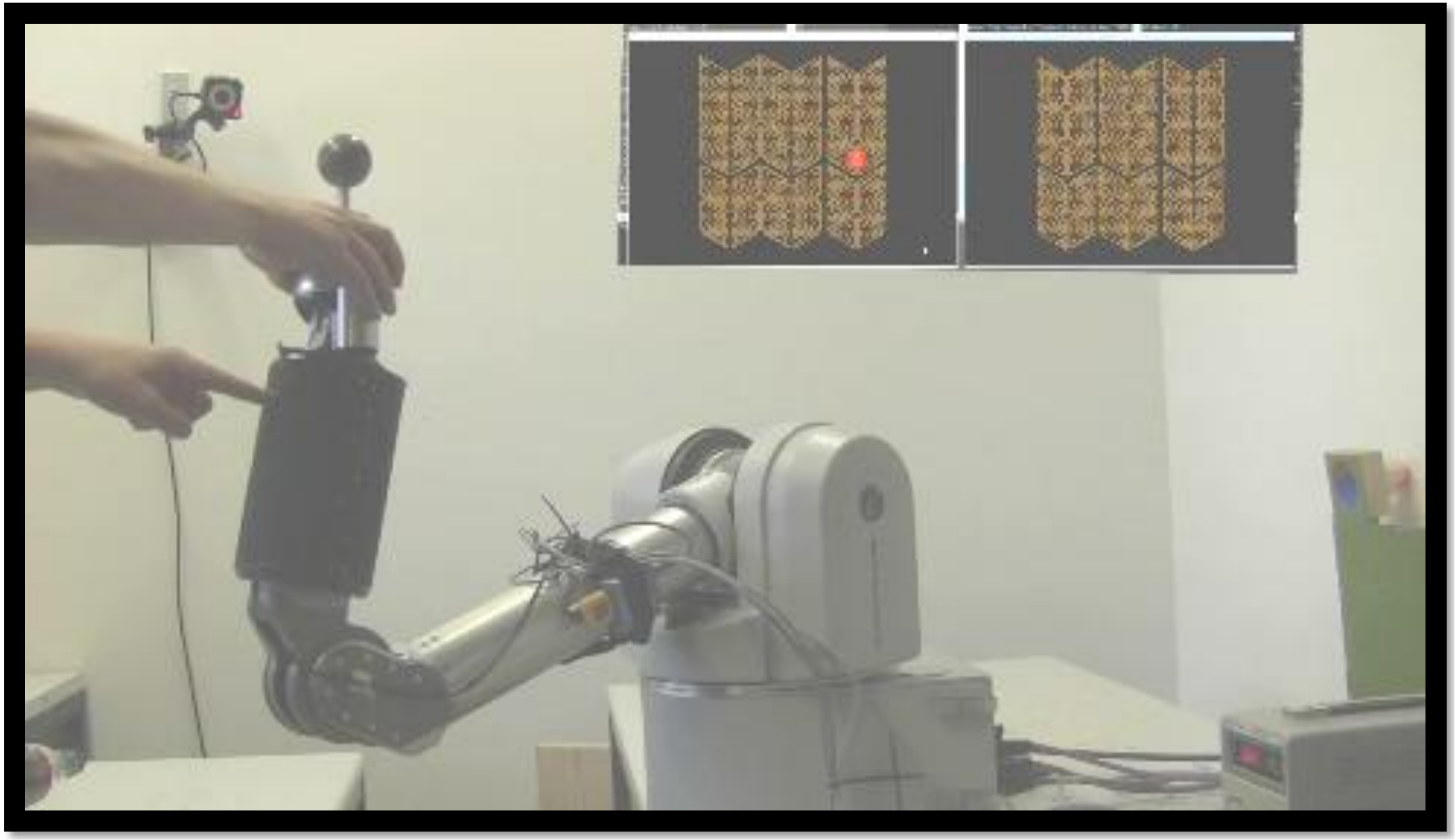
- **Learning VIC from kinesthetic teaching**:
- An operator physically interacts with the robotic to adjust the desired stiffness
- A higher perturbation amplitude results in less stiffness
- The stiffness profile can be learned/adjusted online



Kronander and Billard, Robot Learning from Demonstration for Compliant Manipulation, ICRA 2012

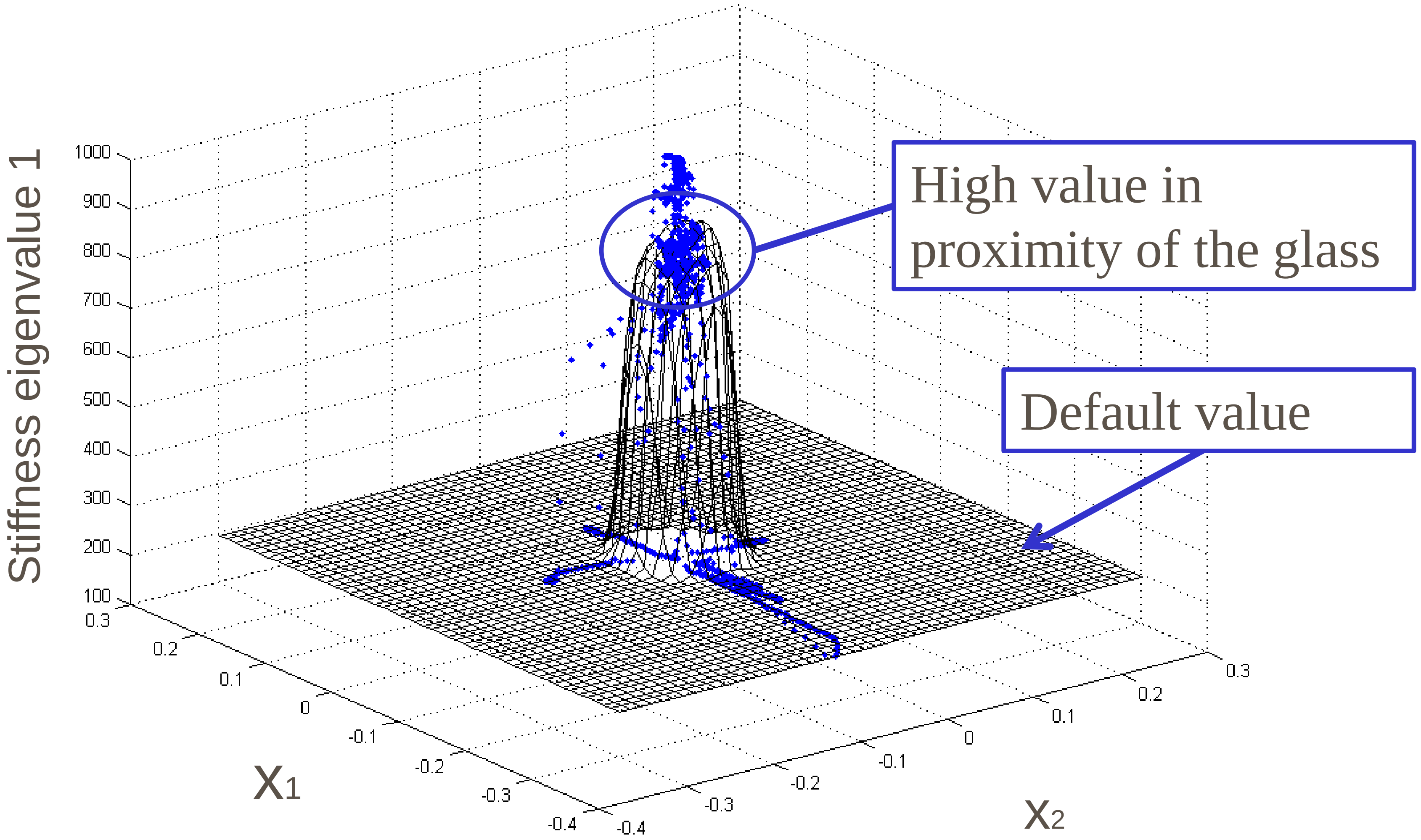
Tactile information

Artificial skin for detecting the grasp pressure



Teaching how and when to increase stiffness

Teaching how and when to increase stiffness



Learning the Desired Impedance Profiles

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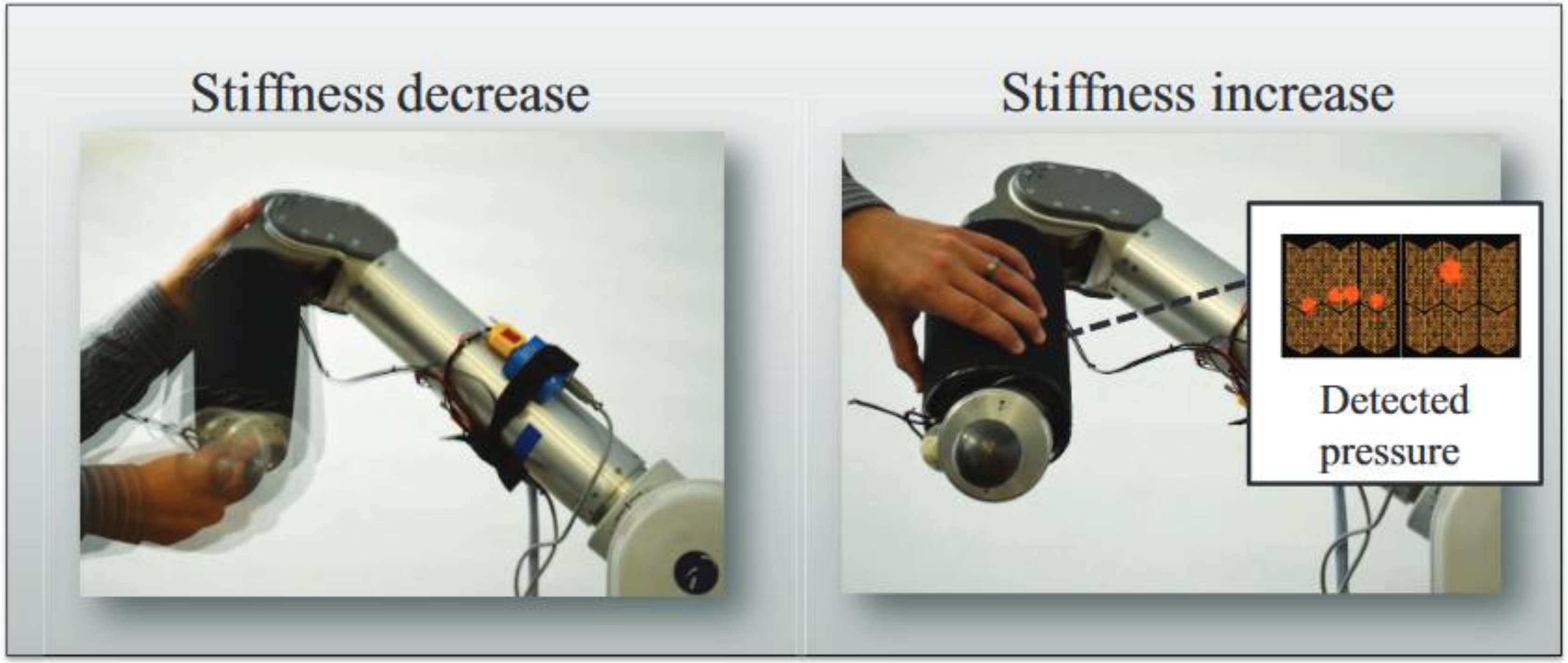
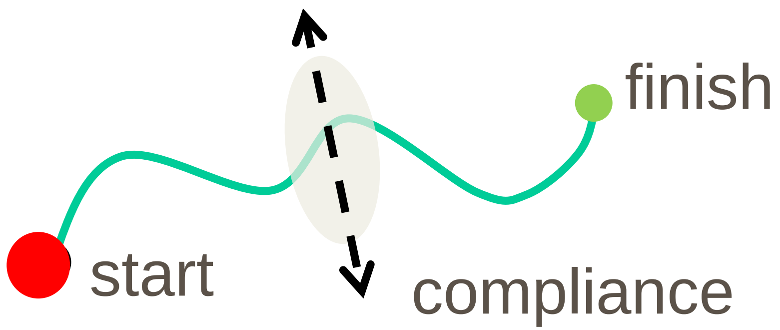


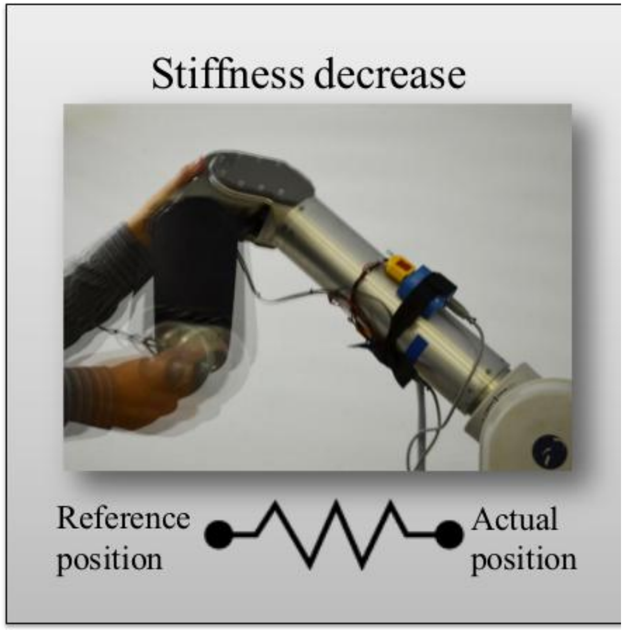
Kronander and Billard, Robot Learning from Demonstration for Compliant Manipulation, ICRA 2012

Teaching stiffness profile

- Start from a known task with strong stiffness
- To reduce the stiffness, the teacher wiggles the robot during task execution.



Teaching range of tolerance - stiffness



Eigenvalue Decomposition of Covariance of Data (perturbation over a time window):

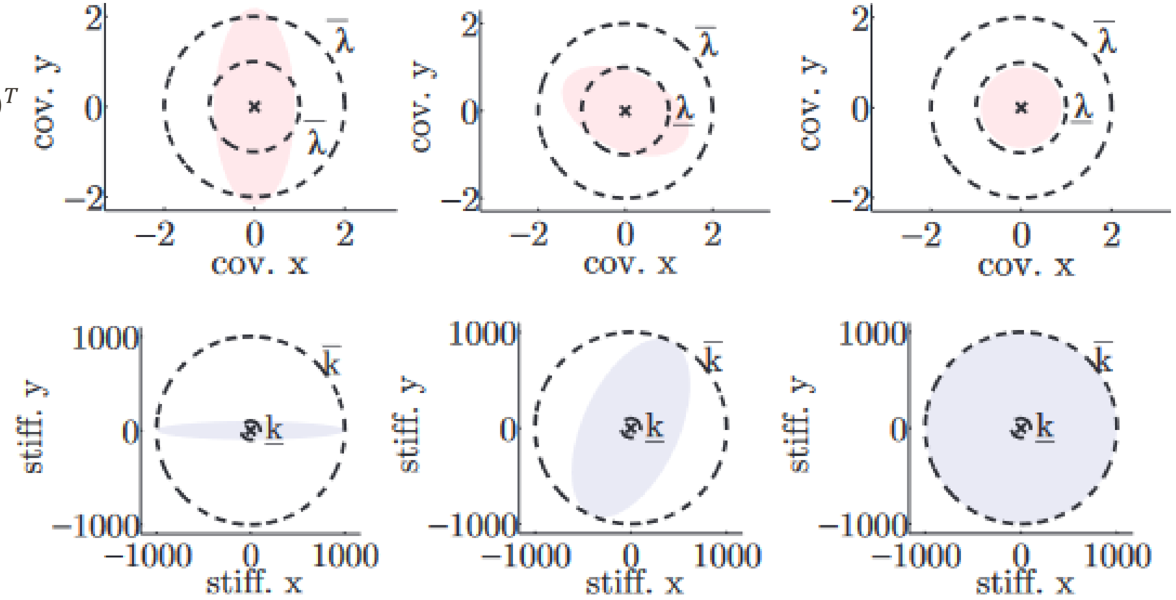
$$\frac{1}{M}\sum_{t-S}^{t}(x_t-\mu_t)(x_t-\mu_t)^T = U\Lambda U^T$$

Stiffness aligned with main axes of perturbation

$$K = U\Lambda^{-1}U^T$$

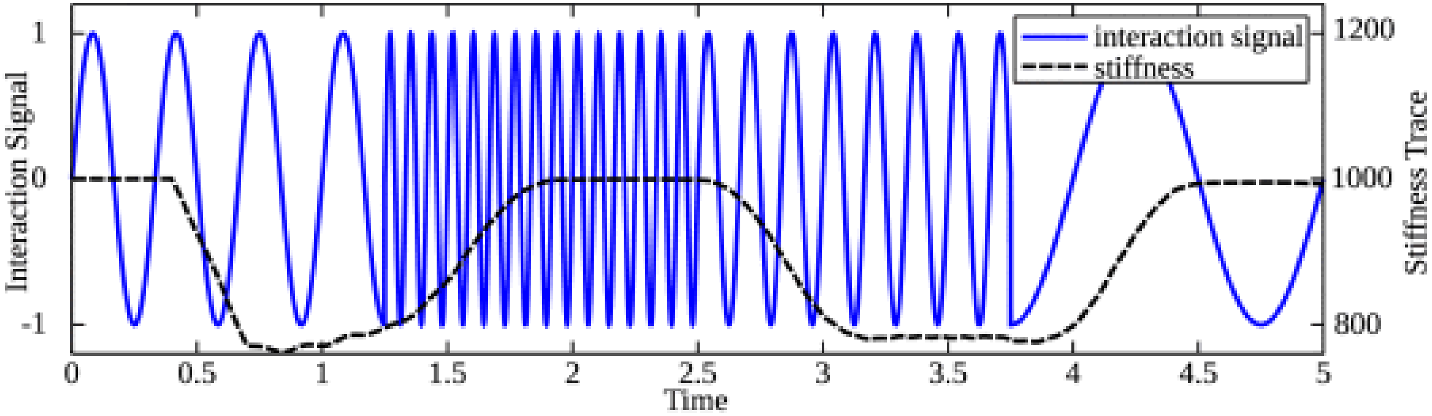
$$\frac{1}{M}\sum_{t-S}^{t}(x_t-\mu_t)(x_t-\mu_t)^T$$

K



Eigenvalues inversely proportional to stiffness

Modeling and learning state-dependent varying stiffness



Example of a sinusoidal time-varying signal

Change in frequency are converted into stiffness changes

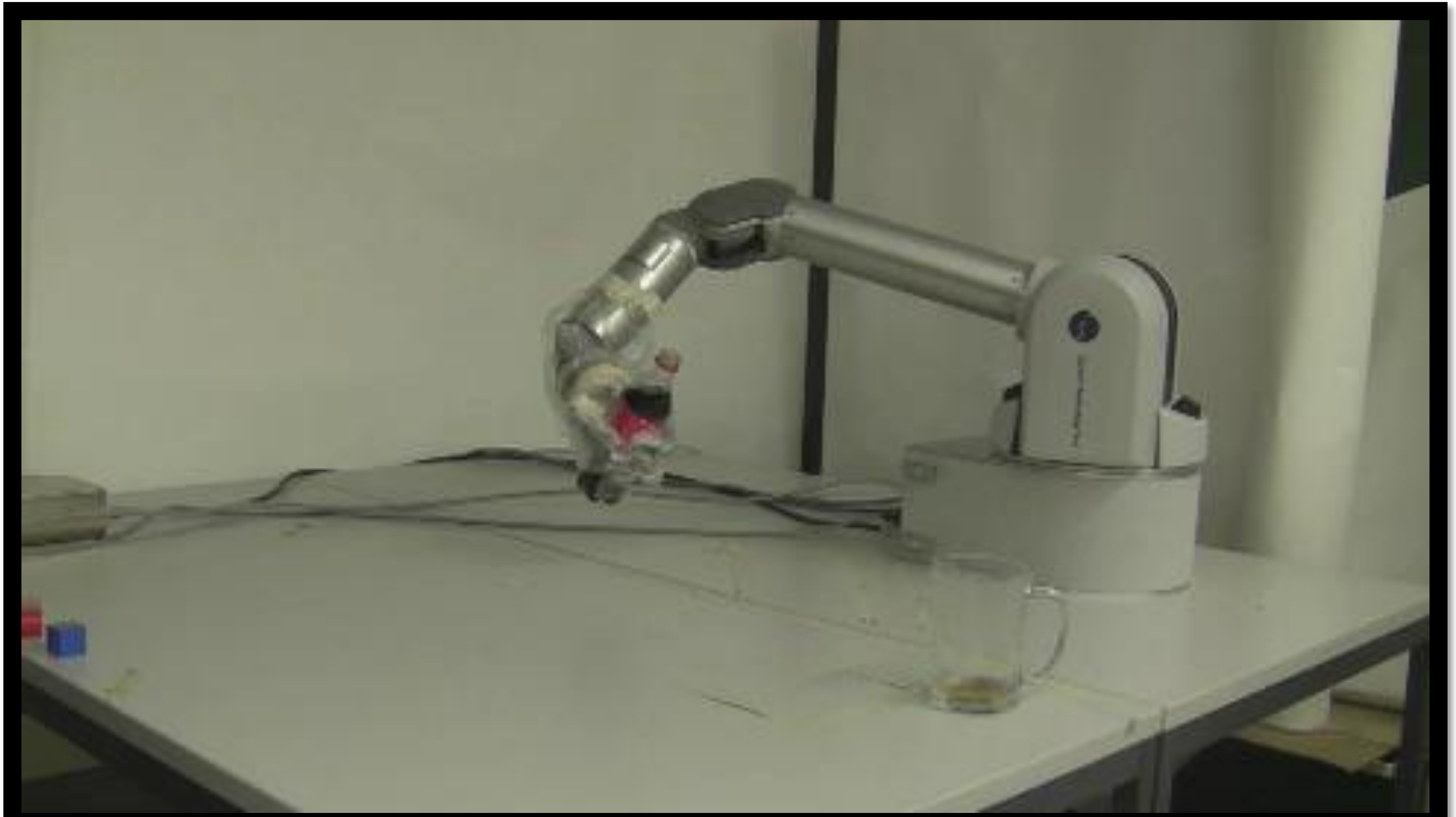
The time-varying stiffness is converted into a state-dependent varying stiffness *K(x)*.
To learn the varying stiffness, one learns a dependency between the position *x* and the Cholesky factor *L* through **Gaussian Mixture Model,** using the variance of the data during demonstration.

$$p(x,L)=\sum_{k=1}^{K} p\left(x,L;\mu^k,\Sigma^k\right),\ \mu^k,\Sigma^k:\ \text{Gauss means and covariance matrices}$$

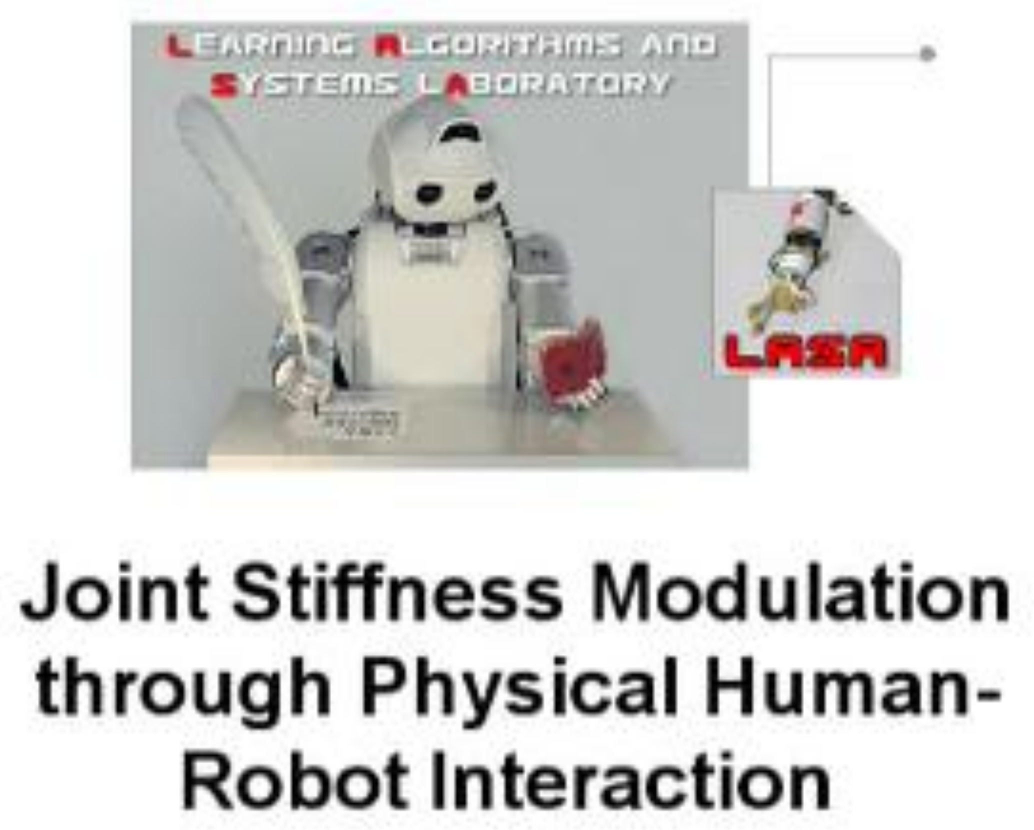
Stiffness matrix is expressed through Cholesky's decomposition $K(x) = L(x)L(x)^T$ **Why ?**

At run time, for a query point *x**, the stiffness matrix is obtained through Gaussian Mixture regression:
$\Lambda^{-1}(x^*)=L(x^*)L(x^*)^T,\ L(x^*)=E\{p(L|x^*)\}$

Reproduction with correct compliance



Teaching Right Amount of Stiffness for Lighting up a match



Kronander, Klas, and Aude Billard. "Learning compliant manipulation through kinesthetic and tactile human-robot interaction." *IEEE transactions on haptics* 7.3 (2013): 367-380.

Teaching Right Amount of Stiffness for Lighting up a match

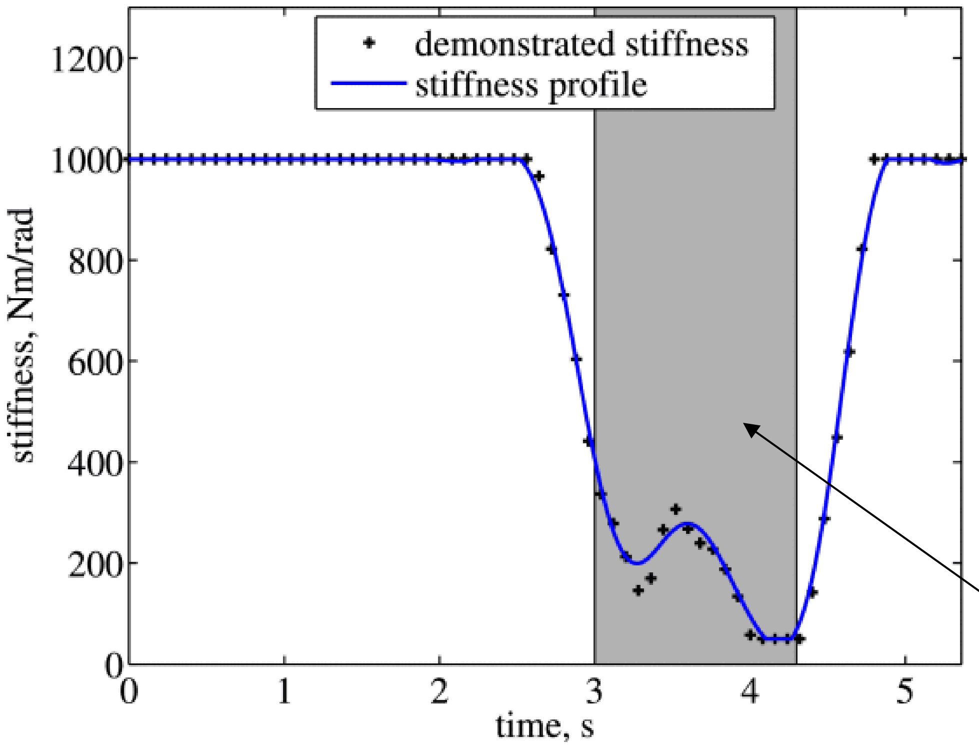
- High stiffness needed for accurate match positioning before the striking motion.
- Low stiffness is necessary to reduce contact forces in the striking phase.

- Joint torque sensors used to measure the interaction.
- Teach a local reduction of the stiffness in the striking phase.

Teaching Right Amount of Stiffness for Lighting up a match

Teaching Stiffness at Joint Level

Teaching Right Amount of Stiffness for Lighting up a match



Sole the stiffness profile for the elbow joint is taught.

The remaining six joints of the robot had a constant stiffness of 1,000 Nm/rad

Striking phase

Here the stiffness profile for each joint is learned through Gaussian Process Regression (GPR).

Lighting a match: results from 20 trials

	Broke	Broke and lit	Not lit	Lit	Success rate
Constant high stiffness	4	11	2	3	15%
Constant low stiffness	0	3	14	3	15%
Learned varying stiffness	0	2	1	17	85%

Lighting a match: results from 20 trials

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Constant high stiffness	4	11	2	3	15%
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Learned varying stiffness	0	2	1	17	85%

Why do you think the constant low stiffness profile resulted in numerous *Not lit* cases?

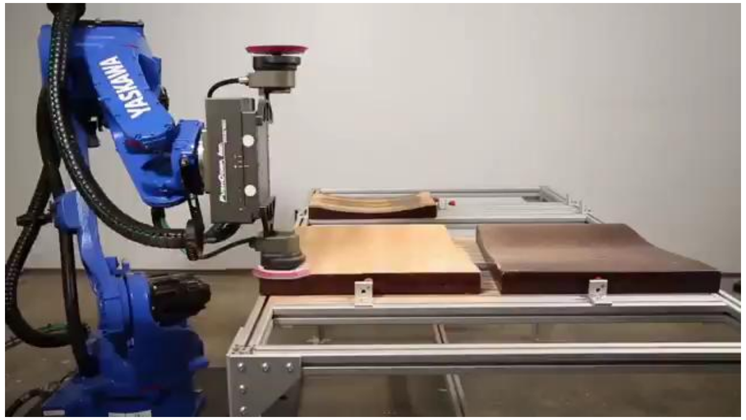
Summary

Why compliant control?

- Compliant control is crucial to enable robots to interact safely with their environment and in particular with humans.

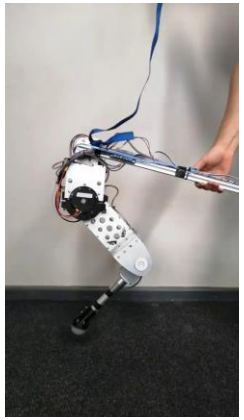
Where is compliant robot control used?

- Constant interaction with the environment



Image/video credits: READY Robotics

Image/video credits: FRANKA EMIKA



Image/video credits: Electronic Systems Laboratory

Image/video credits: Merewether

Summary

Research in Impedance control?

Learning Force Control for Legged Manipulation, ICRA 2024

Current-Based Impedance Control for Interacting with Mobile Manipulators, IROS 2024

Passivity-Based Adaptive Force-Impedance Control for Modular Multi-Manual Object Manipulation, RAL 2024

Summary

Why compliant control?

- Compliant control is crucial to enable robots to interact safely with their environment and in particular with humans.

How to program robots to become compliant?

- Compliance is usually obtained by controlling the robot through impedance control.
- By setting the impedance parameters (stiffness and damping), one can modulate the response of the robot to external forces.
- As the compliance depends on the task and may also vary along the task, it is important to set stiffness and damping as *varying parameters*, that varies with time or state of the system (see exercises).

How to teach robots the right compliance ?

- Kinesthetic teaching can be used to train the robot to stiffen or unstiffen, using the robot's tactile and force sensing.
- State-dependent stiffness profiles can be learned through standard machine learning for regression.