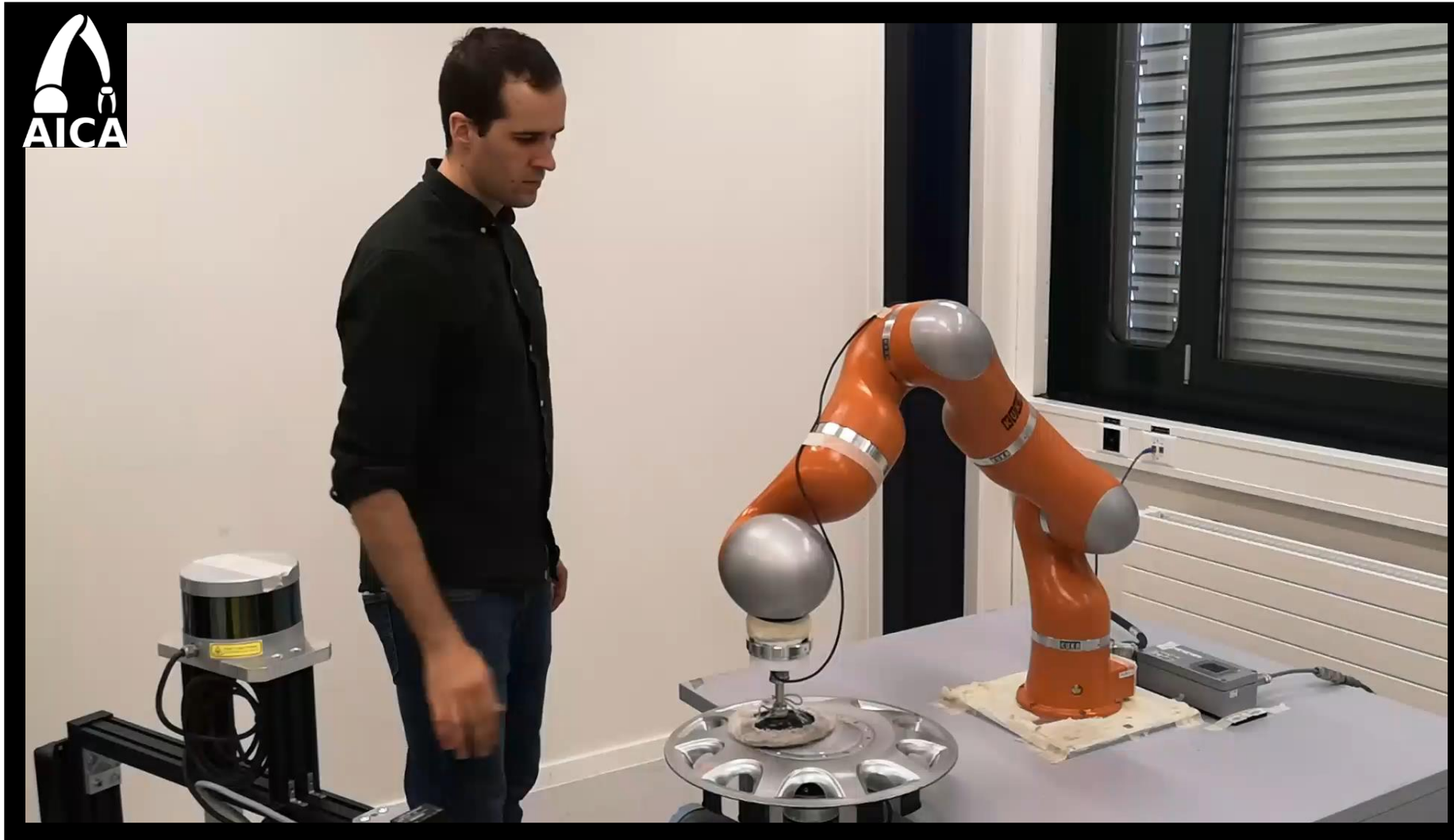
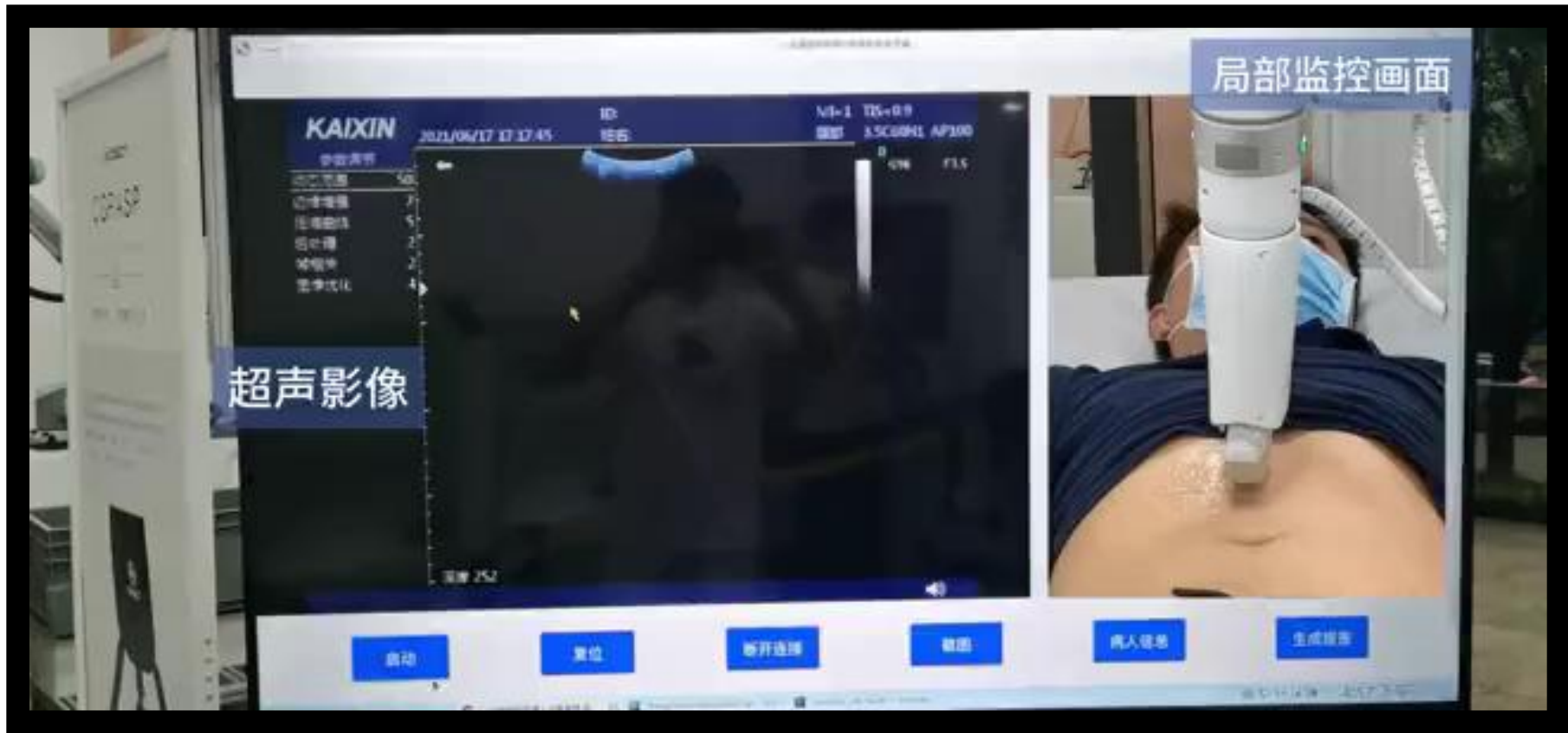


## Force Control with Dynamical Systems

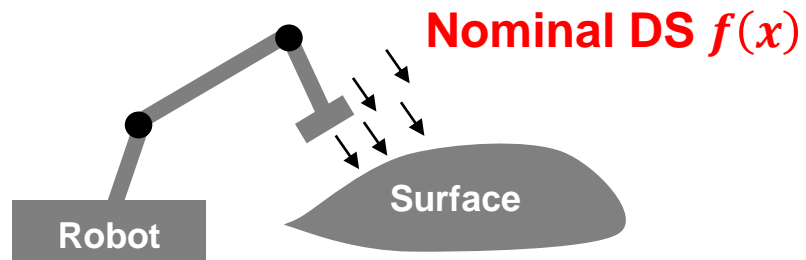
## Controlling Robot's Forces when in Contact



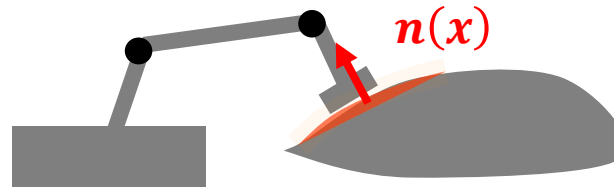
## Controlling Robot's Forces when in Contact



## Controlling for force at contact



## Controlling for force at contact: Principle

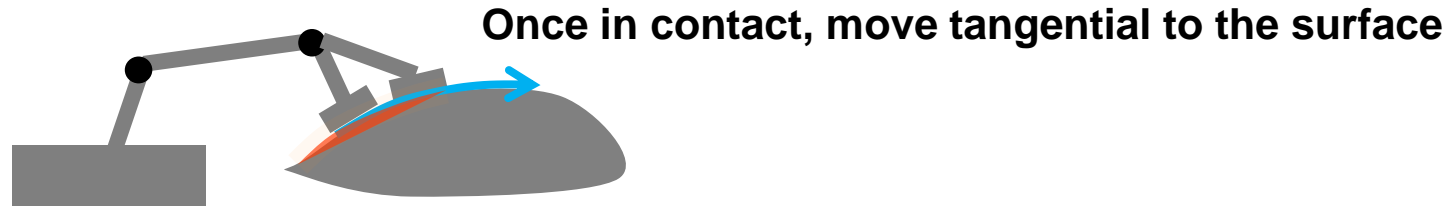


$n(x)$  normal vector to the surface

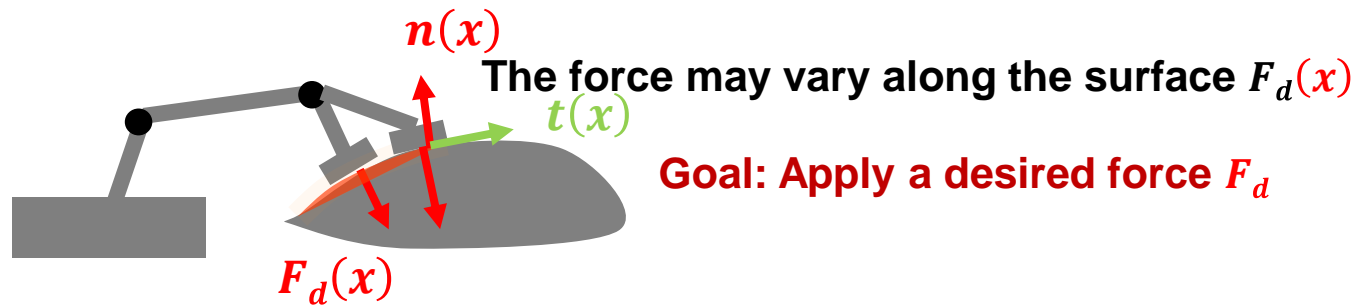
Make contact

Assumption: Surface is impenetrable

## Controlling for force at contact: Principle



## Controlling for force at contact: Principle

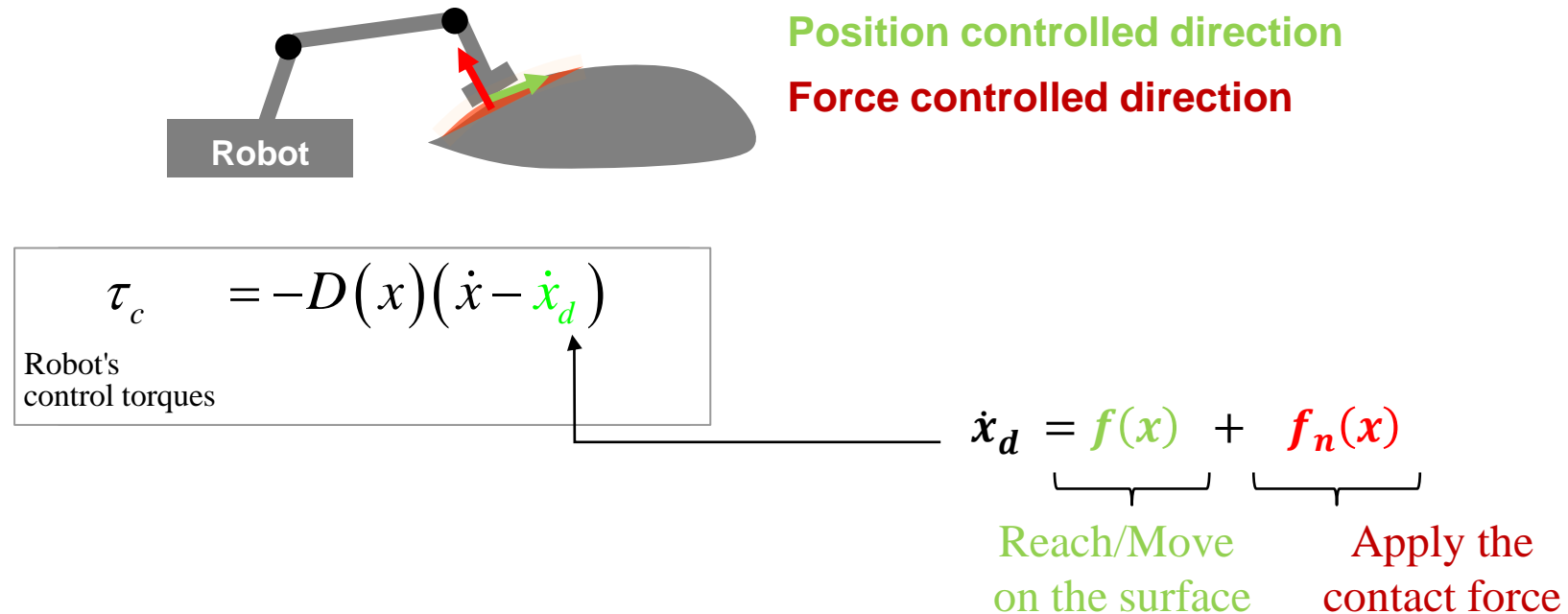


**Idea:** If we can project the control torques onto the surface, we simplify the computation.

→ We assume that we can compute the normal to the surface when in contact and we project the control onto [a frame of reference moving along the surface in direction of motion](#).

## Passive-DS for Controlling Forces on the Surface

To generate forces, we need to control the robot's torques. We use the passive DS approach.



To separate control of force and control of motion, we decompose the nominal DS into two components:

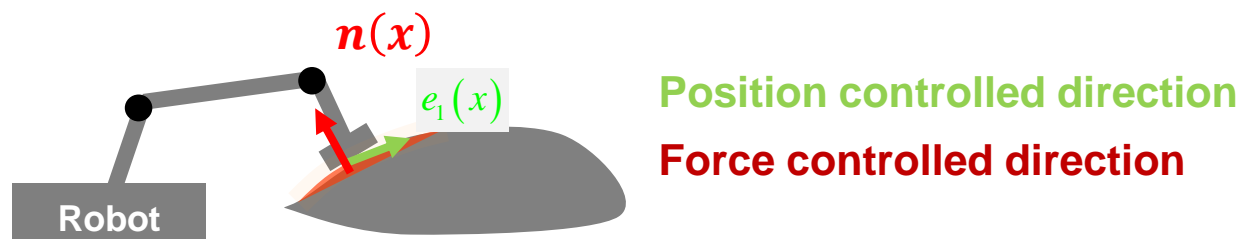
$$\dot{x}_d = f(x) + f_n(x)$$

Force is applied only once in contact, hence  $f_n(x) = 0$  in free space



## Passive-DS for Controlling Forces on the Surface

To generate forces, we need to control the robot's torques. We use the passive DS approach.



$$\tau_c = \lambda_1 f(x) + \lambda_1 f_n(x) - D(x) \dot{x}$$

Robot's  
control torques

Eigencomposition of  $D(x)$

$$D(x) = Q(x) \Lambda(x) Q(x)^T$$

$$Q(x) = [e_1(x) \ e_2(x)]$$

$$e_1(x) = \frac{f(x)}{\|f(x)\|}$$

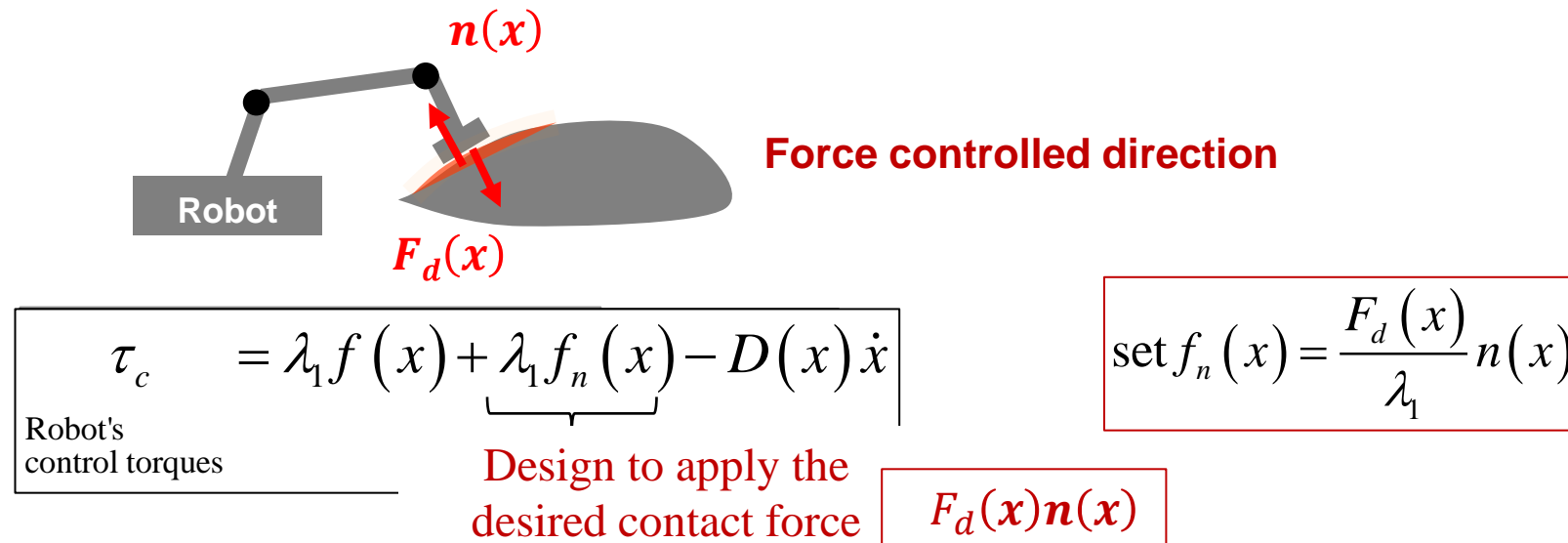
$$e_2(x) = n(x)$$

Fixed impedance

$$\Lambda = \begin{bmatrix} \lambda_1 & \\ & \lambda_2 \end{bmatrix}$$

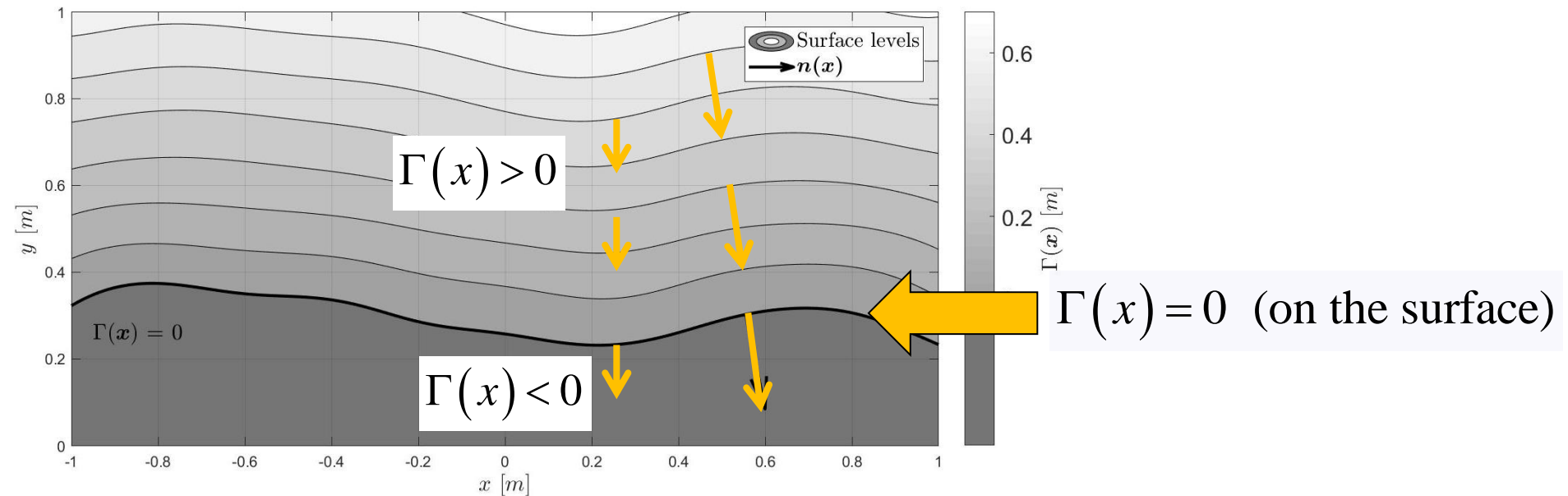
## Passive-DS for Controlling Forces on the Surface

To generate forces, we need to control the robot's torques. We use the passive DS approach.



## Modeling the Surface

We model the surface through a continuous function  $\Gamma(x)$ , that measures the distance to the surface.  $\Gamma(x)$  is continuously differentiable such that we can compute the normal vector  $n(x)$  at any position  $x$

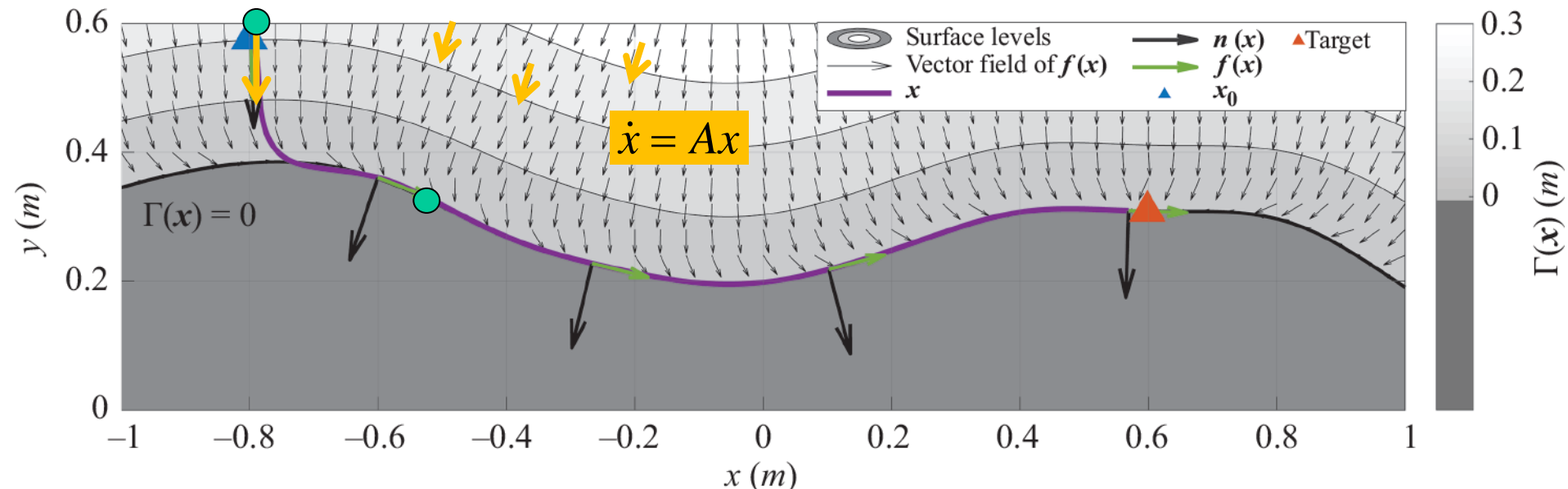


## Nominal DS

We define a linear DS moving downwards towards the surface.

We modulate this DS to force it to move along the surface.

To stop the motion at a target on the surface, we can set this as the attractor of the DS.



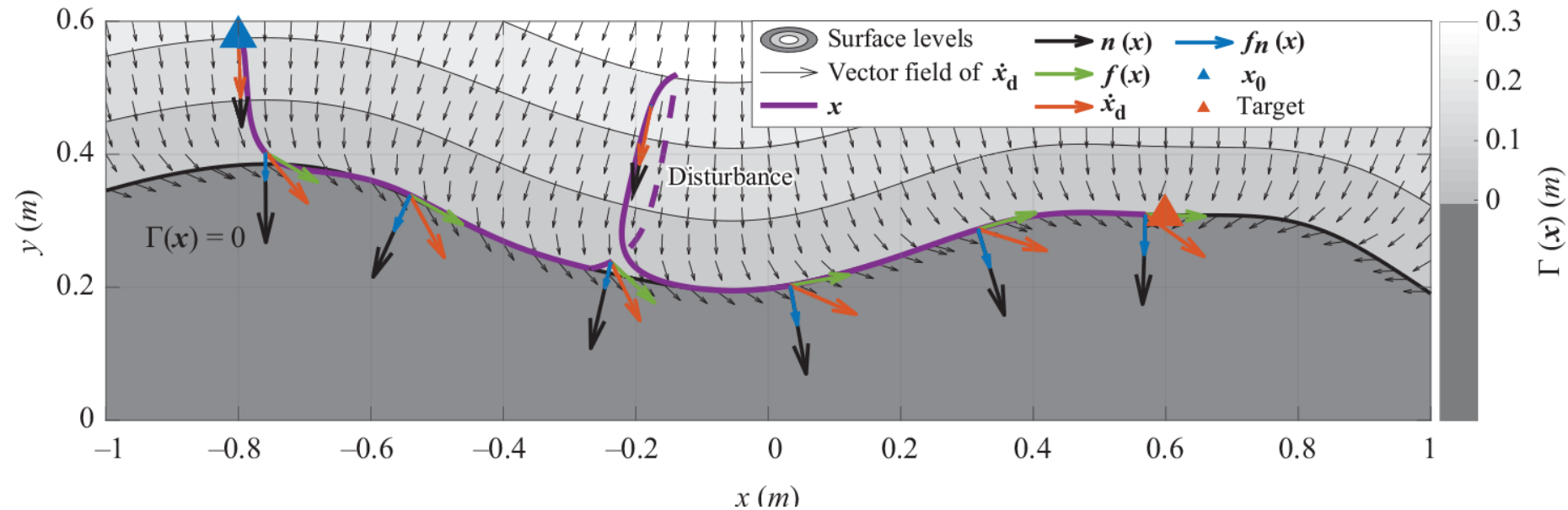
$$f(x) = R(x)Ax$$

$R(x)$ : Rotation to align to the surface once in contact  
(same as in obstacle avoidance with constant velocity)

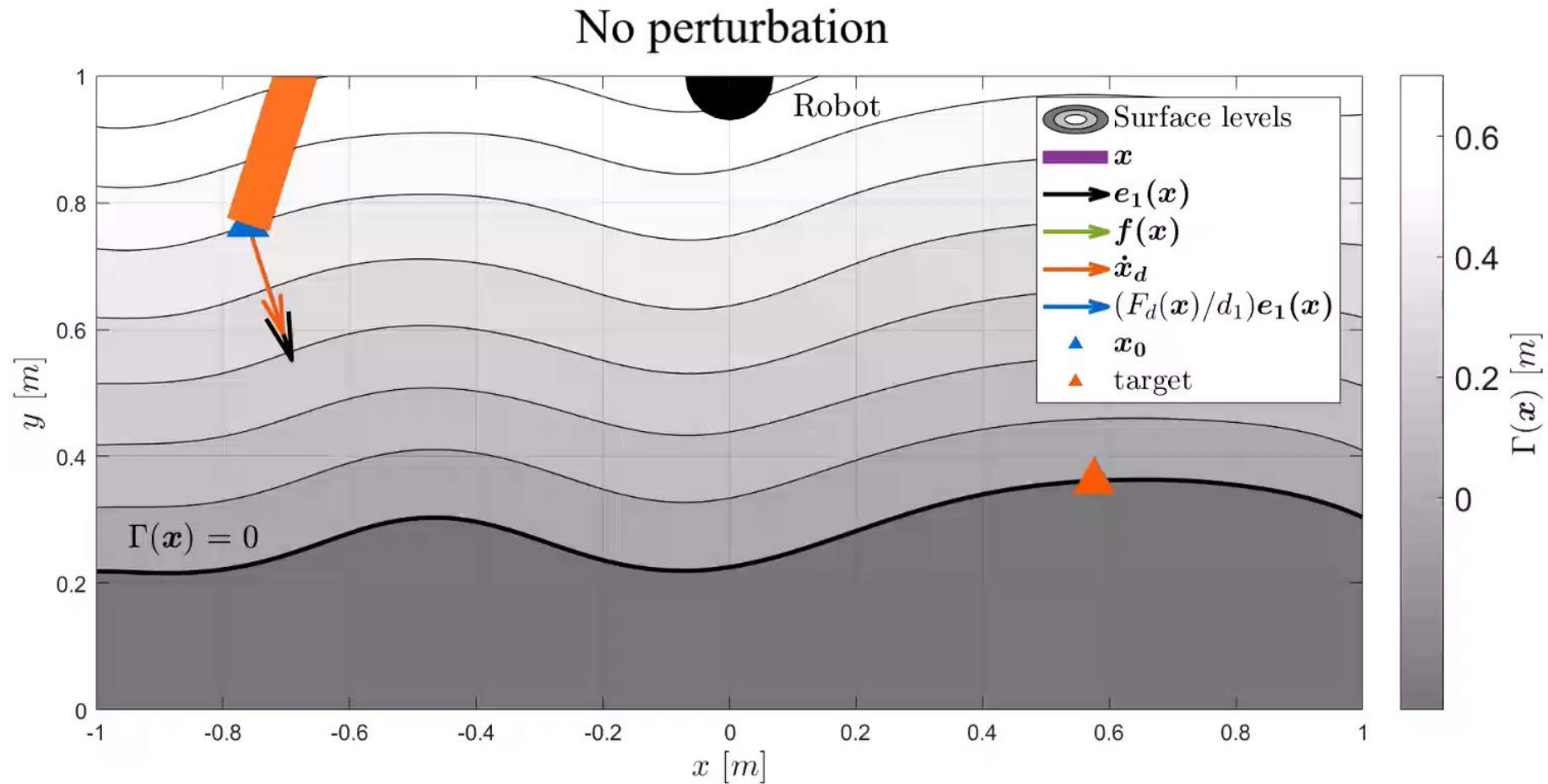
## Visualization of the Decomposition

To separate control of force and control of motion, we decompose the nominal DS into two components:

$$\dot{x}_d = f(x) + f_n(x) \quad f_n(x) = 0 \quad (\text{in free space})$$

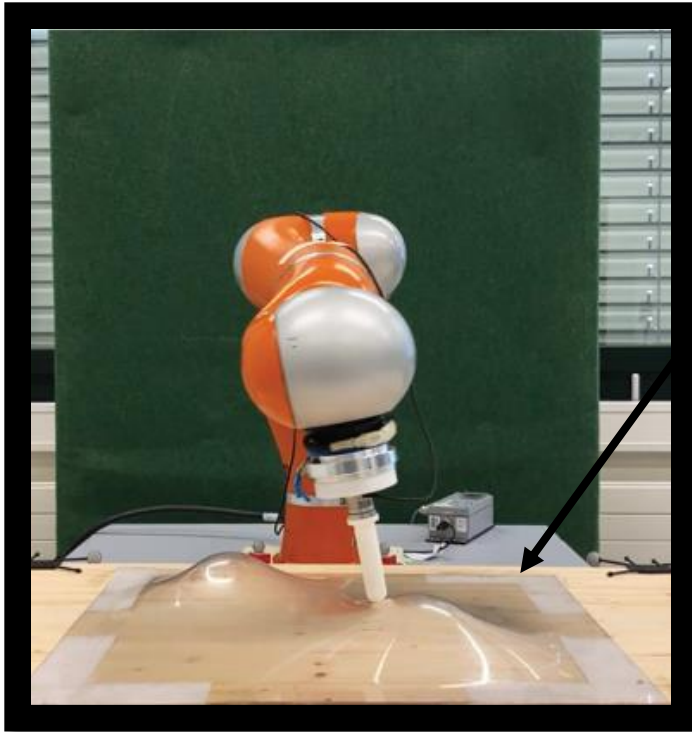


## Animation of Principle



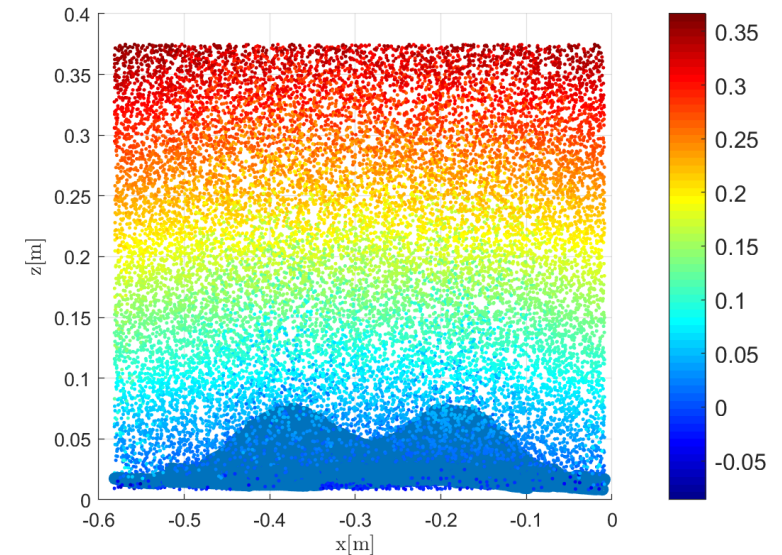
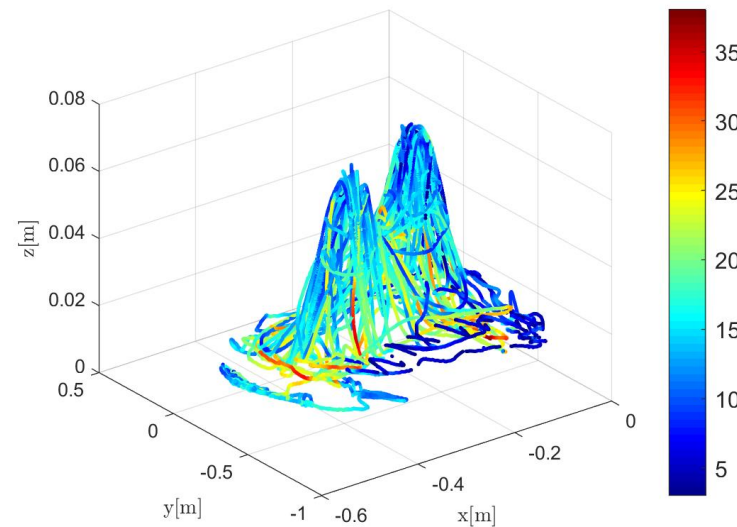
## Robotic Demonstration

**Task:** The robot must polish in circular motion the surface applying a constant force of 20N.



Learn a model of the surface  
using Support Vector Regression

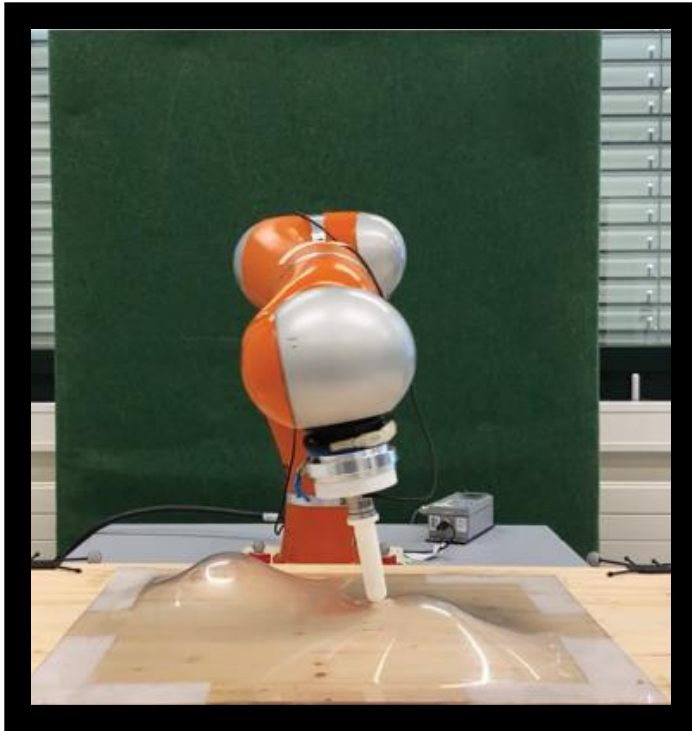
Input dataset to learn the surface model:



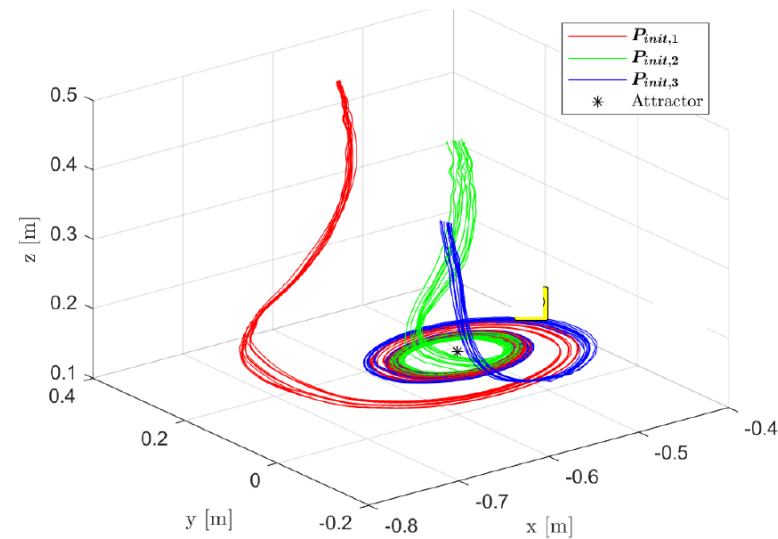


## Robotic Demonstration

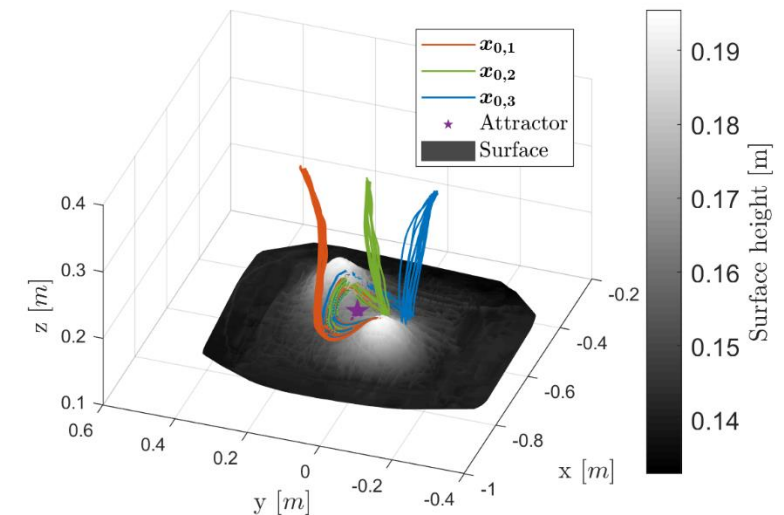
**Task:** The robot must polish in circular motion the surface applying a constant force of 20N.



Define a nominal DS that creates a limit cycle on the surface.

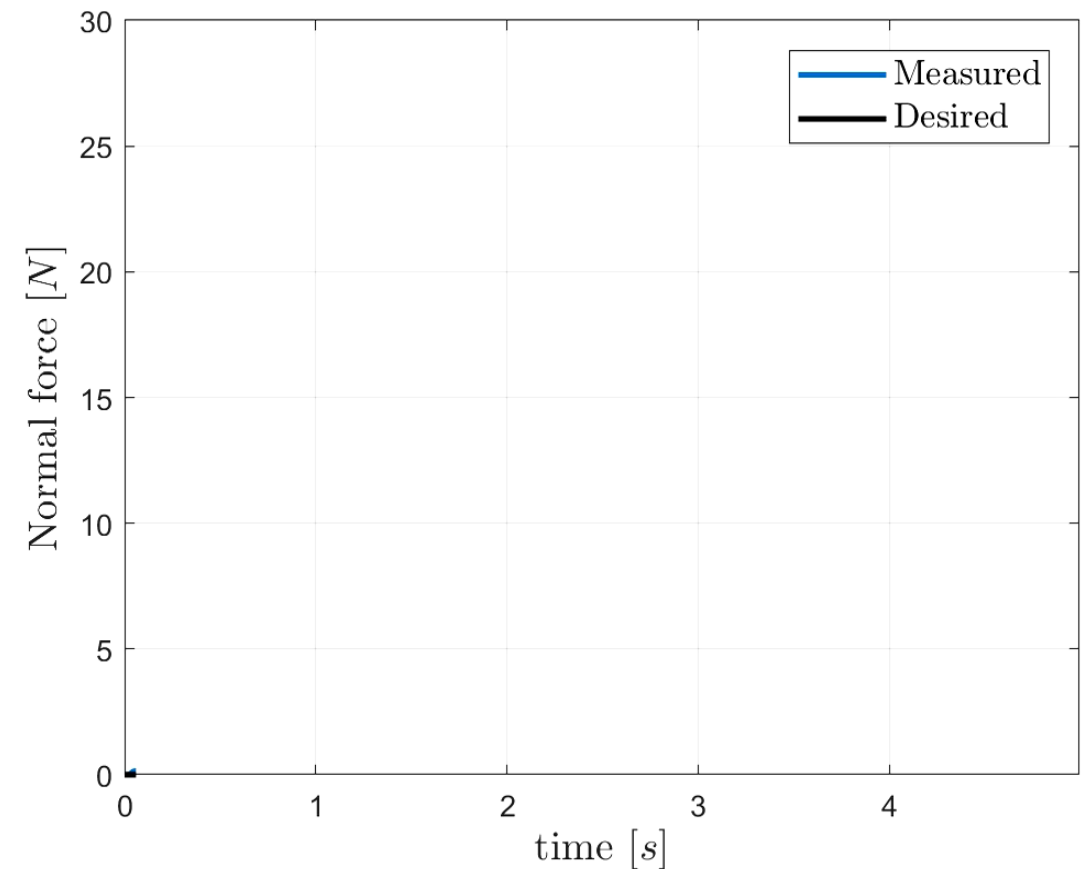
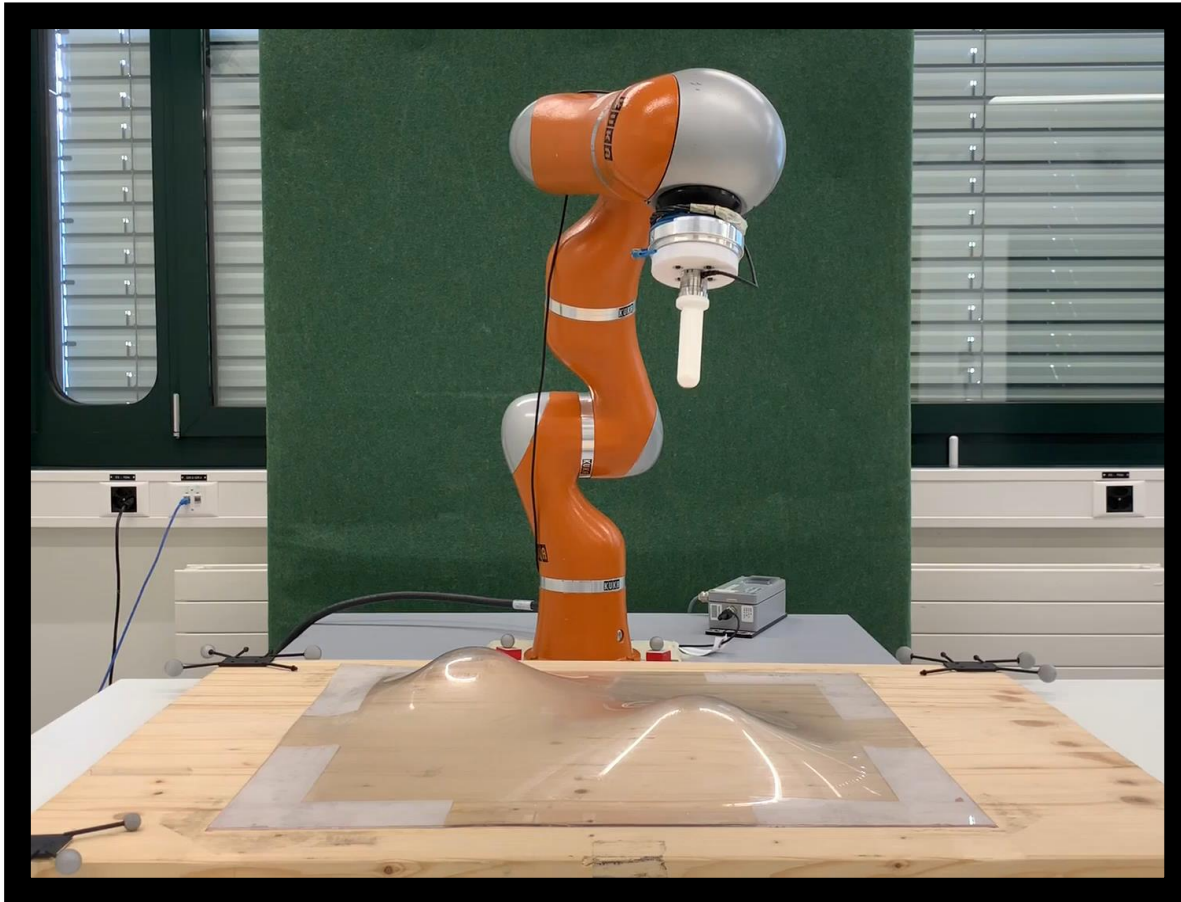


Project on the surface.





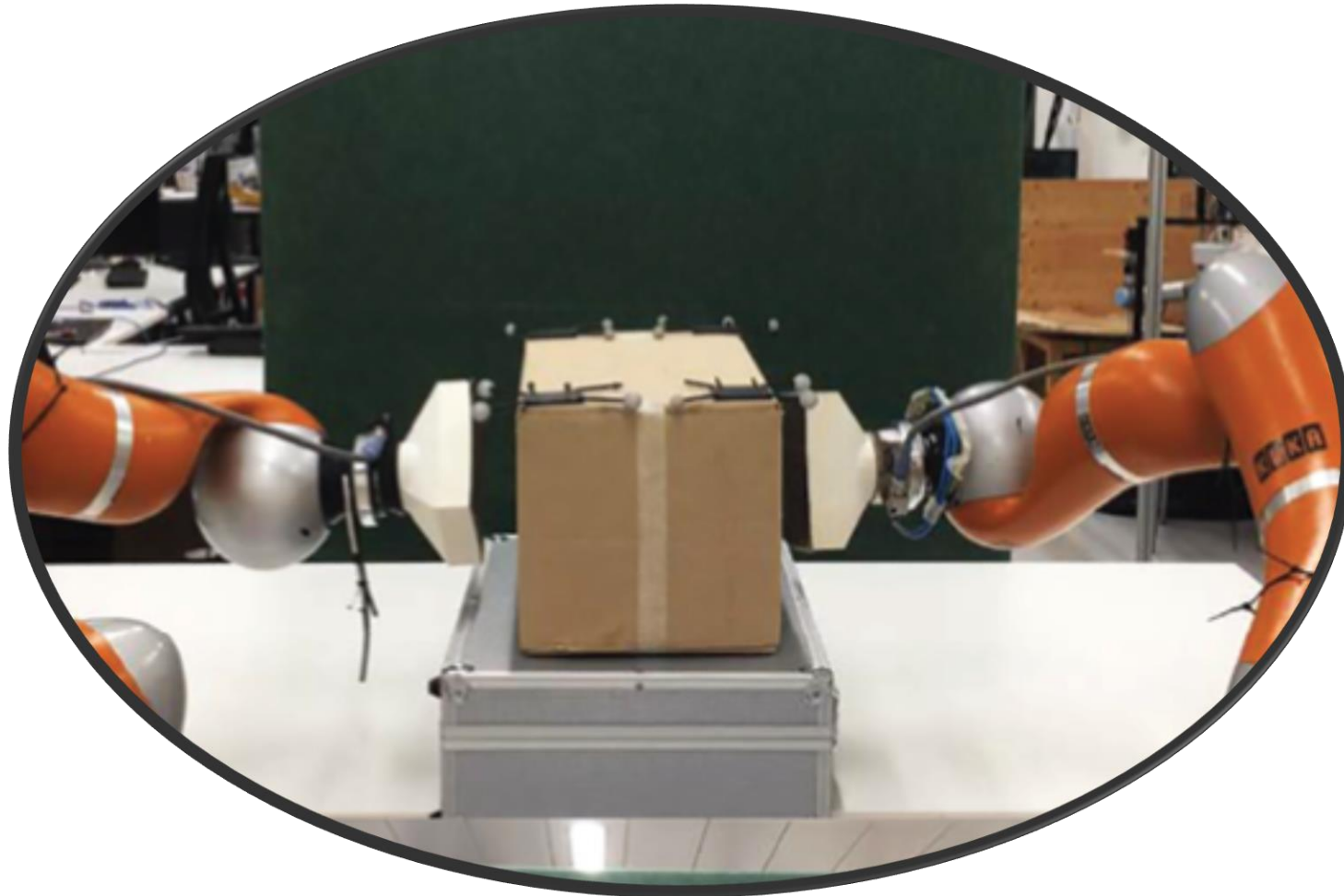
## Robotic Demonstration – Robustness to Various Disturbances



Passivity see exercise session

## Extension to Control Bimanual Platform

**Task:** The robots must reach either side of the box and apply enough force to support the box's weight.



## Extension to Control Bimanual Platform



## Variables to Control Bimanual Platform

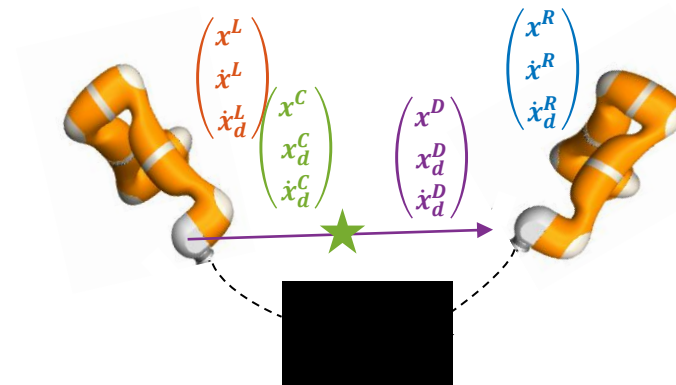
- Robots' center position and distance vector

$$\begin{cases} x^C = \frac{x^L + x^R}{2} \\ x^D = x^R - x^L \end{cases}$$

- Robots' nominal dynamics

$$\begin{cases} f^R(x^C, x^D) = \dot{x}_d^C + \frac{\dot{x}_d^D}{2} \\ f^L(x^C, x^D) = \dot{x}_d^C + \left(-\frac{\dot{x}_d^D}{2}\right) \end{cases}$$

To simplify control and ensure coordination, compute control in relative position



Positioning + grasping dynamics

# Nominal Dynamics for Bimanual Platform

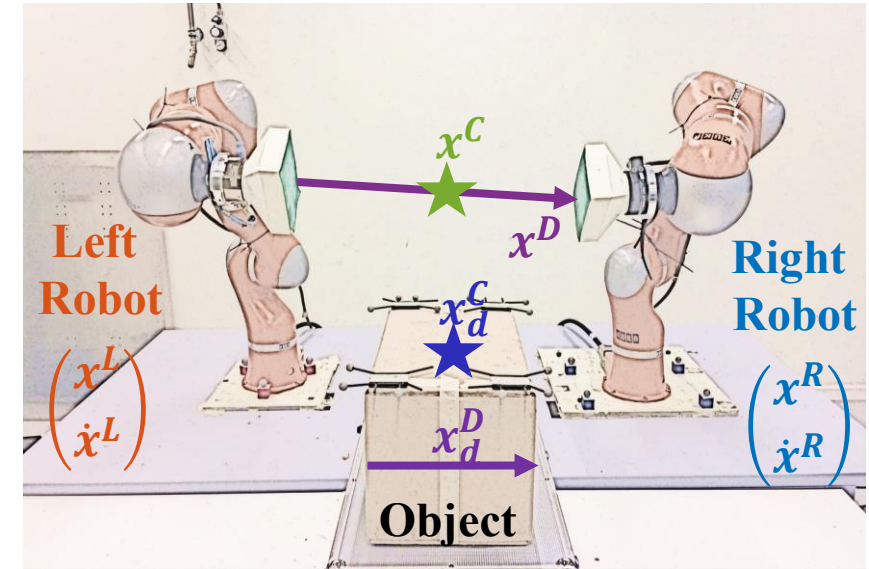
- Desired robots' center position and distance vector **dynamics**:

$$\begin{cases} \dot{x}_d^C = A_C(x_d^C - x^C) & \text{Center positioning dynamics} \\ \dot{x}_d^D = A_D(x_d^D - x^D) & \text{Closing dynamics} \end{cases}$$

with:  $A_C, A_D \geq 0$

- Robots' **nominal dynamics**:

$$\begin{cases} f^R(x^L, x^R) = \dot{x}_d^C + \frac{\dot{x}_d^D}{2} \\ f^L(x^L, x^R) = \dot{x}_d^C + \left(-\frac{\dot{x}_d^D}{2}\right) \end{cases} \quad \begin{matrix} \text{Center positioning} \\ + \\ \text{Closing dynamics} \end{matrix}$$





## Force Desired for Bimanual Platform

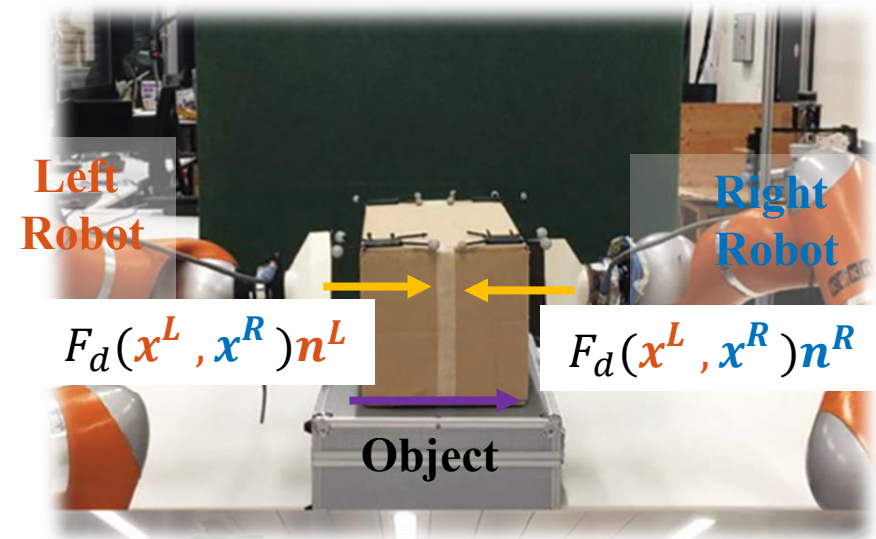
- Robots' **desired modulated dynamics**:

$$\begin{cases} \dot{x}_d^R = f^R(x^L, x^R) + \frac{F_d(x^L, x^R)}{\lambda_1^R} n^R \\ \dot{x}_d^L = f^L(x^L, x^R) + \frac{F_d(x^L, x^R)}{\lambda_1^L} n^L \end{cases}$$

with:  $n^R = -n^L = \frac{x_d^D}{\|x_d^D\|}$  **Grasping direction**

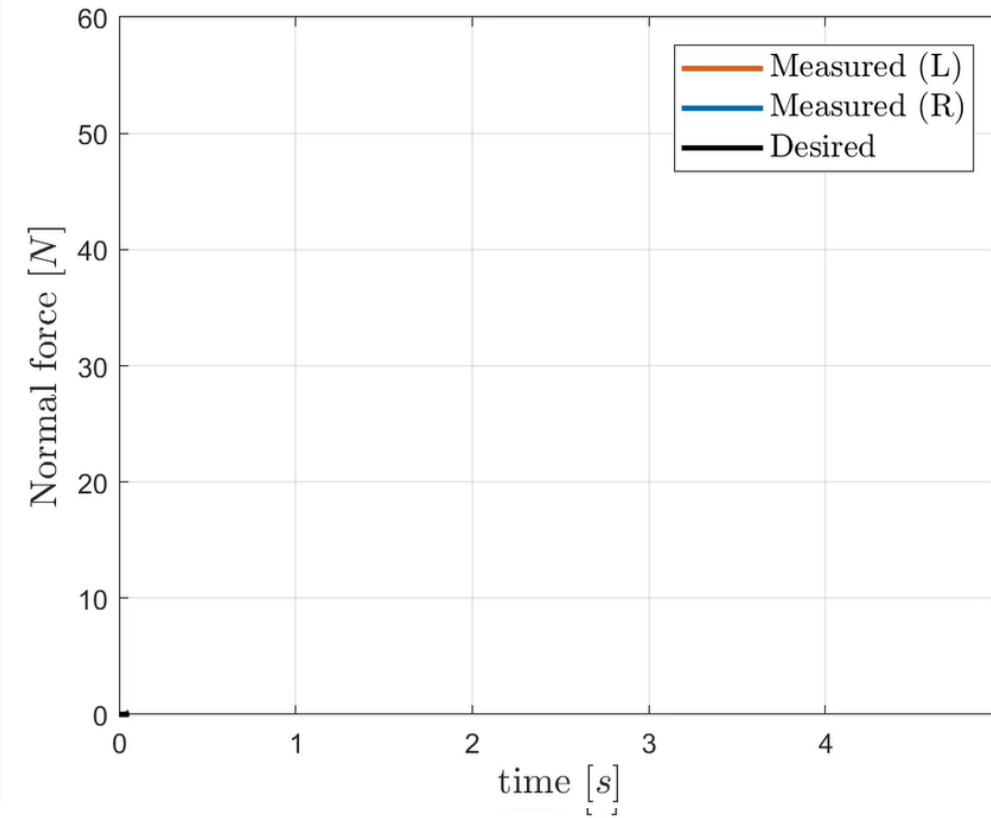
$\bar{F}_d(x^L, x^R)$ : **Desired contact force**

$\lambda_1^L, \lambda_1^R$ : **Impedance gains**



- Ensures the passivity through a tank for energy of both arms

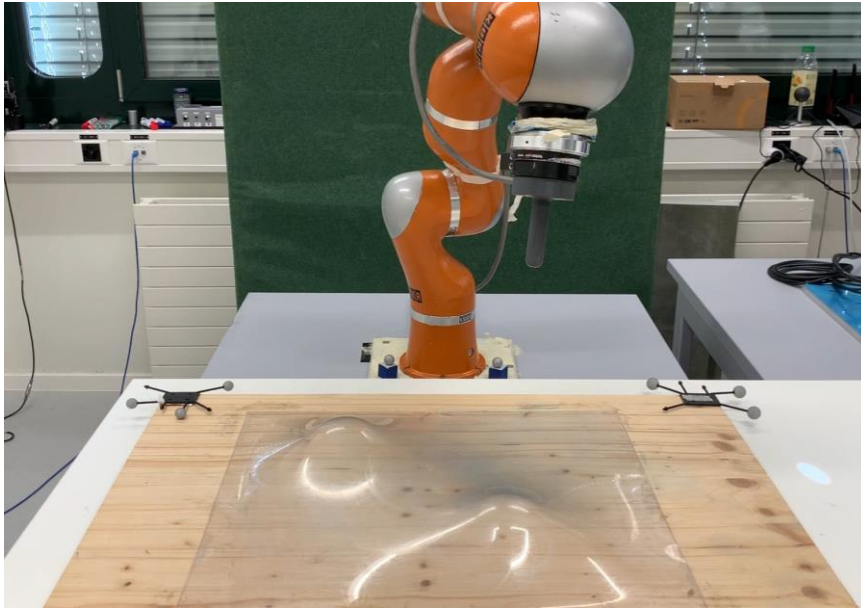
## Robot Demonstration



## Learning Force Adaptation

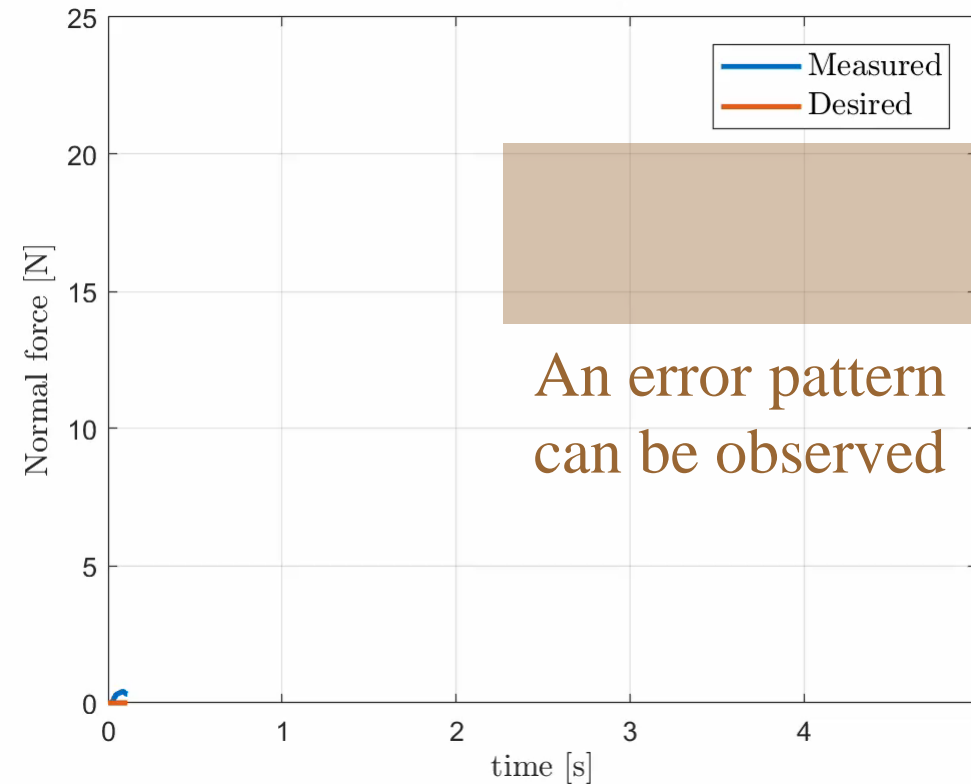


## Imprecise Force Generation



### Force tracking errors result from:

- Uncertainties in the surface
- Uncertainties in the robot model
- Measurement noises



**Goal:** Exploit the **adaptability** and **robustness** of the time-invariant **DS framework** to learn force compensation models

# State-Dependent Force Correction

- ❖ Force generation with DS:

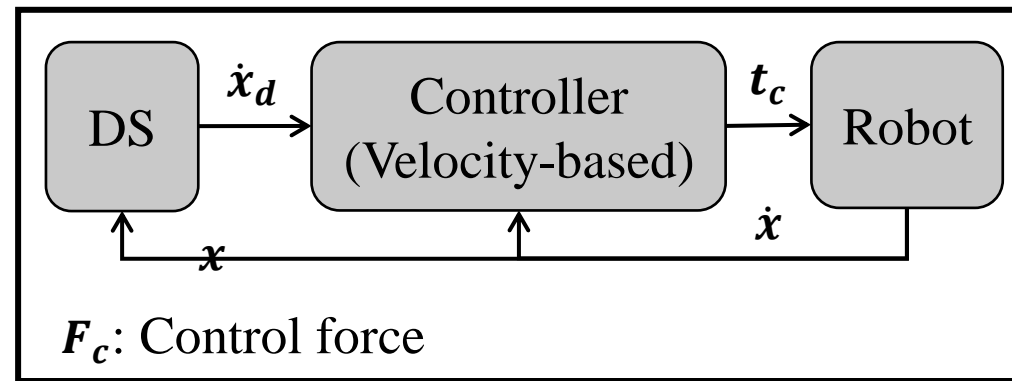
$$\dot{x}_d = f(x) + f_n(x)$$

Nominal DS  
(responsible for motion)
Modulation term  
(responsible for contact force)

- ❖ Introduction of a state-dependent force correction model  $\tilde{F}_d(\mathbf{x}, \boldsymbol{\theta})$ :

$$f_n(x) = \frac{F_d(x) + \tilde{F}_d(x, \theta)}{\lambda_1} n(x)$$

DS in closed-loop configuration



$$\dot{x}_d \in \mathbb{R}^3$$

Desired dynamics

$$f(x) \in \mathbb{R}^3$$

Nominal DS

$$f_n(x) \in \mathbb{R}^3$$

Normal modulation term

$$F_d(x) \geq 0$$

Desired force profile

$$n(x) \in \mathbb{R}^3$$

Normal direction to the surface

$$\lambda_1 \geq 0$$

Impedance gain

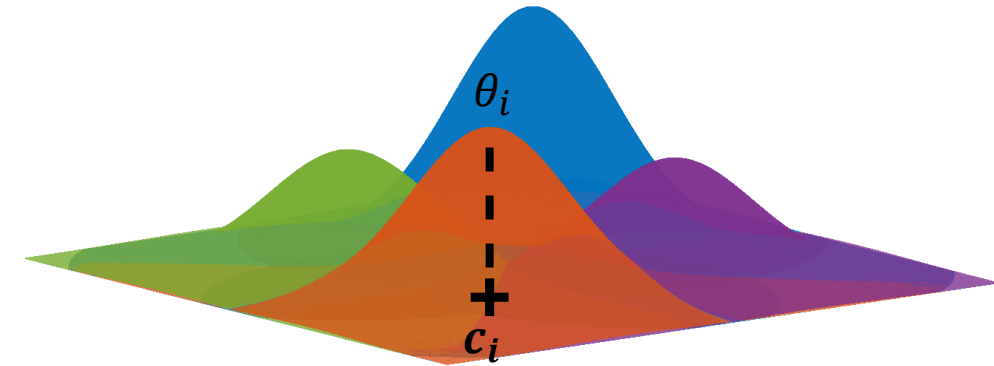
## State-Dependent Force Correction

- ❖ Design  $\tilde{F}_d(\mathbf{x}, \boldsymbol{\theta})$  using Gaussian Radial Basis kernel functions (RBFs):

$$\tilde{F}_d(\mathbf{x}, \boldsymbol{\theta}) = \frac{\sum_{i=1}^K \theta_i \varphi(\mathbf{x} - \mathbf{c}_i)}{\sum_{j=1}^K \varphi(\mathbf{x} - \mathbf{c}_j)} \quad \text{with: } \underbrace{\varphi(\mathbf{x}) = \exp \frac{-\|\mathbf{x}\|^2}{2\sigma^2}}_{\text{Gaussian kernel}}$$

Hyper-parameters

$\left\{ \begin{array}{l} K: \text{Number of Gaussian} \\ \mathbf{c}_i: \text{Center position of Gaussian } i \\ \sigma: \text{Kernel width} \end{array} \right.$



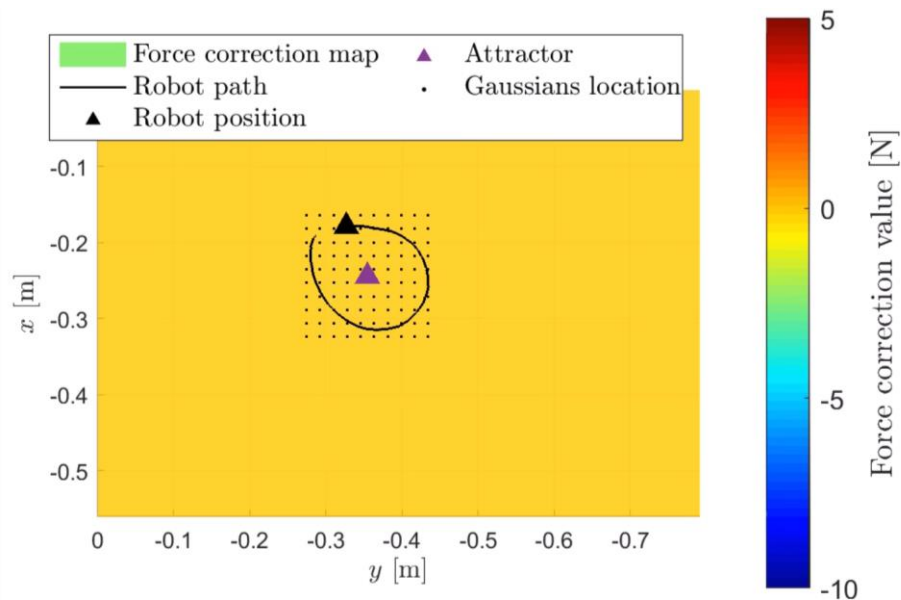
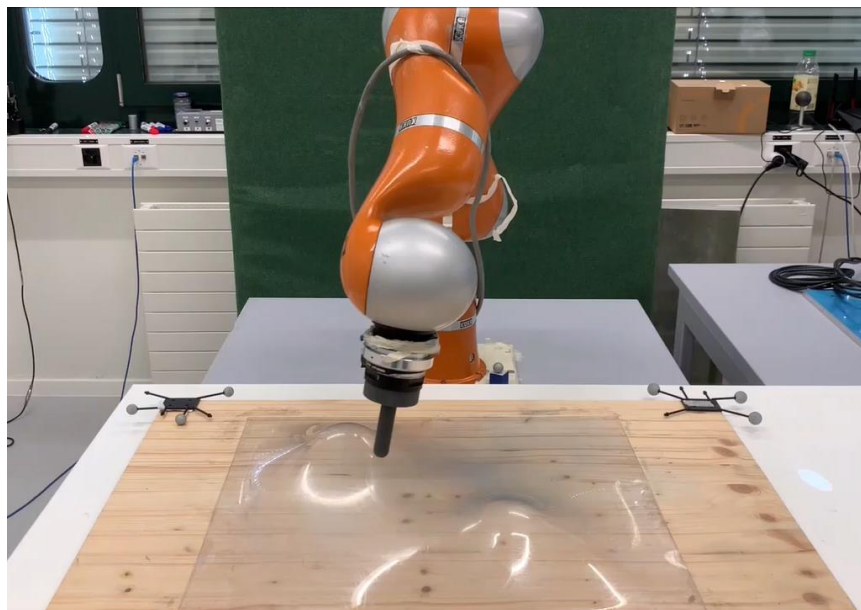
- ❖ Update the weights  $\boldsymbol{\theta}$  to minimize the normal force error  $F_e$ :

$$F_e = F_d(\mathbf{x}) - \mathbf{n}(\mathbf{x})^T \mathbf{F}_m$$

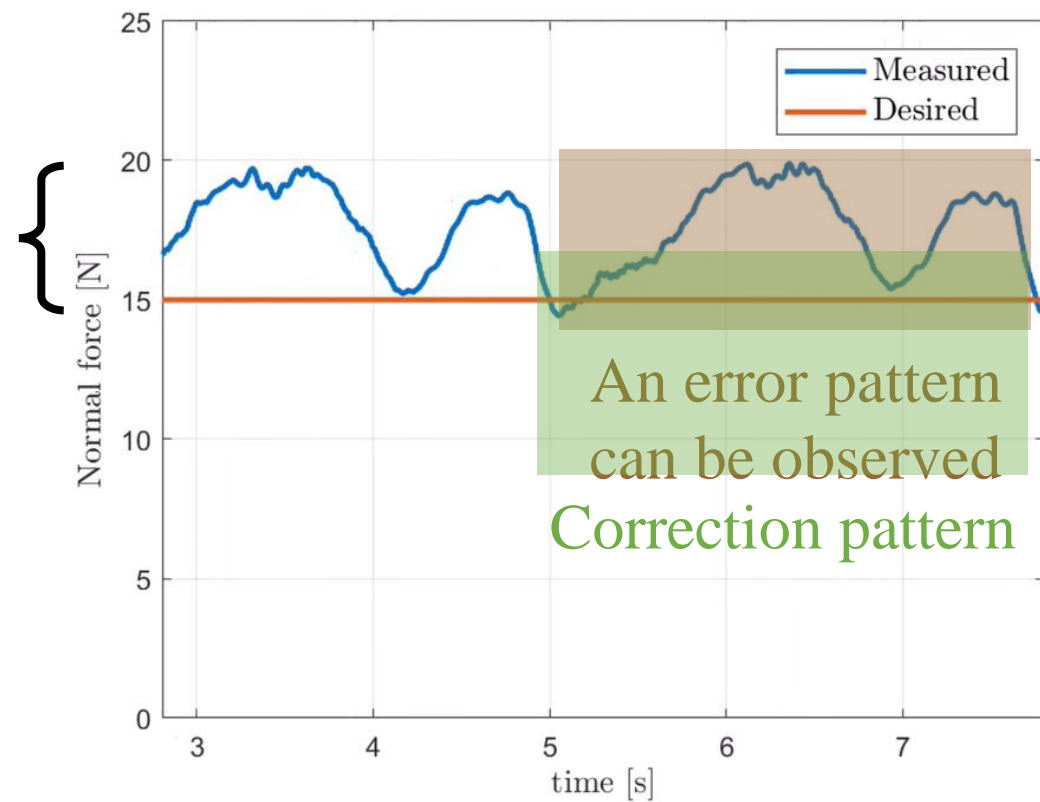
$F_d(\mathbf{x})$ : Desired contact force

$\mathbf{n}(\mathbf{x})$ : Normal vector to the surface

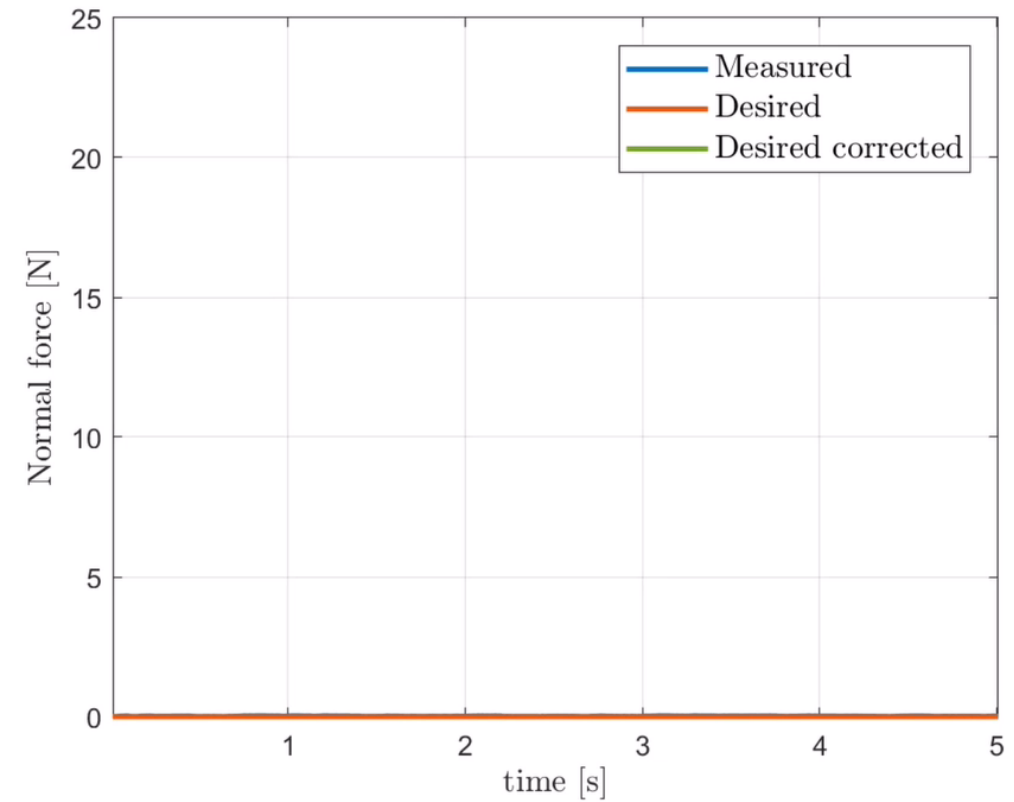
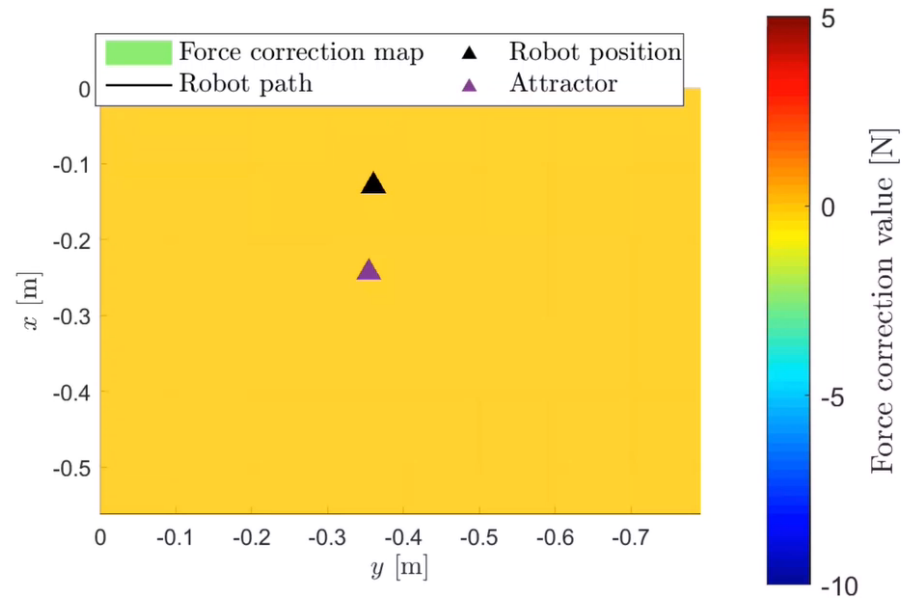
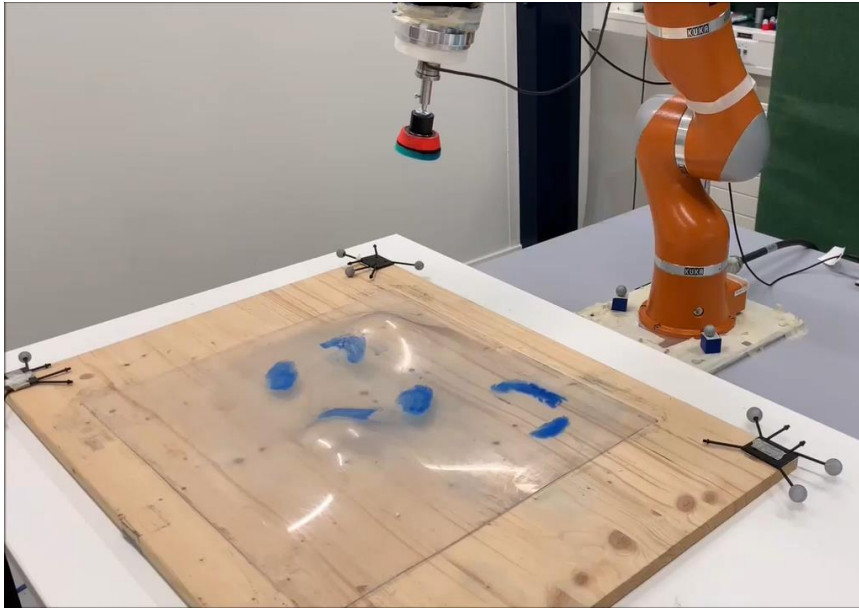
$\mathbf{F}_m$ : Measured contact forces



Initial error  
range



4x



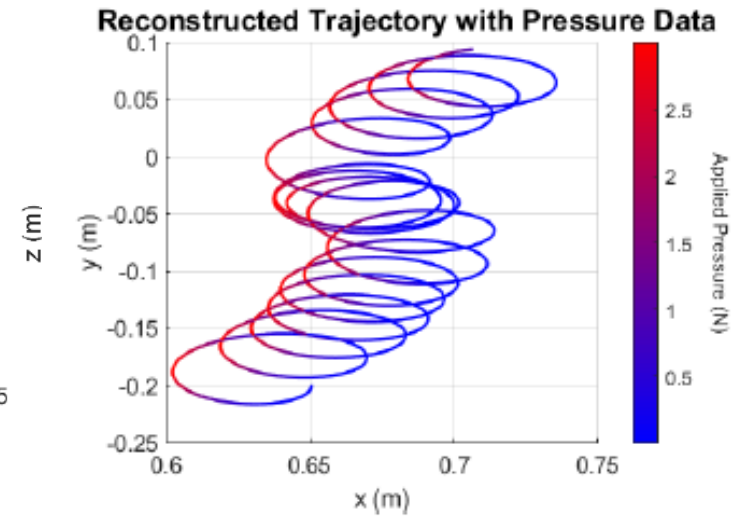
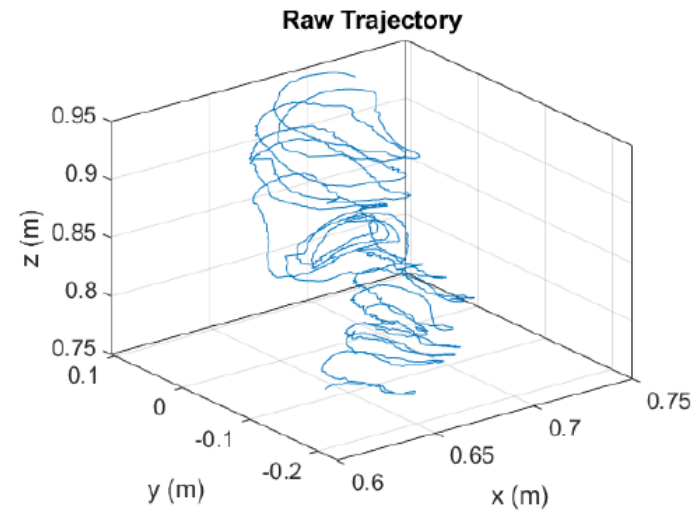
4x

## Other Examples of Learned-Force Based Control with DS

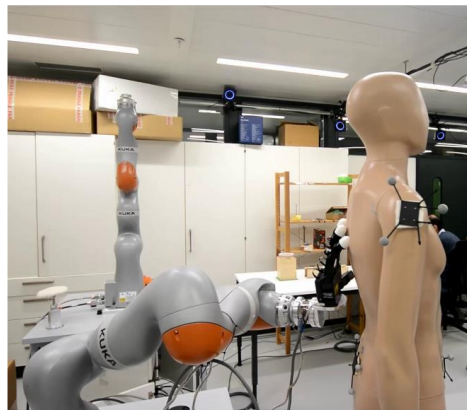
## Example of application: Learning Motion and Force Pattern to Massage



**Human recordings**



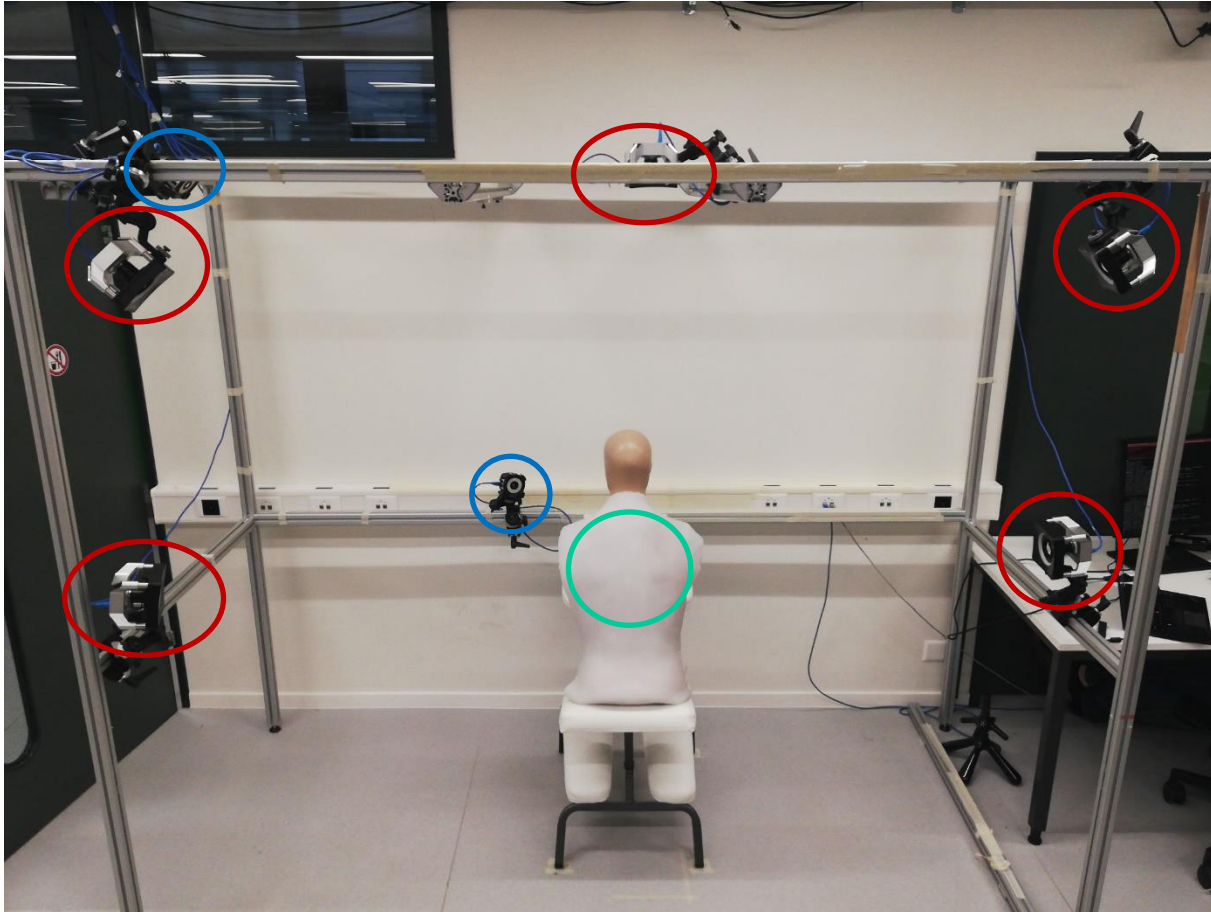
**Modeling**



**Robotic massage**



## GATHERING DATA FOR TRAINING – EXPERIMENTAL SET-UP

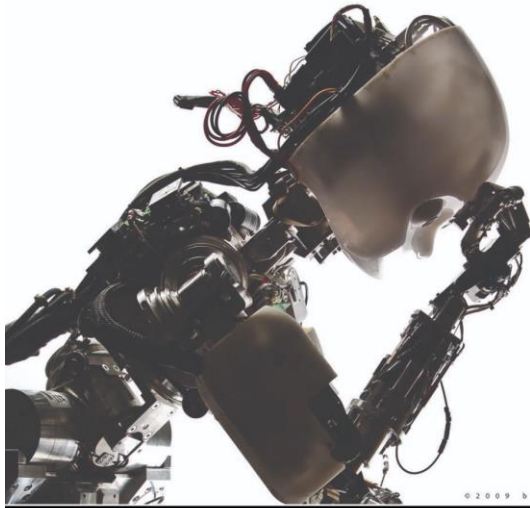


- ❖ Mannequin on massage chair
- ❖ Silicone sheet placed over the mannequin
  - ❖ Represent human tissue for the robot

- 7 Optitrack cameras
  - 5 x Prime 17w
  - 2 x S250:E
    - Limited to 125 Hz
- Infrared marker positions (X, Y, Z) captured



## Demonstration by Massage Therapist



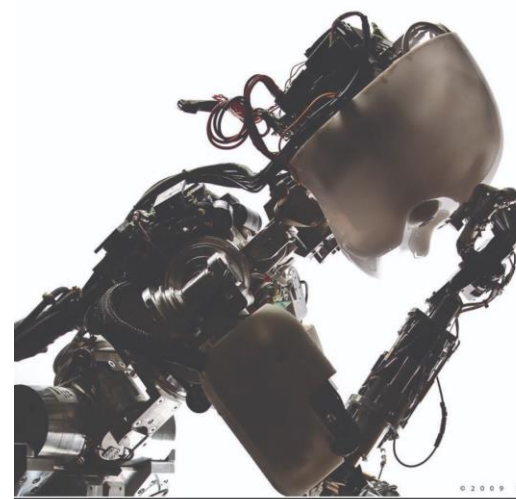
# LASA

Learning Algorithms and  
Systems Laboratory



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FÉDÉRALE DE LAUSANNE

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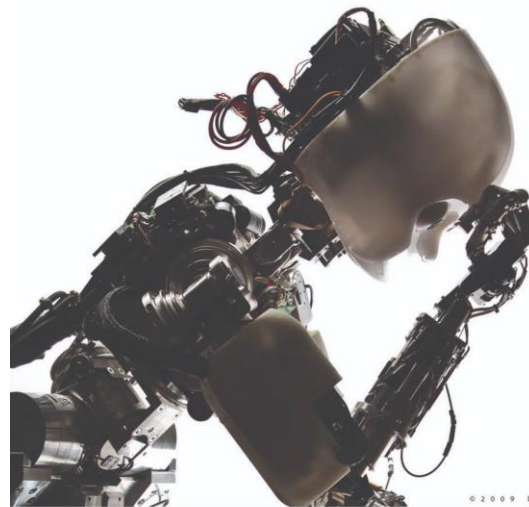
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# LASA

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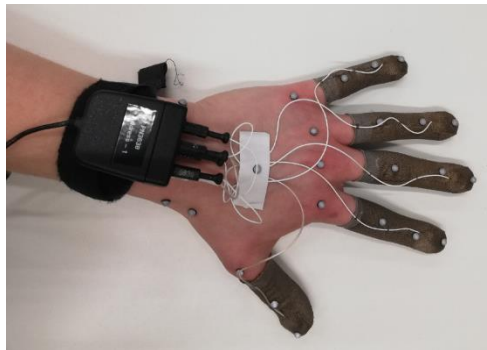


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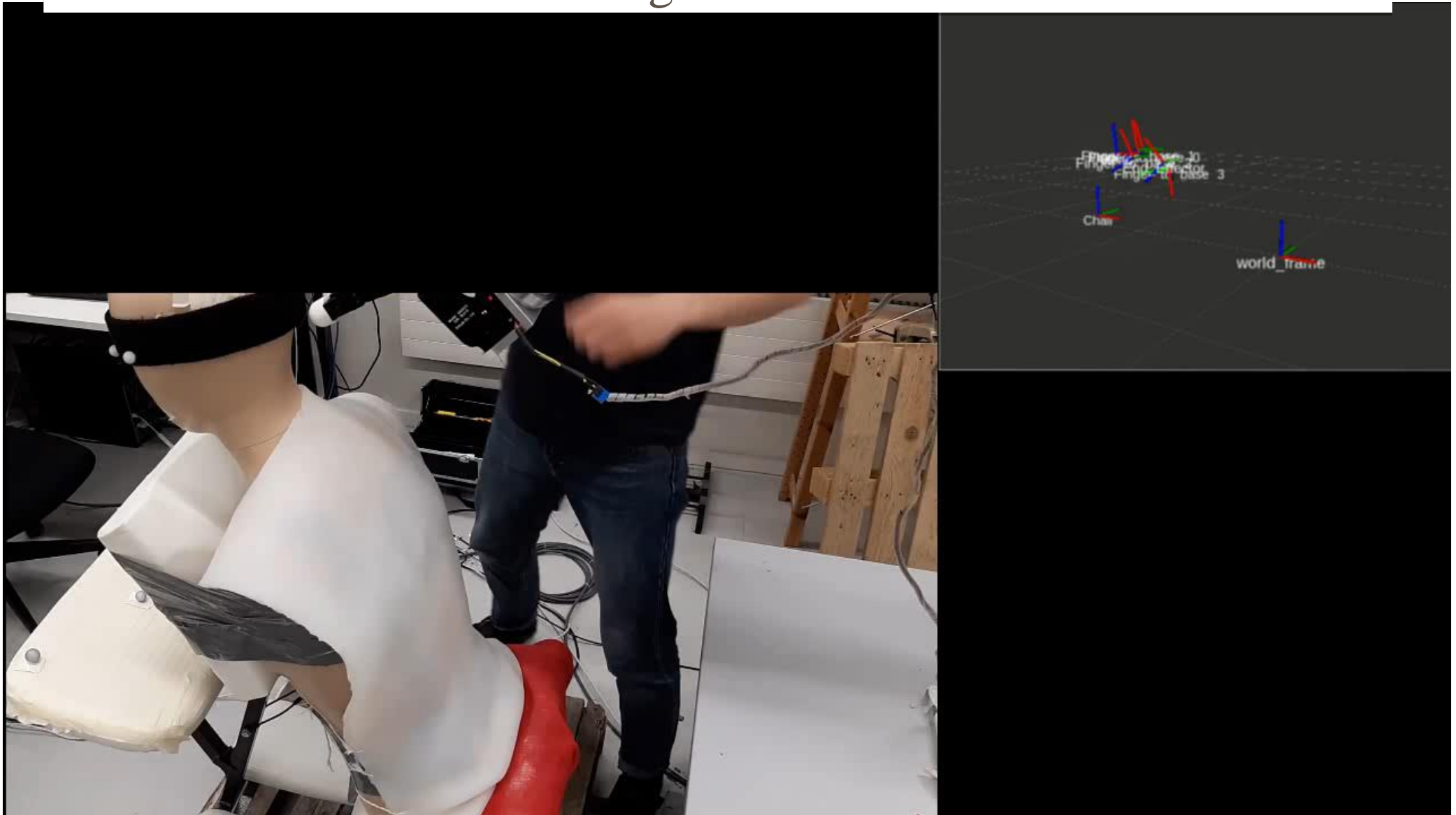
©2009 BASTIEN HAYET

## FORCE MEASUREMENT

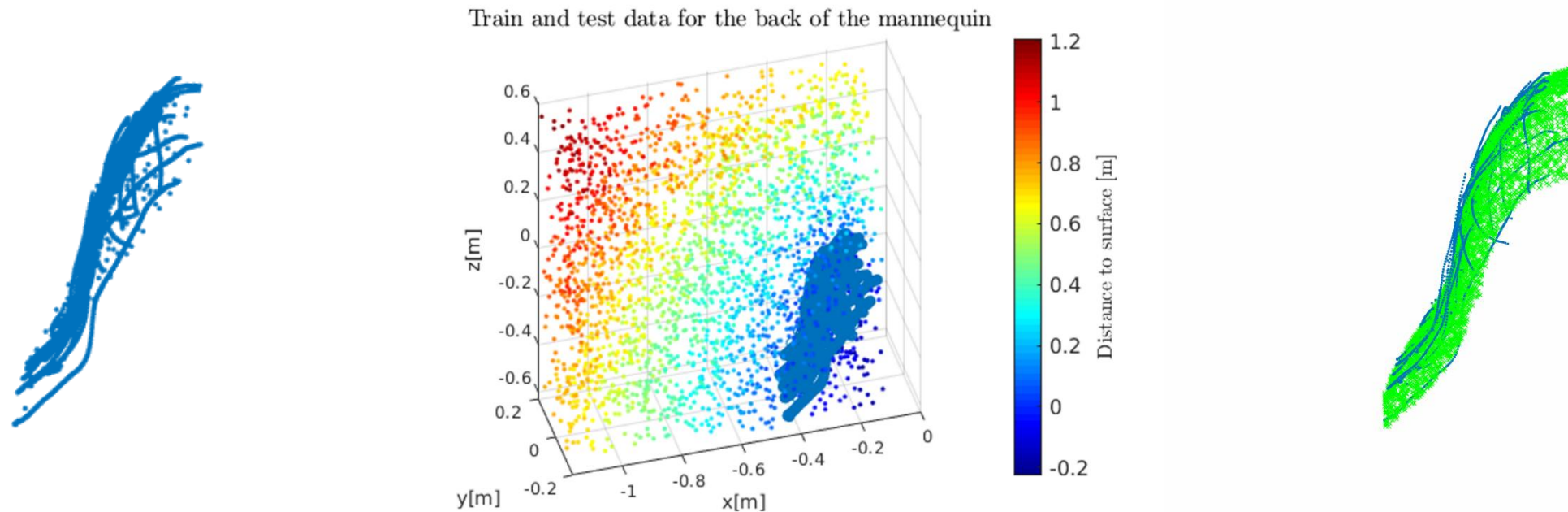
- Pressure Profiles Systems, FingerTPS
  - + Sensors have multiple sensing areas
  - + Multiple sizes
  - + Fabric covered sensor
  - Force mapped across entire surface
  - 40 Hz sampling rate



## Modeling Back Surface

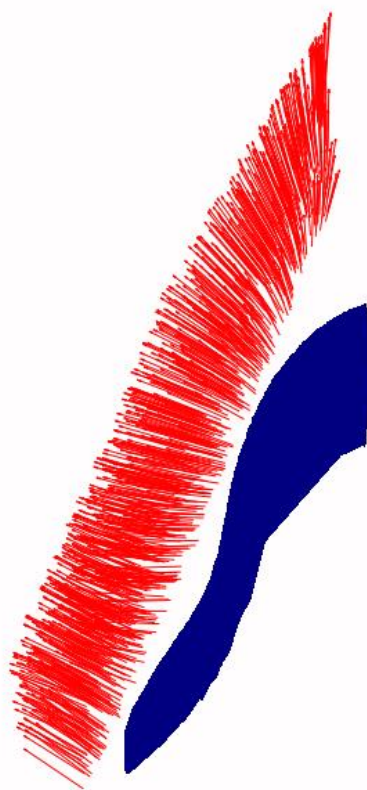


## Modeling Back Surface

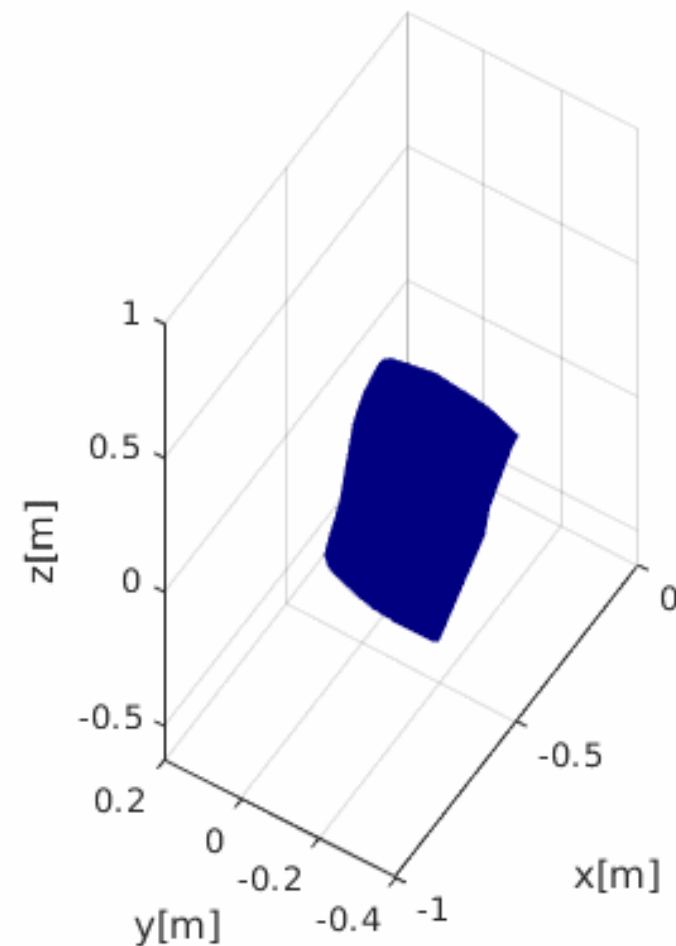


- **Cross-validation used for model validation with 5 folds.**
- **Grid search to find optimal  $C$  and  $\sigma$  for the SVR.**
- **Best testing accuracy achieved with a RMSE of 0.52 [cm] for points on the back**

## Reconstructed surface & gradients for the trained SVR

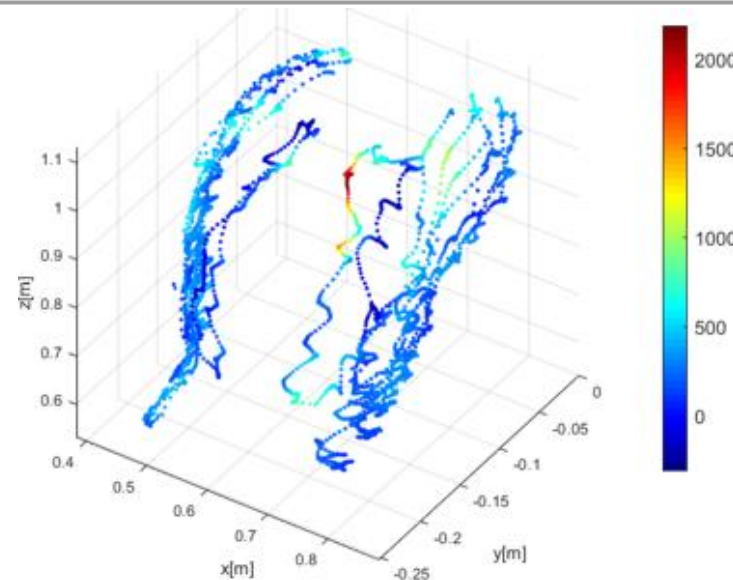


Simulated sample trajectories for the trained SVR

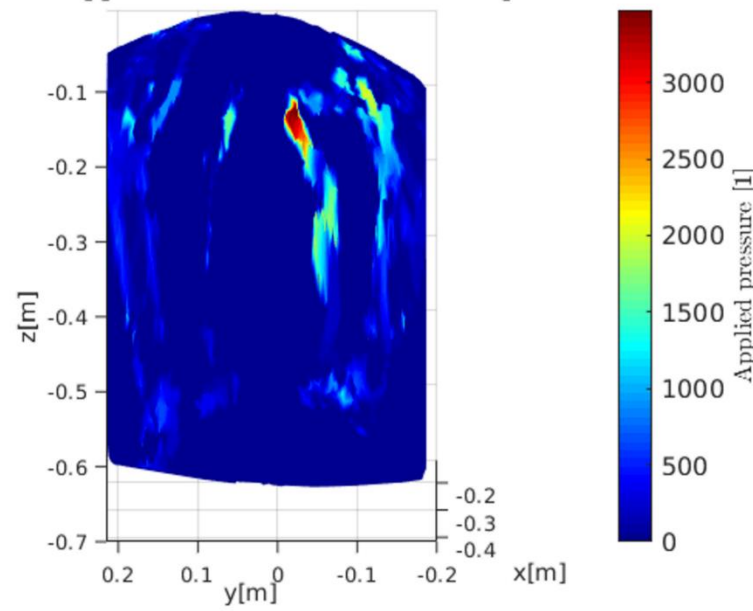




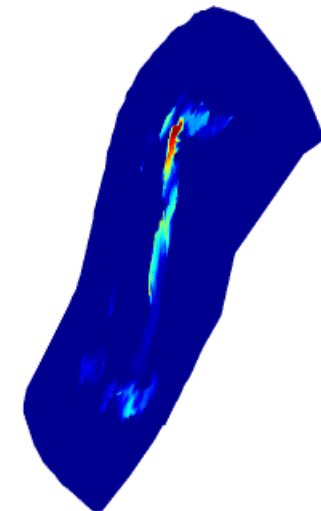
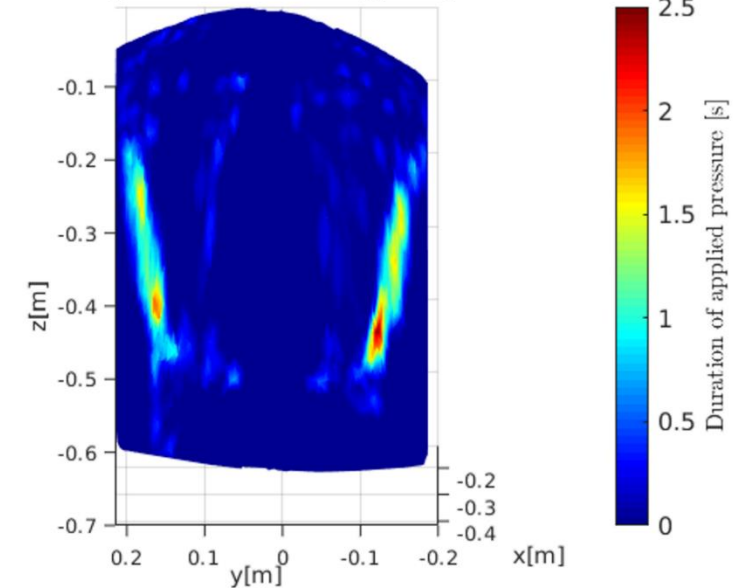
## Test data from movements on the back of the mannequin



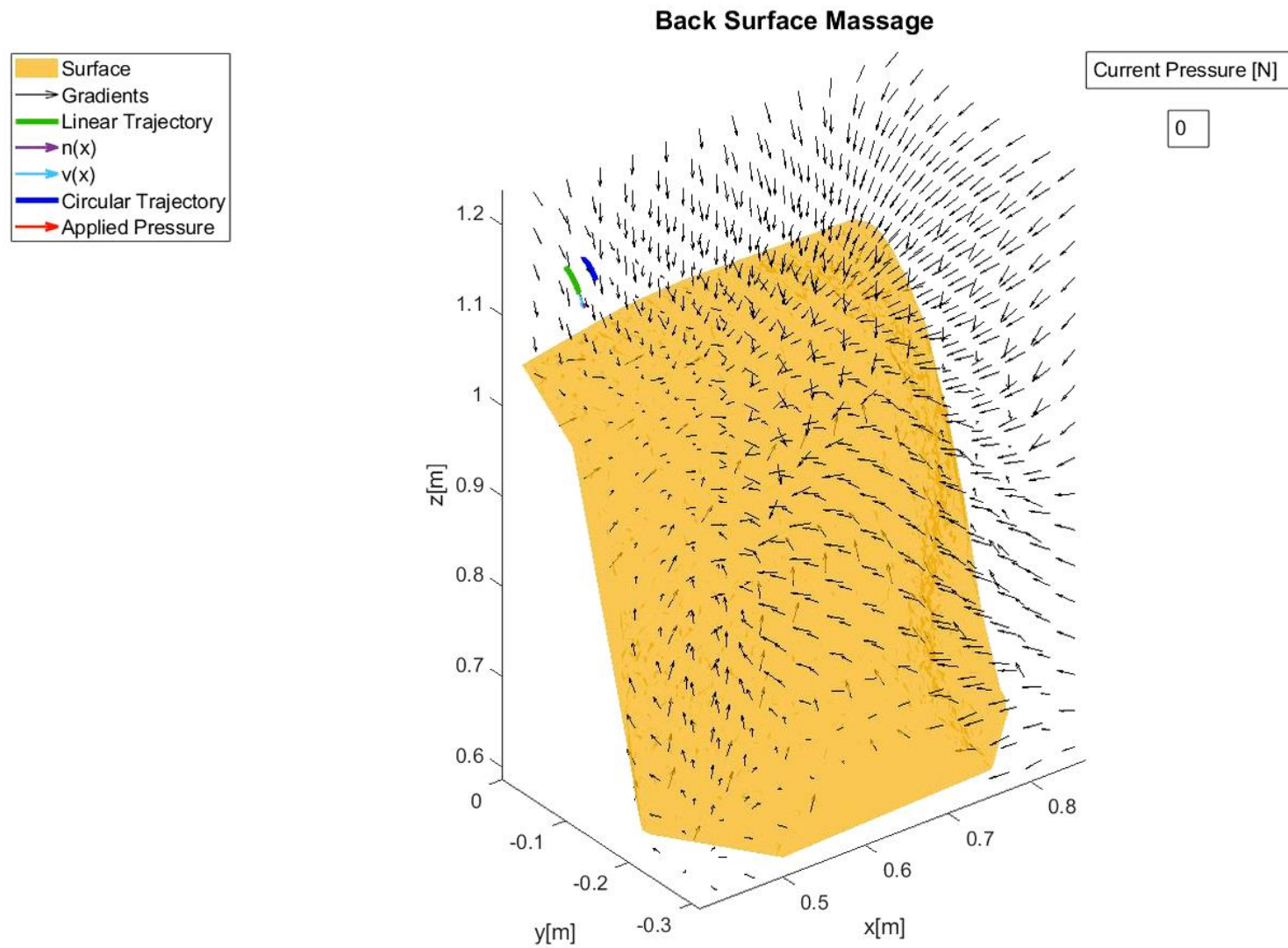
Force applied on the back of the mannequin



Colormap over duration of applied pressure



## Reconstruction of Motion on Back Surface

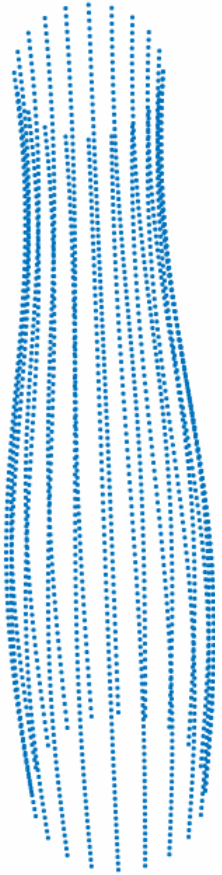


## Learning Arm Massage

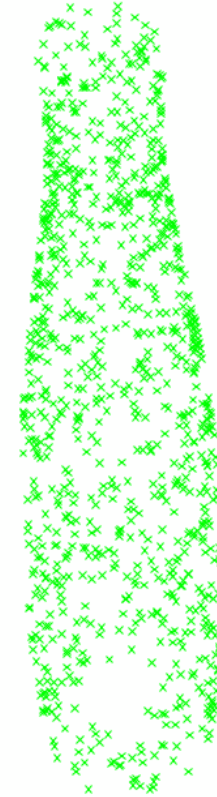




Create artificial data for the arm



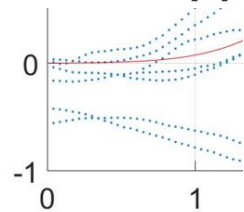
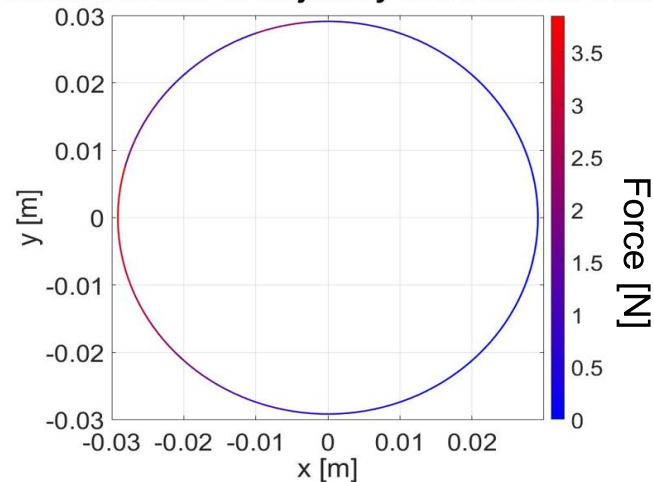
Train SVR and predict the surface



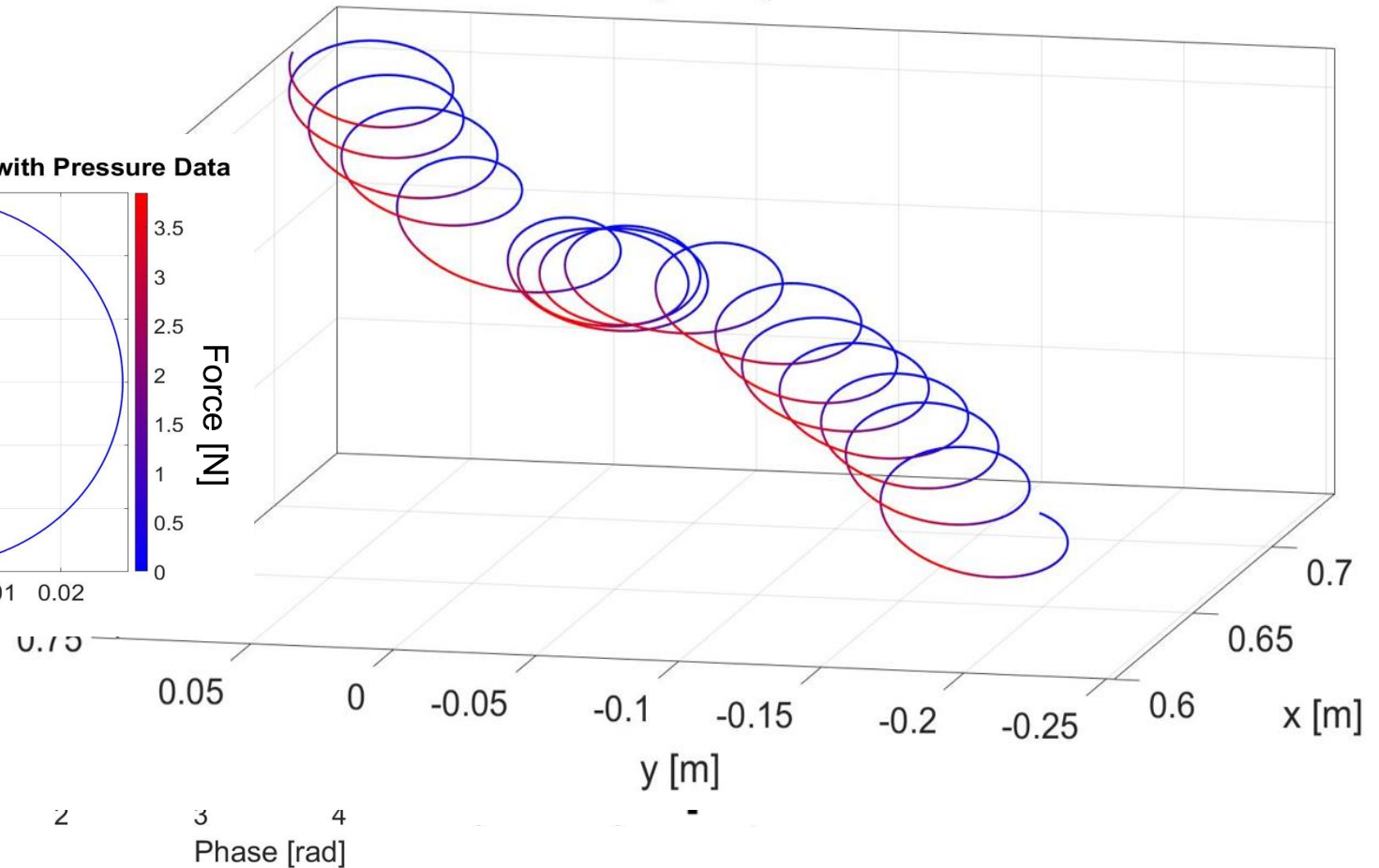
Step 2:

- Use the model to reconstruct the behavior

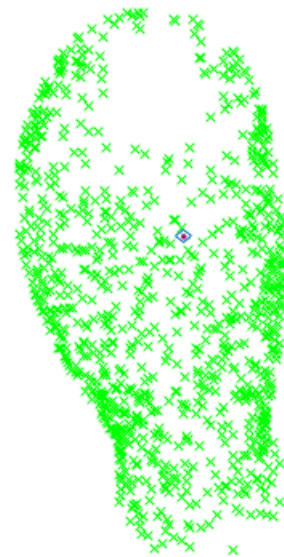
Stacked Radius of Trajectory with Pressure Data



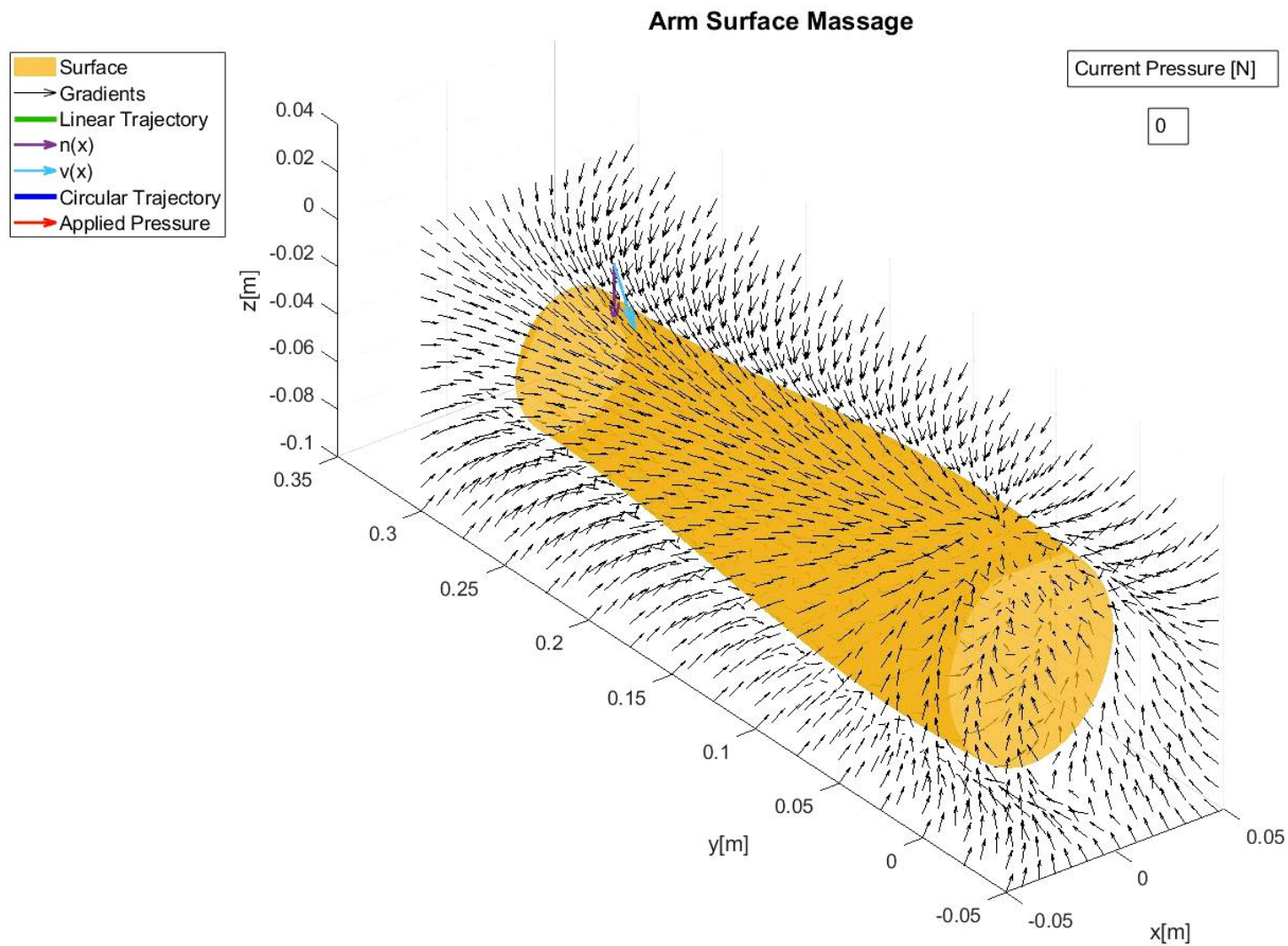
Reconstructed Trajectory with Pressure Data

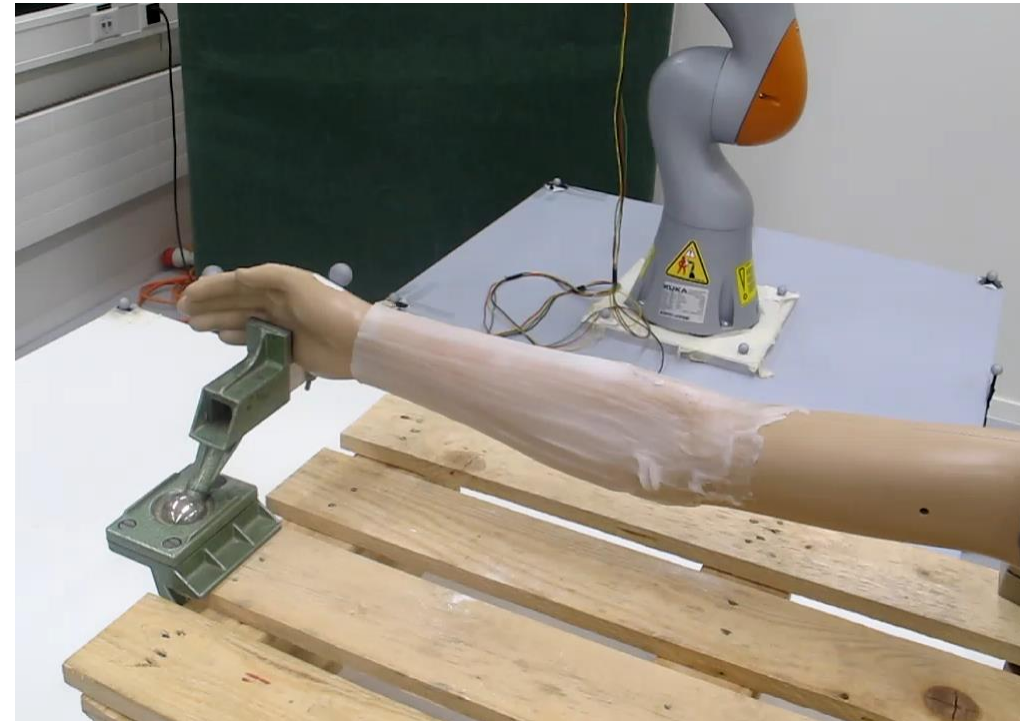
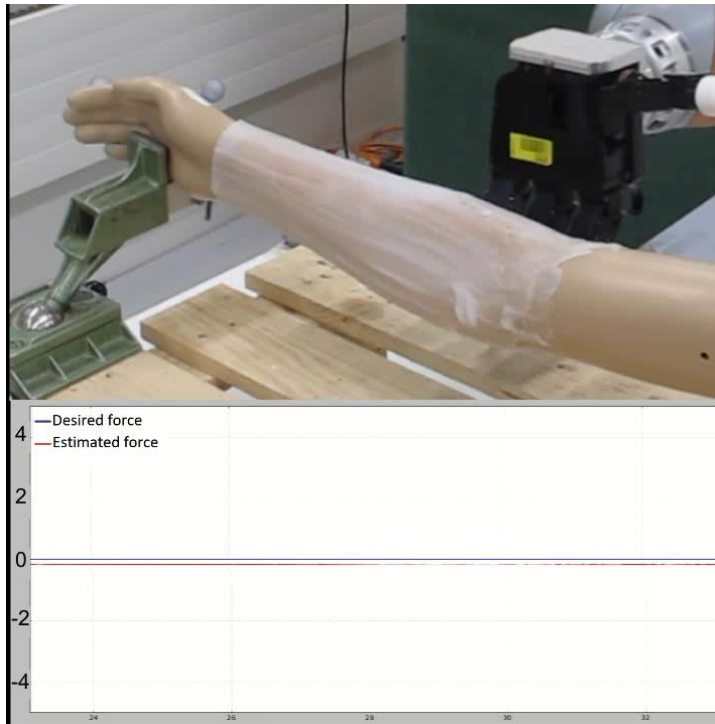
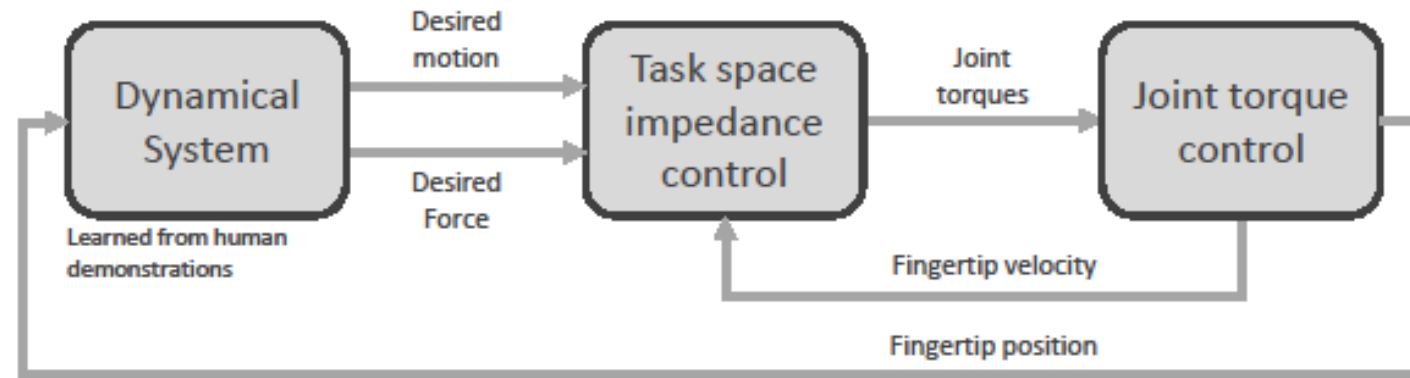


## Reconstruction of Motion on Arm Surface



## Reconstruction of Motion on Arm Surface





## Summary

- ❑ Introduced a means to **use Passive-DS to perform force control**.
  - The DS is decomposed into two parts, one controlling for motion along the surface, the other controlling for force.
  - The control is simplified by introducing a **function  $\Gamma$  that determines the distance and normal to the surface** everywhere (akin to principle used in obstacle avoidance) → This allows **decoupling control of force from motion along two orthogonal axes**.
- ❑ Machine learning (e.g nonlinear regression through SVR) can be used to model the function  $\Gamma$ .
- ❑ A force-based model associated to position is used for modulate the force along the DS
- ❑ To show that the system remains **passive**, as the DS is not necessarily conservative, the tank is required.
- ❑ Pattern of force can be **learned**
  - ❑ To compensate for unmodeled interaction forces (nonlinear friction, other dynamics of robot poorly modelled)
  - ❑ To learn a position-dependent pattern of force. Showed an example of application to model massage