TP–E1

Noise in Electronic Devices and Measurement Systems

(1 session)

CONTENTS

1. General Description

1.1. Purposes

The goal of this practical is to better understand the origin of the noise in electronic and optoelectronic measurement systems. We will measure intrinsic noise sources (in particular the thermal noise in resistors and the shot noise in photodiodes). In order to measure these intrinsic noise sources we will build an appropriate amplification chain (followed by an appropriate analog-to-digital converter and a digital signal processing algorithm). This will help in elucidating if and how the amplification chain contributes to the overall measured noise. We will also implement a synchronous modulation/demodulation scheme (lock-in detection) as a mean to improve the signal-to-noise ratio of an optical sensor in presence of 1/f noise and interferences at low frequency.

1.2. Mathematical Description

1.2.1. Noise in the Time Domain

Let's consider a signal *v(t)*, such as the voltage at the output of a sensor. It may be defined as:

(1) $$v(t) = v_s(t) + v_N(t)$$ [V]

The voltage *v(t)* carries both the information of the quantity to be measured by the sensor $v_s(t)$ and the voltage noise associated to the noise $v_N(t)$. Generally, the noise is a stochastic process whose mean value is zero while its root-mean-square (rms) value $v_{N,RMS}$ (and "power" $\overline{v_N^2}$) are not zero. The rms value and the "power" of the noise $v_N(t)$ computed over a time interval *T*) are usually defined as:

(2) $$\overline{v_N^2} = \frac{1}{T}\int_0^T [v_N(t)]^2\, dt$$ [V²]

(3) $$v_{N,RMS} = \left(\overline{v_N^2}\right)^{1/2}$$ [V]

1.2.2. Noise in the Frequency Domain

By means of the Fourier transform, we may describe noise as

(4) $$V_N(f) = \int_{-T/2}^{T/2} [v_N(t)] \cdot \exp(-j2\pi f t)\, dt$$ [V/Hz]

Let's define the voltage power spectral density of *v(t)* as

(5) $$S_{N,V}(f) = \frac{1}{T}\left|V_N(f)\right|^2$$ [V²/Hz]

and the reduced spectral density as

(6) $$S_{N,V}^V(f) = \sqrt{S_{N,V}(f)}$$ [V/Hz^1/2]

It is now possible to compute noise voltage rms from the spectral density (power theorem):

(7) $$v_{N,RMS} = \left(\int_0^\infty S_{N,V}(f)\, df\right)^{1/2}$$ [V]

The following table summarizes the physical dimensions mentioned so far:

$v_N(t)$	$v_{N,RMS}$	$\overline{v_N^2}$	$S_{N,V}(f)$	$S_{N,V}^V(f)$
V	V	V^2	V^2/Hz	V/$Hz^{1/2}$

1.2.3. Sum of uncorrelated noise sources

When the contribution of two uncorrelated noise sources $v_{n1}(t)$ and $v_{n2}(t)$ sum up, the overall result is given by:

$$v_n(t) = v_{n1}(t) + v_{n2}(t)$$

In the case of uncorrelated noise sources we have $\overline{v_{n1} v_{n2}} = 0$ and hence:

$$\overline{v_n^2} = \overline{v_{n1}^2} + \overline{v_{n2}^2} \qquad \text{and} \qquad v_{n,rms} = \sqrt{\overline{v_n^2}} = \sqrt{\overline{v_{n1}^2} + \overline{v_{n2}^2}} = \sqrt{(v_{n1,rms})^2 + (v_{n1,rms})^2}$$

In conclusion, one should consider the summation of the power contributions and not the noise voltages!

$$v_{n,rms} \neq v_{n1,rms} + v_{n1,rms}$$

1.3. Noise Sources

Noise sources in the electrical systems can be *intrinsic, extrinsic or induced*.

1.3.1. Intrinsic noises

They come from the intrinsic nature of the electrical components and it is usually impossible to cancel them. The most important types of intrinsic noise are the following ones:

- **Thermal noise** (a.k.a. Johnson noise), caused by the thermal agitation of the electrons.
- **Shot noise**, caused by the discrete nature of the carriers (electrons). It arises whenever the current results limited by the crossing of a potential barrier by the carriers.
- **Generation/Recombination noise**, caused by the formation/recombination of electrons/holes pairs.
- **1/f noise** (a.k.a. flicker noise), is a conductivity modulation whose origin is currently still unknown. In semiconductors mobility fluctuations and charge carrier densities fluctuations are considered as the main causes of the 1/f noise. 1/f noise is present also in many non-electronics processes (e.g. earthquakes, biological processes, etc.).

1.3.2. Extrinsic noises

They are induced from the external environment to the measurement system itself. The extrinsic noises are often due to the electromagnetic coupling. For instance, some causes may be the high voltage lines, mains, cathodic tubes, mobile phones, and computers. In this context, adding a shielding on both cables and equipment may be crucial to minimizing the extrinsic noise.

1.3.3. Induced noises

Further noise contributions are induced by analog-to-digital converters (i.e. quantization), amplification stages, conditioning systems, aliasing filters, and any other electrical subunit added on the signal path. Those blocks generate additional noise and reshape the noise produced by the previous subsystems.

1.4. Noise from a resistor

The intrinsic noise contributions in a resistor are the thermal noise and the 1/f noise.

Thermal noise:

The noise voltage power spectral density across a resistor R is given by:

(8) $$S_{NV}(f) = 4kTR$$ [V²/Hz]

$$S^{V}_{N,V}(f) = \sqrt{4kTR}$$ [V/Hz^{1/2}]

The noise current power spectral density across a resistor R is given by:

(9) $$S_{NI}(f) = 4kT/R$$ [A²/Hz]

$$S^{I}_{N,I}(f) = \sqrt{4kT/R}$$ [A/Hz^{1/2}]

where k is the Boltzmann constant and T is the absolute temperature (in K).

1/f noise:

In general, the voltage and current spectral density associated to the 1/f noise happen to be:

(10) $$S_{NV}(f) = C_1 V^2 \frac{1}{f}$$ [V²/Hz]

(11) $$S_{NI}(f) = C_1 I^2 \frac{1}{f}$$ [A²/Hz]

where, C_1 depends on the structure of the electrical component. For some devices, the 1/f noise is given by the Hooge's empirical law:

(12) $$S_{NV}(f) = V^2 \frac{\alpha}{N} \frac{1}{f}$$ [V²/Hz]

(13) $$S_{NI}(f) = I^2 \frac{\alpha}{N} \frac{1}{f}$$ [A²/Hz]

where α depends on the structure of the electrical component and N is the amount of carriers.

Shot noise:

It is modelled by a current source placed in parallel with the ideal component (i.e. noiseless). The corresponding power spectral density is:

(14) $$S_{NI}(f) = 2eI$$ [A²/Hz]

(15) $$S^{I}_{NI}(f) = \sqrt{2eI}$$ [A/Hz^{1/2}]

where I is the mean current flowing through the component. In electronics, the main sources of shot noise are the PN junctions while it transforms to thermal noise in the case of resistances (i.e. thermal diffusion dominates).

1.4.1. Noise Measurement across a Resistor

We will study resistors as a source of intrinsic noise. That is an important issue since the corresponding thermal noise significantly limits the sensitivity (i.e. the minimum detectable signal variation) of any sensor with resistive elements (such a several magnetic sensors, inductive sensors, strain sensors, chemical sensors, light sensors, temperature sensors, etc. etc.). In this framework, the system adopted to measure the noise coming from different resistors is depicted in Figure 8 and the corresponding noise sources are the amplification chain and the analog-to-digital converter (See the appendix for circuit details).

Let's now recall that the thermal noise for a given resistor is

$$S_{N,V}^{V} = \sqrt{4kTR} \qquad [\mathrm{V/Hz^{1/2}}]$$

The white noise induced by an amplifier may be split in a voltage noise independent from the source impedance and a current noise proportional to the source impedance. The manufacturer typically provides the voltage spectral density e_n in nV/Hz$^{1/2}$ as well as the current spectral density i_n in either pA/Hz$^{1/2}$ or fA/Hz$^{1/2}$. The overall induced noise at the input of the amplifier is then:

(14) $$S_{N,V_{ind}}^{V} = \sqrt{(e_n)^2 + (R \cdot i_n)^2} \qquad [\mathrm{V/Hz^{1/2}}]$$

Consequently, the equivalent total noise at the input is:

(15) $$S_{N,V_{tot}}^{V} = \sqrt{4kTR + (e_n)^2 + (R \cdot i_n)^2} \qquad [\mathrm{V/Hz^{1/2}}]$$

Such a situation may be modelled in terms of noise generators, an ideal resistor and an ideal operational amplifier, as in the following schematic:

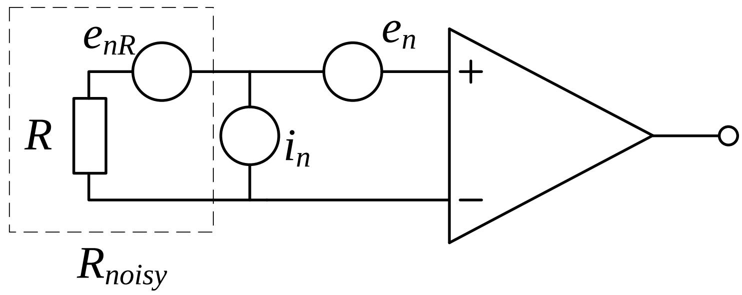

Figure 1. Noise model of an amplifier and a resistor R at the input

1.5. Synchronous modulation/demodulation

In order to improve the SNR of a measurement system in presence of a 1/f component in the noise or low frequency extrinsic noise (such as external interferences), the adoption of a synchronous modulation/demodulation (or "synchronous detection" or lock-in detection") can be very efficient. The useful signal is effectively "transferred" from a frequency region with a very large noise to a frequency region with much less.

To better understand the working principle of the synchronous detection, let's consider an optical barrier composed of a light emitting diode (LED) and a photodiode. An optical barrier can be employed to count the number of people at the entrance of a store (see Figure 2). Instead of using a continuous light beam, we use a "chopped" beam (i.e., an amplitude modulated light beam with modulating frequency f_0). This "chopping" can be performed by: (1) driving electrically the LED with an AC voltage at frequency f_0 or (2) by placing a mechanical chopper (e.g., a rotating disk with openings) in front of the LED powered with a DC voltage. In this TP we will use a mechanical chopper placed in front of a LED powered with a DC voltage. The light beam $s_{DC}(t)$ transits through the chopper which cuts the beam at regular intervals, and then modulates the signal from low frequency (DC) to a sufficiently high f_0, where both the 1/f noise and the environmental interferences become negligible (only the white noise remains relevant).

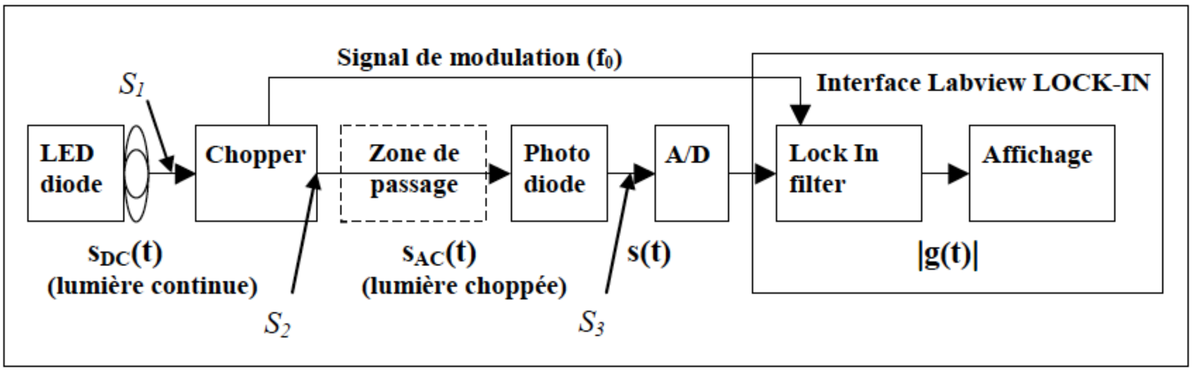

Figure 2: ***Synchronous Detection: Schematic showing the working principle.***

After that, the signal $s_{AC}(t)$ must be demodulated so as to only preserve the AC signal at f_0. Such a task is performed by means of a lock-in amplifier (Figure 3) which applies a narrow band-pass filter at f_0. In particular, we will use an analog-to-digital converter (A/D) to digitalize the signal and will perform a "software" demodulation. The bandwidth Δf is determined by the inverse of low pass filter in the software (which is actually a simple averaging time). A larger bandwidth would allow to follow faster variation of input signal (i.e., counting people crossing very fast the barrier). On the other hand, lower bandwidth reduces the noise. At the output of the lock-in, we will recover module and phase of the previously filtered signal.

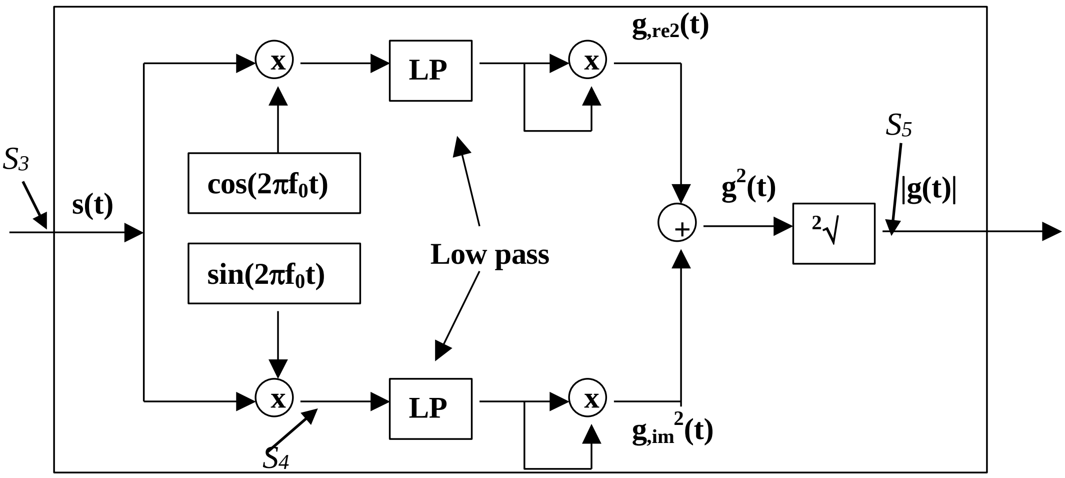

Figure 3: ***Schematic showing the working principle of a lock-in amplifier.***

A **Lock-In** allows creating a very narrow (with width easily adjustable) band-pass filter about the modulation/demodulation frequency (plus the possibility to reject noise which is out-of-phase with respect to the useful signal). The measured input signal $s(t)$ is split in two branches and multiplied by a cosine and a sine at the same frequency f_0 (i.e. the apparatus needs a reference signal at f_0). The signal multiplication corresponds to the demodulation and leads to a signal displacement in the frequency domain (i.e. from f_0 to DC). Finally, it is sufficient to filter out the high frequency components so as to recover the useful components and cancel a significant part of the noise with the other environmental perturbations. Figure 4 points out the benefits of a synchronous detection.

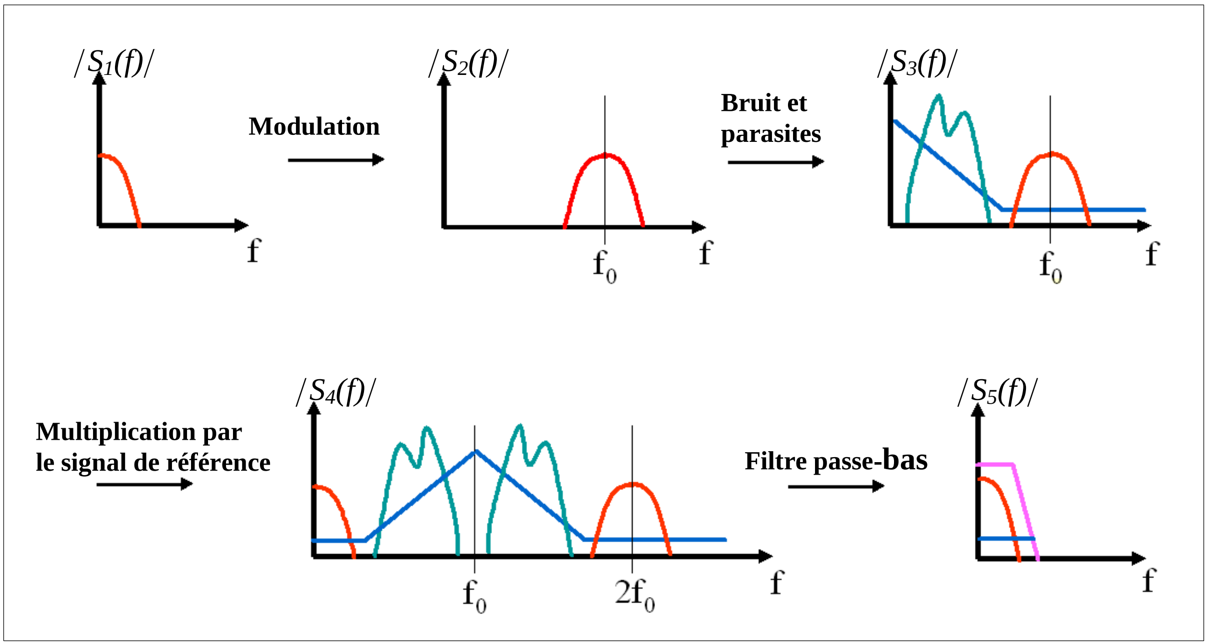

Figure 4: ***Synchronous Detection: Spectral Analysis.***

2. Questions

Question 1

Which are the values of e_n and i_n for the amplifiers INA217 and OPA604? (gain equal to 100 and frequency of interest higher than 1 kHz). Electrical specifications available at:

INA217: http://focus.ti.com/docs/prod/folders/print/ina217.html

OPA604: http://focus.ti.com/docs/prod/folders/print/opa604.html

Question 2

The setup to measure the noise of some resistors is composed of a resistor connected to two amplifiers connected in series. Which is the necessary gain of the first amplifier so as to neglect the noise coming from the second amplification stage?

Question 3

Assume you have a 12-bit digital acquisition board with an input dynamic range of ±0.1 V. Which is the minimum necessary gain to measuring a 1 nV signal at the input?

Question 4

Investigate the figure in Appendix 5.2. This is the measurement setup for the noise measurements of resistors. Create a table with the following values in V/Hz$^{1/2}$ for each resistance (10 Ω, 30 Ω, 100 Ω, 300 Ω … 3 MΩ, 10 MΩ): (You can use the simplified figure below to understand which component to consider in the calculations. Figure 5(a) corresponds to question (a), and so on…)

a) Intrinsic white noise ($S^V_{N,V}$);

b) Total intrinsic white noise (resistance to be measured together with resistances R_{b1} and R_{b2});

c) Total equivalent *input* noise with only INA217 amplifier ($S^V_{N,Vtot1}$);

d-1) Total equivalent *input* noise with INA217 as a first stage ($S^V_{N,Vtot}$);

d-2) Total equivalent *input* noise with OPA604 as a first stage ($S^V_{N,Vtot}$).

Question 5

Which amplifier would you choose as first amplification stage for a sensor whose resistance is low (<1 kΩ)? And if the resistance is high (>100 kΩ)?

Question 6

Which is the total noise rms (in V) at the output of the amplifier INA217 (gain equal to 100) with a sensor of 10 kΩ for a bandwidth of 10 Hz about 10 kHz?

Question 7

Let's consider a photodiode with a current of 1 µA. Which is the shot-noise that you expect at the output (in A/Hz$^{1/2}$)? If you connect the photodiode to a transimpedance amplifier having a gain of 10^7 V/A, which is the spectral density you expect at the output (in V/Hz$^{1/2}$)?

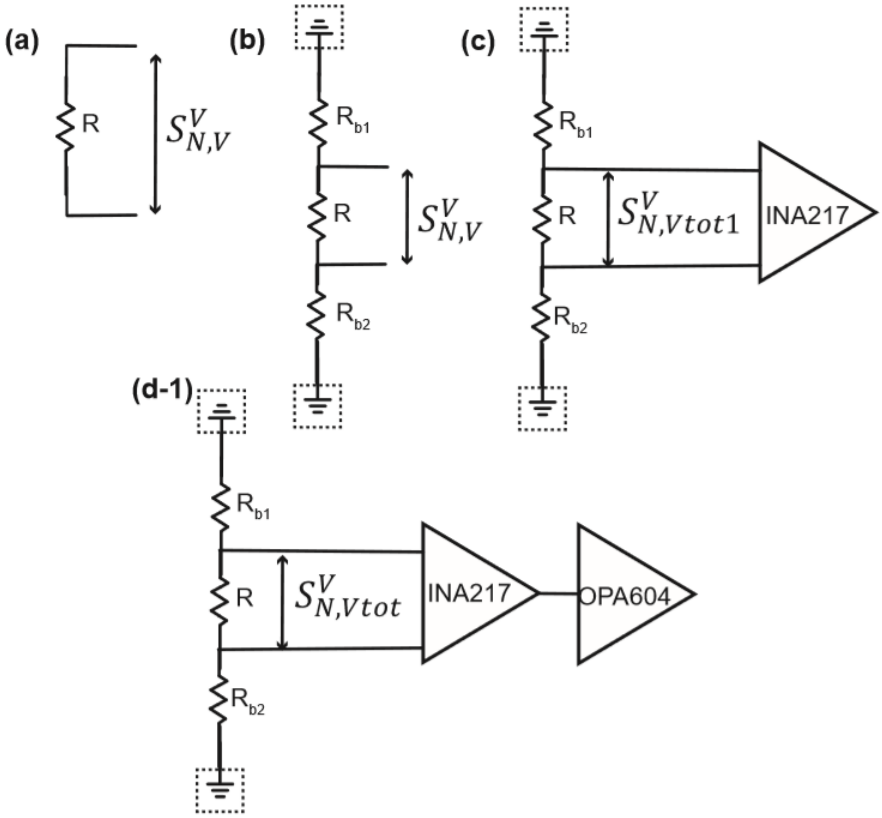

Figure 5 : Representation of the configurations of Question 4

Question 8

Prove that the amount of 1/f noise power is the same regardless of the given frequency decade.

Question 9

The 1/f noise is present in many non-electronics systems. Give some examples.

3. Experiment 1: Noise from Resistances

3.1. Setup

In this experiment ("Noise from Resistances" and "Synchronous Detection"), we have:

- PC with 3 programs Labview (Spectrum Analyser_2025, Simple Digital Multimeter_2025, Lock in Amplifier_2025): see «TP NOISE» on the desktop.
- Digital Acquisition Board (16 bits, max. 300 kSamples/s) controlled by the PC.
- Preamplifier based on the INA217.
- Preamplifier based on the op-amp OPA604.
- Commercial transimpedance amplifier EG&G5182
- Adjustable DC supply voltage (±12V 18mA for the amplifiers, 4V 17mA for the LED)
- LED
- Photodiode
- Mechanical light chopper (with electronic control)
- Oscilloscope
- Multimeter
- Resistances (values within 10 Ω and 10 MΩ).

3.2. Noise measurement without amplifier

1. Plug a resistor (e.g. 10 kΩ) on the support whose the two ends are connected to the input of the ADC controlled by the PC (see Figure 6, CH0).
2. Detect the noise level with the Labview program «Spectrum Analyser_2025.vi», and then compare the measured value with the theoretical one.
3. Which is the shape of the spectral density? Can you see any white noise there? Which is the value?
4. Why can't the thermal noise be correctly measured in this way? How can you make the thermal noise non-negligible with respect to the other noise contributions?

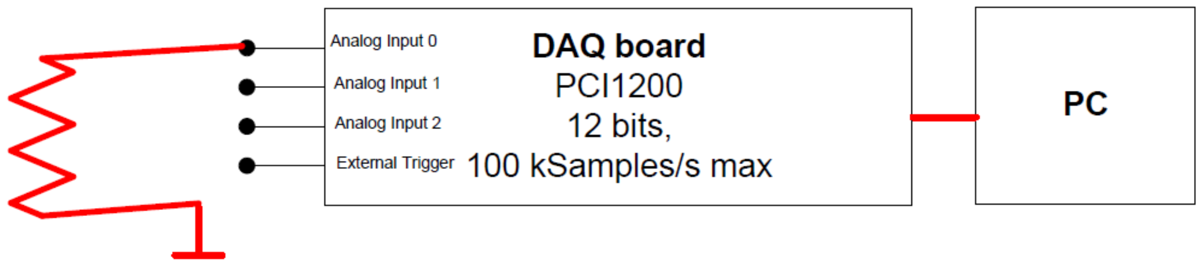

Figure 6 : ***In order to measure the resistor's noise without amplifier, the resistor is directly connected to the digital acquisition board.***

3.3. Noise measurement with an amplifier

Introducing an amplification chain is essential to make the thermal noise voltage produced by the resistors measureable. Accordingly, we are going to connect two amplification stages between the resistor and the PC (Figure 8). The two amplifiers introduce additional noise, and the corresponding contributions have to be taken into account (and measured).

3.3.1. Measurement of the amplifiers' noise

The amplifiers (implemented using INA217 and OPA604) have a fixed gain equal to 100 (101 to be more precise). In order to measure the noise induced by the preamplifier, we have to short-circuit its input ports (Figure 7) (the short circuit has a very low resistance, which produce a negligible thermal noise with respect to those of the amplifiers). The overall gain of the amplification chain is 10000 (10201 to be more precise).
The equivalent voltage noise at the input of the INA 217 given by the manufacturer is 2 nV/Hz$^{1/2}$ while the one corresponding to the OPA604 is 10 nV/Hz$^{1/2}$ (see datasheets).

Which value are you obtaining?

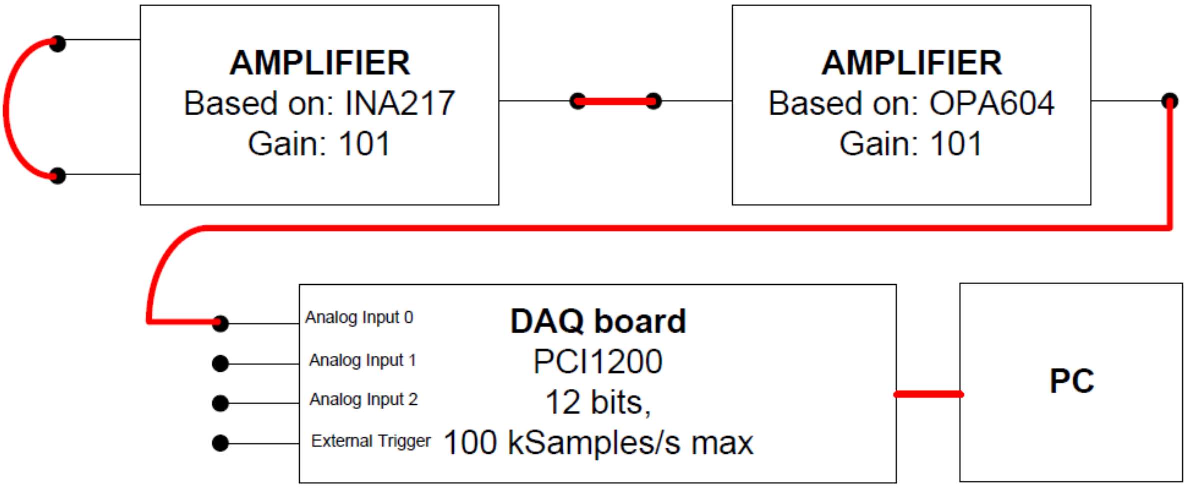

Figure 7 : ***Block diagram to show the required connections for the noise measurement of the amplifiers***

What is the noise induced in the measurement system by the second amplifier with respect to the noise introduced by the first one? In the following, we will neglect the noise coming from the second stage? Is that a reasonable assumption?

Save your measurements to have data for the report!

3.3.2. Measurement of the voltage noise from resistors

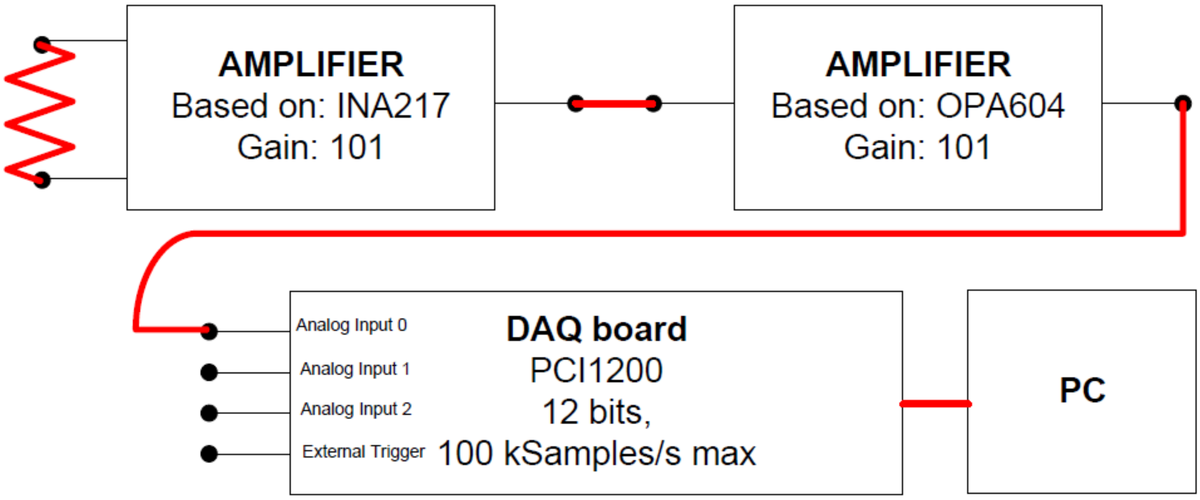

Figure 8 : ***Block diagram to show the required connections for a noise measurement with the amplifiers.***

As described throughout the general description, resistors are a source of thermal noise. The ideal characteristic is given by

$$S_{N,V}^{V} = \sqrt{4kTR} \qquad [\mathrm{V/Hz^{1/2}}]$$

However, other noise sources sum up on the signal path (i.e. voltage noise coming from the INA217 and OPA604, see Figure 1 for amplifier noise representation). Consequently, the total noise is:

$$S_{N,V\,tot}^{V} = \sqrt{4kTR_p + (e_n)^2 + (R_p \cdot i_n)^2} \qquad [\frac{\mathrm{V}}{\sqrt{\mathrm{Hz}}}]$$

with:
$$R_p = R//(R_{b1} + R_{b2})$$

where R is the resistance to be measured while $R_{b1,2}$ are the two resistances at the input of the amplifier (see Appendix 5.2).

In case of small resistances, the associated thermal noise gets hidden by the voltage noise of the amplifier. On the contrary, in case of big resistances the dominant noise source is the current noise of the amplifier.

1. Connect the resistance as shown in Figure 8.
2. Perform a measurement to verify the proper working condition of the system (use a 10 kOhm and check with the theoretical value). Save the measurements to have data for your report.
3. Choose a set of resistances in the range of [50 Ω – 10 MΩ] so as to track a complete characteristic $S_{N,V}^{V}$ (R). Save the measurements to have data for your report.
4. Analyze the frequency behavior:
    a. Compare the measurements with those performed without amplifier.
    b. In general, what effects can you observe? Analyze:
        i. Low frequency noise and its cause
        ii. The presence of a flat region at intermediate frequency
        iii. The high frequency behavior
        iv. Possible peaks and their causes
5. Track the level of white noise for each resistance and compare those values with the theoretical ones computed so far. Analyze the following aspects:

   a. Behavior for small resistances. Causes?
   b. Behavior for big resistances. Causes?
   c. Behavior for “medium” resistances. Are there values of resistance for which the intrinsic thermal noise of the resistor is dominant?
6. Why for small and large resistor there is deviation from the expected behavior? How can you model such a behavior at the block diagram level of the amplifier? (Hint: take a look back at figure 5 (d-1))

4. Experiment 2: Synchronous detection

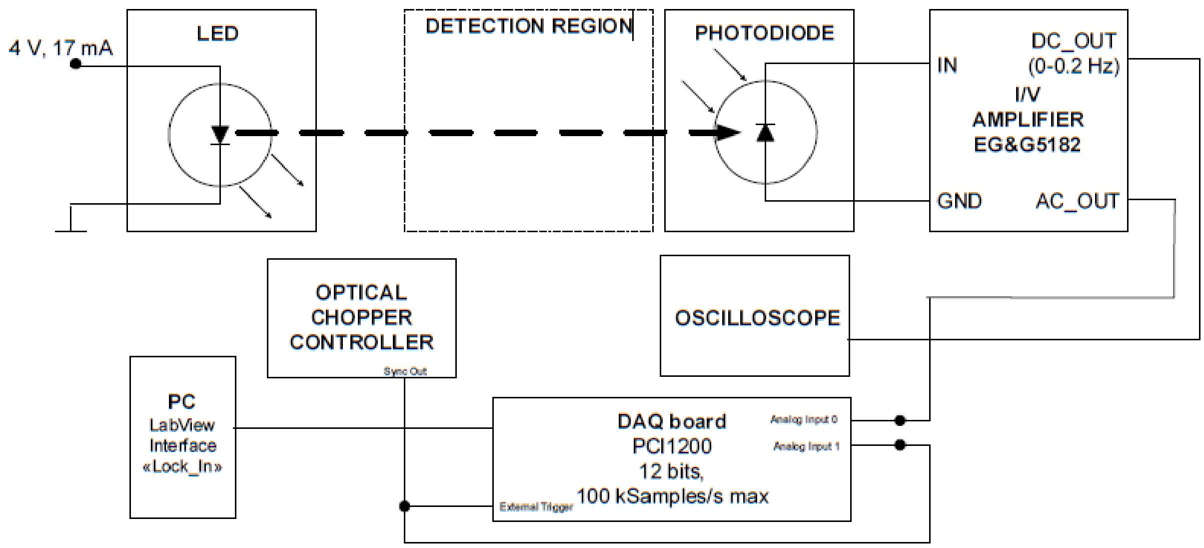

Figure 9 : ***Block diagram concerning the connection of the passage sensor***

Prepare the setup as described in Figure 9, and then simulate the entrance of a customer in a store by moving your hand through the light beam. Observe the signal variation on the oscilloscope. (Be careful not to increase the LED voltage more than ~4.5V to avoid breaking it)

Is the passage detectable? Which is the voltage change at the output of the transimpedance amplifier and which is the current variation in the photodiode upon the passage of a person? Switch off the lamp above the set-up and/or the room light, and measure again the signal variation. What happens? What is the sensitivity of the system with respect to the ambient light? Is the system suitable for a real application? Is it still meaningful outdoor (with the natural variations of the sunlight, cars' headlights, …..)?

The majority of those perturbations play a role only at low frequency, so finding out a way to use the photodiode at higher frequency will be crucial to make the system robust (count properly the number of persons crossing the barrier).
First of all, you have to measure the noise spectrum in order to identify the best possible frequency region where you should perform the modulation (i.e., the modulation frequency f_0). Plug the AC output of the transimpedance amplifier to the channel 0 of the digital acquisition board (Figure 9). In which regions can you observe the minimum level for noise (use the program Spectrum Analyser_2025.vi)? That will be crucial to determining the modulation frequency.

Prepare the setup as shown in Figure 10 (you can now put the chopper in place) and observe the signal behavior after the synchronous detection (use the program Lock In Amplifier_2025vi).

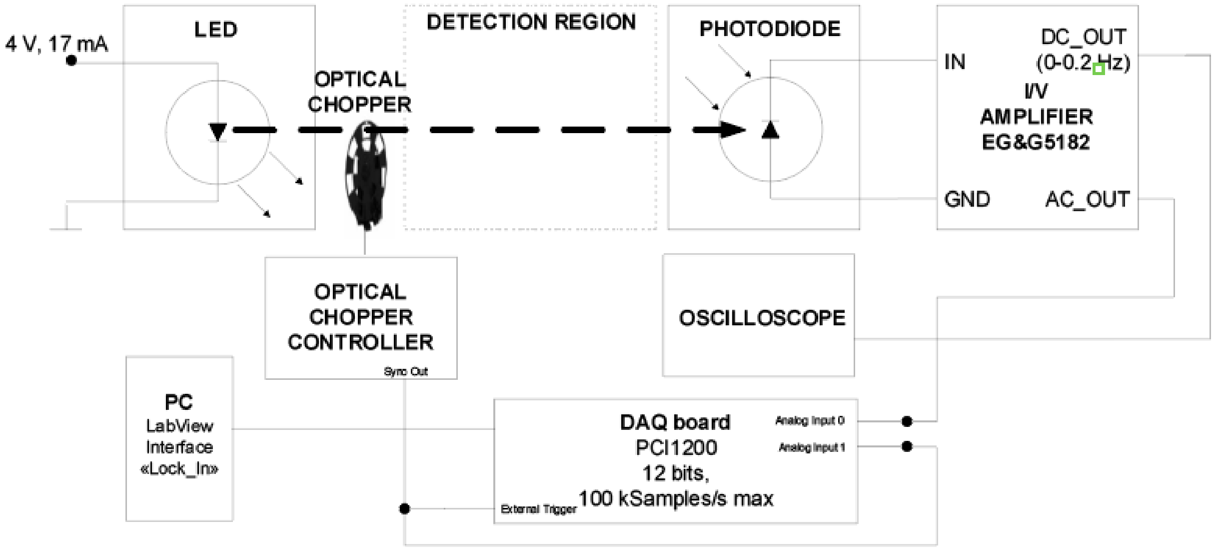

Figure 10 : ***Block diagram of the optical detector with a synchronous detection.***

1. By means of this method, is it possible to track the number of customers entering the supermarket without errors due to the ambient light variations?
   Firstly try with an AC gain of 10^8 V/A (gain DC equal to 10^7 V/A). After, try again with an AC gain of 10^7 V/A (gain DC equal to 10^6 V/A). Why can't you properly determine the passage of a person and the variation of the ambient light with a gain too high?
2. Which position of the chopper does it ensure a proper immunity to the maximum noise? (close to LED, close to the photodiode, in between, …)
3. We could also use a very narrow band-pass filter to isolate the frequency of interest. In this case, which is the drawback with respect to the use of a lock-in amplifier?
4. In the Labview program « Lock-In », you can adjust the acquisition time/time constant. Which is the best compromise for a maximum immunity to noise (long or short)? What about detecting rapid passages?
5. Why does the lock-in multiply the measured signal *s(t)* with both a cosine and a sine (not just a single one)? Which problem may eventually arise if only one of the two is used?
6. We have studied the synchronous detection in the case of an optical detector. Is it also suitable for other kind of sensors? Provide some examples.

We will now move forward by measuring the shot noise of a photodiode. In order to arrange the setup, follow the block diagram in Figure 11 (you can now put the cardboard box on top of the LED/photodiode to block all light except the LED's).

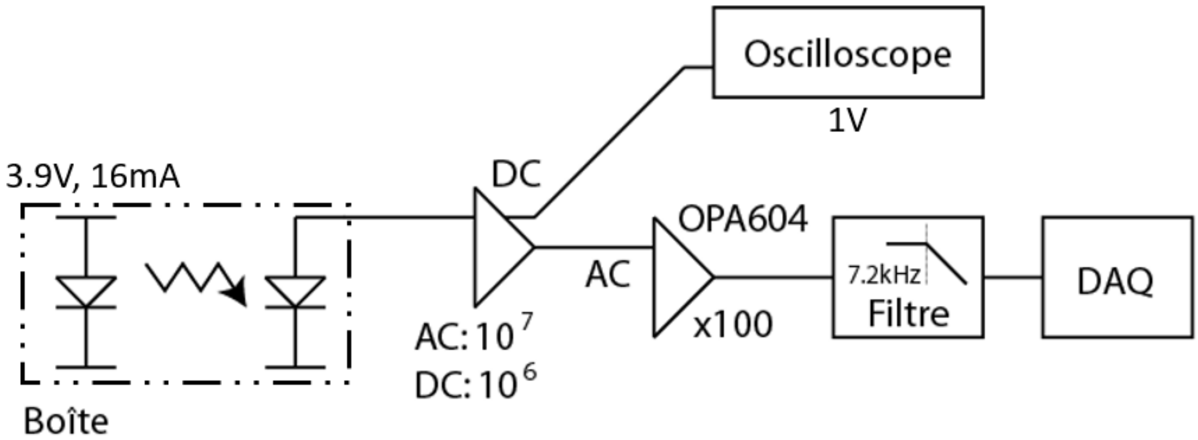

Figure 11 : ***Block diagram to show the setup for the measurement of the shot noise.***

1. Compare the value of the measured shot noise with the theoretical one.
2. Decrease I_{DC} as seen on the oscilloscope by a factor 2 by lowering the LED's voltage and verify that the shot noise goes down by a factor $\sqrt{2}$.
3. Measure the noise of the amplifier EG&G 5182 (open circuit), and then check whether it corresponds to the value provided by the manufacturer.
   (Datasheet easily available on the Internet).

Notes

All your suggestions to improve the TP guidelines are welcome.

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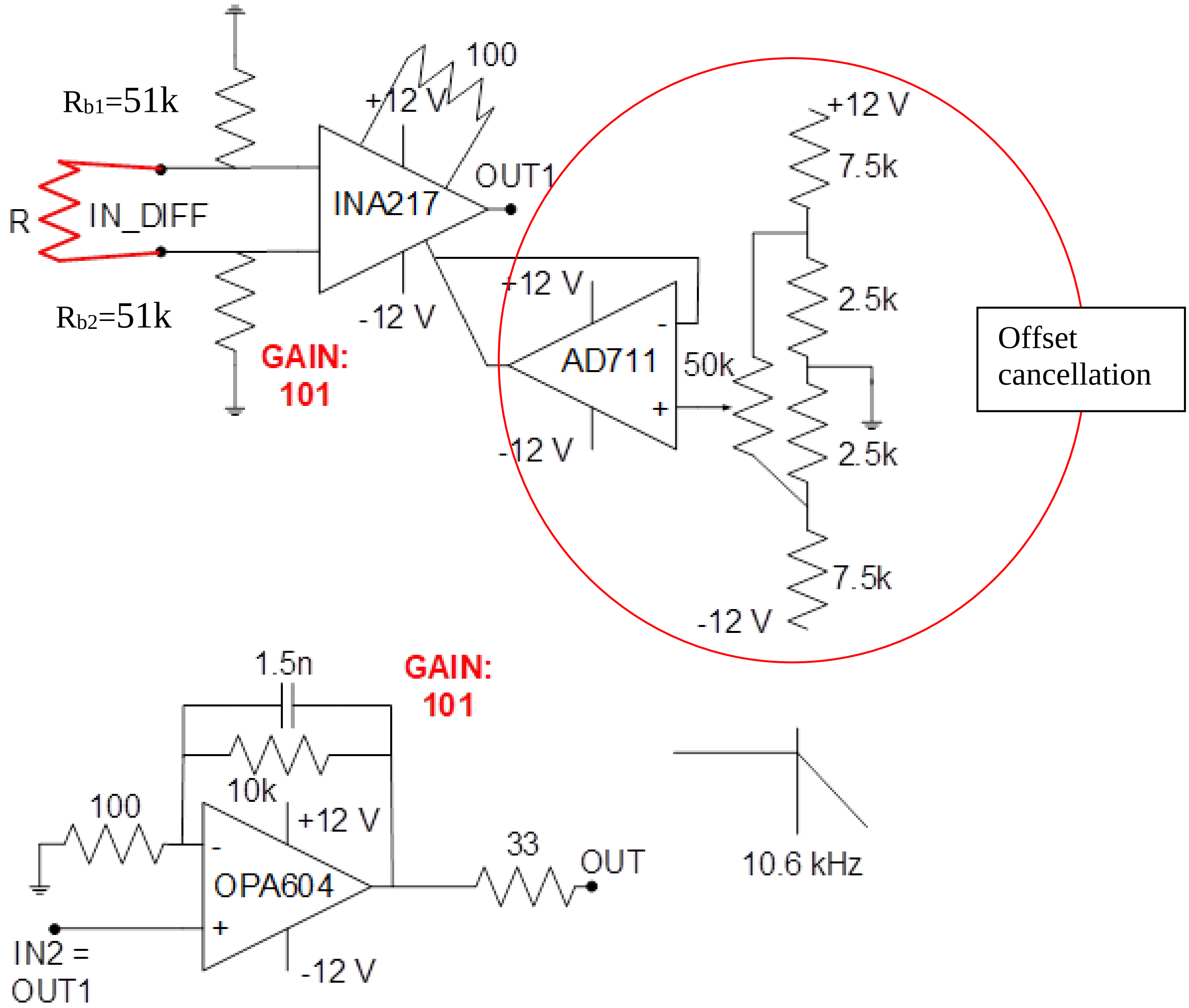
5. Appendix

5.1.Access the programs:

In the desktop:

1. Access from Labview the programs in the folder TP_NOISE
2. Use the following programs:
   - Simple Digital Multimeter_2025.vi
   - Spectrum Analyser_2025.vi
   - Lock In Amplifier_2025.vi

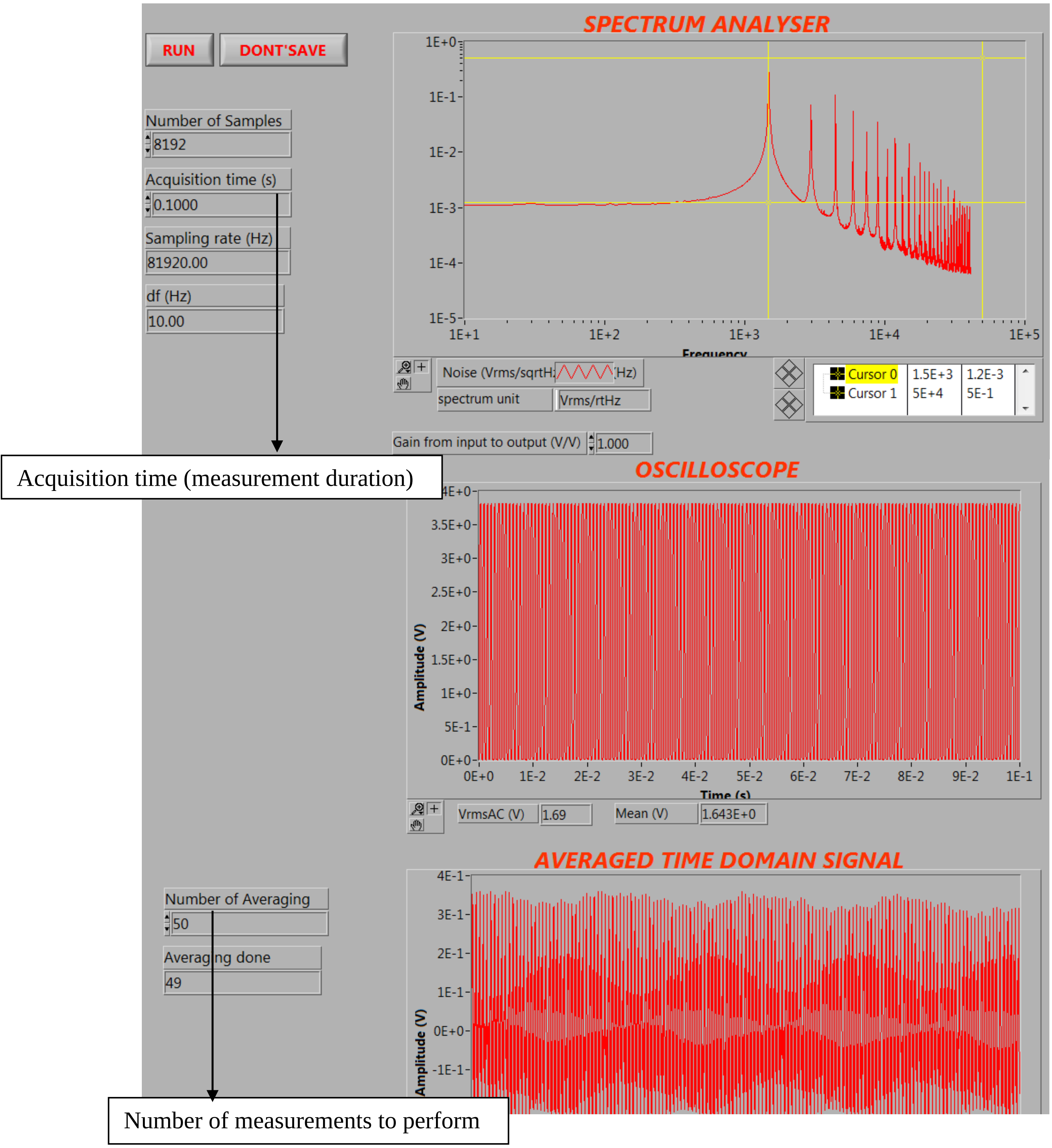
5.2. Detailed schematic concerning the electrical circuits

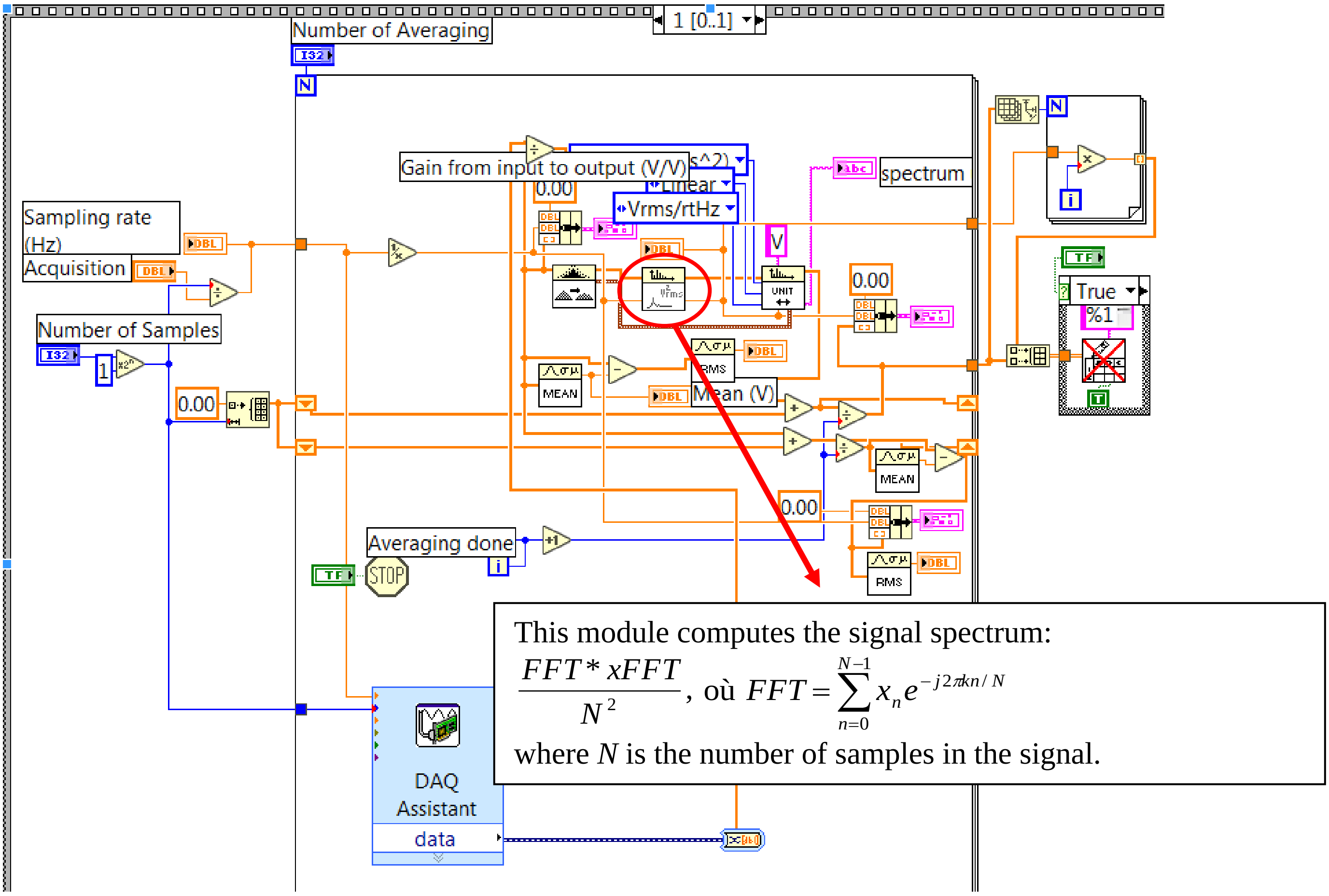


5.3. Labview programs

Warning: Don't edit Labview programs! Always use in "Run Mode" (Operate → Change to Run Mode).

5.3.1. Spectrum Analyser_2025.vi



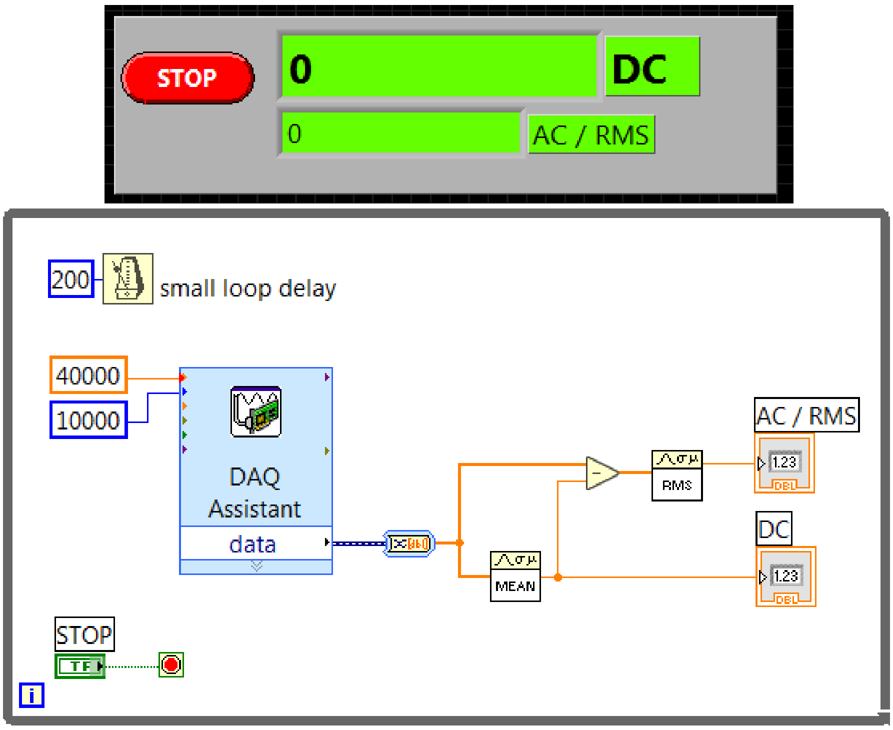


This module computes the signal spectrum:

$$\frac{FFT^* xFFT}{N^2}, \text{ où } FFT = \sum_{n=0}^{N-1} x_n e^{-j2\pi kn/N}$$

where N is the number of samples in the signal.

5.3.2. Simple Digital Multimeter_2025.vi



5.3.3. Lock In Amplifier_2025.vi