Session W1- 23.02.2024, Take home messages

Applications of springs

- Preconstraint backlash (in gears, screws, rack pinions, transmissions in general)
  - Pay attention to the additional effort that has to be compensated by the actuator.
  - Higher is the pretension, the higher is the friction.

- Adjust referencing
- Correct alignment
- Gravity compensation
- Energy storage: can be used as passive sources of effort or put in series with actuators: series elastic actuators

REHAssist

Springs and elasticity.

Part 2- Rigid bodies versus elastic bodies

Rigid bodies versus Elastic bodies

A solid body has:
- **A mass -**
- **A moment of inertia -**
- **A stiffness /rigidity-**

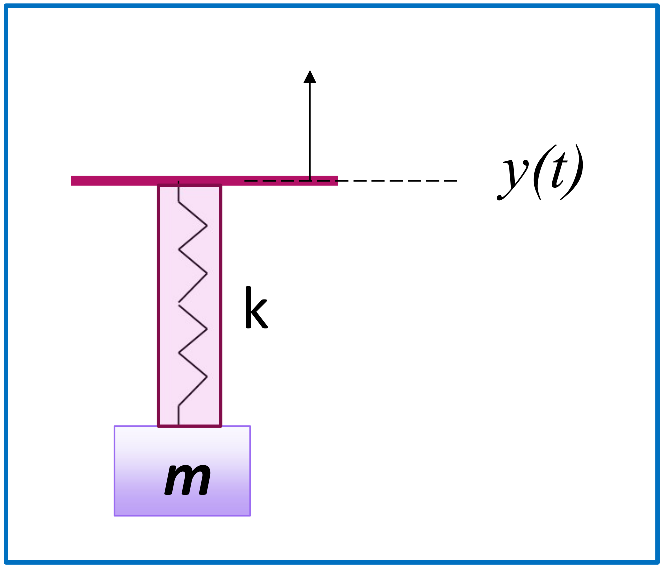
Rigid bodies are never infinitely rigid. This point out **stiffness in different directions**.
- A stiffness is associated with a vibration mode in ifferent directions. These are **eigenmodes**/**eigenfrequencies**.
- The 3 basic modes are bending, pulling and twisting.
- Other secondary modes can be considered according to the shape of the solid body.

Spring-mass model

The basic model of a mass and a spring is thus appropriate

>>> To understand the effects of the stiffness of an element and the type of movement (the trajectory):

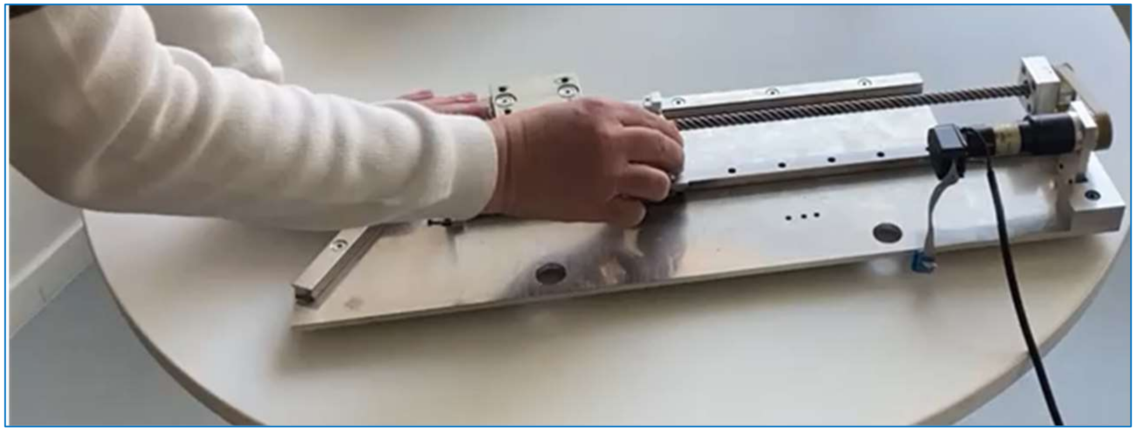
- **On the eigen frequency,**
- **On the dynamic error**
- ¨ …



Rigid bodies versus Elastic bodies

Consider the following screw based robot axis.

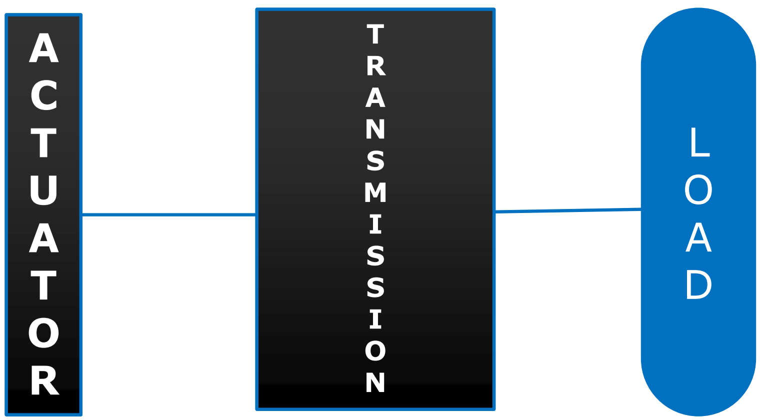
What are the components of this robot axis ?



What are the components of a robot axis?

A robot axis is composed as follows:

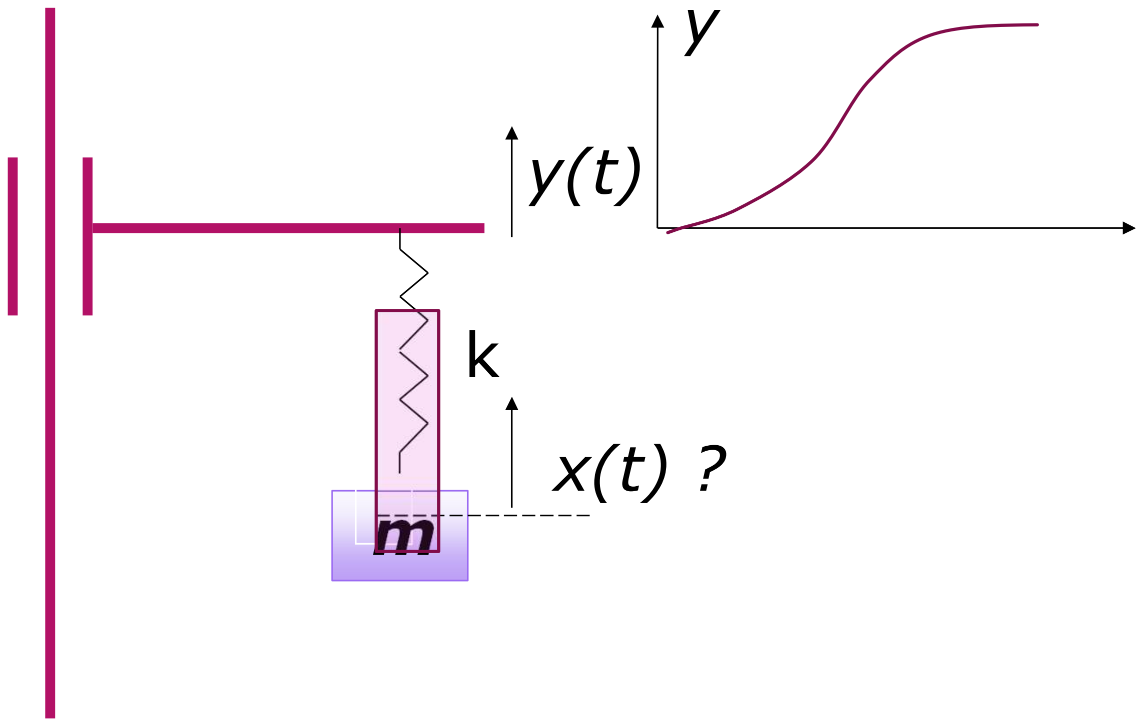
- An actuator that provides the effort to move.
- A transmission
- And a load to be displaced.



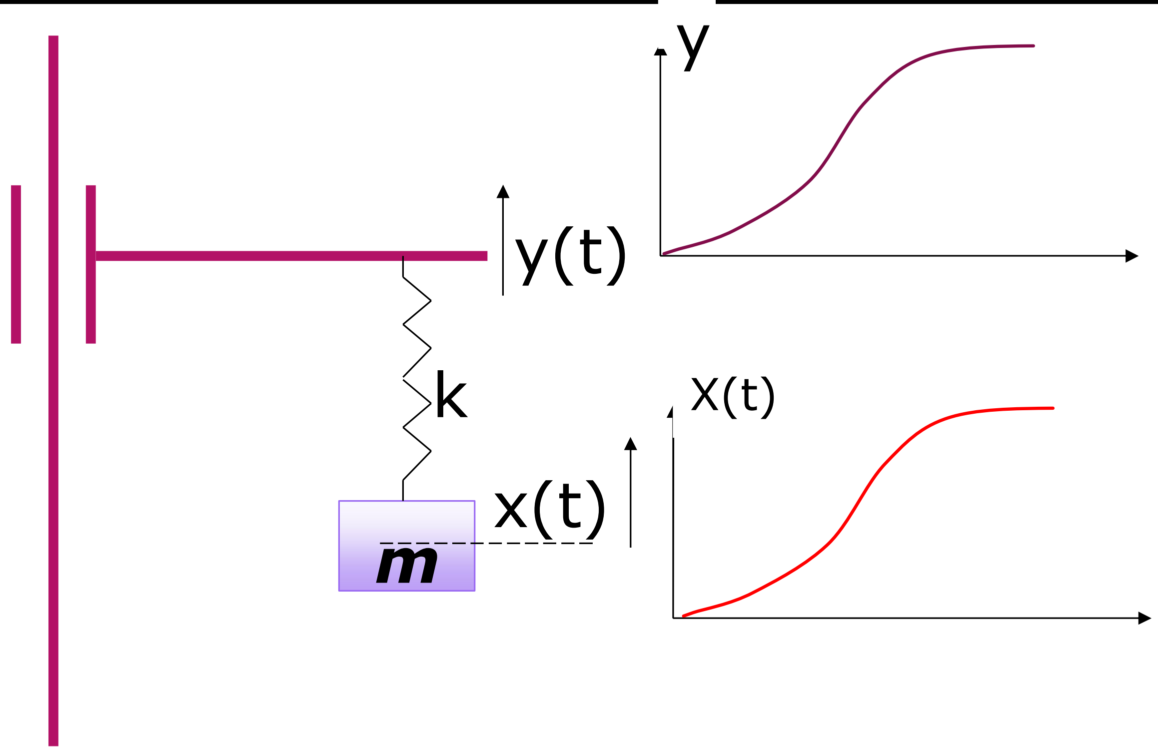
Given the law of motion y(t) of the actuator/motor, what is the law of motion of the load?

Objectif:

Displace the robotic arm input with the law of motion **y(t)** and **deduce the law of motion of the output x(t)**

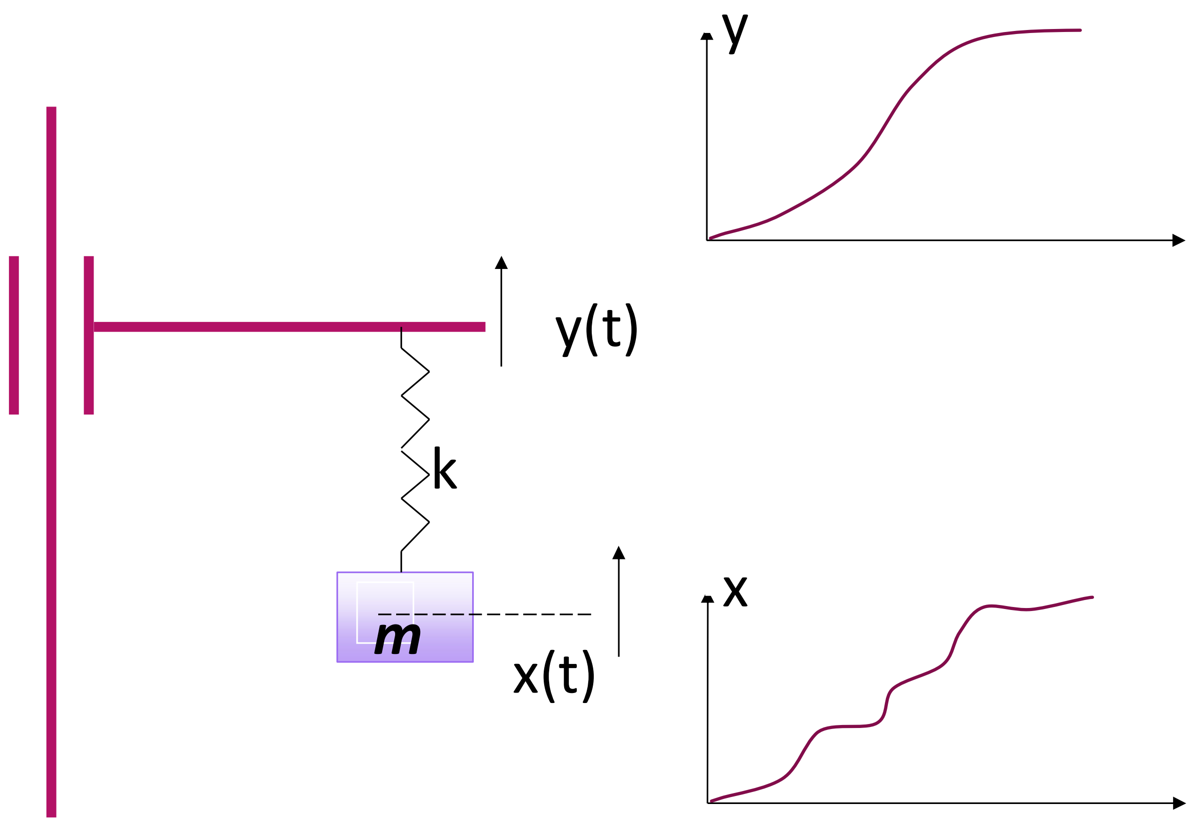


Case 1: consider that the intermediate transmission is infinitely rigid. k is infinite.



if the transmission is infinitely rigid,	the movement of the load m (output) **is the same** as at the actuator side

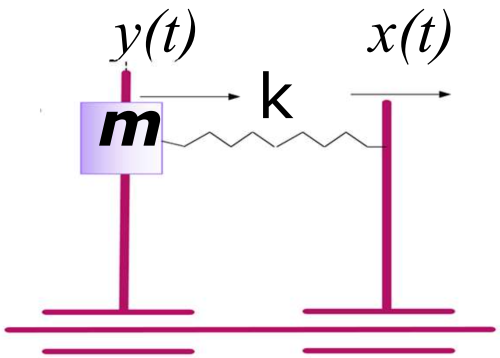
Case 2: the intermediate transmission is not infinitely rigid. k is not infinite.



Exercise:

Simulate the output of a spring mass system.

- Try different trajectories.
- Try different stiffnesses



Analytic solution: Law of motion y versus x

$\ddot{x} + \omega_0^2 x = \omega_0^2 y(t)$ avec $\omega_0^2 = \frac{K}{m}$

General solution of the equation without the second member :

> $\boldsymbol{x = C_1 cos\omega_0 t + C_2 sin\omega_0 t}$

Which position profile is better?

h is the distance to travel.

T is the travel duration.

	Law		Movement profile	Displacement law	Speed shape	Speed ($\dot{y}$)	Speed Factor	Acceleration form	Acceleration ($\ddot{y}$)	Acceleration Factor	Acceleration
1	Slope			$y = \frac{h}{T} t$		$\dot{y} = \frac{h}{T}$	1		$t = 0, \ddot{y} = \infty$ T $t = 0, \ddot{y} = 0$ T	∞	
2	Parabolic or constant acceleration	$\frac{t}{T} \leq 0.5$		$y = 2h(\frac{t}{T})^2$		$\dot{y} = \frac{4ht}{T^2}$	2		$\ddot{y} = \frac{4h}{T^2}$	1	
		$\frac{t}{T} \geq 0.5$		$y = h\left[1 - 2(1 - \frac{t}{T})^2\right]$		$\dot{y} = \frac{4h}{T}(1 - \frac{t}{T})$			$\ddot{y} = -\frac{4h}{T^2}$		
3	Cubic or linear acceleration	$\frac{t}{T} \leq 0.5$		$y = 4(\frac{t}{T})^3 h$		$\dot{y} = \frac{12h}{T}\left(\frac{t}{T}\right)^2$	3		$\ddot{y} = \frac{24h}{T^2}(\frac{t}{T})$	3	
		$\frac{t}{T} \geq 0.5$		$y = h\left[1 - 4(1 - \frac{t}{T})^3\right]$		$\dot{y} = \frac{12h}{T}\left[1 - (\frac{t}{T})^2\right]$			$\ddot{y} = \frac{24h}{T^2}(1 - \frac{t}{T})$		
4	Rectilinear acceleration			$6h\left[\frac{1}{2}(\frac{t}{T})^2 - \frac{1}{3}(\frac{t}{T})^3\right]$		$\dot{y} = 6h\left[\frac{t}{T^2} - \frac{t^2}{T^3}\right]$	1.5		$\ddot{y} = 6h(\frac{1}{T^2} - \frac{2}{T^2}\frac{t}{T})$	1.5	
5	Sinusoid			$y = \frac{h}{2}\left[1 - cos\ cos\ \frac{\pi t}{T}\right]$		$\dot{y} = \frac{\pi h}{2T} sin\ sin\ \frac{\pi t}{T}$	1.57		$\ddot{y} = \frac{\pi^2 h}{2T^2} cos\ cos\ \frac{\pi t}{T}$	1.234	
6	Cycloid			$y = \frac{h}{\pi}(\frac{\pi t}{T} - \frac{1}{2} sin\ sin\ \frac{2\pi t}{T})$		$\dot{y} = \frac{h}{T}(1 - cos\ cos\ \frac{2\pi t}{T})$	2		$\ddot{y} = \frac{\pi^2 h}{T^2} sin\ sin\ \frac{2\pi t}{T}$	1.57	
7	Trapezoidal Acceleration	$t < \frac{T}{8}$		$y = \frac{64}{9} h(\frac{t}{T})^3$		$\dot{y} = \frac{64h}{3T}(\frac{t}{T})^2$	2		$\ddot{y} = \frac{128}{3}\frac{h}{T^2}\frac{t}{T}$	1.33	

REHAssist

What is the law of the movement at the load (output) **x(t)**?

1-What happens with a movement @constant accelerations?

h is the distance to travel.

T is the travel duration.

$$a = \pm \frac{4.h}{T^2}$$

The position error $\Delta = x - y$

$$\Delta = \frac{a}{\omega_o^2}\left[1+\cos(\omega_o \cdot t) - 2\cdot\frac{\omega_o\cdot\cos(\omega_o\cdot\frac{T}{2})\cdot\cos(\omega_o\cdot t)+\sin(\omega_o\cdot\frac{T}{2})\cdot\sin(\omega_o\cdot t)}{\omega_o\cdot\cos^2(\omega_o\cdot\frac{T}{2})+\sin^2(\omega_o\cdot\frac{T}{2})}\right]$$

$$\gg \Delta_{max} = \frac{4.a}{\omega_0^2}$$

By inverting the problem, the necessary eigen pulsation (of the Arm+transmission) which allows to have a error lower than Δ_{max} is given by

$$\omega_0 = 2.\sqrt{\frac{a}{\Delta_{max}}}$$

$\omega_0 = 2.\pi.f_0$ Soit $f_0 = \frac{\omega_0}{2.\pi}$

$$f_0 = \frac{1}{\pi}\sqrt{\frac{a}{\Delta_{max}}} \gg f_0 = \frac{2}{\pi.T}\cdot\sqrt{\frac{h}{\Delta_{max}}}$$

$$f_0 \approx \frac{0.64}{T}\sqrt{\frac{h}{\Delta_{max}}}$$

REHAssist

2-What happens with a movement @sinusoidal accelerations?

The position error is given by:

$$\Delta = \sin\left(\frac{2\pi t}{T}\right)\left(\frac{h}{2\pi} + \frac{\omega_0^2\, h\, T^2}{2\pi(4\pi^2 - \omega_0^2\, T^2)}\right) + \sin(\omega_0 t)\left(\frac{\omega_0 h\, T}{\omega_0^2 T^2 - 4\pi^2} - \frac{h}{T\omega_0}\right)$$

Si $f_0 T = 1 \;\Rightarrow\; \Delta = \frac{h}{2}$

Si $f_0 T > 3 \Rightarrow$ $f_0 T$ May be neglecte $(f_0 T)^3$ error $<12.5\%$.

We obtain:

$$\Delta_{max} \cong \frac{h}{2\pi}\frac{1}{f_0^3\, T^3}$$

By inverting the problem, the necessary eigen pulsation (of the Arm + transmission) which avoids to have an error highest than Δmax is given by

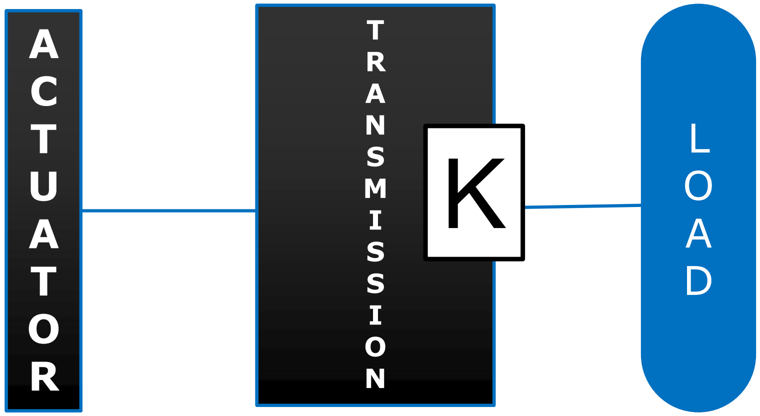
$$f_0 = \frac{1}{T}\cdot\sqrt[3]{\frac{h}{2\,\pi\,\Delta_{max}}}$$

$$f_0 \approx \frac{0.54}{T}\sqrt[3]{\frac{h}{\Delta_{max}}}$$

What are the components of a robot axis?

A robot axis is composed as follows:

- An actuator that provides the effort to move.
- A transmission
- And a load to be displaced.



Given the law of motion y(t) of the actuator/motor, what is the law of motion of the load?

Comparison of the two profiles:

1) Profile @constant acceleration

$$f_0 \approx \frac{0.64}{T}\sqrt{\frac{h}{\Delta_{max}}}$$

2) Profile @sinusoidal acceleration

$$f_0 \approx \frac{0.54}{T}\sqrt[3]{\frac{h}{\Delta_{max}}}$$

Note:

To realize a given path with a specification of the minimum error,

>>> Stiffer is the acceleration profile, stiffer the mechanics has to be.

Examples:

1) h=1m

T=1s

Δ=0.1 mm = 10^{-4} m

Movement @constant acceleration: $\boldsymbol{f_{min} = 64\ Hz}$

Movement @sinusoïdal acceleration : $\boldsymbol{f_{min} = 12\ Hz}$

2) h=0.15m

T=0.1s pégase

Δ=0.5 mm = 0.5 10^{-3} m

Movement @constant acceleration: $\boldsymbol{f_{min} = 110\ Hz}$

Movement @sinusoïdal acceleration : $\boldsymbol{f_{min} = 36\ Hz}$

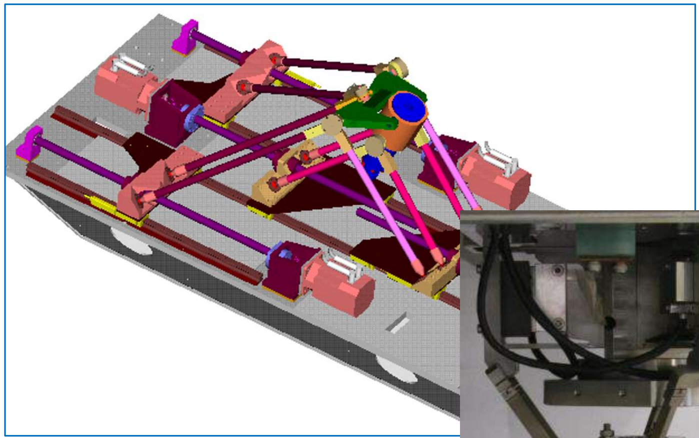
Examples:

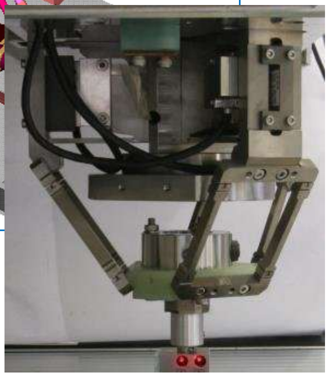
REHAssist

- The eigen-frequency is one of the most important specifications of a robot design
- Higher is the eigenfrequency better is the structure (less vibrations)
- Higher is the eigenfrequency , stronger are the constraints on the mechanical design

< 50 hz

> 75 hz





Ideally > 100 hz

Observations :

- The damping has not been considered⇒ values f_min are therefore too high

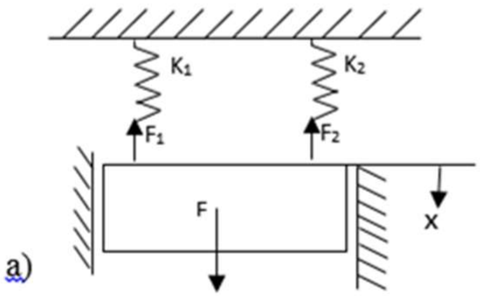
- Case motion with constant acceleration, The Bang-Bang@$T/2$, with no constant velocity between acceleration and deceleration, represents the worst case. The value f_min may then be reduced.

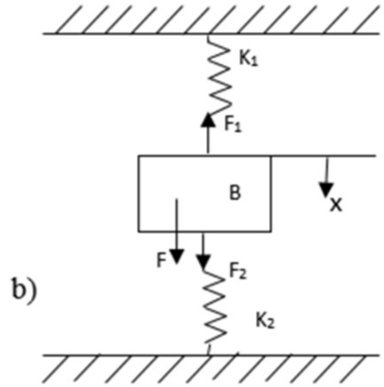
- The model of multi-axes robots is treated in the next section !

What happens in the case of a multi axes robot ?

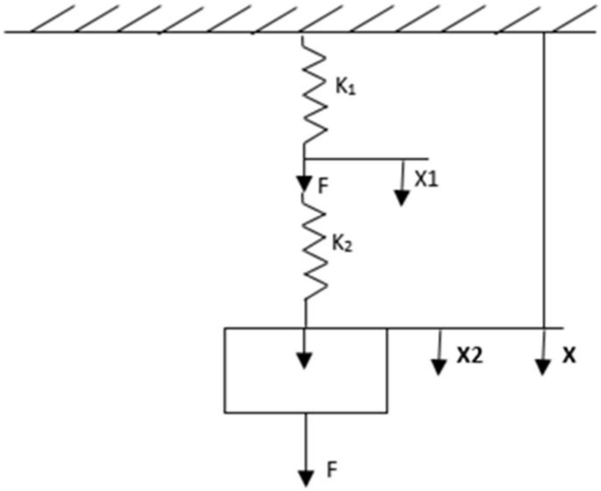
- Is the spring mass model still valid?
- How is the natural frequency of the robot calculated?
- How is the rigidity of the robot calculated?

Multibody equivalent rigidities





En parallèle : $K = K_1 + K_2$



En série :

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \Rightarrow \quad k = \frac{k_1\, k_2}{k_1 + k_2}$$

A Delta robot, for fast (pick and place)

August 2019
0,4mm RMS_error @30 Ge acceleration
pick_and_place

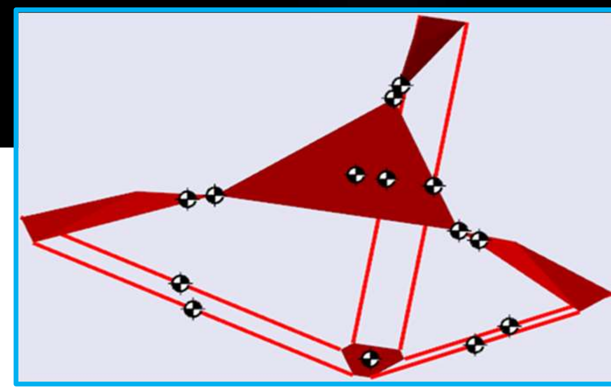
[Patent 2012] Device For Moving And Positioning An Object In Space, Huser M., Tschudi M., Keiffer D., Teklits A., Bouri M., Clavel R., Demaurex MO., Device For Moving And Positioning An Object In Space, reference WO2012152559

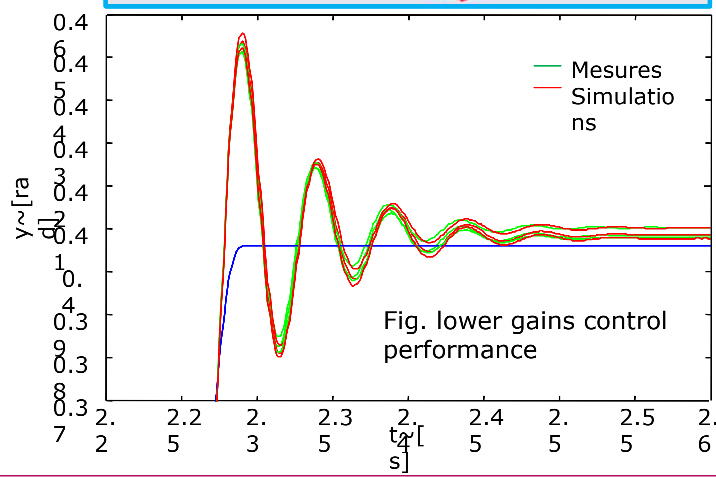
Case of a robot
Closed loop control behaviour

REHAssist

<u>**First simulation model**</u>

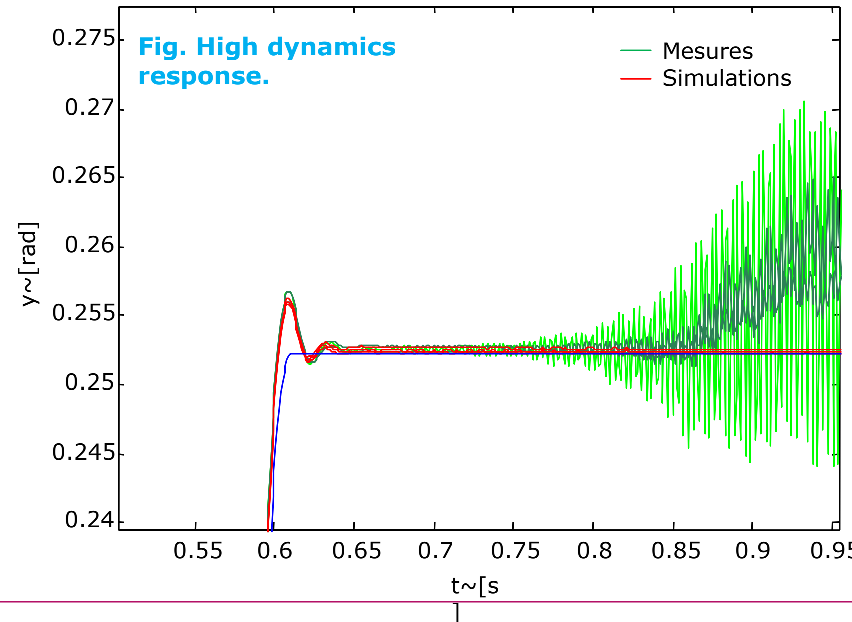
- **Rigid bodies only**
- Integration of the control and the mechanics
- Facilitates the creation and evaluation of new control laws
- Very accurate modeling
- (error <0.3 ° even on dynamic trajectories)





Case of a robot
Closed loop control behaviour
higher control gains

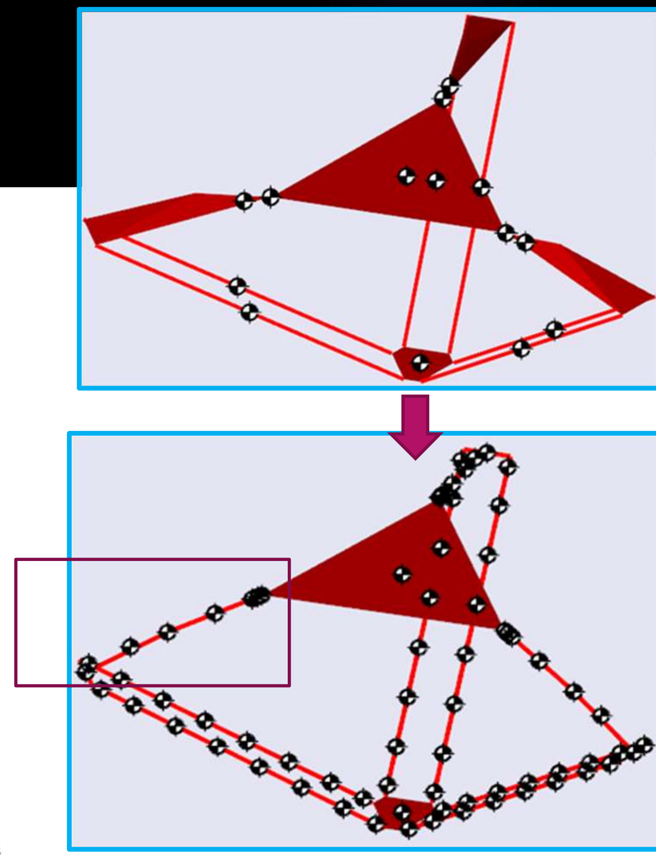
- The rigid-body model is no longer sufficient to take into account the excitation of mechanical vibrating modes.
- Need to take into account the rigidity of parts

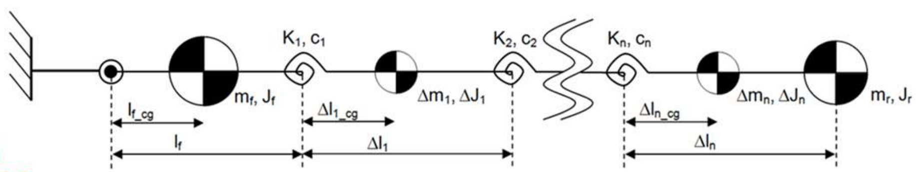


Modification of the rigid bodies' model by considering the eigen-modes

REHAssist

- Discretization of the parts
- Assembly of rigid segments with elastic joints
- Using CAD software to identify the main modes

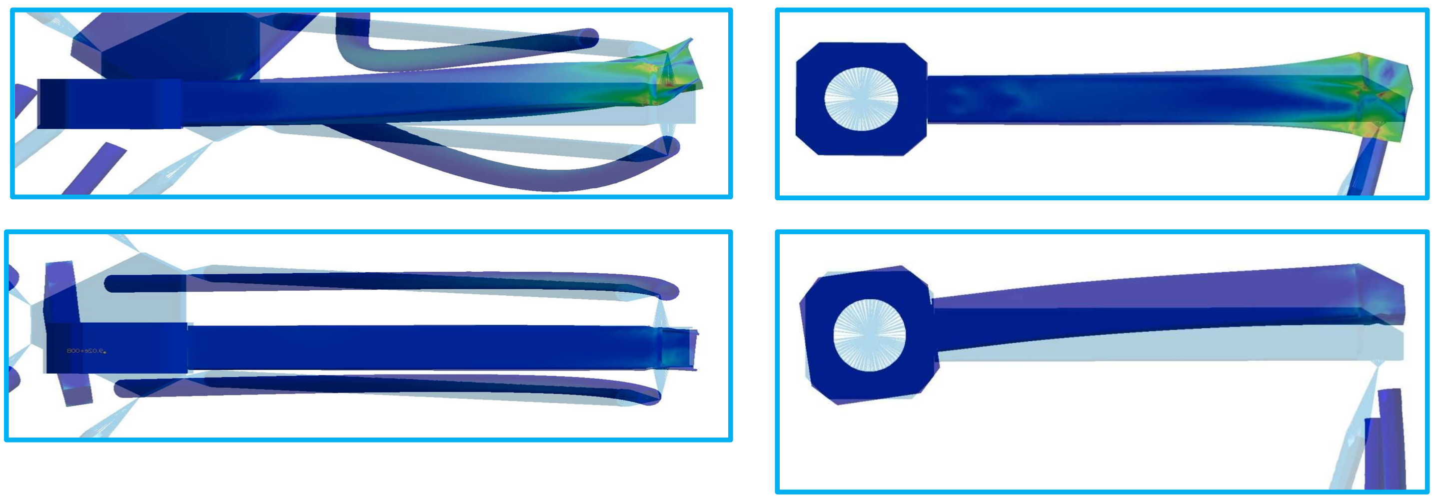




Modification of the rigid bodies' model by considering the eigen-modes

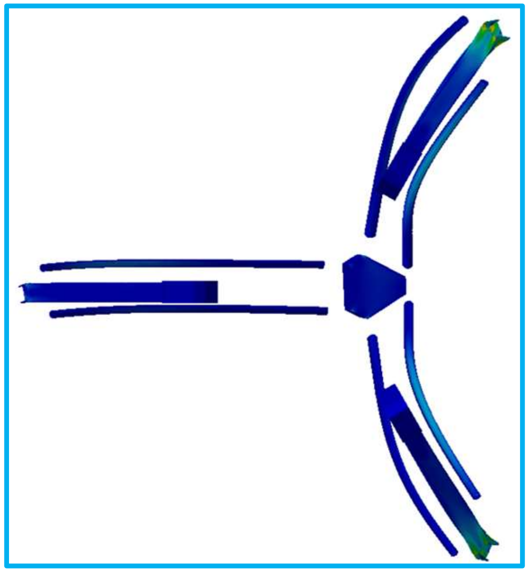
REHAssist

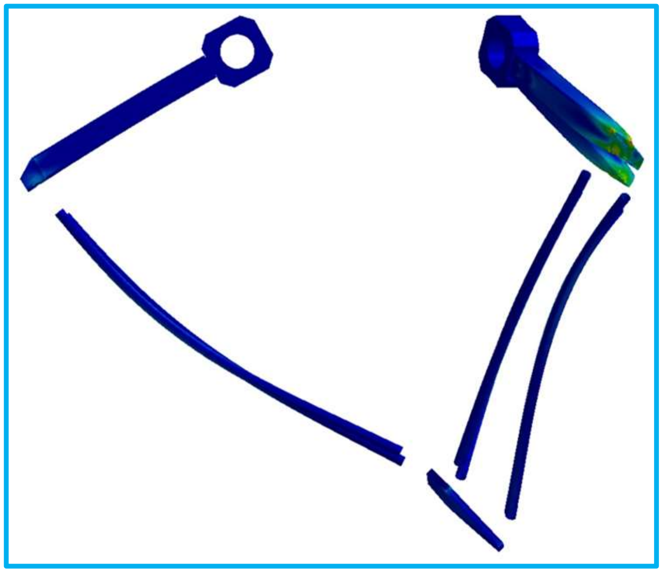
Flexion Mode of the rear arm
(400~450[Hz]



Modification of the rigid bodies' model by considering the eigen-modes

Combined tilting Mode of the mobile plate (end effector)
(280~300 [Hz])

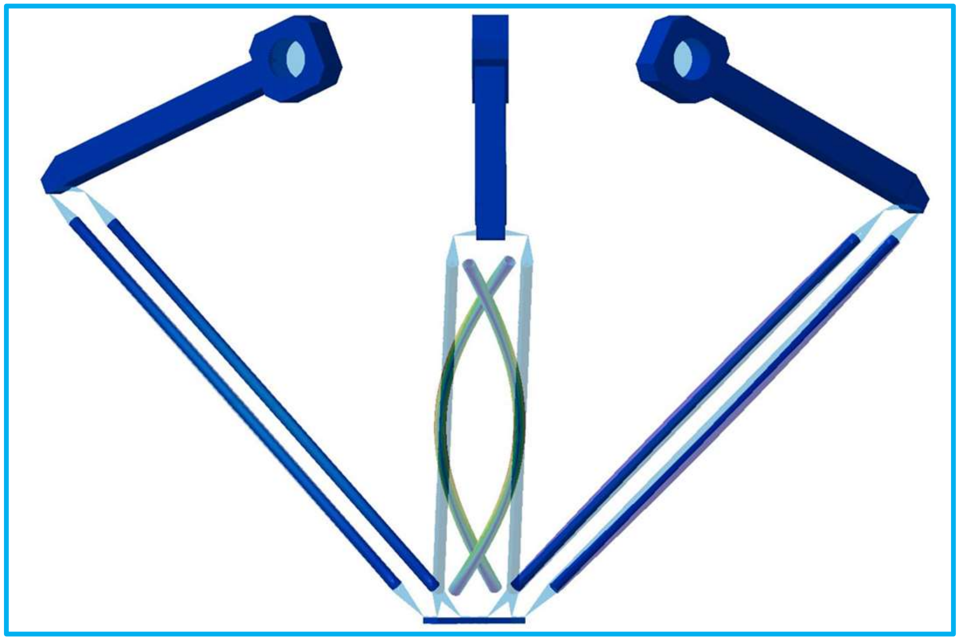




Modification of the rigid bodies' model by considering the eigen-modes

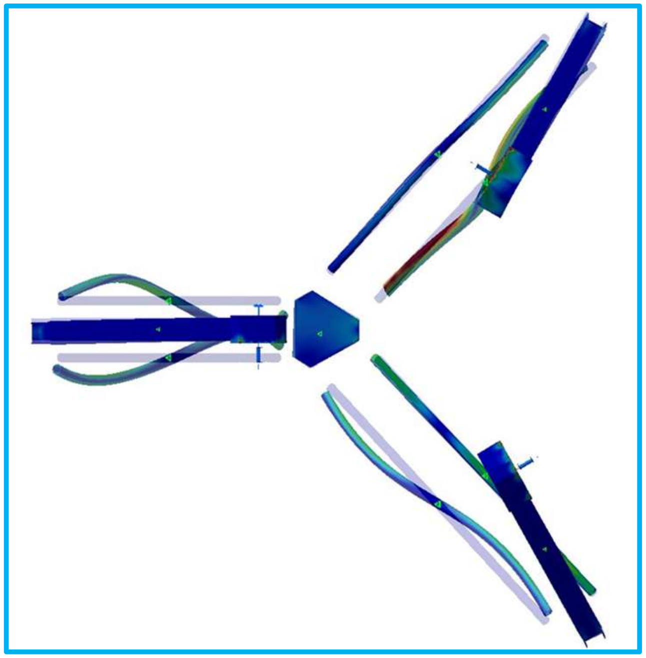
REHAssist

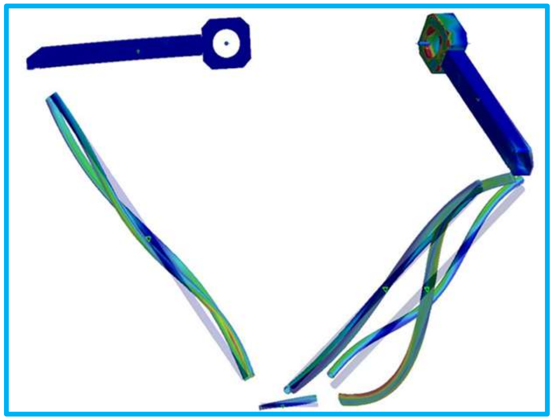
1st mode / Flexion mode of the carbon forearms 120-150 Hz



Modification of the rigid bodies' model by considering the eigen-modes

2nd mode of flexion / 170-200Hz





Anti-Resonance solutions

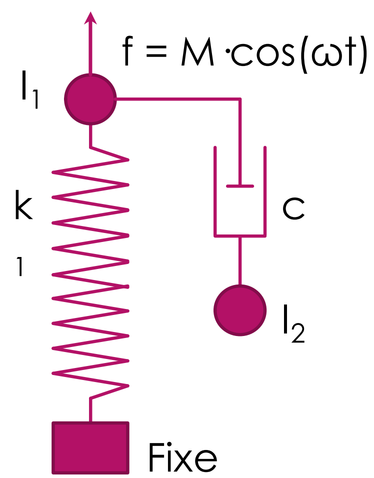
To avoid excitation of the resonant modes, we should :

- Think about the problem as best as we can **when designing the robot** (Acting on the materials and shapes of the mechanical parts to increase the eigen frequencies). The simulation of the entire robot is a very good approach before construction.
- Develop **appropriate control strategies** that bypass excitation of resonant modes.
- Use **anti resonant active filters**: signal filters often used with the most noisy speed signal (this signal reinjected through the control avoids excitation of the eigen modes)
- Use mechanical anti-resonance filters (dampers / shock absorbers)

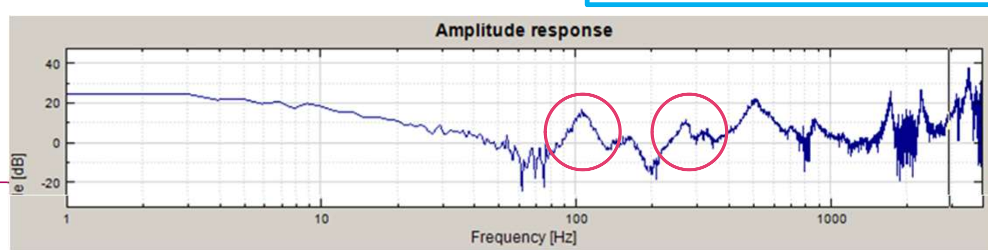
REHAssist

Mechanical Damping on the Robot Arm

Lanchester damper :

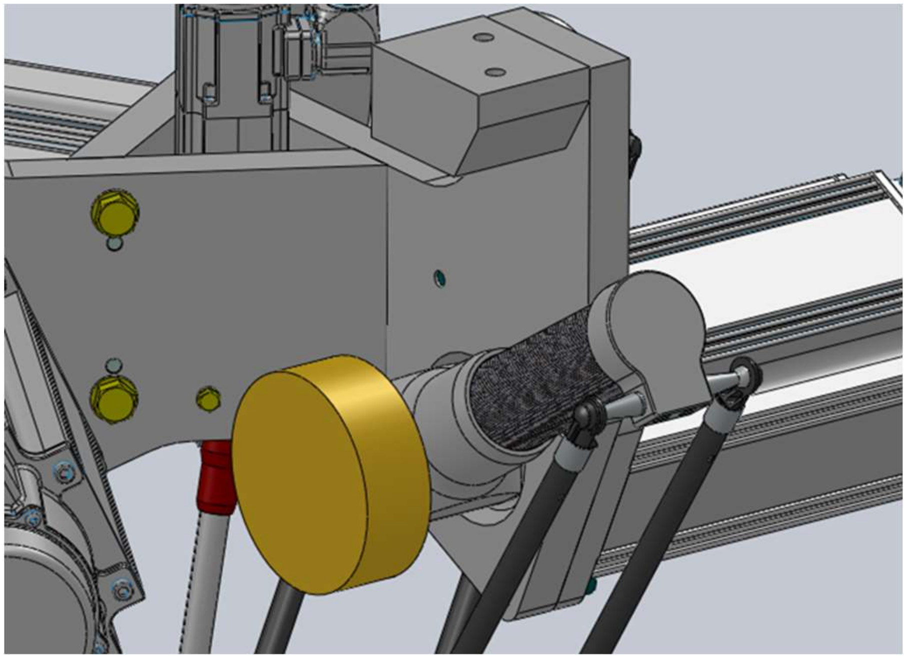


- The ratio of the inertias defines the damping of the resonant frequency.
- The viscous damping / friction coefficient 'c' is defined by the damping frequency.
- In our case, several tests should be performed to determine the damping of the required pulsation frequency.



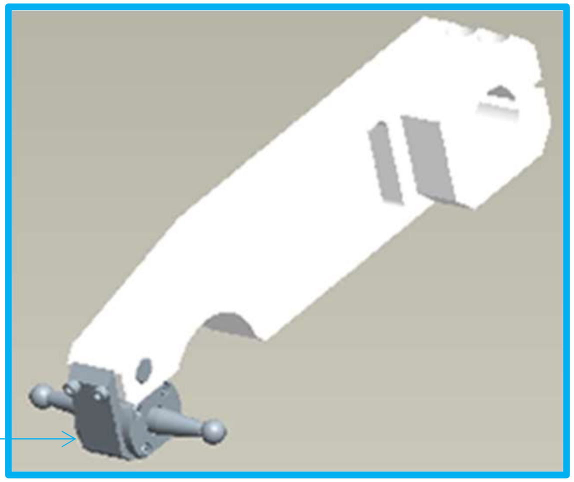
Mechanical Damping on the Robot Arm

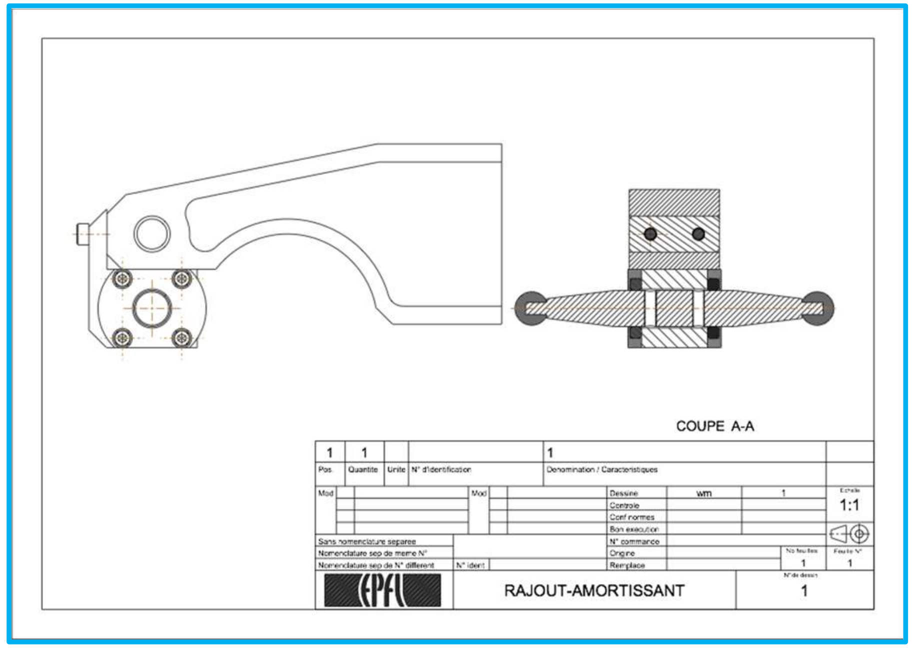
The damper adds half of the total inertia (Motor + Rear Arm) (Ø115mm, H40mm)



REHAssist

Mechanical Damping using rubber rings

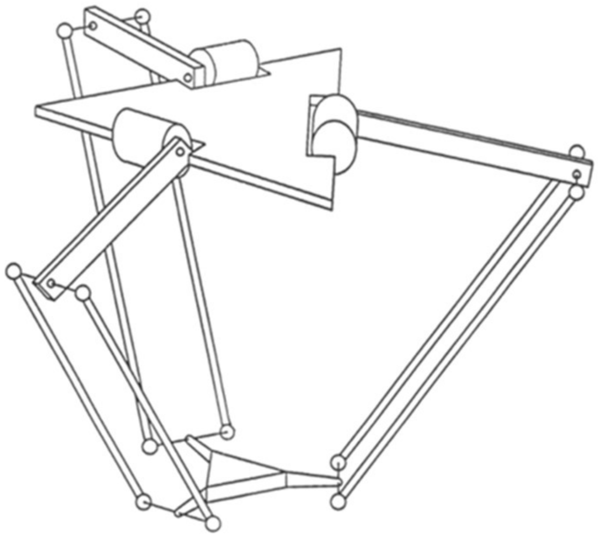




It is possible to use rubber rings.

Moment of Inertias Versus Masses

REHAssist



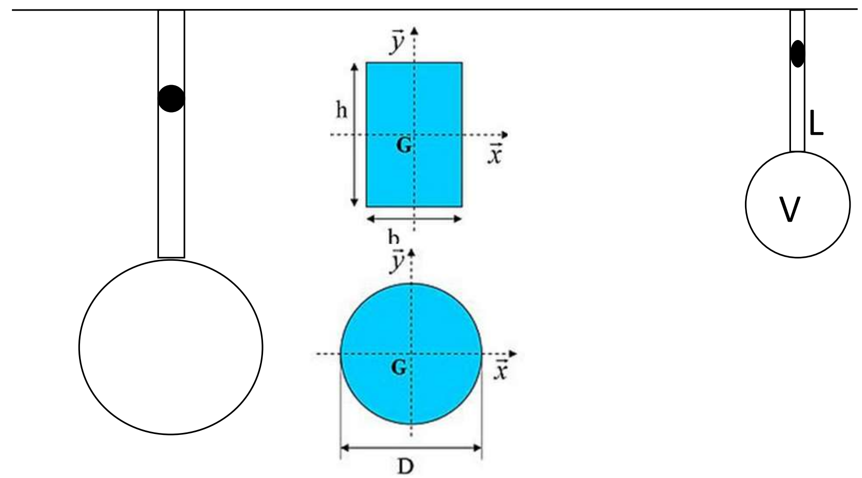
		Masse	Inertie
Moteur		**17.8 kg**	**45.8 10-4 kg*m^2**
Bras	Fix arbre	0.555 kg	11.21 10-4 kg*m^2
	Tube	0.115 kg	17 10-4 kg*m^2
	Fix rotules	0.078 kg	40.15 10-4 kg*m^2
	Rotules	0.024 kg	14.47 10-4 kg*m^2
	Total	**0.772 kg**	**82.83 10-4 kg*m^2**
Avant-bras	(paire)	**0.185 kg**	-
Nacelle	Rb40, base	**0.047 kg**	-
Charge		**0.5 kg**	-

Similarity laws
Big versus **Small**

Consider two solid bodies:
- Of **identical shapes**.
- Of **identical** materials.
- With **a scale factor α** between the two.

S is the section,
V, is the volume,
I, Inertia,
L, thelength

X* corresponds to the homothetic ratios between the variables
L* is the homothetic ratio



Dimensionnal equalities

$S^* = L^{*2}$

$I^* = L^{*4}$ **Examples** : $I_x = \dfrac{\pi d^4}{64}$ *ou* $\dfrac{bh^3}{12}$

$I_G^* = L^{*4}$ $I_G = \dfrac{\pi d^4}{32}$ ou

$V^* = L^{*3}$

$M^* = L^{*3}$

$J^* = L^{*5} = \sum M_i\, r_i^2$

Traction

$$\Delta l = \frac{F}{E.S} l \Rightarrow \frac{F}{\Delta l} = \frac{E.S}{l}$$

$$K_t^* = \frac{L^{*2}}{L^*} (traction) = L^*$$

Flexion

$$f = \frac{Fl^3}{3EI} \Rightarrow \frac{F}{f} = \frac{3EI}{l^3}$$

$$K_p^* = \frac{L^{*4}}{L^{*3}} (flexion) = L^*$$

Torsion

$$\theta = \frac{M.l}{GI_p} \Rightarrow \frac{M}{\theta} = \frac{GI_p}{l}$$

$$K_\theta^* = \frac{L^{*4}}{L^*} (torsion) = L^{*3}$$

Eigen frequency ratios

$$f = \sqrt{\frac{k}{Masse}} \quad f = \sqrt{\frac{k}{Inertie}}$$

$$f_t^* = \left(\frac{L^*}{L^{*3}}\right)^{1/2} = \frac{1}{L^*} \qquad f_f^* = \left(\frac{L^*}{L^{*3}}\right)^{1/2} \qquad f_\theta^* = \left(\frac{L^{*3}}{L^{*5}}\right)^{1/2} = \frac{1}{L^*}$$

- **The quality of the robot depends on its frequential behaviour**
- **The smaller the robot is, the higher are its eigen frequencies**

Quality factor

- The quality of a robot depends on its frequency behavior.
- **The smaller the robot is, the higher are its eigen frequencies**

OK

Question:
How to compare two robots **independently to their sizes** ?

Factor of Quality introduced in the PhD thesis of Marc Olivier Demaurex [EPFL 1978]

The quality factor which is desired independent of the size of the Industrial Robot (IR) is defined as follows (***f*- eigen frequency and** ℓ ***is the extent of the IR***):

$$Q = f \cdot \ell \qquad \text{(m/s)}$$

$$\Rightarrow \quad Q^* = f^* \cdot \ell^* = \frac{1}{L^*} \cdot L^* = 1$$

- A **high quality factor** will allow **good dynamic repeatability** of the position.
- Achieving a high quality factor will require special attention during the design and will generally result in a higher manufacturing cost.

Relation Q - Δ

REHAssist

As a first estimation, consider the mouvement with a constant acceleration / **a= constant**

$$\Rightarrow f_{min} = \frac{2}{\pi T}\sqrt{\frac{h}{\Delta_{max}}}$$

$$\Rightarrow Q = \frac{2l}{\pi T}\sqrt{\frac{h}{\Delta_{max}}}$$

Case of the robot Micro Delta $\ell = 0.175\ m$)

$$Q_{min} = 250 \cdot 0{,}17 \cong 42.5\ \text{m/s}$$

Case of the robot Delta Direct Drive $\ell = 0.4\ m$

$$Q_{min} = 125 \cdot 0{,}4 \cong 50\ \text{m/s}$$

Case of the robot Sigma6 $\ell = 0.2\ m$

$$Q_{min} = 250 \cdot 0{,}2 \cong 50\ \text{m/s}$$

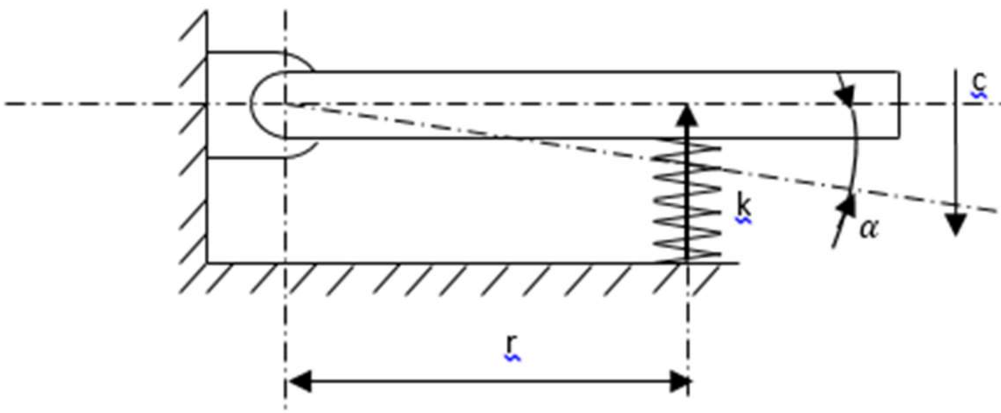
Case of the robot IRB120 $\ell = 1\ m$

$$Q_{min} = 35 \cdot 1 \cong 35\ \text{m/s}$$

$30 \le Q \le 40$ [m/s]	Good realisation
$40 < Q \le 55$ [m/s]	Very good realisation
$55 \le Q$	Your challenge !

REHAssist

Raideur angulaire : influence de la position du ressort :



$M_\alpha = F.r = k.r.\alpha.r = k_\alpha.\,\alpha$

Force

avec $k_\alpha = kr^2$

1	Purposes for a spring	Know the implementations - description examples
2	What are the vibratory modes of a solid body? Rigidity conditions of a robot arm	Traction-compression / flexion / torsion. Choice of materials. Is there a difference between alloy Alu et steel ?
3	Non rigide transmissions - Response to a trajectory	From slide 31 -
4	What are the types of trajectories used in robotics? What are their differences?	Bang bang in acceleration, trapezoid in accel,… example of the difference : modules of resonant excitation. Dynamic error of the vibrational mode.
5	What is the link between the vibratory error of the eigen mode and the acceleration of a robot arm?	Slide 38 and previous - Lesson Springs
6	What is the link between the vibratory error of the eigen mode and the rigidity of a robot arm?	Slide 38 and previous - Lesson Springs
7	In the case of a robot…. How to solve the dynamic error problem according to the trajectory?	Simulation - See Delta example from Spring Lesson.
8	What are the elements that influence the occurrence of resonance in the placement of a robot arm ? Solutions to avoid it !	Resonance: eigen frequency of excitation. Mechanical dampers - Filters anti resonance- reduce the dynamic/ acc
9	Rules of d'homotheties. What for? What effect does it have on the eigen frequencies?	Example of an arm of rectangular section b x h and of length l.
10	Quality Factor. Definition, Purposes. Limits	Lesson definition. Limits. Assume only one solid body.

Lecture :
Industrial and Applied Robotics

2016

41 students:
- 20 MT et MTE
- 19 GM+GME
- 1 Electricité
- 1 EDPR

2017

99 students:
- 73 MT et MTE
- 25 GM+GME
- 1 Electricité

2018

110 students:
- 81 MT et MTE
- 28 GM+GME
- 1 Electricité E

2019

56 students:
- 21 MT et MTE
- 35 GM+GME

2020

56 students:

2021

81 students:
MT_RO : 23
MT: 13
MT_ech/EE/MX: 2/1/1
GM / GM_ech : 38 / 3

2022

47 students:
MT_RO : 16
MT/ MT_ech/EE: : 6 /1/1
GM / GM_ech : 19 / 4

2022

43 students:
MT_RO : 20
MT/MT_ech: 7 /1
GM / GM_ech: 12 / 2

1st question:
- Which trajectory should we apply ?

REHAssist

	Loi		Profil mouvement	Loi déplacement	Forme vitesse	Vitesse ($\dot y$)	Facteur vitesse	Forme accélération	Accélération ($\ddot y$)	Facteur Accélération	Saut Acc	Utilisation
1	Droite inclinée			$y = \frac{h}{T} t$		$\dot y = \frac{h}{T}$	1		$t = \frac{0}{T}, \ddot y = \infty$ $t = \frac{0}{T}, \ddot y = 0$	∞		Très faible Vitesse
2	Parabolique ou à accélération constante	$\frac{t}{T} \leq 0.5$		$y = 2h(\frac{t}{T})^2$		$\dot y = \frac{4ht}{T^2}$	2		$\ddot y = \frac{4h}{T^2}$	1		Vitesse moyenne
		$\frac{t}{T} \geq 0.5$		$y = h\left[1 - 2(1 - \frac{t}{T})^2\right]$		$\dot y = \frac{4h}{T}(1 - \frac{t}{T})$			$\ddot y = -\frac{4h}{T^2}$			
3	Cubique ou à accélération linéaire	$\frac{t}{T} \leq 0.5$		$y = 4(\frac{t}{T})^3 h$		$\dot y = \frac{12h}{T}\left(\frac{t}{T}\right)^2$	3		$\ddot y = \frac{24h}{T^2}(\frac{t}{T})$	3		Faible vitesse
		$\frac{t}{T} \geq 0.5$		$y = h\left[1 - 4(1 - \frac{t}{T})^3\right]$		$\dot y = \frac{12h}{T}\left[1 - (\frac{t}{T})^2\right]$			$\ddot y = \frac{24h}{T^2}(1 - \frac{t}{T})$			
4	Accélération rectiligne			$6h\left[\frac{1}{2}(\frac{t}{T})^2 - \frac{1}{3}(\frac{t}{T})^3\right]$		$\dot y = 6h\left[\frac{t}{T^2} - \frac{t^2}{T^3}\right]$	1.5		$\ddot y = 6h(\frac{1}{T^2} - \frac{2}{T^2}\frac{t}{T})$	1.5	-12h/T3	Faible vitesse
5	Sinusoïde			$y = \frac{h}{2}\left[1 - \cos\frac{\pi t}{T}\right]$		$\dot y = \frac{\pi h}{2T}\sin\frac{\pi t}{T}$	1.57		$\ddot y = \frac{\pi^2 h}{2T^2}\cos\frac{\pi t}{T}$	1.234		Vitesse moyenne
6	Cycloïde			$y = \frac{h}{\pi}(\frac{\pi t}{T} - \frac{1}{2}\sin\frac{2\pi t}{T})$		$\dot y = \frac{h}{T}(1 - \cos\frac{2\pi t}{T})$	2		$\ddot y = \frac{\pi^2 h}{T^2}\sin\frac{2\pi t}{T}$	1.57		très grande vitesse
7	Accélération trapézoïdale	$t < \frac{T}{8}$		$y = \frac{64}{9}h(\frac{t}{T})^3$		$\dot y = \frac{64h}{3T}(\frac{t}{T})^2$	2		$\ddot y = \frac{128}{3}\frac{h}{T^2}\frac{t}{T}$	1.33		Grande vitesse
		$\frac{T}{8} \leq t \leq \frac{3T}{8}$		$y = h\left[\frac{8}{3}(\frac{t}{T})^2 - \frac{1}{3}\frac{t}{T} + \frac{1}{72}\right]$		$\dot y = (\frac{16t}{3T^2} - \frac{1}{3T})$			$\ddot y = h(\frac{16}{3\pi^2})$			
		$\frac{3T}{8} \leq t \leq \frac{T}{2}$		$y = h\left[-\frac{64}{9}(\frac{t}{T})^3 + \frac{32}{3}(\frac{t}{T})^2 - \frac{10}{3}\frac{t}{T} + \frac{7}{18}\right]$		$\dot y = h(-\frac{64}{3T}\left(\frac{t}{T}\right)^2 + \frac{64}{3T}\frac{t}{T} - \frac{10}{3t})$			$\ddot y = h(\frac{128}{3T^2}\frac{t}{T} + \frac{64}{3t^2})$			

2nd question: How to compare these profiles?

important factors:

- the velocity factor
- the acceleration factor.

$$v = \frac{vitesse\ max\ du\ mouvement\ considéré}{vitesse\ constante\ équivalente\ au\ cas\ 1}$$

the velocity factor

$$\boldsymbol{v} = \frac{\boldsymbol{y'}_{\boldsymbol{max}}}{\boldsymbol{h/T}}$$

The velocity factor compares the maximal velocities reached with the different types of movements

$$\alpha = \frac{accélération\ max.\ du\ mouvement\ considéré}{accélération\ d'un\ mouvement\ à\ accélération\ constante}$$

the acceleration factor

$$\alpha = \frac{y''_{max}}{\frac{4.h}{T^2}}$$

The acceleration factor compares the maximal accelerations reached with the different types of movements