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MICRO-450
Bases de la robotique

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Durée : 2 heures

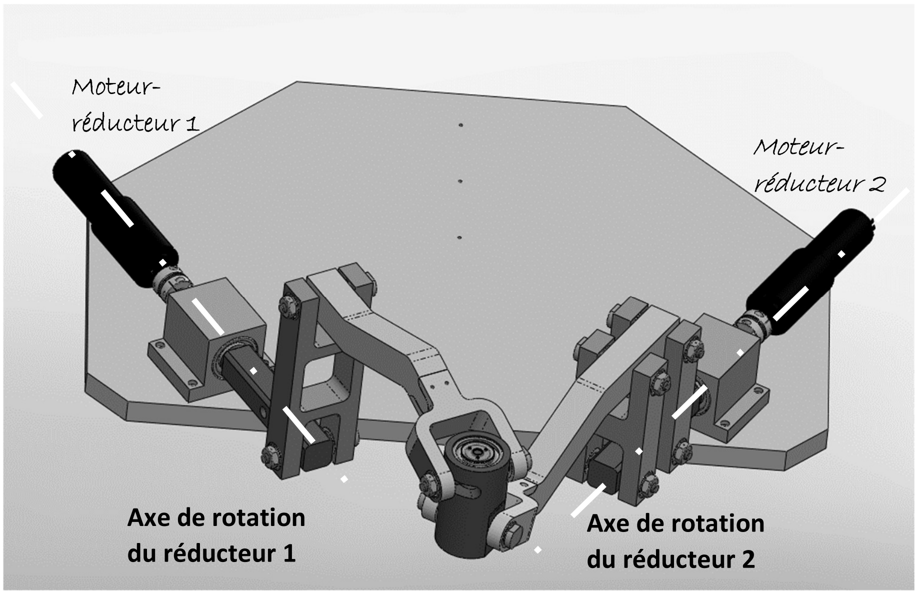
Exercise. 1 : True or false ? (15 pts)

Please note (V) or (F) on the prepared page.

1.1	A parallel robot is a structure characterized by a closed kinematic loop.	**T**
1.2	In general, a parallel robot is more rigid than a serial one.	**T**
1.3	The role of the derivative gain of a PD controller is to cancel the static error.	**F**
1.4	The proportional gain of a PD controller reduces the static error.	**T**
1.5	The dynamic model of a robot relates joint positions and joint torques.	**T**
1.6	The optimal reduction gear ratio (corresponding to optimal fitting of motor-reduction gear-load) allows to minimize energy losses	**T**
1.7	The optimal reduction gear ratio (corresponding to optimal fitting of motor-reduction gear-load) allows maximum output acceleration	**T**
1.8	Redundancy of an industrial robot allows to increase the number of degrees of freedom	**F**
1.9	The Jacobian of a robot relates the applied force at the tool with joint torques.	**T**
1.10	The Jacobian of a robot relates joint positions with joint angles.	**F**
1.11	Inverse kinematics gives position & orientation of the end-effector in function of joint angles.	**F**
1.12	Forward kinematics of a serial link robot can have multiple mathematical solutions	**F**
1.13	Inverse kinematics of a parallel-link robot has only one mathematical solution.	**F**
1.14	The trace of the direction cosine matrix is always = 1	**F**
1.15	Calculation of an orientation quaternion gives { ½ , 1/3 , 1/3, 1/3 } Can this be correct?	**F**

Exercise. 2 (21 pts)

The figure below shows a two-axis pantograph kinematics:



These two drive trains are designed as follows:

Motor **ECI52X,**	Reduction gear **GP52X,**
• Moment of inertia **Jm = 160 gcm²**	• Gearing ratio n= **160** • Moment of inertia**, J_red = 10 gcm²** **measured at the gear input.**

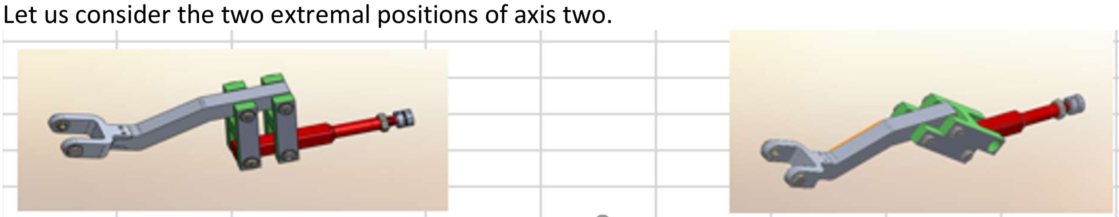
2.1 The motor inertia as seen from the load is:

(A) 0.04 kgm² **(B) 0.4 kgm²** **(C) 4 kgm²** **(D) 0.02 kgm²**

2.2 The reduction gear inertia as seen from the load is:

(A) 0.025 kgm² **(B) 0.25 kgm²** **(C) 2.5 kgm²** **(D) 0.0125kgm²**

Let us consider the two extremal positions of axis two.



For each configuration, the moment of inertia of the load (arm+fore arm) at the output gear shaft side are given as follows:

Configuration A	Configuration B
$I_{axis2A} = 0.1\ kg.m^2$	$I_{axis2B} = 0.04\ kg.m^2$

2.3 In configuration A , the total inertia seen from the load is:

(A) 0.165kgm² **(B) 0.525 kgm²** **(C) 6.6 kgm²** **(D) 0.1325kgm²**

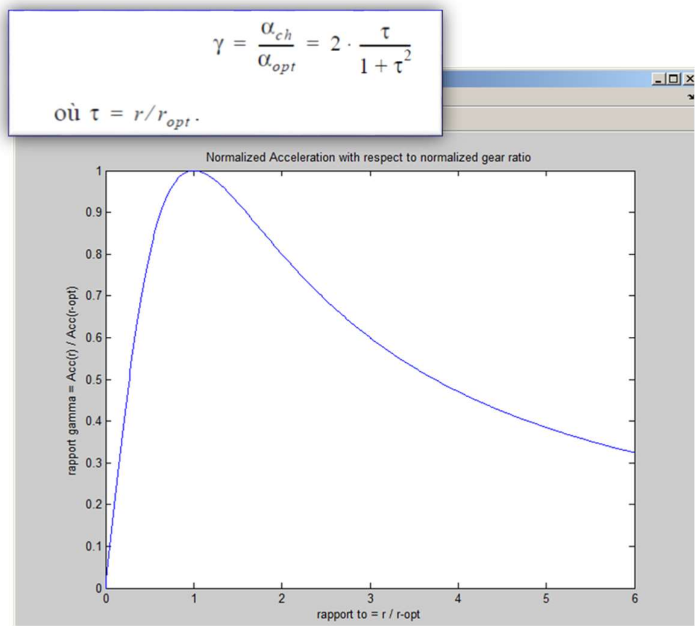
2.4 In configuration B , the total inertia seen from the load is:

(A) 0.465 kgm² **(B) 0.69 kgm²** **(C) 6.54 kgm²** **(D) 0.0725kgm²**

2.5 Choice of an optimal reduction gear ratio : What will it be for configuration A?

(A) 40 **(B) 80** **(C) 120** **(D) 160**

Here is a plot of relative output acceleration over optimal acceleration $\gamma(\tau)$ in function of the relative reduction gear ratio tau (r optimal corresponds to tau = 1, optimal acceleration corresponds to $\gamma=1$)



2.6 In configuration A, with r optimal, the given drive train would allow which percentage of optimal acceleration?

(A) ~ 30% **(B) ~ 60%** **(C) ~ 80%** **(D) ~ 100%**

2.7 In configuration B, with r optimal, the given drive train would allow which percentage of optimal acceleration?

(A) ~ 30% **(B) ~ 60%** **(C) ~ 80%** **(D) ~ 100%**

Exercise. 3 (10 pts)

We design a linear type Delta robot. A first proposal is to use a rotational motor, a belt transmission and a ball-screw linear drive. The belt transmission consists of two pulleys with **8mm** diameter on the motor side and **20 mm** diameter on the side of the ball-screw. The lead (pitch) of the screw is **10mm**.

Motor specs: **Inertia Jm** = 150 g.cm^2. **Nominal speed** = 8000 rpm. **Nominal torque** = 0.2 Nm

Incremental encoder with pitch of **1000 / full revolution.**

3.1 The best position resolution of the linear axis is:

(A) 1 micron **(B) 5 microns** **(C) 10 microns** **(D) 2.5 microns**

Linear_resolution = 10/(1000*4*2,5) = 1 micron

3.2 The controller sampling frequency is 1 kHz and the output velocity is computed from derivation over a single sampling period. The resolution for velocity is

(A) 1 mm/sec **(B) 5 mm/sec** **(C) 10 mm/sec** **(D) 2.5 mm/sec**

3.3 The output velocity is :

(A) 0.5 m/sec **(B) 1 m/sec** **(C) 2.5 m/sec** **(D) 10 m/sec**

Vel_out_nom_lin = (8000 / 2.5) * 10 /60 = 0.533 m/s

3.4. The transmission belt and the ball screw have losses **of 10% each**. What is the nominal force available at the linear output?

(A) 240 N **(B) 300 N** **(C) 375** **(D) 400 N**

Hint: Think of power transmission.

Force_out_nom_lin = (0,2 * 8000 * 2Pi/60)/ 0,5 * 0,9 = 301 N

3.5 What power should be specified for an equivalent direct drive linear actuator? (in place of the motor, belt-drive and ball-screw)?

(A) 120 Watt **(B) 150 Watt** **(C) 190 Watt** **(D) 210 Watt**

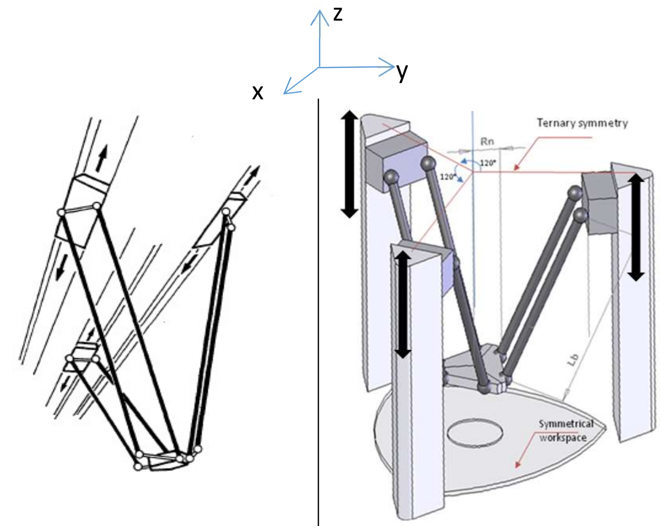
Exercise 4 (25 pts)

There are several versions of linear Delta robots.

We will consider two versions:

1) with all linear drives horizontal

2) with all linear drives in vertical arrangement



1- Horizontal linear Delta

2- Vertical linear Delta

4.1 What is the number of degrees of freedom of the horizontal version?

(A) 2 **(B) 3** **(C) 4** **(D) 1**

4.2 What is the number of degrees of freedom of the vertical version?

(A) 2 **(B) 3** **(C) 4** **(D) 1**

For the control of these linear axes, we make the following assumptions:

- m_h is the equivalent mass seen at each horizontal linear actuation axis. For simplification, we will consider it as independent of the robot position. Neglect moments of inertia.
- m_v is the equivalent mass seen at each vertical linear actuation axis. For simplification, we will consider it as independent of the robot position. Neglect moments of inertia.
- Neglect dry friction in both cases. The damping coefficient for each linear axis is k_v.

4.3 Concerning the Jacobian matrix, Which one of the following statements is correct?

(A) The Jacobian of the horizontal version does not depend on the robot position
(B) The Jacobian of the vertical version does not depend on the robot position
(C) The Jacobian of both versions is function of the robot position (since the direct geometric model is non linear and function of the robot position)
(D) The Jacobian of the vertical version corresponds to Matrix "Identity"

4.4 Consider the dynamic model of both robot versions, with the simplifying assumption of constant equivalent masses m_h and m_v . Which one of the following statements is correct?

(A) The dynamic models of both versions are decoupled

Assuming that all the masses in movement are simplified as stated, using m_h and m_v . constant masses respectively for the horizontal and the vertical versions.

(B) The dynamic model of each robot depends on the end effector position

(C) The dynamic model of both versions are identical

(D) Only the dynamic model of the horizontal version is decoupled

4.a The motors are controlled in torque. Which type of minimal controller for position control of a linear axis of the horizontal version would be feasible: **P , PI , PD or PID ? Explain** (2 pts).

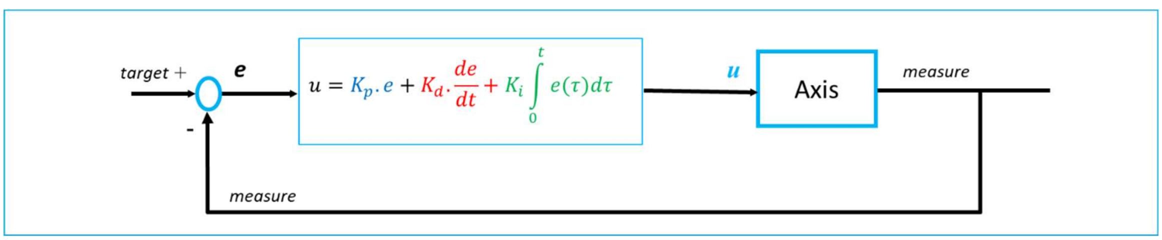
4.b The motors are controlled in torque. Which type of minimal controller for position control of a linear axis of the vertical version would be feasible: **P , PI , PD or PID ? Explain** (2 pts)

For the Horizontal version, if we do not consider the gravity, the PD will suffice to control each axis.

For the vertical axis, we require to use a PID to control the robot.

The derivative action is required to add damping and accordingly stabilize the position control.

4.c Give the controller equation for the case 4b. Give a controller block diagram showing the closing of the control loop and explicit the variables involved. (3pts)



u (or u_{PID}) is the applied torque

e is the position error

$u_{PID} = K_p.e + K_d.\frac{de}{dt} + K_i \int_0^t e\, dt$

Kp is the proportional gain control coefficient

Kd is the derivative gain control coefficient

Ki is the integral gain control coefficient

e is the position error, **e = position_desired – position_measured**

4.d Give the expression for the inverse dynamics of one of the horizontal axes (1pt)

To obtain the dynamic model, we write

$$\sum F = m_h.\ddot{q} = F_h - k_v\dot{q}$$

Which leads to: $\boldsymbol{F_h = m_h.\ddot{q} + k_v\dot{q}}$

q is the translation joint coordinate of the linear axis.

4.e Give the expression for the inverse dynamics of one of the vertical axes (2pts)

$\sum F = m_v.\ddot{q} = F_v - k_v\dot{q} - m_v.g$

Hint about the sign of gravity, in a steady state ($\ddot{q} = 0$), the vertical force should balance the gravity force. $F_v - m_v.g = 0$ ie. $\boldsymbol{F_v = m_v.g}$

This leads to:

$$F_v = m_v.\ddot{q} + k_v\dot{q} + m_v.g$$

q is the translation joint coordinate of the linear axis.

g is the gravity acceleration.

4.f What are the expressions of the a priori generalized torques for a horizontal axis and for a vertical axis? (3 pts)

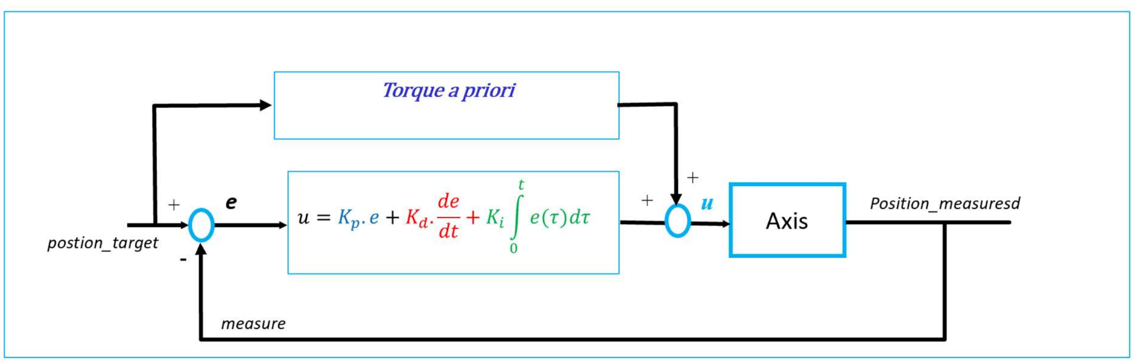
$$F_{h_{ap}} = m_h.\ddot{q}_d + k_v\dot{q}_d$$

$$F_{v\_ap} = m_v.\ddot{q}_d + k_v\dot{q}_d + m_v.g$$

q_d is the desired translation joint position of the linear axis.

g is the gravity acceleration.

4.g Give a block-diagram of answer **(4.c)** with the generalized a-priori torque. What is the complete expression for the generalized control torque? (3pts)



For the horizontal axis, the controller expression is

$$\boldsymbol{u_h} = u_{PID} + u_{ap} = K_p.e + K_d.\frac{de}{dt} + K_i \int_0^t e(\tau)d\tau + \boldsymbol{m_h}.\ddot{\boldsymbol{q}}_{\boldsymbol{d}} + \boldsymbol{k_v}\dot{\boldsymbol{q}}_{\boldsymbol{d}}$$

For the vertical axis, the controller expression is

$$\boldsymbol{u_v} = u_{PID} + u_{ap} = K_p.e + K_d.\frac{de}{dt} + K_i \int_0^t e(\tau)d\tau + \boldsymbol{m_v}.\ddot{\boldsymbol{q}}_{\boldsymbol{d}} + \boldsymbol{k_v}\dot{\boldsymbol{q}}_{\boldsymbol{d}} + \boldsymbol{m_v}.\boldsymbol{g}$$

q, and $\mathbf{q_d}$ are respectively the position_measured and position_desired for the concerned axis horizontal or vertical.

4.h In case of using an a priori control path (as in the previous question), what would be the minimal controller **P , PI, PD ou PID ? Justify your choice!** (1 pt)

In case of using an a priori control, the integrator may no more be required for gravity compensation, the **PD controller will suffice**.

In presence of unknown dry friction, that can not be included in the a priori, the integral action will be required to compensate for that friction and thus cancel the steady state error.

Exercise 5 (14 pts)

Given the robot shown to the right ***(Figure 5.1)****, with base coordinates*
θx, θy et θz are rotation angles with respect to these coordinate axes.
All joints are simple pivots, shown as introduced in the lecture.

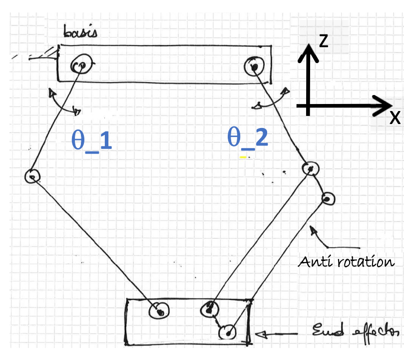


Figure 1

5.a *Give vector [X] of the operational (world) coordinates of this robot. Not necessarily all place holders need to be filled (1 pt).*

X = [x, z]

The parallel bar mechanism is used to compensate for the rotation around the axis y

5.b Give vector [q] of the generalized coordinates of this robot and explain what they represent (or show it in the figure 5.1 above). *Not necessarily all place holders need to be filled* (2 pts)

$q = [\theta\_1, \theta\_2]$

θ_1 is the rotation of the arm 1 around the basis horizontal axis x Ref figure.	θ_2 is the rotation of the arm 2 around the basis horizontal axis x. Ref figure			

5.c We aim to add a tool rotation axis around z without adding a motor on the end effector. **Propose a possible kinematic design** to realize such a motion. (3pts)

Multiple choice questions (refer to **Figure 5.1**)

5.1 The funcion X= ϕ(q) is :

- **(A) Forward Kinematics (Modèle géométrique direct)**
- **(B) Inverse Kinematics (Modèle géométrique inverse)**
- **(C) Inverse Jacobian**
- **(D) None of the above**

5.2 How many degrees of freedom for the structure of Figure 5.1?

(A) 2 **(B) 3** **(C) 4** **(D) 6**

5.3 The Robot of Figure 5.1 has the following number of mobilities:

(A) 2 **(B) –2** **(C) -4** **(D) 4**

Mo = 8 – 6. 2 = -4 (8 pivots, 2 loops)

5.4 This structure

- **(A) is over constrained (because there is less Mobilities than DOFs)**
- **(B) has internal mobilities**
- **(C) is redundant**
- **(D) none of the above**

Exercise 6 (8 pts)

6.1 A 4 degrees-of-freedom SCARA robot has a number p of postures.

(A) p = 1 **(B) p = 2** [(C) p = 4] (D) p = 8

6.2 The quaternion { ½, –½,–½,–½ } stands for a rotation of

(A) 60° [(B) 120° or –120°, depending on axis direction]

(C) –60° **(D) is not a unit quaternion**

6.3 What is the main reason to use signals in quadrature for incremental encoders?

(A) To increase resolution **(B) To have an absolute value**

[(C) To determine the direction of increments] **(D) To increase precision**

6.4 An optical absolute position encoder with a resolution of 1/4000 of its full range will need at least

(A) 8 tracks **(B) 10 tracks** [(C) 12 tracks] **(D) 16 tracks**

Exercise 7 (15 pts)

Some easy calculations. No, you do no need a calculator ! **Give results simply in terms of square roots** and fractions, without computing decimals !

7.a Give the quaternion corresponding to a rotation of 90° around z-axis

$$\lambda_0 = \cos(\vartheta/2) \quad \text{and} \quad \underline{\lambda} = \sin(\vartheta/2)\,[x, y, z]^T, \quad ||x, y, z|| = 1 \qquad (11b')$$

Reminder,

If we consider V=(Vx,Vy,Vz), then the associated unity vector is :

$V_u = [\frac{Vx}{||V||}, \frac{Vz}{||V||}, \frac{Vz}{||V||}]^T = \frac{1}{||V||}(V_x, V_y, V_z)$

Qa = $[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\,(0, 0, 1)]^T = [\frac{\sqrt{2}}{2}, 0, 0, \frac{\sqrt{2}}{2}]^T$

7.b Give the quaternion corresponding to a rotation of 90° around axis $[1,1,0]^T$

Qb = $[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \cdot \frac{1}{\sqrt{2}}\,(1,1,0)]^T = [\frac{\sqrt{2}}{2}, \frac{1}{2}, \frac{1}{2}, 0]^T$

7.c Give the quaternion corresponding to the combined rotation consisting of **rotation 7a) followed by rotation 7b)**

Q = Qb . Qa = $\left(\frac{\sqrt{2}}{2}+\frac{1}{2}i+\frac{1}{2}j\right).\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}k\right)$

$$=\left(\frac{1}{2}+\frac{1}{2}k+\frac{\sqrt{2}}{4}i+\frac{\sqrt{2}}{4}ik+\frac{\sqrt{2}}{4}j+\frac{\sqrt{2}}{4}jk\right)=\left(\frac{1}{2}+\frac{1}{2}k+\frac{\sqrt{2}}{4}i-\frac{\sqrt{2}}{4}j+\frac{\sqrt{2}}{4}j+\frac{\sqrt{2}}{4}i\right)$$

$$=\left(\frac{1}{2}+\frac{\sqrt{2}}{2}i+\frac{1}{2}k\right)$$

Thus : Q = $[\frac{1}{2},\frac{\sqrt{3}}{2}\left(\sqrt{\frac{2}{3}},0,\frac{1}{\sqrt{3}}\right)]^T = [\frac{1}{2},\frac{\sqrt{2}}{2},0,\frac{1}{2}]^T$

7.d What is the axis of this combined rotation?

Axis is $\left(\sqrt{\frac{2}{3}},0,\frac{1}{\sqrt{3}}\right)$

7.e What is the rotation angle of this combination?

Cos(θ/2) = $\frac{1}{2}$ and $sin(\theta/2) = \frac{\sqrt{3}}{2}$

Angle is 120°.

7.f What are axis and angle if the combination is done in reversed order, i.e. **first 7b then 7a** ?

Q = Qa . Qb = $\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}k\right)\left(\frac{\sqrt{2}}{2}+\frac{1}{2}i+\frac{1}{2}j\right)$

$$=\left(\frac{1}{2}+\frac{\sqrt{2}}{4}i+\frac{\sqrt{2}}{4}j+\frac{1}{2}k+\frac{\sqrt{2}}{4}ki+\frac{\sqrt{2}}{4}kj\right)=\left(\frac{1}{2}+\frac{\sqrt{2}}{4}i+\frac{\sqrt{2}}{4}j+\frac{1}{2}k+\frac{\sqrt{2}}{4}j-\frac{\sqrt{2}}{4}i\right)$$

$$=\left(\frac{1}{2}+\frac{\sqrt{2}}{2}j+\frac{1}{2}k\right)$$

Thus : Q = $[\frac{1}{2},\frac{\sqrt{3}}{2}\left(0,\sqrt{\frac{2}{3}},\frac{1}{\sqrt{3}}\right)]^T = [\frac{1}{2},0,\frac{\sqrt{2}}{2},\frac{1}{2}]^T$

Axis is $\left(0,\sqrt{\frac{2}{3}},\frac{1}{\sqrt{3}}\right)$

Angle is 120°

__Exercise 8__ (10 pnts)

8.a Give the matrix of direction cosines for the composed operation of a first rotation around z-axis followed by a second rotation of 90° around axis $[1, 1, 0]^T$

The corresponding unity vector of $[1, 1, 0]^T$ is $\frac{1}{\sqrt{2}}[1,1,0]^T$, thus $[\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0]^T$

Formula of Rodriguez is (where $[x,y,z]^T$ is the rotation unity vector

$$\mathbf{R} = (1-\cos\vartheta)\begin{bmatrix} xx & xy & xz \\ xy & yy & yz \\ xz & yz & zz \end{bmatrix} + \cos\vartheta \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \sin\vartheta \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}$$

The rotation around the $[1, 1, 0]^T$, same as around the unity vector $[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0]^T$ is

$$R_v = (1-\cos(90))\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \cos(90)\begin{bmatrix} \; \end{bmatrix} + \sin(90)\begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \mathbf{0} \end{bmatrix}$$

$$\mathbf{R}_z = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The resulting rotation is

$$\mathrm{R} = R_v . R_z = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \mathbf{0} \end{bmatrix} . \begin{bmatrix} \boldsymbol{c} & -\boldsymbol{s} & \mathbf{0} \\ \boldsymbol{s} & \boldsymbol{c} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(\boldsymbol{c}+\boldsymbol{s}) & \frac{1}{2}(\boldsymbol{c}-\boldsymbol{s}) & \frac{1}{\sqrt{2}} \\ \frac{1}{2}(\boldsymbol{c}+\boldsymbol{s}) & \frac{1}{2}(\boldsymbol{c}-\boldsymbol{s}) & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}(\boldsymbol{c}+\boldsymbol{s}) & \frac{1}{\sqrt{2}}(\boldsymbol{c}-\boldsymbol{s}) & \mathbf{0} \end{bmatrix}$$