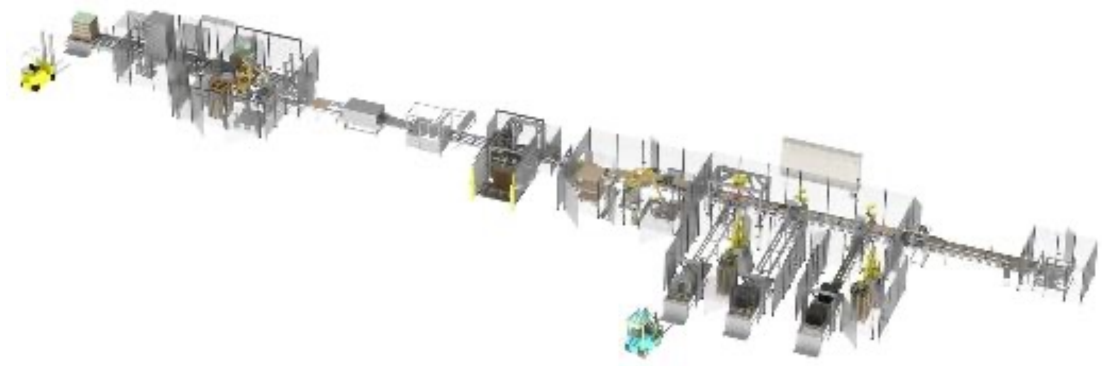


MANUFACTURING SYSTEMS AND SUPPLY CHAIN DYNAMICS

Chapter 8: Cooperative Flow Dynamics in Production Lines

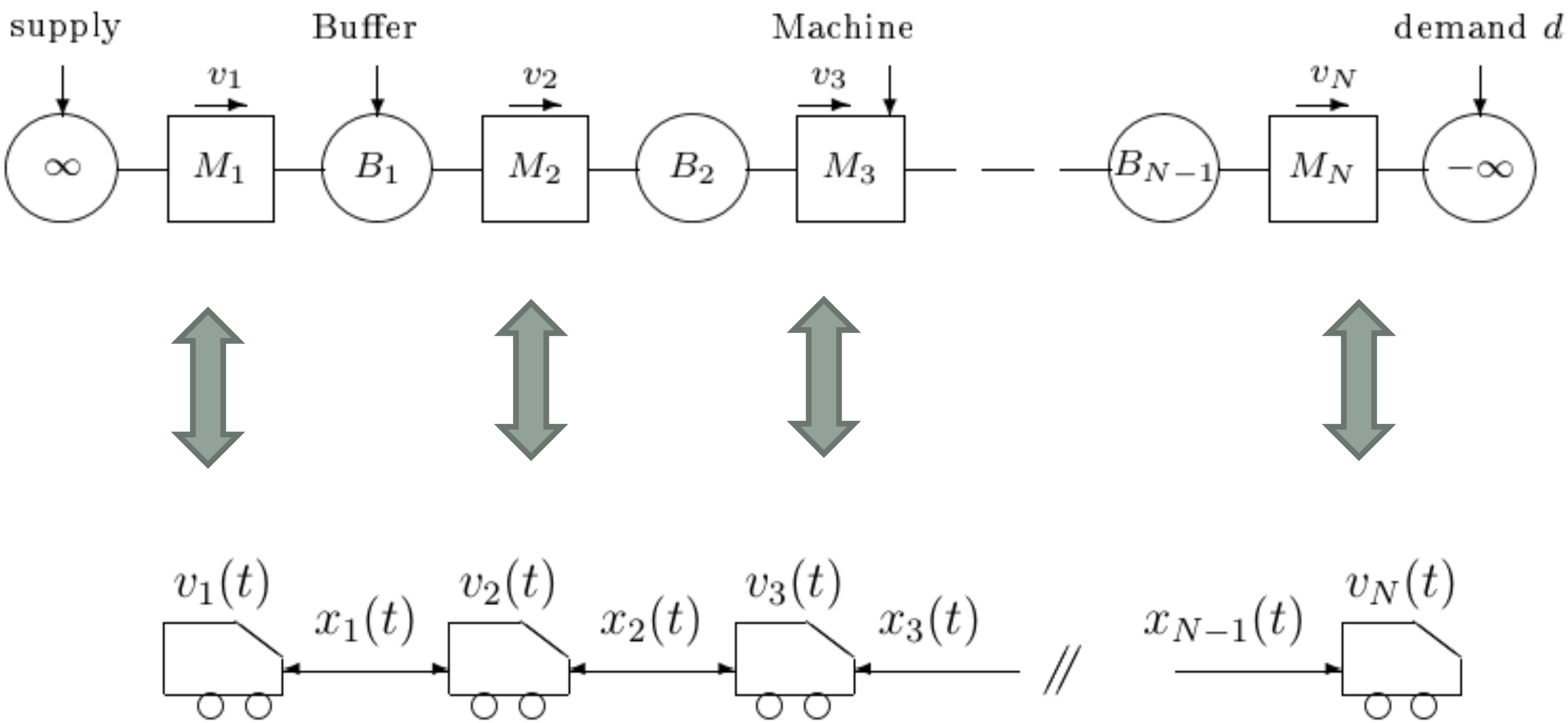
EPFL, Master MT

Roger Filliger (BFH), Olivier Gallay (UniL)

Course Content

Goal: analogy to gain dynamic (non stationary) insights!

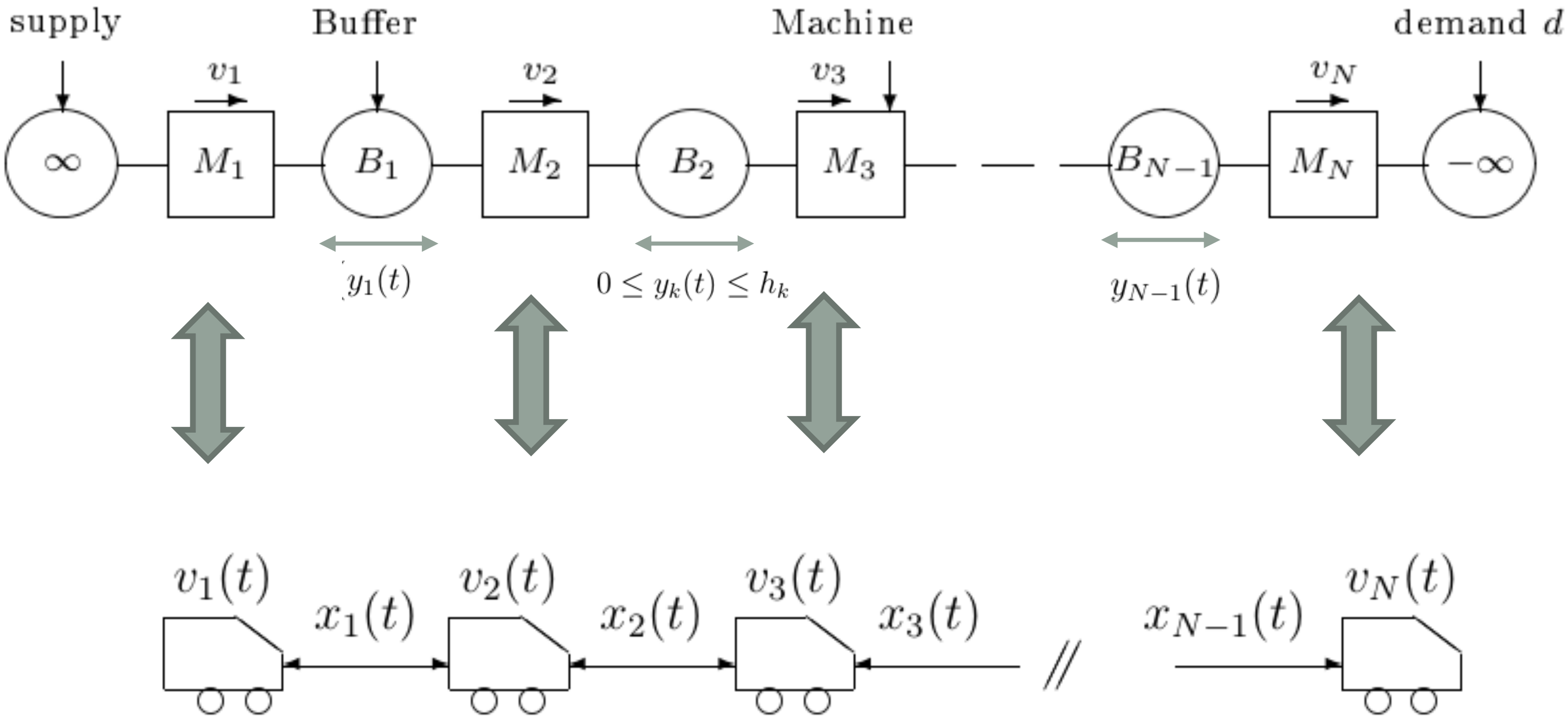
Buffered production line



[One lane traffic]()

Goal: analogy to gain dynamic (non stationary) insights!

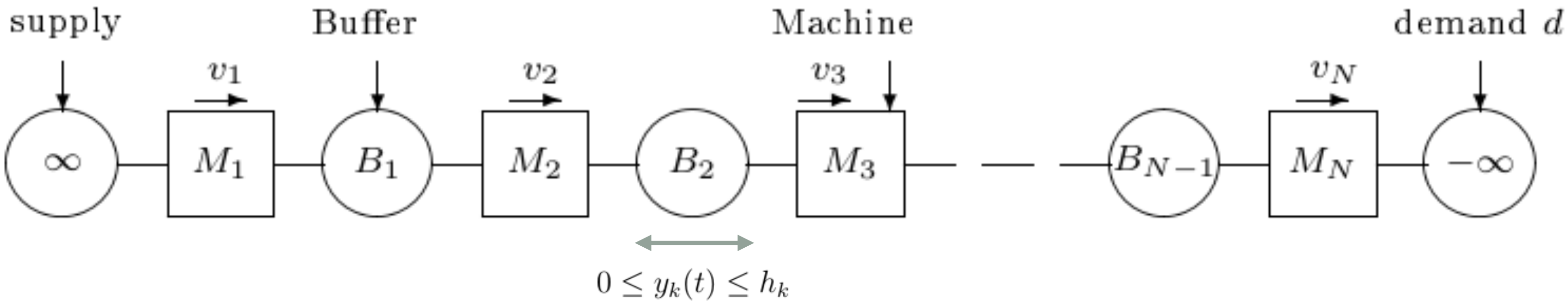
Buffer content: $0 \leq y_k(t) \leq h_k$



Headway: $x_k(t) > 0$

Goal: analogy to gain dynamic (non stationary) insights!

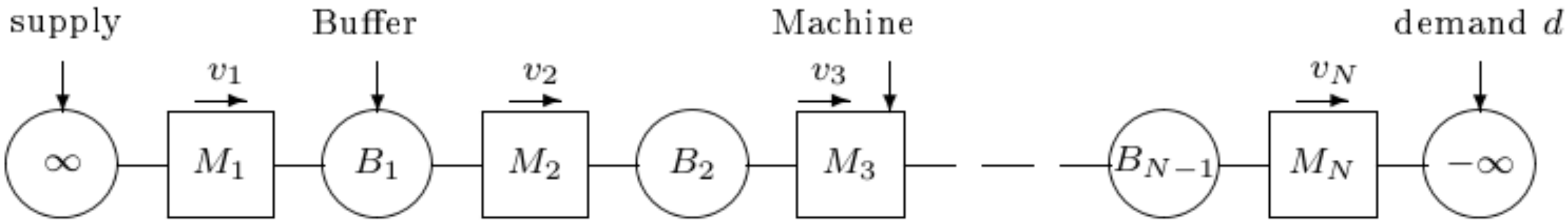
Stochastic evolution equations



$$\frac{\mathrm{d}y_k(t)}{\mathrm{d}t} = v_k(t)I_k(t) - v_{k+1}(t)I_{k+1}(t), \quad y_k(0) = y_k, \quad k = 1, \ldots, N,$$

Goal: analogy to gain dynamic (non stationary) insights!

Stochastic evolution equations



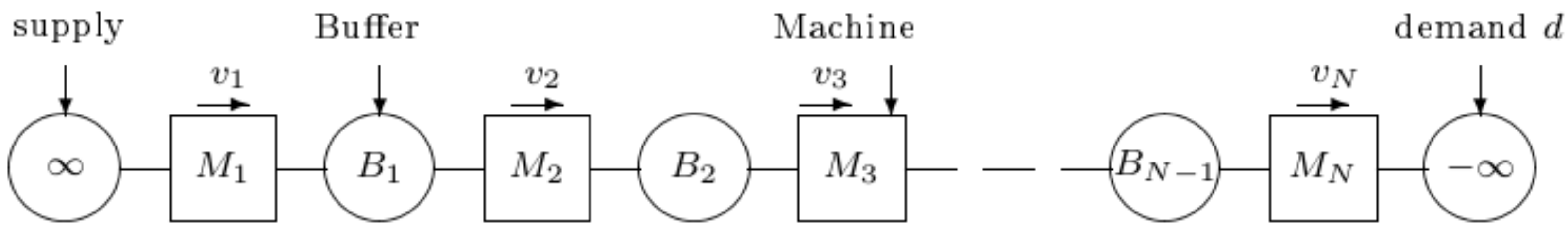
$$\frac{\mathrm{d}y_k(t)}{\mathrm{d}t} = v_k(t)I_k(t) - v_{k+1}(t)I_{k+1}(t), \quad y_k(0) = y_k, \quad k = 1, \ldots, N,$$

$I_k(t)$ is an alternating renewal process with states $\{0, 1\}$

M_k is up whenever $I_k(t) = 1$ and M_k is down when $I_k(t) = 0$.

Goal: analogy to gain dynamic (non stationary) insights!

Stochastic evolution equations



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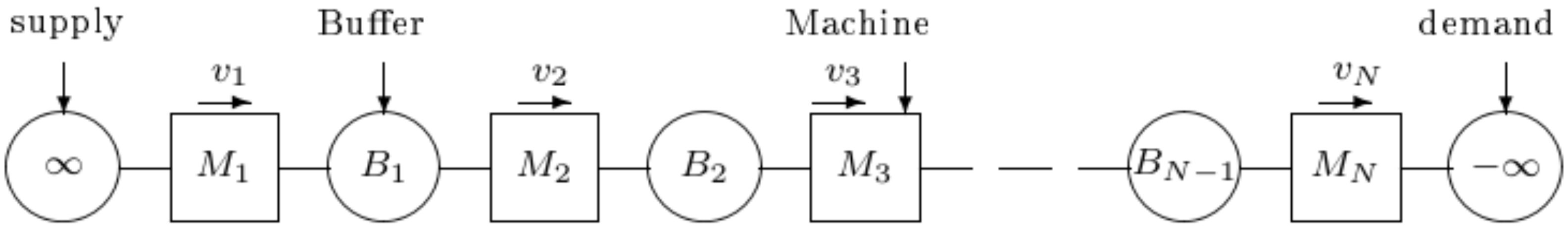
M_k is up whenever $I_k(t) = 1$ and M_k is down when $I_k(t) = 0$.

$$(y_1(t), \ldots, y_{N-1}(t), y_N(t)) \in [0, h_1] \times \ldots \times [0, h_{N-1}] \times \mathbb{R},$$

Assumptions

Stochastic evolution equations



A1) $y_k(t) = 0 \Rightarrow v_{k+1}(t) \leq v_k(t),\ k = 1, ..., N-1$

A2) $y_k(t) = h_k \Rightarrow v_k(t) \leq v_{k+1}(t),\ k = 1, ..., N-1$

$$(v_1(t), ..., v_N(t)) \in [0, V_{\max,1}] \times ... \times [0, V_{\max,N}].$$

A3) Transport time of items from M_k to B_k and from B_k to M_{k+1}, $k = 1, ..., N-1$, are assumed to be short and are neglected.

A4) Machine M_1 is never starved (enough raw material) and M_N is not influenced by the market (enough demand).

8.2.2 Optimal-Velocity Car Traffic Model

We consider N cars $\{M_k\}_{k=1..N}$ travelling on a single lane as showed in Figure 8.2. For $k = 1, ..., N-1$, denote by $x_k(t) > 0$ the headway between the

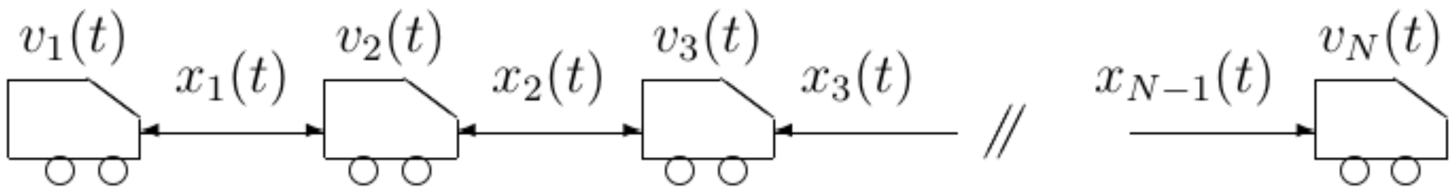


Figure 8.2: N cars on a single-lane.

cars M_k, M_{k+1} and for $k = 1, ..., N$ denote by $v_k(t) \in [0, V_{\max,k}]$ the speed of M_k, where $V_{\max,k}$ is the maximal velocity of M_k.

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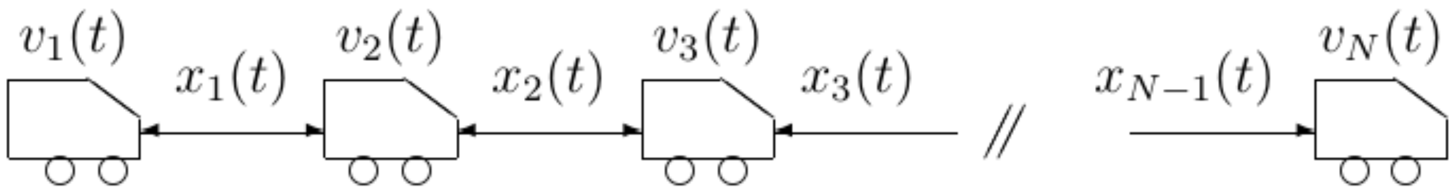

The Optimal-Velocity (OV) traffic model [15] states the existence of an optimal velocity function $\mathcal{V}_k$ which depends on the headways x_k, x_{k-1} and the presence of a response delay time τ_k, required for a driver of M_k to adjust its speed, such that:

$$\mathcal{V}_k(t) \stackrel{\text{not.}}{=} \mathcal{V}_k\big(x_{k-1}(t), x_k(t)\big) = v_k(t + \tau_k). \tag{8.4}$$

8.2.2 Optimal-Velocity Car Traffic Model

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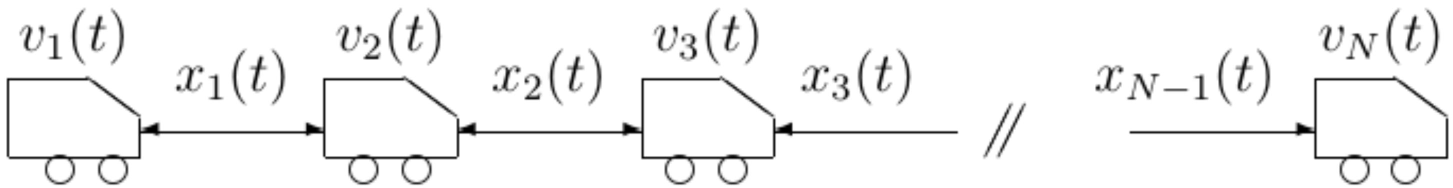


Expanding Eq. (8.4) up to first order, adding the corresponding headway variations and specifying the optimal velocity yields the following class of OV-models:

$$\begin{cases} \frac{\mathrm{d}x_k(t)}{\mathrm{d}t} &= v_{k+1}(t) - v_k(t), \quad k = 1, \ldots, N, \\ \frac{\mathrm{d}v_k(t)}{\mathrm{d}t} &= \alpha_k\Big(\mathcal{V}_k(t) - v_k(t)\Big), \quad k = 1, \ldots, N, \end{cases} \tag{8.5}$$

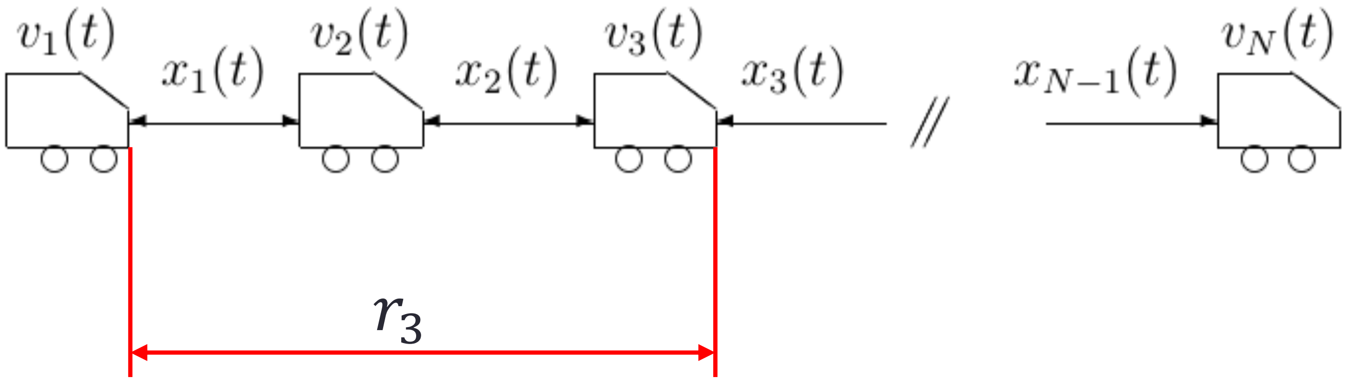
where $\alpha_k = \tau_k^{-1}$ and where the optimal velocity of M_k at time t is of the form:

$$\mathcal{V}_k(t) = V_{\max,k} \cdot F_k\Big(x_{k-1}(t), x_k(t)\Big). \tag{8.6}$$

8.3 Linear Stability Analysis

Obviously, a steady state for cars in a line is given when all of them run orderly with the same constant optimal velocity $\mathcal{V}_k = v^e$ and with constant headway x_k^e, such that:

$$V_{\max,k} F_k\left(x_{k-1}^e, x_k^e\right) = v^e \quad k = 1, \ldots, N-1. \tag{8.11}$$



$$r_k(t) := \sum_{j=0}^{k-1} x_j(t) \quad k = 1, \ldots, N, \quad x_0 \equiv 0.$$

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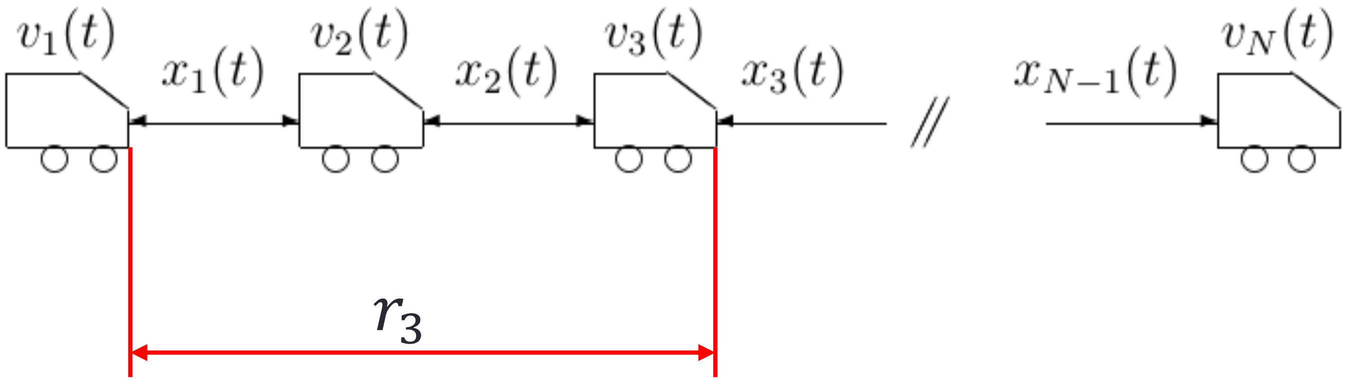


Dynamics with the new variables:

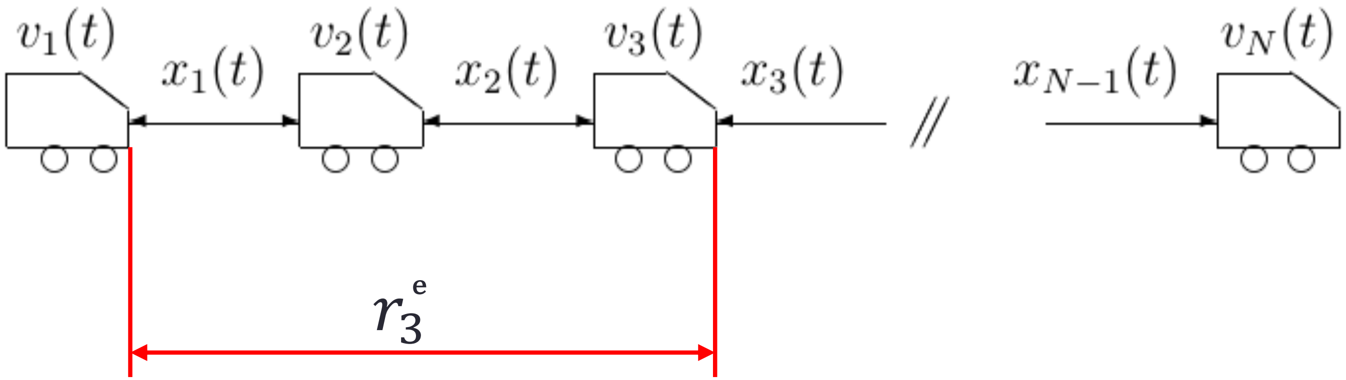
$$\frac{\mathrm{d}r_k(t)}{\mathrm{d}t} = v_k(t), \;\; \text{k=1,...,N}$$

$$\frac{\mathrm{d}v_k(t)}{\mathrm{d}t} = \alpha_k \Big(V_{\max,k} F_k \left[r_k(t) - r_{k-1}(t), r_{k+1}(t) - r_k(t) \right] - v_k(t) \Big)$$

Steady State:

$$r_k^e(t) = \sum_{j=1}^{k-1} x_j^e + v^e t,$$
$$v_k^e(t) = \mathcal{V}_k(t) = v^e,$$

all cars with equal (optimal) velocity and (optimal) distances

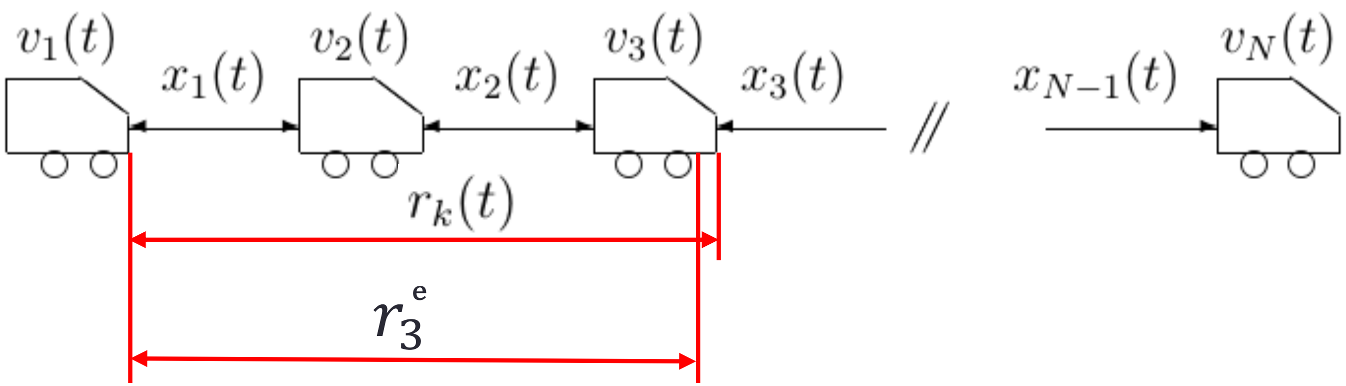


$$\frac{\mathrm{d}r_k(t)}{\mathrm{d}t} = v^e \qquad k=1,\ldots,N$$
$$\frac{\mathrm{d}v_k(t)}{\mathrm{d}t} = 0$$

Perturbation of Steady State solution:

$$\delta r_k(t) := r_k(t) - r_k^e(t).$$

not all cars with equal (optimal) velocity and (optimal) distances



$$\frac{dr_k(t)}{dt} \neq v^e \qquad k=1,\ldots,N$$

$$\frac{dv_k(t)}{dt} \neq 0$$

Perturbation of Steady State solution:

$$\delta r_k(t) := r_k(t) - r_k^e(t).$$

Study the evolution of the perturbation!

$$\frac{\mathrm{d}^2 \delta r_k(t)}{\mathrm{d}t^2} = \alpha_k \left[V_{\max,k} \left(\delta r_{k+1} \partial_y F_k + \delta r_k (\partial_x F_k - \partial_y F_k) - \delta r_{k-1} \partial_x F_k \right) - \frac{\mathrm{d}\delta r_k}{\mathrm{d}t} \right] \tag{8.15}$$

where

$$\partial_x F_k := \left. \frac{\partial F_k(x,y)}{\partial x} \right|_{x=x_{k-1}^e} \quad \text{and} \quad \partial_y F_k := \left. \frac{\partial F_k(x,y)}{\partial y} \right|_{y=x_k^e}. \tag{8.16}$$

Ansatz for the solution:

$$\delta r_k(t) := \frac{1}{N}\sum_{j=0}^{N-1} c_j e^{2\pi i \cdot j \frac{k}{N}} e^{\left(\lambda(j) - i\omega(j)\right)t}, \quad k \in \{1, ..., N\},$$

Study the evolution of the perturbation!

$$\frac{\mathrm{d}^2 \delta r_k(t)}{\mathrm{d}t^2} = \alpha_k \left[V_{\max,k} \left(\delta r_{k+1} \partial_y F_k + \delta r_k \left(\partial_x F_k - \partial_y F_k \right) - \delta r_{k-1} \partial_x F_k \right) - \frac{\mathrm{d}\delta r_k}{\mathrm{d}t} \right] \tag{8.15}$$

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Stability is given when

$$\frac{\alpha_k(\partial_y F_k - \partial_x F_k)}{V_{\max,k} \cdot (\partial_x F_k + \partial_y F_k)^2} > 2 \;\; \forall k \in \{1, ..., N\}.$$

where

$$\partial_x F_k := \left.\frac{\partial F_k(x,y)}{\partial x}\right|_{x=x_{k-1}^e} \quad \text{and} \quad \partial_y F_k := \left.\frac{\partial F_k(x,y)}{\partial y}\right|_{y=x_k^e}. \tag{8.16}$$

Transcription for production line:

$$\begin{array}{lcl} \text{machines in a flow shop} & \leftrightarrow & \text{cars in a single lane,} \\ \text{free buffer space } h_k - y_k & \leftrightarrow & \text{headway } x_k, \\ \text{production rate} & \leftrightarrow & \text{car velocity,} \end{array}$$

$$\phi_k(x,y) := F_k\big(h_{k-1} - x, h_k - y\big)$$

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$$\phi_k(x,y) := F_k\big(h_{k-1} - x, h_k - y\big)$$

Stability is given when

$$\mathcal{Z}_k > 2, \;\; \forall k = 1, \ldots, N$$

$$\mathcal{Z} := \frac{\alpha(\partial_x \mathcal{V} - \partial_y \mathcal{V})}{(\partial_x \mathcal{V} + \partial_y \mathcal{V})^2}$$

A simple pull production control:

$$\phi_k\big(x_{k-1}(t), x_k(t)\big) = \phi\big(x_k(t)\big) = \begin{cases} 1 & \text{if} \quad x_k(t) = 0, \\ 1 - \frac{x_k(t)}{h_k} & \text{if} \quad 0 < x_k(t) < h_k, \\ 0 & \text{if} \quad x_k(t) \geq h_k. \end{cases}$$

We find:

$$\mathcal{Z}_k := \frac{\alpha_k h_k}{V_{\max,k}} > 2 \;\; \forall k \in \{1, ..., N\}.$$

or equivalently

$$\frac{1}{2}\frac{h_k}{V_{\max,k}} > \frac{1}{\alpha_k} = \tau_k \;\; \forall k \in \{1, ..., N\}.$$

Using the dimensionless performance measure from stationary analysis:

$$\mathcal{F} = \frac{\mu h}{V_{\max}}$$

(with μ the breakdown rate of a machine), we have established a direct relation with the stability analysis:

$$\frac{1}{\mathcal{F}} = \frac{\alpha_k}{\mu_k}\frac{1}{\mathcal{Z}_k} < \frac{\alpha_k}{\mu_k}\frac{1}{2}.$$