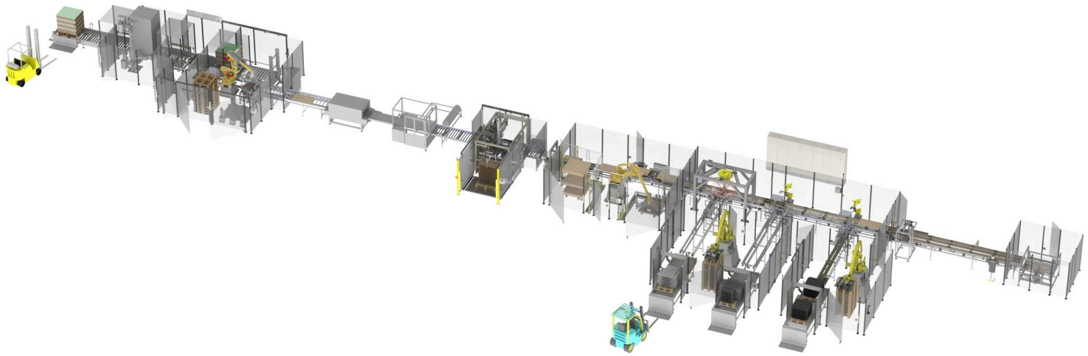

MANUFACTURING SYSTEMS AND SUPPLY CHAIN DYNAMICS

Chapter 7: Production Lines and Aggregation

EPFL, Master MT

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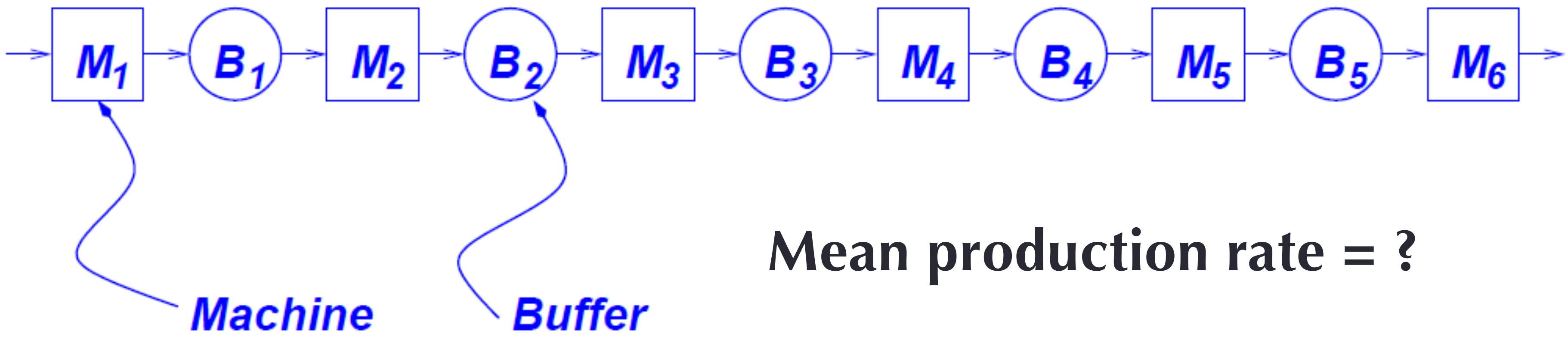
Course Content

1. *Introduction*
2. *Inventory Theory*
3. *Safety Stock in Manufacturing Systems*
4. *Elements of Queueing Theory*
5. *Productions Flows*
6. *Production Dipole*
7. ***Production Lines and Aggregation***
8. *Cooperative Flow Dynamics*
9. *Introduction to Queueing Networks*
10. *Supply Chain Analysis*
11. *Elements of Reliability Analysis*
12. *Maintenance Policies*

Linear Production Lines

Transfer line:

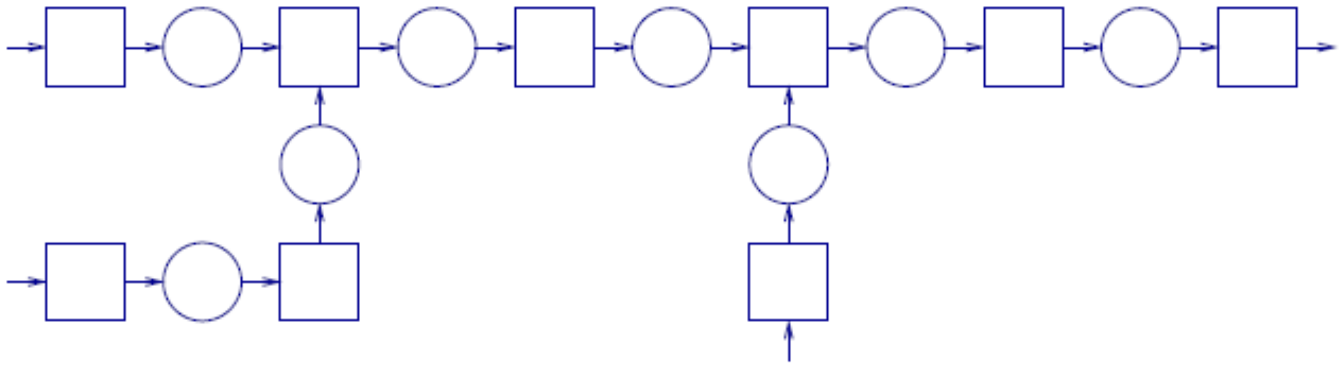
- Linear network of ***k*** **workstations or machines** (M_1, ..., M_k)
- **Separated by buffer storages** (B_1, ..., B_{k-1})



Mean production rate = ?

Study the **interactions between manufacturing stages**, and their (partial) **decoupling by using buffers**.

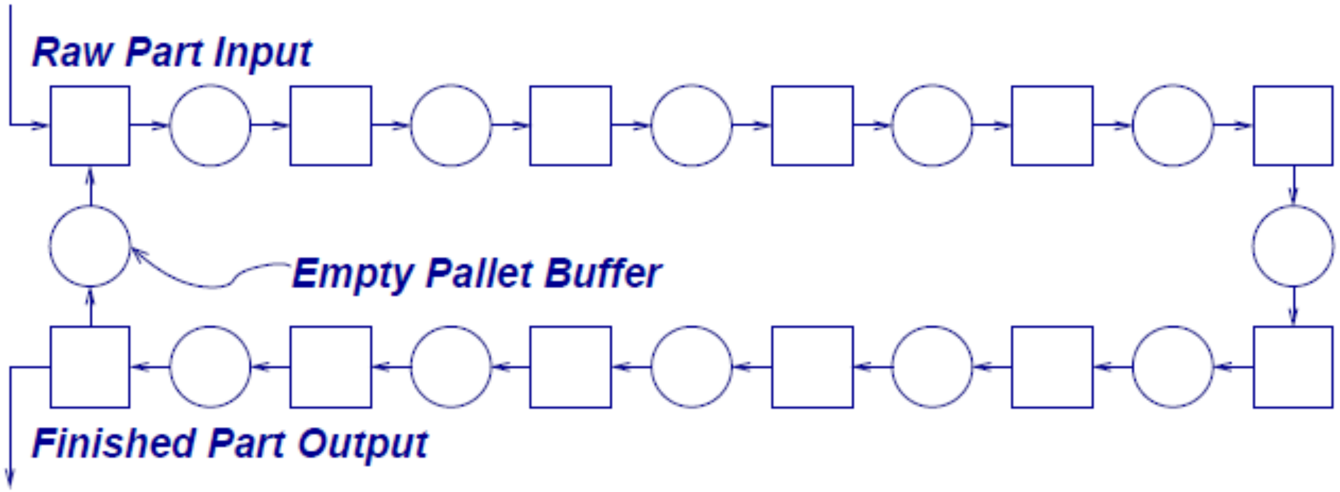
More Complex Production Lines



Assembly systems are *trees,* and may involve *thousands* of parts.

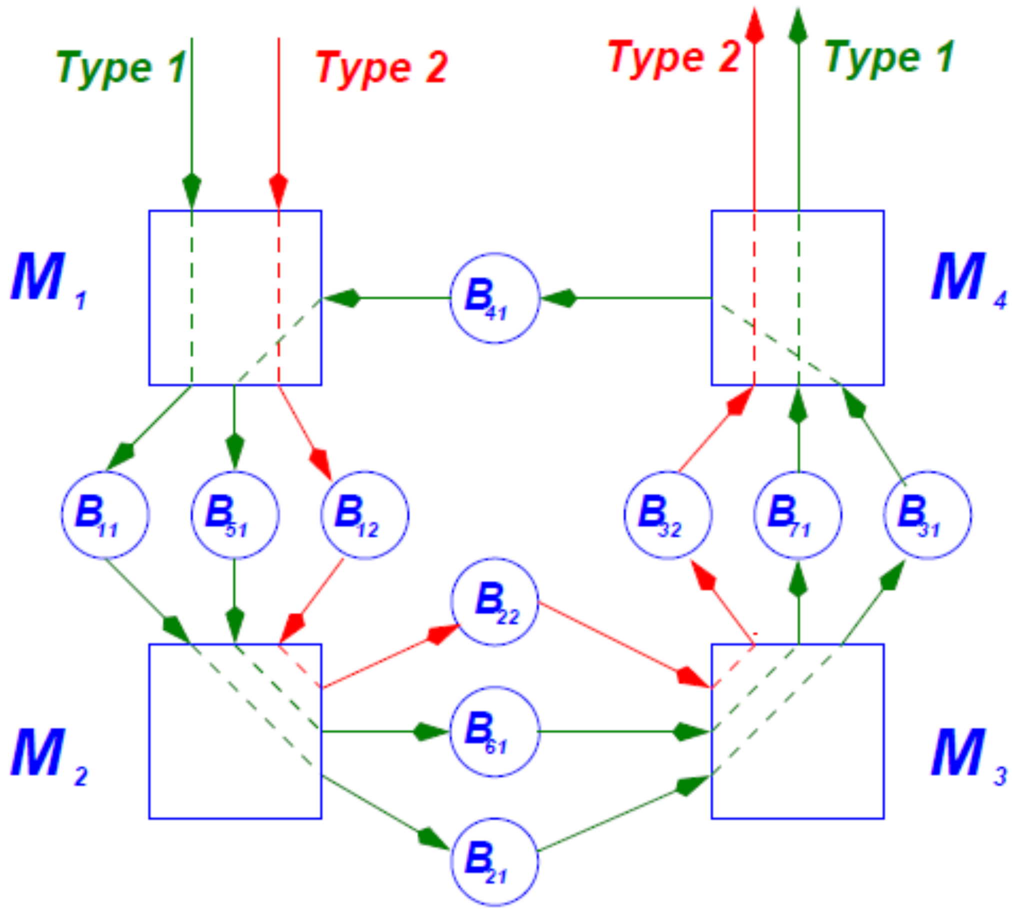
More Complex Production Lines

Limited number of pallets or fixtures:

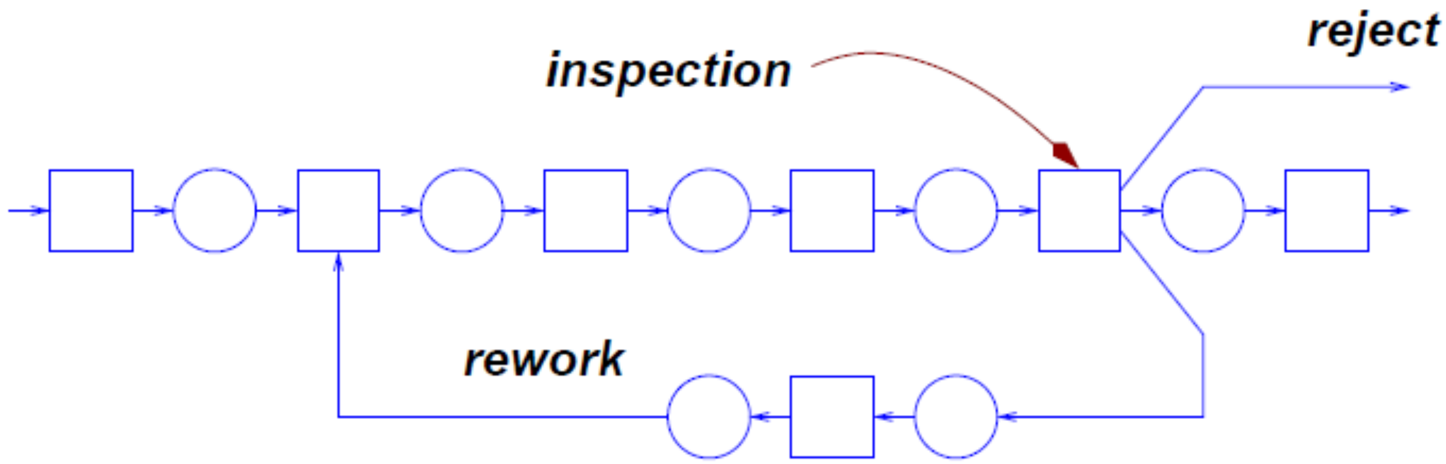


More Complex Production Lines

System with reentrant flow and two part types



More Complex Production Lines



Routes are random. The number of parts in the loop varies.

Linear Production Lines: Aggregation

General idea: iteratively replace two machines by an effective machine by using the formulae for the indisposabilty and the productivity of a production dipole

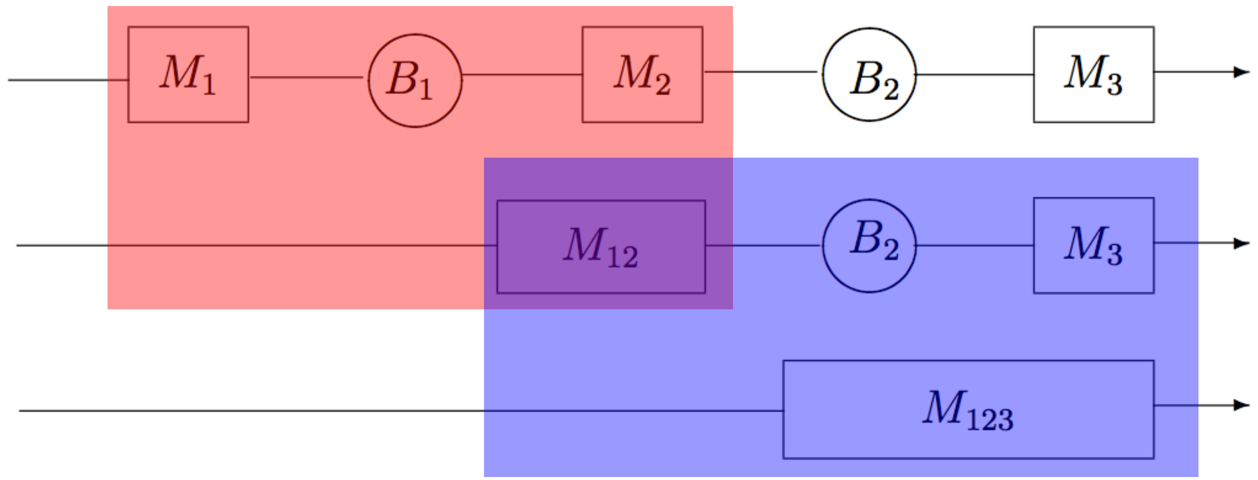
Two different **heuristics** to estimate the productivity of the production line:

- **Aggregation from upstream** (start with the two machines upstream, and so on)
- **Aggregation from downstream** (start with the two machines downstream, and so on)

Linear Production Lines: Aggregation from Upstream

Consider a production line with **3 workstations** (M_1, M_2 and M_3) with similar parameter set *{U,p,r}*, *I = p/r (indisposability)*, which are separated by **2 buffers** B_1 and B_2 with capacity h_1 and h_2 respectively.

- Start with the two machines upstream (M_1 and M_2) and the first buffer upstream B_1 => replace by effective machine with dipole characteristics (see Chapter 6).
- Then, replace the effective machine just created upstream and the remaining downstream machine M_3 and the second buffer B_2 by an effective machine characterizing the whole production line



Reminder: Production Dipole, Mean Production Rate

Under **stationary conditions**, the mean production rate of a production dipole with a buffer *B* of size h:

$$\langle U(h) \rangle = \frac{U}{1 + I_{dip}(h)}$$

$I_{dip}(h)$ is **the effective indisposability of the production dipole**:

$$I_{dip}(h) = I_1 \left\{ \frac{\left(\frac{\alpha}{\beta}\right)^2 e^{\Gamma \cdot h} - 1}{\frac{\alpha}{\beta} e^{\Gamma \cdot h} - 1} \right\} \quad \text{if } \alpha \neq \beta$$

$$I_{dip}(h) = I_1 \left\{ 1 + \frac{1}{1 + \frac{\alpha}{\alpha+1} F \cdot (1 + I_1)} \right\} \quad \text{if } \alpha = \beta$$

where:

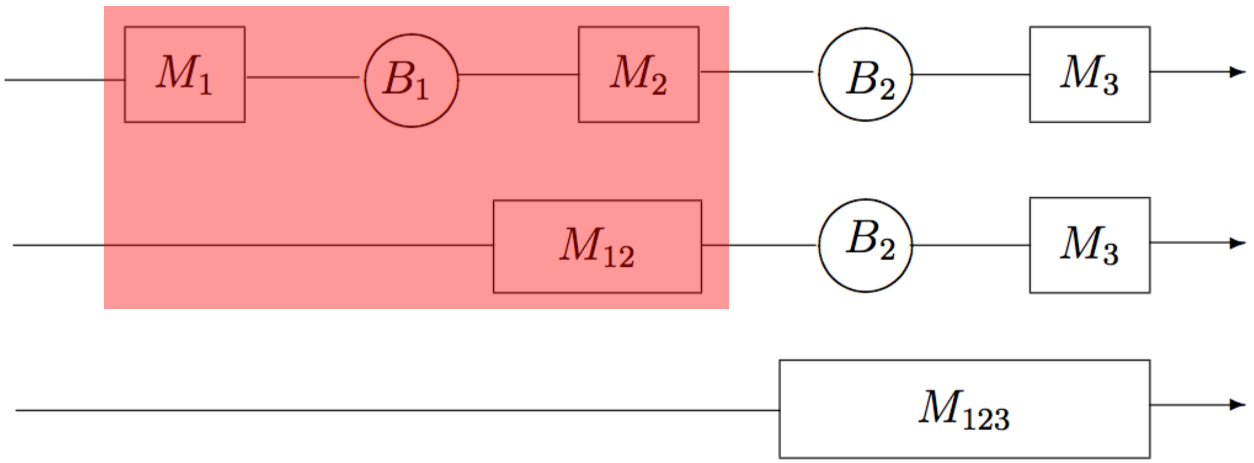
$$\Gamma = \frac{\alpha - \beta}{U} \left\{ \frac{r_1}{1+\alpha} + \frac{p_1}{1+\beta} \right\} \quad [1/\text{pce}]$$

$$F = \frac{r_1 \cdot h}{U}$$

Linear Production Lines: Aggregation from Upstream

- $I_{12}(h_1)$: effective indisposability of the dipole constituted of machines M_1 and M_2 and buffer B_1 (capacity h_1) => This dipole is identified as an effective machine M_{12} coupled via buffer B_2 to machine M_3
- We get: $I_{12}(h_1) = \quad I \cdot \left\{1 + \frac{1}{1+\frac{1}{2}F_{12}\cdot(1+I)}\right\} =: I \cdot \alpha_{12}$

$$F_{12} = \frac{r \cdot h_1}{U}$$

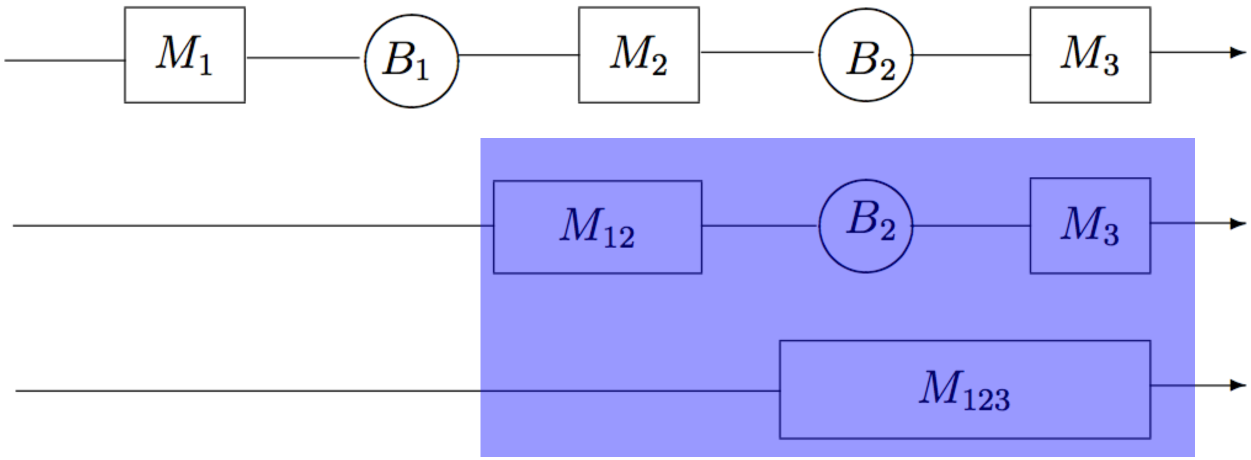


Linear Production Lines: Aggregation from Upstream

- $I_{123}(h_1, h_2)$: effective indisposability of the whole production line, constituted of effective machine M_{12}, machine M_3 and buffer B_2 (capacity h_2)
- We get:

$$\Gamma_{123} = \frac{\alpha_{12} - 1}{U}\left\{\frac{r}{1+\alpha_{12}} + \frac{p}{2}\right\}$$

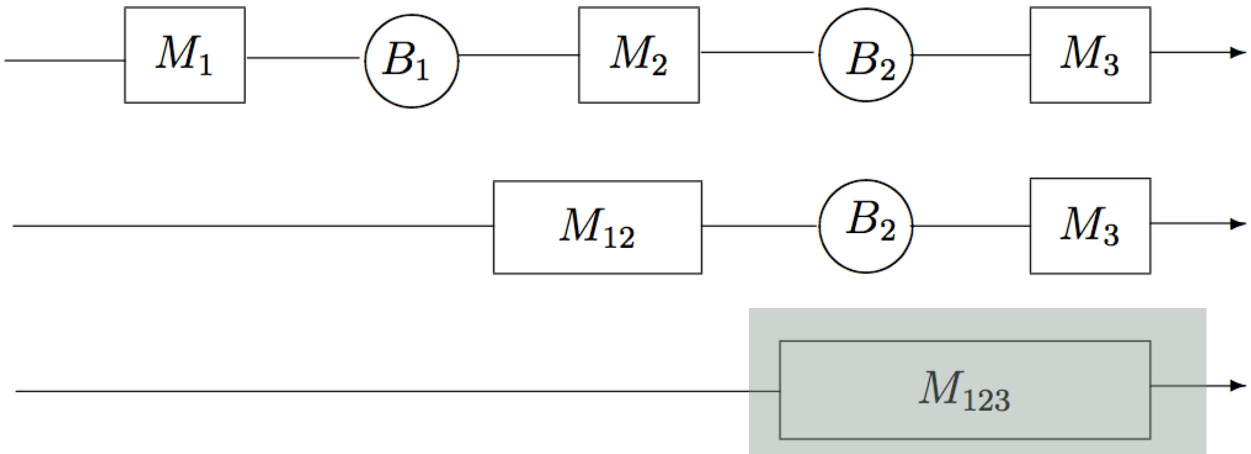
$$I_{123}(h_1, h_2) = I\left\{\frac{\alpha_{12}^2 e^{\Gamma_{123}\cdot h_2} - 1}{\alpha_{12} e^{\Gamma_{123}\cdot h_2} - 1}\right\}$$



Linear Production Lines: Aggregation from Upstream

- Under stationary conditions, the mean production rate of the production line can hence be by:

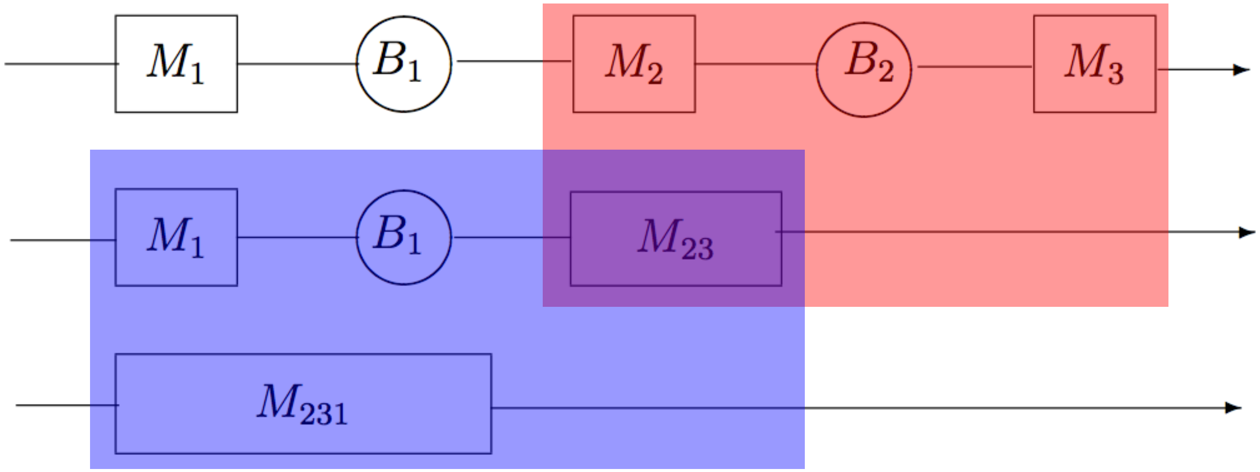
$$\langle U(h_1, h_2) \rangle = \frac{U}{1 + I_{123}(h_1, h_2)}$$



Linear Production Lines: Aggregation from Downstream

Consider a production line with **3 workstations** M_1, M_2 and M_3) with similar parameter set *{U,p,r}*, *I = p/r (indisposability)*, which are separated by **2 buffers** B_1 and B_2 with capacity h_1 and h_2 respectively.

- Start with the two machines downstream (M_2 and M_3) and the first buffer downstream B_2 => replace by effective machine with dipole characteristics (see Chapter 6).
- Then, replace the effective machine just created upstream and the remaining upstream machine M_1 and the firstbuffer B_1 by an effective machine characterizing the whole production line



Reminder: Production Dipole, Mean Production Rate

Under **stationary conditions**, the mean production rate of a production dipole with a buffer *B* of size h:

$$\langle U(h) \rangle = \frac{U}{1 + I_{dip}(h)}$$

$I_{dip}(h)$ is **the effective indisposability of the production dipole**:

$$I_{dip}(h) = \quad I_1 \left\{ \frac{\left(\frac{\alpha}{\beta}\right)^2 e^{\Gamma \cdot h} - 1}{\frac{\alpha}{\beta} e^{\Gamma \cdot h} - 1} \right\} \qquad \text{if } \alpha \neq \beta$$

$$I_{dip}(h) = \quad I_1 \left\{ 1 + \frac{1}{1 + \frac{\alpha}{\alpha+1} F \cdot (1 + I_1)} \right\} \qquad \text{if } \alpha = \beta$$

where:

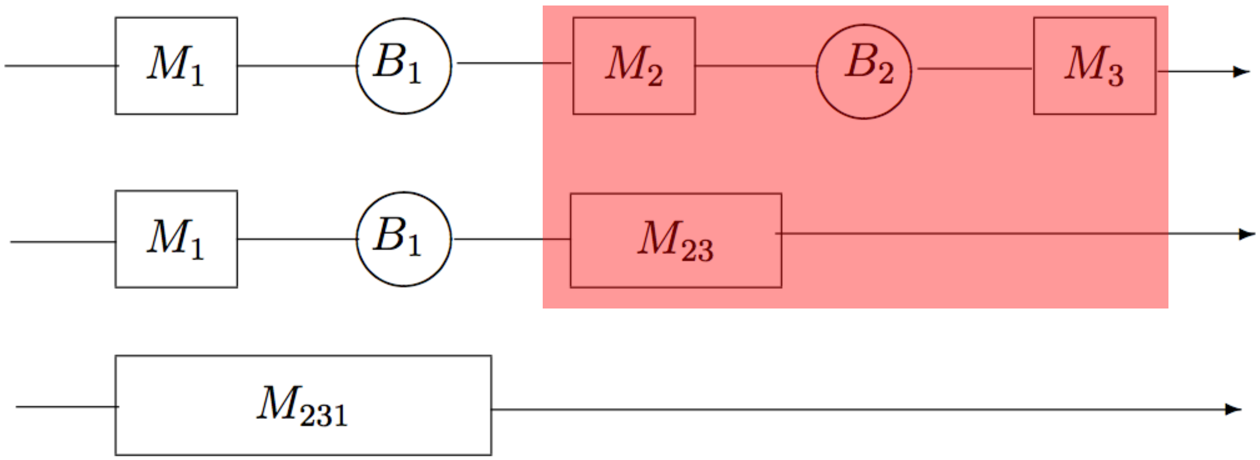
$$\Gamma = \frac{\alpha - \beta}{U} \left\{ \frac{r_1}{1 + \alpha} + \frac{p_1}{1 + \beta} \right\} \quad [1/\text{pce}]$$

$$F = \frac{r_1 \cdot h}{U}$$

Linear Production Lines: Aggregation from Downstream

- $I_{23}\ (h_2)$: effective indisposability of the dipole constituted of machines M_2 and M_3 and buffer B_2 (capacity h_2) => This dipole is identified as an effective machine M_{23} coupled via buffer B_1 to machine M_1
- We get: $$I_{23}(h_2) = \quad I \cdot \left\{1 + \frac{1}{1+\frac{1}{2}F_{23}\cdot(1+I)}\right\} =: I \cdot \alpha_{23}$$

$$F_{23} = \frac{\mu \cdot h_2}{U}$$

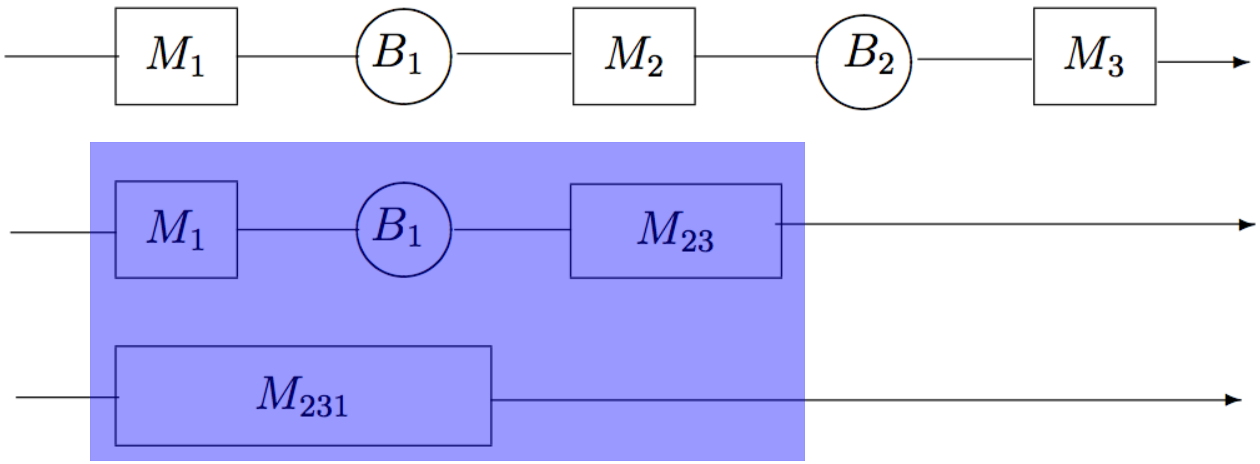


Linear Production Lines: Aggregation from Downstream

- $I_{231}(h_1, h_2)$: effective indisposability of the whole production line, constituted of effective machine M_{23}, machine M_1 and buffer B_1 (capacity h_2)
- We get:

$$\Gamma_{231} = \frac{\alpha_{23} - 1}{U} \left\{ \frac{r}{1 + \alpha_{23}} + \frac{p}{2} \right\}$$

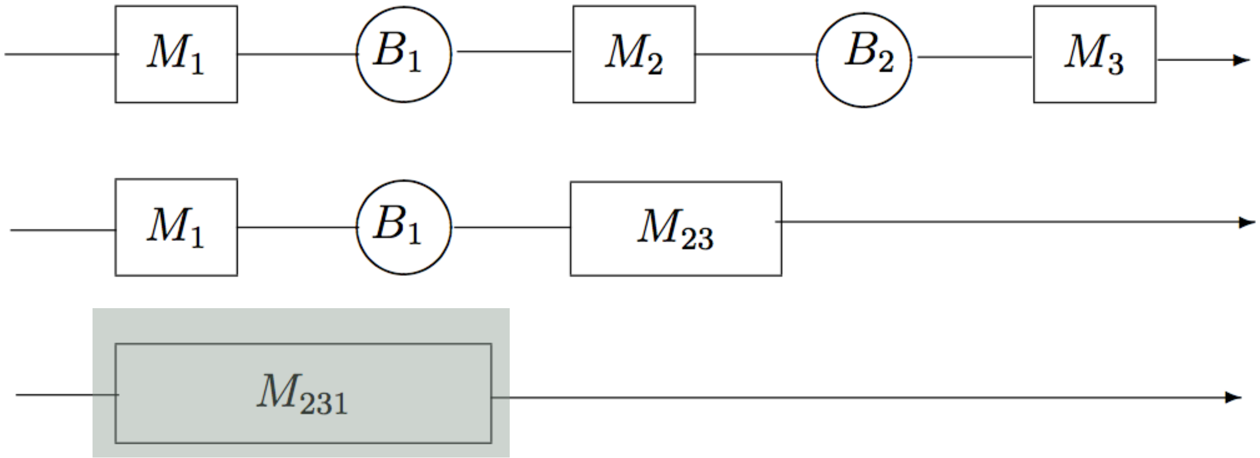
$$I_{231}(h_1, h_2) = I \left\{ \frac{\alpha_{23}^2 e^{\Gamma_{231} \cdot h_1} - 1}{\alpha_{23} e^{\Gamma_{231} \cdot h_1} - 1} \right\}$$



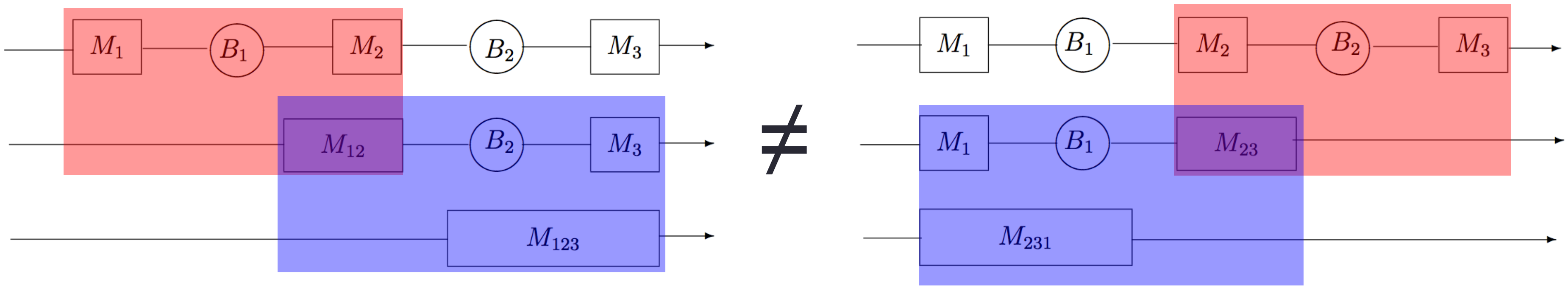
Linear Production Lines: Aggregation from Downstream

- Under stationary conditions, the mean production rate of the production line can hence be by:

$$\langle U(h_1, h_2) \rangle = \frac{U}{1 + I_{231}(h_1, h_2)}$$



Linear Production Lines: Aggregation Order



- In general, aggregation from upstream and from downstream do not yield the same results: **aggregation is order-dependent**
- Following [Terracol & David, 1981], the best approximation is reached **when machines are regrouped in the order of increasing buffer stock capacities** (we first regroup the two adjacent machines which are separated by the buffer with smallest capacity, and so on)

Linear Production Lines: Aggregation

Example 8 We consider the above production line composed of 3 work-stations with similar parameter set $\{U, p, r\}$, when $h_1 = 0$ (i.e., no buffer between machines M_1 and M_2). For the upstream aggregation, the above formulae become:

$$
\begin{aligned}
I_{12}(0) &= 2I \quad (\text{i.e., } \alpha_{12} = 2) \\
\Gamma_{123} &= \frac{1}{U}\left\{\frac{r}{3} + \frac{p}{2}\right\} \\
I_{123}(0, h_2) &= I\left\{\frac{4e^{\Gamma_{123} \cdot h_2} - 1}{2e^{\Gamma_{123} \cdot h_2} - 1}\right\}
\end{aligned}
$$

Note that, in this situation, we obtain an exact result (not an approximation).

For the downstream aggregation, we obtain the following results:

$$
\begin{aligned}
F_{23} &= \frac{r \cdot h_2}{U} \\
\alpha_{23} &= \left\{1 + \frac{1}{1 + \frac{1}{2}F_{23} \cdot (1 + I)}\right\} \\
I_{231}(0, h_2) &= I\left\{\alpha_{23} + 1\right\} > I_{123}(0, h_2)
\end{aligned}
$$

which overestimates the indisposability. Differences are depicted in Figure 7.4

Linear Production Lines: Aggregation

We consistently have:

$$\lim_{h_2 \to \infty} I_{123}(0, h_2) = \lim_{h_2 \to \infty} I_{231}(0, h_2) = 2I$$
$$\lim_{h_2 \to 0} I_{123}(0, h_2) = \lim_{h_2 \to 0} I_{231}(0, h_2) = 3I$$

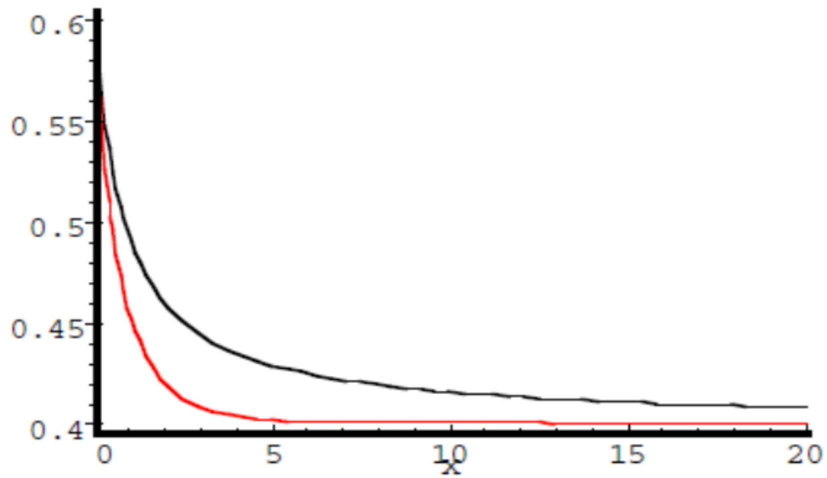


Figure 7.4: Aggregation of 3 identical machines with $I = 0.2$, when $h_1 = 0$. The lower curve corresponds to the indisposabilty when using upstream aggregation and is (in this case) the correct result (as it is exact). The upper curve corresponds to the downstream aggregation, and (in this case) overestimates the indisposability.

Linear Production Lines: AnyLogic

Exercise 27 Explore the *AnyLogic* model entitled "*Activity Based Costing Analysis*", found in the Example Section entitled *Manufacturing*. Change the buffer capacities, and observe the influence on in-process-inventory, backlog (i.e., shortage), and costs. ⊙

Exercise 28 Simulate with *AnyLogic* a production line composed of 3 identical machines, and observe the mean production rate of the production line, and its variance over time. ⊙