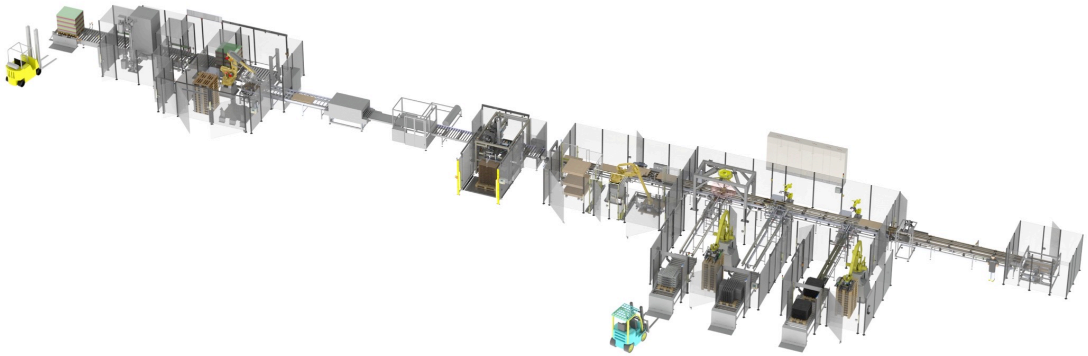

MANUFACTURING SYSTEMS AND SUPPLY CHAIN DYNAMICS

Chapter 6: Production Dipole

EPFL, Master MT

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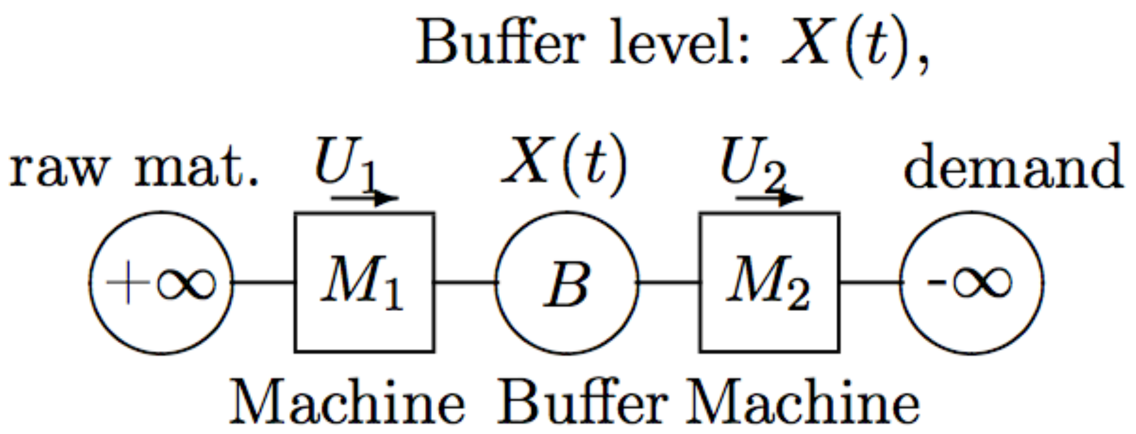
Introduction

Preceding chapters: manufacturing system with single workstation

Now, manufacturing production line composed **of two successive machines**

- M_1 with production capacity U_1 (when operating) and indisposability I_1
- M_2 with production capacity U_2 (when operating) and indisposability I_2

and an intermediate buffer B of capacity h

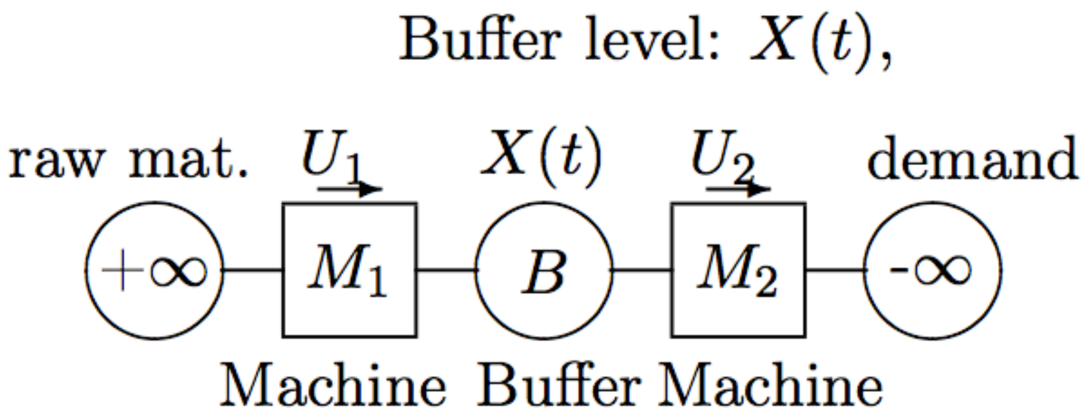


Introduction

Such a manufacturing system if called **a production dipole** or a **two-stage transfer line**

Assumptions: **permanent supply** of raw material and **permanent absorbing demand**

Question: **what is the mean and variance of the production output of such a production line?**



Intermediate Buffer – Practical Illustrations

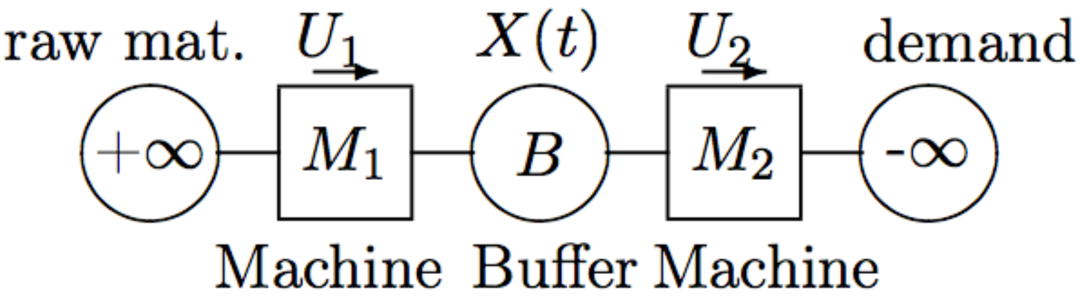
Production Dipole

Isolated mean production rate of $\boldsymbol{M_1} = \frac{\boldsymbol{U_1}}{\boldsymbol{1+I_1}}$

Isolated mean production rate of $\boldsymbol{M_2} = \frac{\boldsymbol{U_2}}{\boldsymbol{1+I_2}}$

Production rate of the dipole *M*, if machines M_1 and M_2 are linked via a buffer?

Buffer level: $X(t)$,

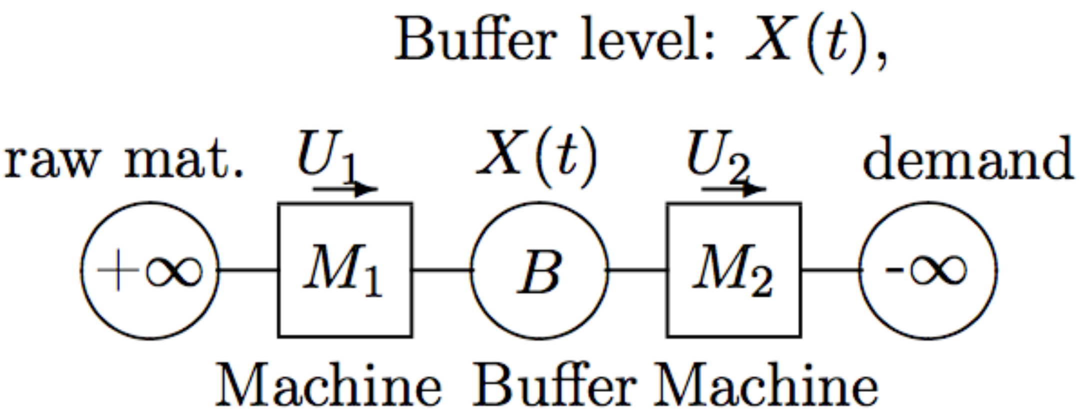


Production Dipole

In general: no closed formula for the mean production rate of the dipole M

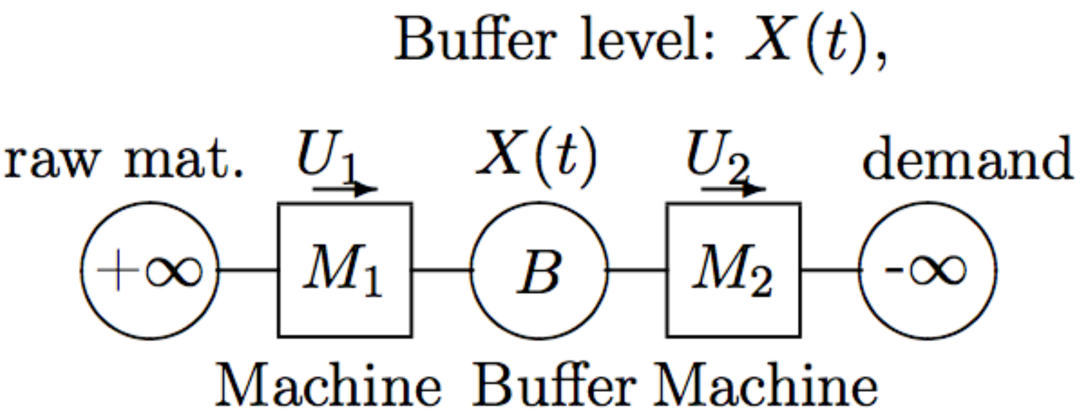
We look for an **approximation** of the form:

$$\text{mean production rate of } \boldsymbol{M} \approx \frac{\boldsymbol{U_d}}{\boldsymbol{1+I_d}}$$



Production Dipole: AnyLogic

Exercise 22 Using *AnyLogic*, simulate the behavior of a production dipole for two identical machines, with constant cycle time $\tau_c = 1$ minute, MTTF $= 80$ minutes, and MTTF $= 10$ minutes. Observe, for different buffer sizes, the mean cumulated production (over a fixed time horizon, *e.g.* 10000 minutes), the mean production rate, and its variance over time. ⊙



Production Dipole: Hypothesis

(H1) The **hydrodynamic assumption** applies (i.e., the mean-time-to-failure MTTF and the mean-time-to-repair MTTR are much longer than the cycle times of the two machines).

(H2) If the buffer B is empty and M_1 is down, M_2 is starved and cannot produce. If the buffer is full and M_2 is down, M_1 is blocked and cannot produce.

(H3) Blocked or starved machines cannot break down.

(H4) Conveyor time between buffer and machines is negligible.

Production Dipole: Hypothesis

(H5) Time-to-failure TTF of machines M_1 (resp. M_2) is exponentially distributed with parameter p_1 (resp. $\boldsymbol{p_2 = \alpha p_1}$).

(H6) Time-to-repair TTR of machines M_1 (resp. M_2) is exponentially distributed with parameter r_1 (resp. $\boldsymbol{r_2 = \beta r_1}$).

(H7) We suppose that the **production capacities** U_1 and U_2 of the two machines are **identical**.

We introduce the following notations:

$$p_2 = \alpha p_1, \quad r_2 = \beta r_1 \Rightarrow I_2 = \frac{\alpha}{\beta} I_1, \quad U_1 = U_2 = U.$$

Production Dipole: Mean Production Rate

Under **stationary conditions** and the above hypothesis (H1-H7), mean production rate of a production dipole with a buffer *B* of size *h*:

$$\langle U(h) \rangle = \frac{U}{1 + I_{dip}(h)}$$

$I_{dip}(h)$ is **the effective indisposability of the production dipole**:

$$I_{dip}(h) = \quad I_1 \left\{ \frac{\left(\frac{\alpha}{\beta}\right)^2 e^{\Gamma \cdot h} - 1}{\frac{\alpha}{\beta} e^{\Gamma \cdot h} - 1} \right\} \qquad \text{if } \alpha \neq \beta$$

$$I_{dip}(h) = \quad I_1 \left\{ 1 + \frac{1}{1 + \frac{\alpha}{\alpha+1} F \cdot (1 + I_1)} \right\} \qquad \text{if } \alpha = \beta$$

where:

$$\Gamma = \frac{\alpha - \beta}{U} \left\{ \frac{r_1}{1+\alpha} + \frac{p_1}{1+\beta} \right\} \quad [1/\text{pce}] \qquad F = \frac{r_1 \cdot h}{U}$$

Production Dipole

Exercise 23 Consider the situation of a production dipole when there is **no intermediate buffer** between machines M_1 and M_2 ($h = 0$). Calculate $I_{dip}(0)$ and interpret the result. ⊙

Exercise 24 Consider the situation of a production dipole when there is an **intermediate buffer of unlimited size** between machines M_1 and M_2 ($h = \infty$). Calculate $\lim_{h \to \infty} I_{dip}(h)$ in both of the following cases:

- $\alpha > \beta$ (i.e., $I_2 > I_1$)
- $\alpha < \beta$ (i.e., $I_2 < I_1$)

to show that

$$I_{dip(\infty)} = \max\{I_1, I_2\}$$

Interpret the results. ⊙

Production Dipole: Observations

- The most unavailable machine always determines the global effective production flow of the whole production dipole.

- $I_{dip}(h)$ is decreasing with h => **mean production rate of the production dipole increases when the size h of the intermediate buffer gets larger**

$$I_{dip}(\infty) \leq I_{dip}(h) \leq I_{dip}(0) \Leftrightarrow \langle U(0) \rangle \leq \langle U(h) \rangle \leq \langle U(\infty) \rangle$$

- **$I_{dip}(h)$ lies between $I_1 + I_2$ (when h=0) and max{I_1, I_2} (when h=∞)**

- Increasing the buffer capacity h has beneficial influence on the productivity. However, this will inevitably also **increase in-process inventory** and therefore also increase costs.

Production Dipole

Exercise 25 In a production facility producing spare parts for the IT industry, a production manager has to decide on the size h of an intermediate buffer B to be implemented between the two workstations making the two successive production steps to produce a given product. The production capacity of both machines is 2 [*pieces*/*minute*]. The time-to-failure of both machines is exponentially distributed with parameters $p_1 = 1/60$ [$1/minute$] and $p_2 = 1/80$ [$1/minute$]. The time-to-repair of both machines is exponentially distributed with parameters $r_1 = 1/5$ [$1/minute$] and $r_2 = 1/4$ [$1/minute$]. Every additional space in the intermediate buffer costs 1000 CHF to implement. Every item that goes out of the production facility has a market value of 1 CHF.

From an economic perspective, what buffer size h_1 or h_2 should the production manager favor if s/he considers a time horizon of one year for the return on investment for the implementation of the intermediate buffer?

Hint: One year is composed of 250 working days, and each working day is composed of 8 hours. We suppose that there is enough demand for the items that go out of the production facility. We do not consider the costs related to in-process inventory. ⊙

Production Dipole: Fluctuations Around the Mean

Approximation: consider the dipole as an isolated machine M_{dip} with a mean production rate $\langle U(h) \rangle$, and a mean cumulated productivity for a time horizon H given by:

$$\langle \Sigma(\mathcal{H}) \rangle = \langle U(h) \rangle \mathcal{H} = \frac{U\mathcal{H}}{1 + I_{dip}(h)}$$

Accordingly, we find:

$$\sigma^2_{\Sigma(\mathcal{H})}(h) = 2\frac{\langle U(h) \rangle^2 I_{dip}(h)}{r_{dip}(1 + I_{dip}(h))^3} \quad [pce^2]$$

With: $r_{dip} = \min\{r_1, r_2\}$

Production Dipole: Fluctuations Around the Mean

Exercise 26 Compare two production dipoles with identical machines ($\alpha = \beta = 1$) but with respective indisposabilities $I = 0.1$ and $I = 0.15$.

(i) As a function of h, varying from 0 to 1000, plot the values $I_{dip}(h)$ for fixed $r/U = 0.002$.

(ii) As a function of h, varying from 0 to 1000, plot the values $\sigma^2_{\Sigma(\mathcal{H})}(h)$ for fixed $r/U = 0.002$ and $U = 50$ $[pce/sec]$ and $\mathcal{H} = 3 \cdot 3600$ $[sec]$.

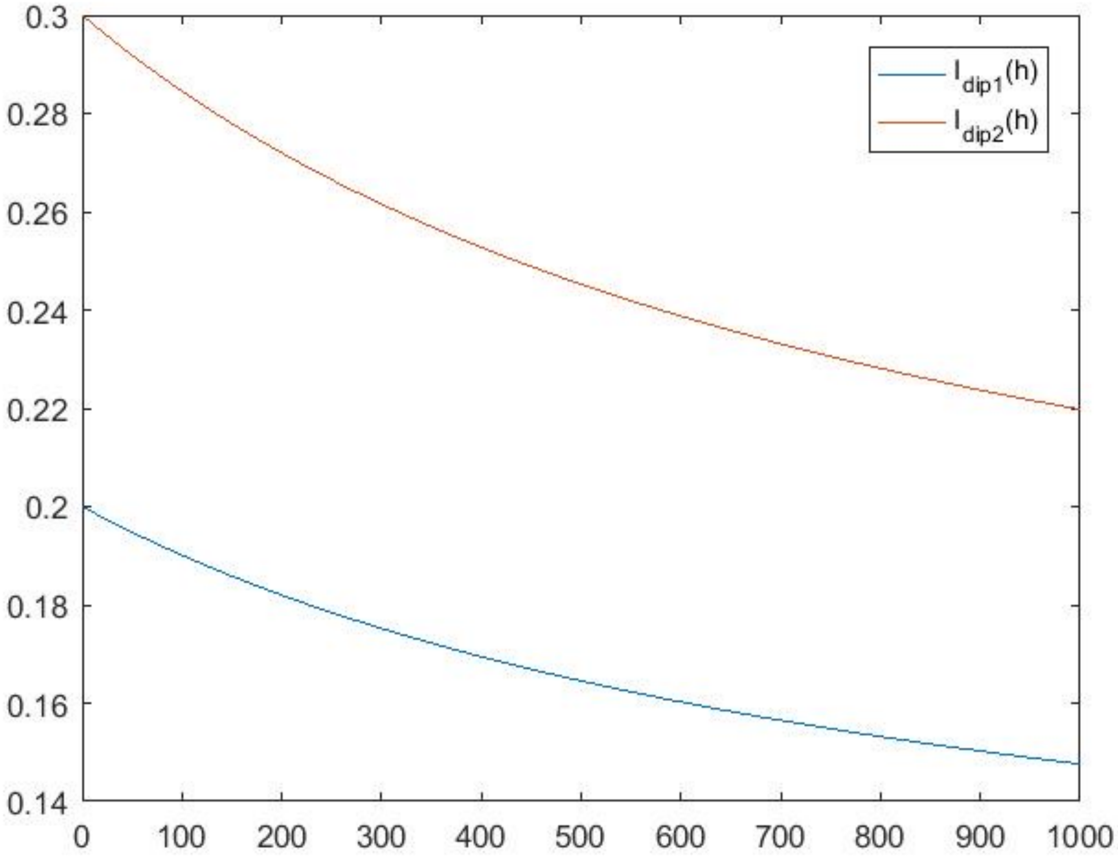
Observe that for $I_{dip}(h) < 0.1$:

- **Increasing** the buffer capacity ***h* reduces the variance** of the cumulated production of the production dipole
- **Increasing** the buffer capacity ***h* increases the mean** production cumulated production of the production dipole

Production Dipole

Production dipole with **identical machines** with:

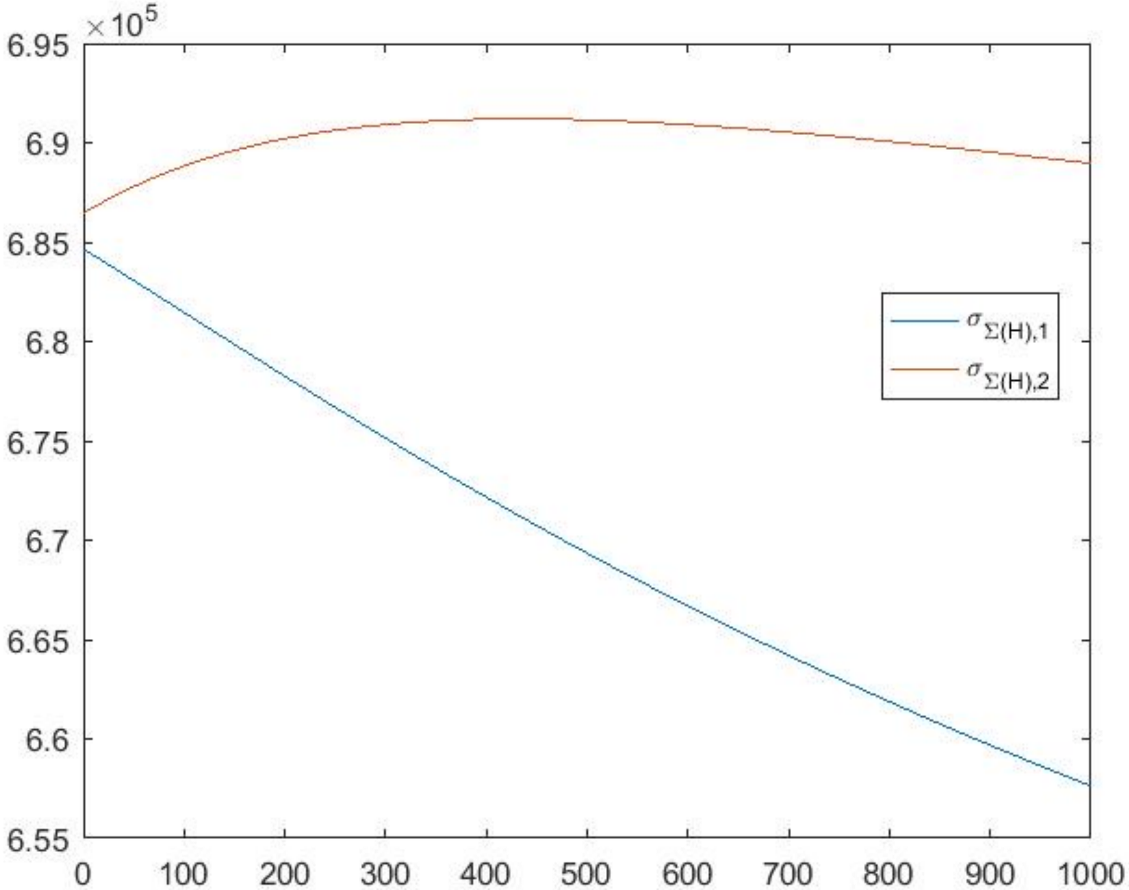
$l_1 = l_2 = 0.1$ and $l_1 = l_2 = 0.15$



Production Dipole: Fluctuations Around the Mean

Production dipole with **identical machines** with:

$I_1 = I_2 = 0.1$ and $I_1 = I_2 = 0.15$



Production Dipole: Different Machines

The effective indisposability $I_{dip}(h)$ of the production dipole is only valid when $\boldsymbol{U_1 = U_2 = U}$

When $\boldsymbol{U_1 \neq U_2}$ **:** we can renormalize p_1 by **equalizing the mean production of M_1 and M_2** and define $\tilde{p}_1$ as:

$$\frac{U_1}{1+I_1} = \frac{U_1}{1+\frac{p_1}{r_1}} = \frac{U_2}{1+\frac{\tilde{p}_1}{r_1}} = \frac{U_2}{1+\tilde{I}_1}$$

Real machine $M_1 : \{U_1, p_1, r_1\}$

Effective machine $\tilde{M}_1 : \{U_2, \tilde{p}_1, r_1\}$

The real and the effective machine have **same mean productivity**.
The instantaneous production rate of $\tilde{M}_1$ is equal to U_2.
The results exposed in this chapter are now applicable for the machines $\tilde{M}_1$ and M_2