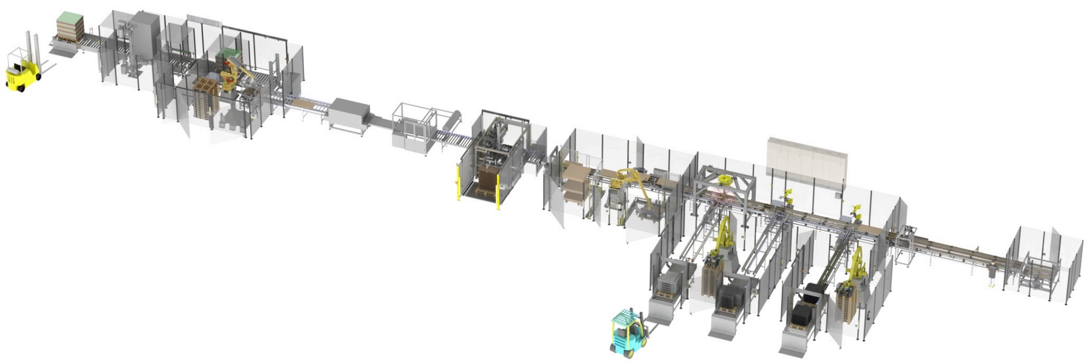

MANUFACTURING SYSTEMS AND SUPPLY CHAIN DYNAMICS

Chapter 5: Production Flows

EPFL, Master MT

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Course Content

Introduction

M

We consider a **single production unit M**

- Machine
- Worker

Constant cycle time of τ_c seconds ($U = 1/\tau_c$ is the production rate)

We consider a **time horizon** $H \gg \tau_c$ seconds, during which the production unit will be operated

During this time horizon H, the productivity of M can undergo **random (or non-random) failures** of diverse nature.

Characterization of a Machine (for production flows)

Is the machine M **operational** ?

If operational, at what **rate** does M produce ?

What is the **maximum production rate** of M ?

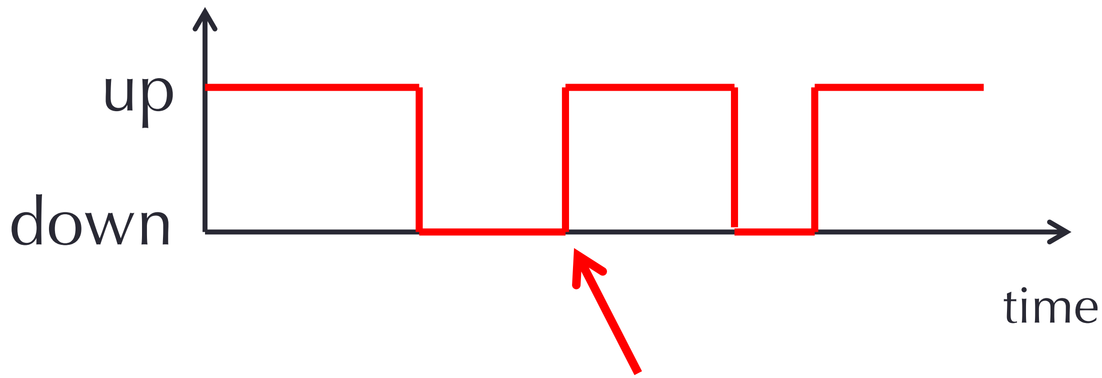
Operational state of M: **up or down**

Production rate of M: **number of parts per time unit**

Capacity of M: **maximum production rate *U***

Characterization of a Machine (for production flows)

Operational state of M:



(often) random

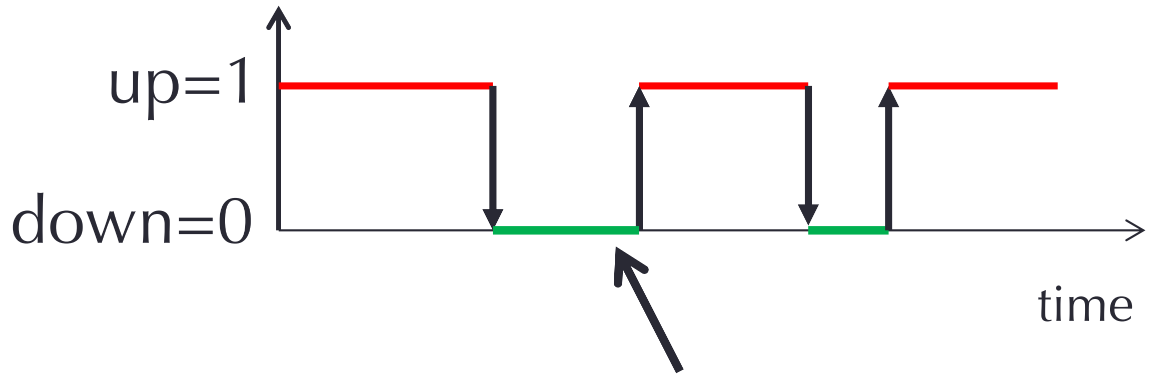
0 ≤ Production rate *u* of M ≤ Capacity *U*

(often) controllable

(often) fixed

Characterization of a Machine (for production flows)

Operational state of M:



Breakdown distribution

Repair distribution

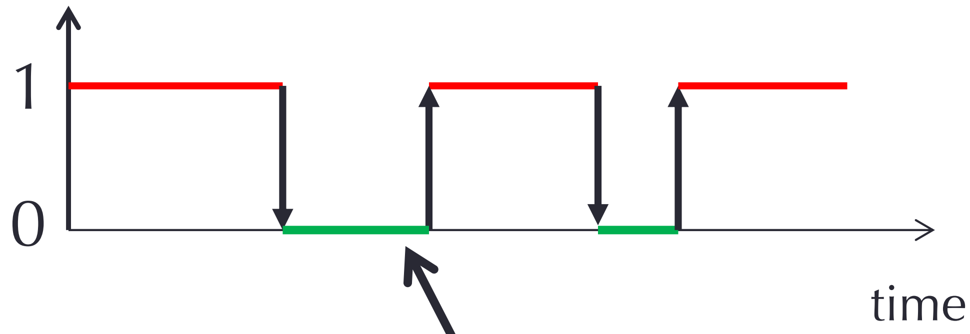
0 ≤ Production rate *u* of M ≤ Capacity *U*

controllable

[parts/time]

Characterization of a Machine (for production flows)

$\chi(t)$ = **Operational state** of M:



Breakdown: f(t)
Repair: g(t)

0 ≤ Production rate *u* of M ≤ Capacity *U*

For simplicity:
production rate = *U* × χ (t)

[parts/time]

Characterization of a Machine (for production flows)

$$\mathbf{M} = \{U, f, g\}$$

U denotes the **capacity**

f defines the (random) **time-before-failure**

g defines the (random) **time-(needed)-to-repair**

Characterization of a Machine (for production flows)

f and *g* hard to derive → **mean values 1/*p*** and **1/*r*** (are measurable)

M={*U*, *p*, *r*}

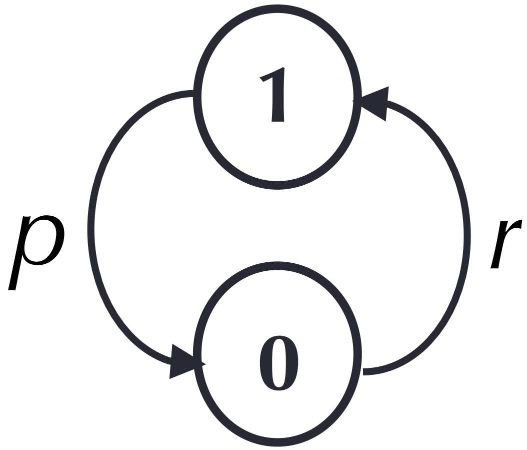
$\frac{1}{p}$:= mean time before failure $= \int_0^\infty t f(t) dt$

$\frac{1}{r}$:= mean time to repair $= \int_0^\infty t g(t) dt$

Characterization of a Machine (for production flows)

$$\mathbf{M=\{\mathit{U}, \mathit{p}, \mathit{r}\}}$$

Time-continuous
Markov chain



U denotes the **capacity**

p is the **rate of breakdown**

r is the **rate of repair**

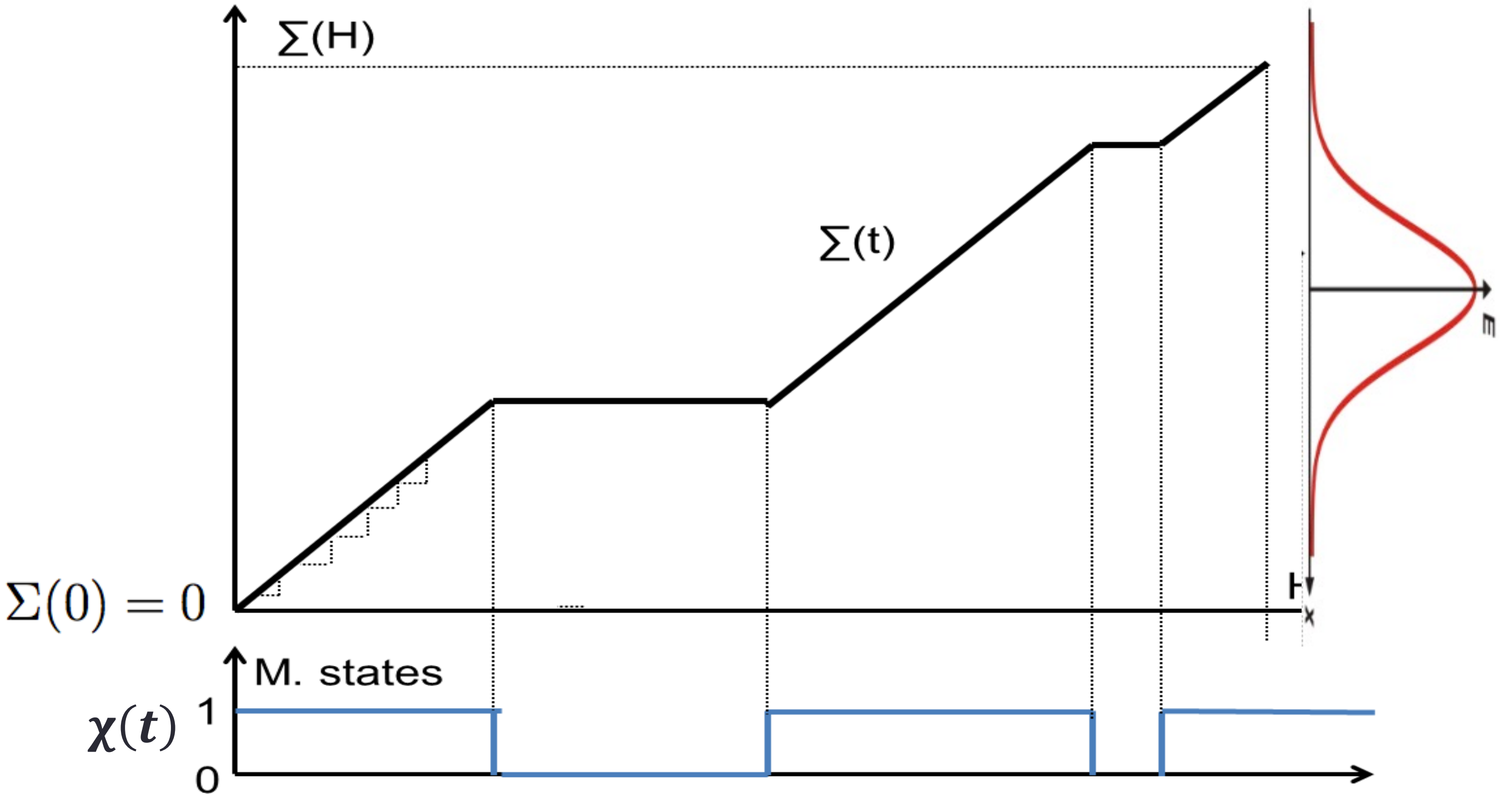
Hydrodynamic Assumption for M = {U, p, r}

$$\frac{1}{U} \ll min\left\{\frac{1}{p},\frac{1}{r}\right\}$$

Time needed to produce one part (cycle time) is much smaller than the mean-time-before-failure and the mean-time-to-repair

=> The behavior of production flow is like that of a **fluid** (no discrete parts)

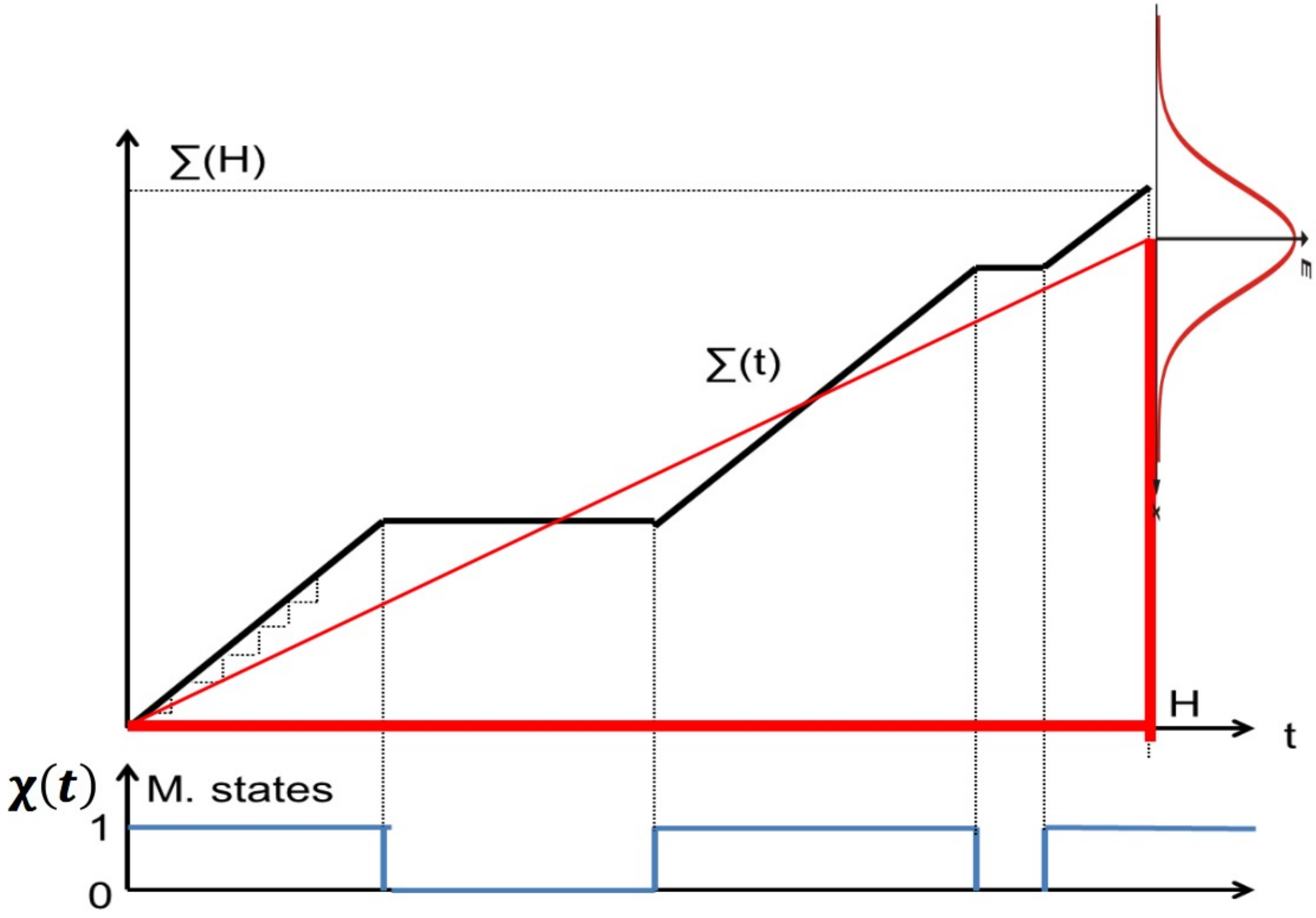
Hydrodynamic Assumption for M = {*U*, *p*, *r*}



$\Sigma(t)$: cumulated production up to time t

$$\Sigma(\mathcal{H}) = \int_0^{\mathcal{H}} U \cdot \chi(t)\, dt$$

Mean Production Rate (for Large Time Horizon *H*)

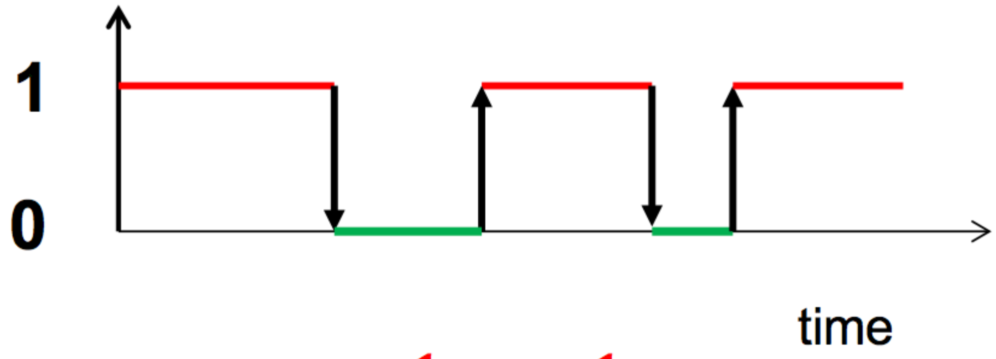


$$m = lim_{H\to\infty}\frac{\Sigma(H)}{H} = lim_{H\to\infty}\frac{1}{H}\int_0^H U \times \chi(t)dt$$

Hypothesis: *U* is Constant

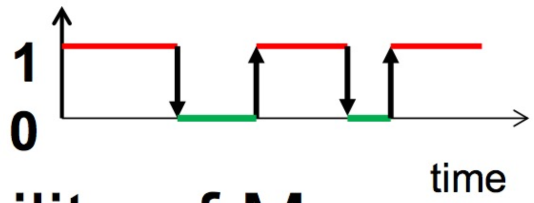
$$\mathbf{m} = lim_{H\to\infty}\frac{1}{H}\int_0^H U \times \chi(t)dt = U \times lim_{H\to\infty}\frac{1}{H}\int_0^H \chi(t)dt$$

$$lim_{H\to\infty}\frac{1}{H}\int_0^H \chi(t)dt = \langle\chi\rangle = \frac{r}{r+p}$$



$$\frac{r}{r+p} = \frac{\frac{1}{p}}{\frac{p+r}{rp}} = \frac{\frac{1}{p}}{\frac{1}{p}+\frac{1}{r}} = \frac{MTF}{MTF+MTR}$$

Indisposability of M



Define I, the indisposability of M:

$$\mathrm{I} := \frac{MTR}{MTF} = \frac{1/r}{1/p} = \frac{p}{r}$$

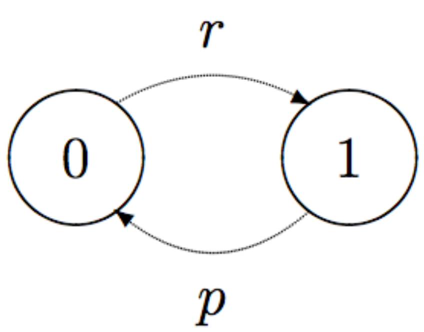
With this definition we have

$$lim_{H\to\infty} \frac{1}{H}\int_0^H \chi(t)dt = \frac{r}{r+p} = \frac{r}{r(1+\frac{p}{r})} = \frac{1}{1+I}$$

(in applications, I should be smaller than 0.2)

Indisposability of M: Derivation

Example 35 Suppose that $X_t \in \{0,1\}$ reflects the state of a machine (1 means up, 0 means down). If the machine time-to-failure (passing from 1 to 0) is exponentially distributed with mean $1/p$ hours and if the machine time-to-repair (passing from 0 to 1) is exponentially distributed with mean $1/r$ hours, we obtain the following Markov chain (note that the links are labelled with rates and that there are no self-loops):



The balance equations for $\pi_0 = \lim_{t\to\infty} P(X_t = 0)$ and $\pi_1 = \lim_{t\to\infty} P(X_t = 1)$ are:

$$\begin{aligned} r\pi_0 &= p\pi_1 \\ p\pi_1 &= r\pi_0 \end{aligned}$$

Indisposability of M: Derivation

Using probability normalization ($\pi_0 + \pi_1 = 1$), we find:

$$\pi_0 = \frac{p}{r+p} = \frac{I}{1+I}, \quad \pi_1 = \frac{r}{r+p} = \frac{1}{1+I}$$

where, like in the discrete case, $I = \frac{p}{r}$ is the **indisposability** of the machine. π_1 represents the fraction of time the machine is operational. If U is the production rate of the machine when the machine is operational, $U\pi_1$ represents the mean production rate of the machine and is an important performance measure. $\diamond$

Indisposability of M: Intuition

N: number of on-off cycles of machine M during the time horizon *H*

For large *H*, we have approximately: $\mathcal{H} \approx N\left(\frac{1}{r} + \frac{1}{p}\right)$

N/p: time-span during which M is operational

N/r: time-span during which M is down

The **efficiency of the machine** the relative fraction of time during which M was operational:

$$\text{Efficiency} = \frac{N\frac{1}{p}}{N\left(\frac{1}{r} + \frac{1}{p}\right)} = \frac{1}{1+I}$$

Indisposability of M: Practical Computation

Exercise 19 Explain how you would quantify the indisposability factor I in a practical situation. $\odot$

Mean Production Rate

$$\boldsymbol{m} = \boldsymbol{lim}_{\boldsymbol{H}\to\infty} \frac{\sum(\boldsymbol{H})}{\boldsymbol{H}} = \frac{\boldsymbol{U}}{\boldsymbol{1}+\boldsymbol{I}}$$

mean cumulated production $\langle\sum(\boldsymbol{H})\rangle$ (for large $\boldsymbol{H}$):

$$\langle\sum(\boldsymbol{H})\rangle = \boldsymbol{mH} = \frac{\boldsymbol{U}}{\boldsymbol{1}+\boldsymbol{I}}\,\boldsymbol{H}$$

with indisposability I of M={U,p,r}: $\mathrm{I} = \frac{\boldsymbol{p}}{\boldsymbol{r}}$

Production Line with Successive Workstations

Manufacturing system composed of several successive workstations

- All the workstations must have (approximately) the **same mean stationary production rate** => manufacturing system is **balanced**
- **Unbalanced manufacturing systems** operate **sub-optimally** even in absence of failures and in presence of large internal buffers to absorb variability.
- In an unbalanced manufacturing system, the **workstation with the smallest mean stationary production rate** is the **bottleneck** and will define the production rate of the whole line

AnyLogic

Using ***AnyLogic***: Simulate the mean cumulated production and compare simulated values with theoretical ones.

Mean Production Rate

$$\boldsymbol{m = lim_{H \to \infty} \frac{\Sigma(H)}{H} = \frac{U}{1 + I}}$$

However, tells nothing about the fluctuations around the mean value. However, **fluctuations are real and important**!

Variance of the Production Rate

O: operating time of the machine M before failure
R: repair time

Variances:

$$\sigma_O^2 := E\big((O - E(O))^2\big) = \int_0^\infty \left(t - \frac{1}{p}\right)^2 f(t)\, dt$$

$$\sigma_R^2 := E\big((R - E(R))^2\big) = \int_0^\infty \left(t - \frac{1}{r}\right)^2 g(t)\, dt$$

Squared **coefficient of variation**: $SCV = \dfrac{\text{variance}}{\text{mean}^2}$

$SCV_O = \sigma_O^2 p^2$ (operating time) $\quad SCV_R = \sigma_R^2 r^2$ (repair time)

Variance of the Production Rate

Example 7: exponential operating and repair times

$$\sigma_O^2 = \int_0^\infty \left(t - \frac{1}{p}\right)^2 f(t)\,dt = \frac{1}{p^2}$$

$$\sigma_R^2 = \int_0^\infty \left(t - \frac{1}{r}\right)^2 g(t)\,dt = \frac{1}{r^2}$$

Then: $SCV_O = SCV_R = 1$

Variance of the Production Rate

Result valid for **large time horizons *H*** (using central limit theorem):

$$P\Big(x \leq \Sigma(\mathcal{H}) \leq x + dx\Big) = \frac{1}{\sqrt{2\pi\sigma^2_{\Sigma(\mathcal{H})}\mathcal{H}}} \exp\Big\{-\frac{(x - m\mathcal{H})^2}{2\sigma^2_{\Sigma(\mathcal{H})}\mathcal{H}}\Big\}dx$$

With: $\sigma^2_{\Sigma(\mathcal{H})} = (SCV_O + SCV_R)\dfrac{U^2 I}{r(1+I)^3}$

We are hence able to estimate the probability to produce more than a given number of items during a fixed time horizon *H*.

Variance of the Production Rate

Exercise 20 For the production of electronic components, we are given the following data for a specific production step of these components:

$$\frac{1}{p} = 1000\ [sec] \quad \text{and} \quad \frac{1}{r} = 200\ [sec]$$

The production rate is given by $U = 12\ \frac{pce}{sec}$ and the time horizon is 3 hours.

Suppose that O (operating time) and R (repair time) are exponentially distributed (implying that $SCV_O + SCV_R = 2$). Calculate:

(1) The mean cumulated production $\langle \Sigma(\mathcal{H}) \rangle$

(2) The stationary production fluctuations at time $\mathcal{H}$ (i.e., $\sigma^2_{\Sigma_\mathcal{H}}$)

(3) The probability that we can ship at least 108′100 components after 3 hours

⊙

Time *T* Needed to do a Given Job

We fix the batch size *B* to be produced

Question: **Are we able to ship this batch of products before a given time horizon *H* ?**

τ_B : **time needed to complete a job of batch size *B*** (random variable, as the production flow is random because of the machine failures), with density $q_B(t)$

$$P(\tau_B \leq T) = \int_0^T q_B(t)\, dt$$

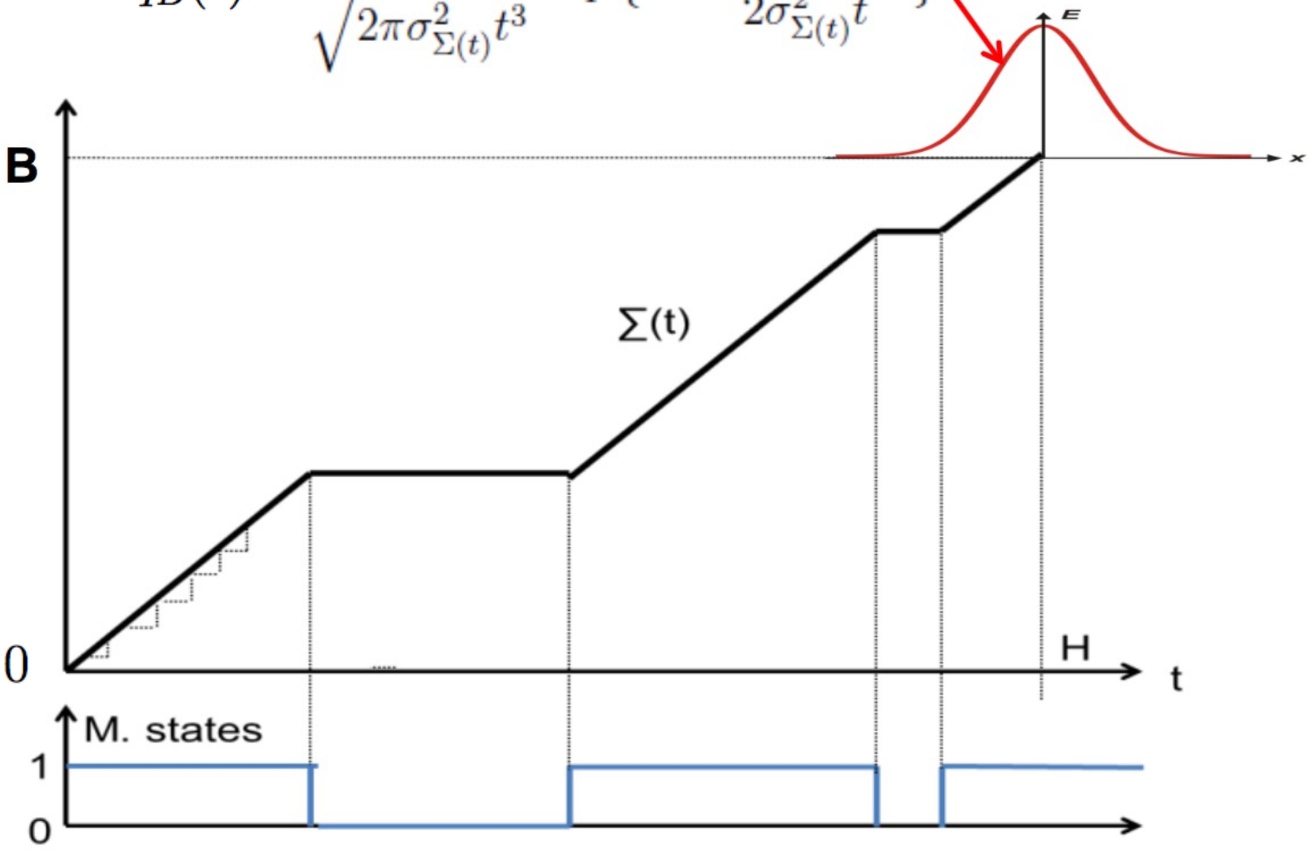
Time *T* Needed to do a Given Job

$$P(\tau_B \leq T) = \int_0^T q_B(t)\, dt$$

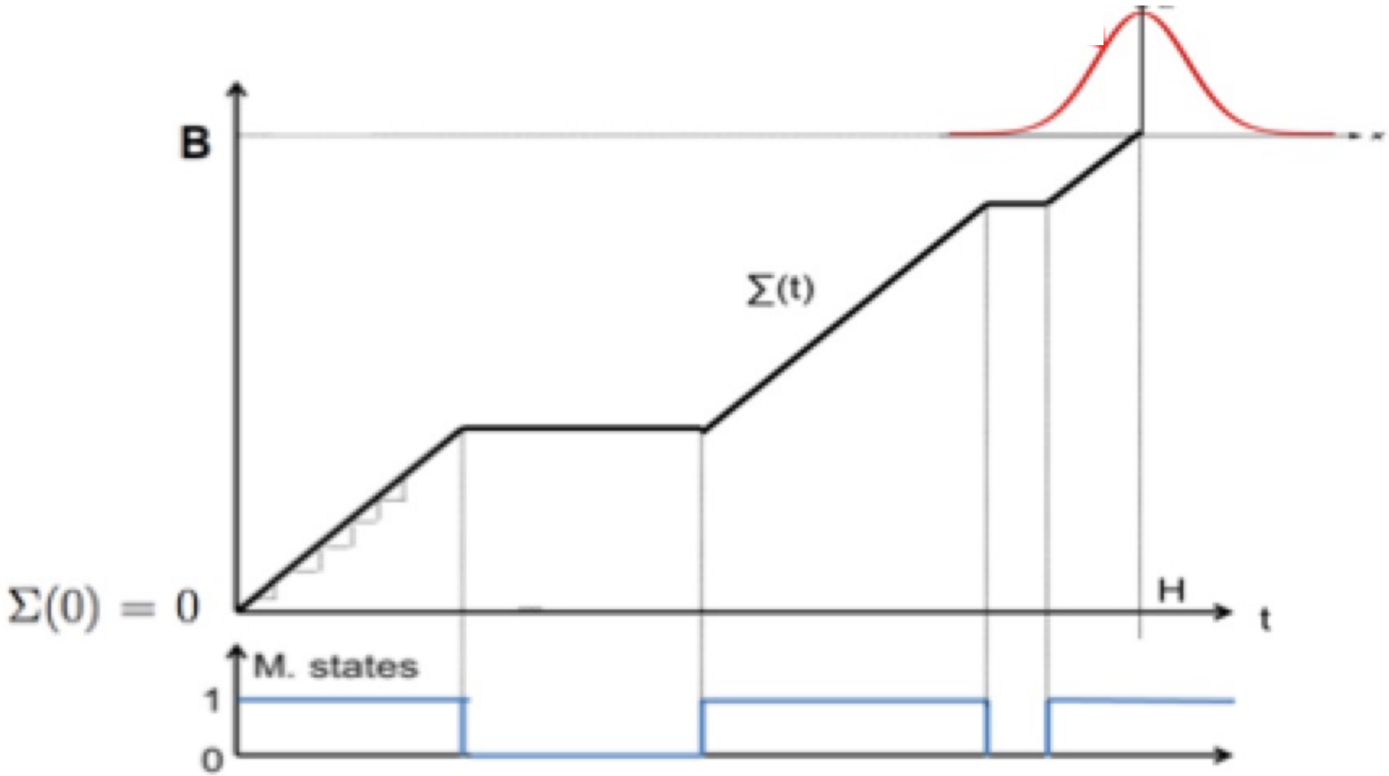
$$q_B(t) = \frac{B}{\sqrt{2\pi\sigma^2_{\Sigma(t)} t^3}} \exp\Big\{ - \frac{(B - mt)^2}{2\sigma^2_{\Sigma(t)} t} \Big\}$$

(inverse Gaussian law)

$\Sigma(0) = 0$



Time *T* Needed to do a Given Job



Mean : $$\langle \tau_B \rangle = \int_0^\infty t q_B(t)\, dt = \frac{B}{U}(1+I) = \frac{B}{m}$$

Variance : $$\sigma^2_{\tau_B} = \frac{(CV_O + CV_R)BI}{rU}$$

Time T Needed to do a Given Job

Exercise 21 In the context of Exercise 20, calculate the probability to finish a job of size $B = 1.2 \cdot 10^5$ within 3 hours. ⊙