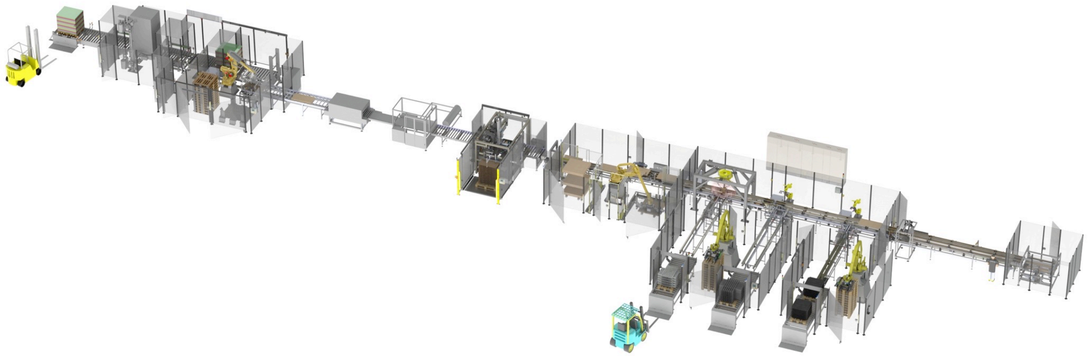

MANUFACTURING SYSTEMS AND SUPPLY CHAIN DYNAMICS

Chapter 3: Safety Stock in Manufacturing Systems

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Course Content

Safety Stock

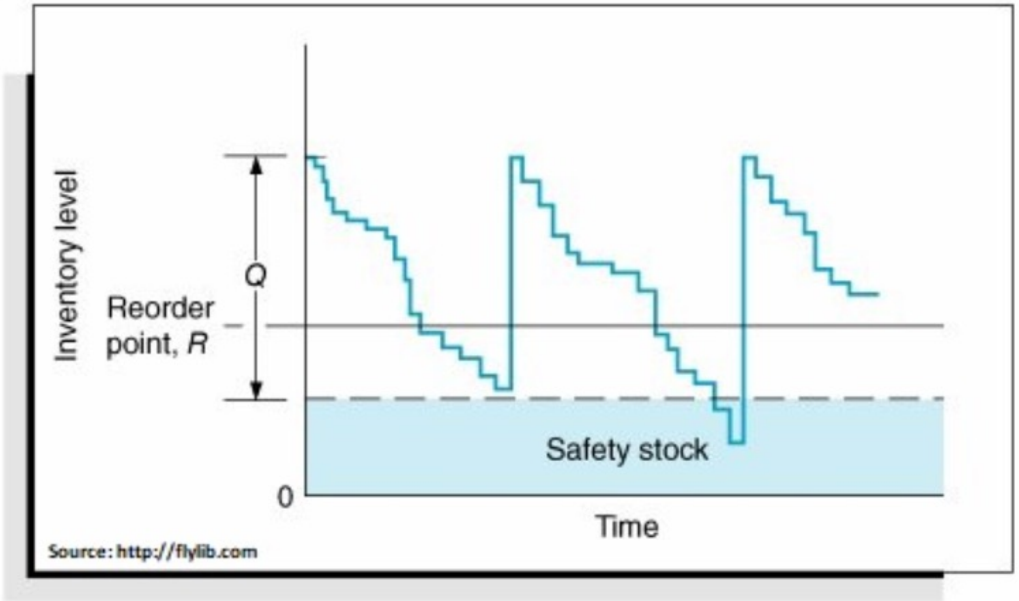
A scheduling policy aims at regulating the downstream inventory of finished goods (Chapter 2)

- Downstream inventory too small => risks of shortage or lost sales
- Downstream inventory too large => high holding costs

Safety stock: target inventory level for the downstream buffer

Needed because of the:

- **randomness** affecting the manufacturing system
- **capacity constraints** that limit the ability to respond to unexpected demand

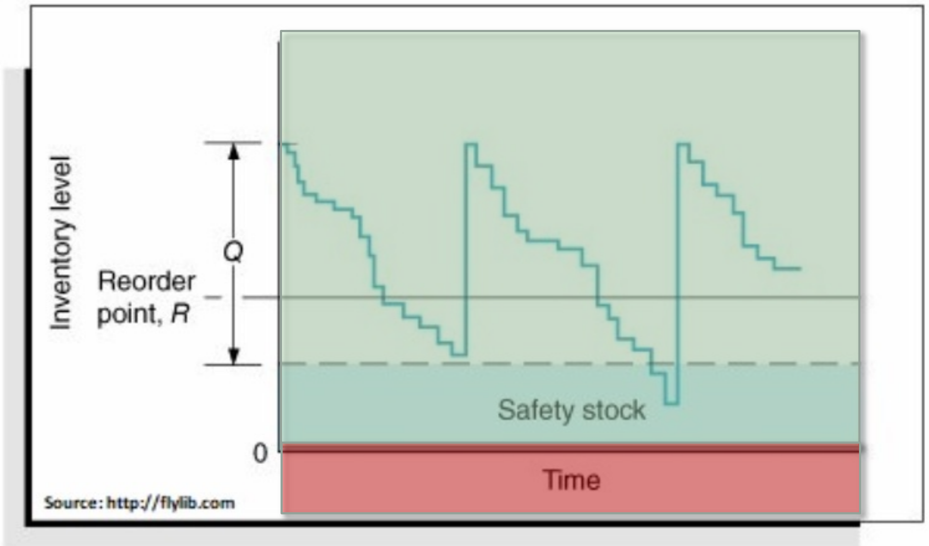


Safety Stock and Service Level

Index policy:

- Threshold together with a time varying index
- If the index exceeds the threshold, an appropriate action is triggered
- Easy to implement in practice

Service level: probability of no stock-out



No stock-out

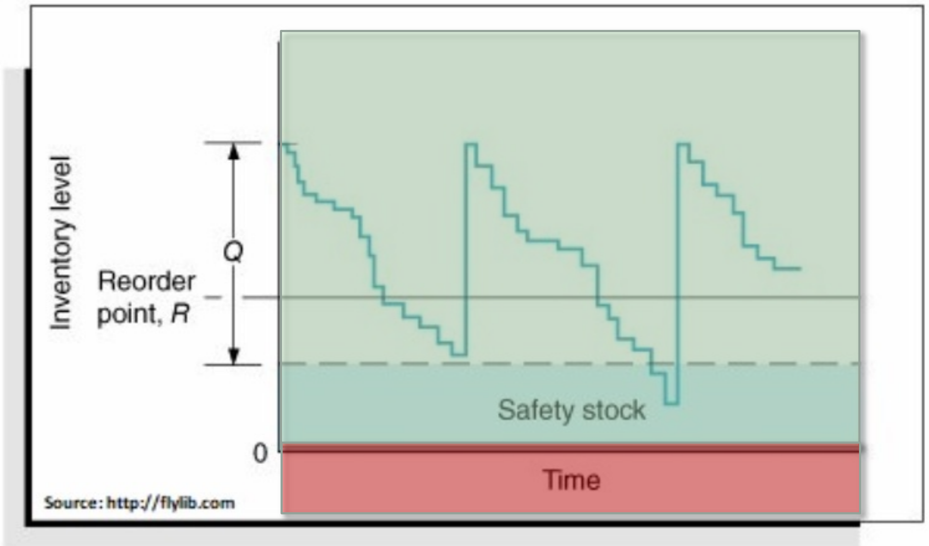
Stock-out

Safety Stock and Service Level

Order Quantity: take the same as EOQ model with planned shortages (approximation[1])

$$Q^* = \sqrt{\frac{2dK}{h}}\sqrt{\frac{p+h}{p}}$$

[1] *S. Axsäter, "Using the Deterministic EOQ Formula in Stochastic Inventory Control," Management Science, 42: 830–834, 1996*



No stock-out

Stock-out

Safety Stock and Service Level

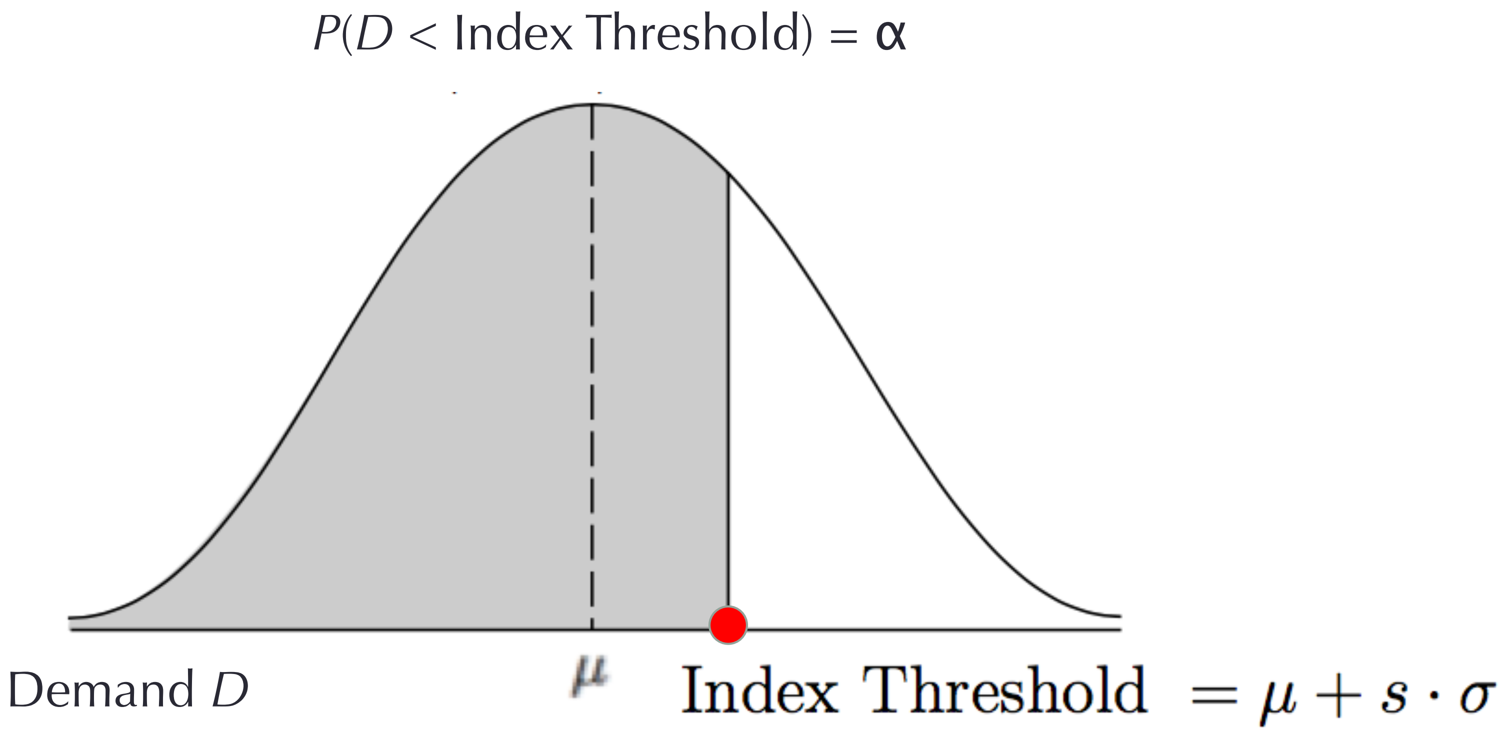
- We suppose the demand follows a **normal distribution** $\mathcal{N}(\mu, \sigma^2)$
- The production manager fixes the service level **α** (e.g. 0.9 = 90%)
- To achieve this service level, we need to add to the mean demand μ a safety stock of size s·σ, such that Φ(s) = α, where:

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy$$

- The index threshold (re-order point) is given by:

$$\text{Index Threshold } = \mu + s \cdot \sigma$$

Safety Stock and Service Level



Safety Stock and Service Level

Example 4:

- The daily demand for an electric switch is modelled as a **normal distribution** $\mathcal{N}(\mu = 105, \sigma^2 = 144)$
- The target service level is fixed to **α** = 90%
- Table of the standard normal law gives: ***s* = 1.28**
- **Index Threshold (Re-Order Point) = 105 + 1.28 · 12 = 121**
- The index policy states that when downstream inventory of finished goods drops below 121, the production manager must react and increase production capacity in order to produce more items.

Safety Stock and Service Level

Other distribution for the demand.

Example: **Uniform** distribution in $[a, b]$

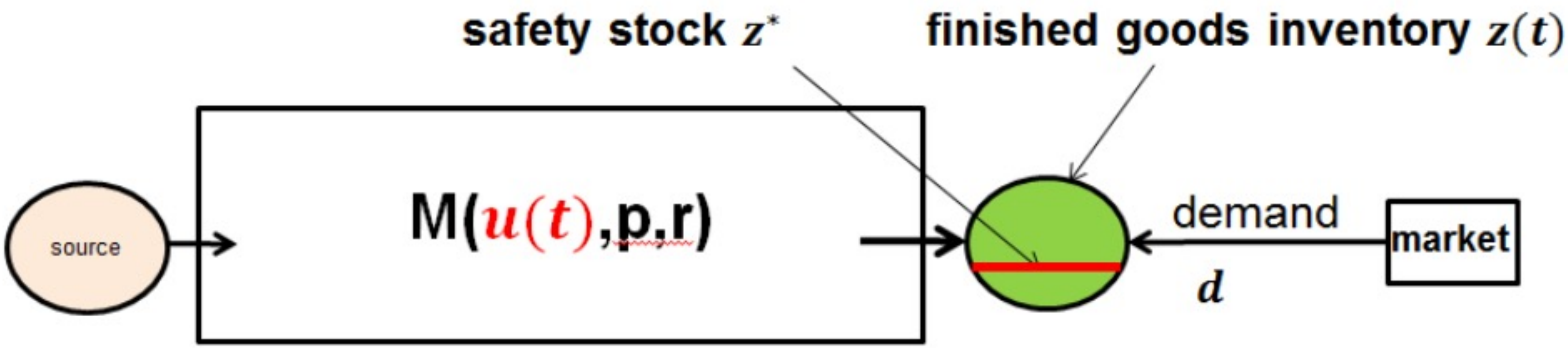
Index Threshold = $a + \alpha\ (b\text{-}a)$

Resulting safety stock (expected inventory level just before the order quantity is received):

Safety stock = Index Threshold – $\mu = a + \alpha\ (b\text{-}a) - (a+b)/2 = (\alpha\text{-}1/2)\ (b\text{-}a)$

Safety Stock as Hedging Point Problem

Prone-to-failure machine



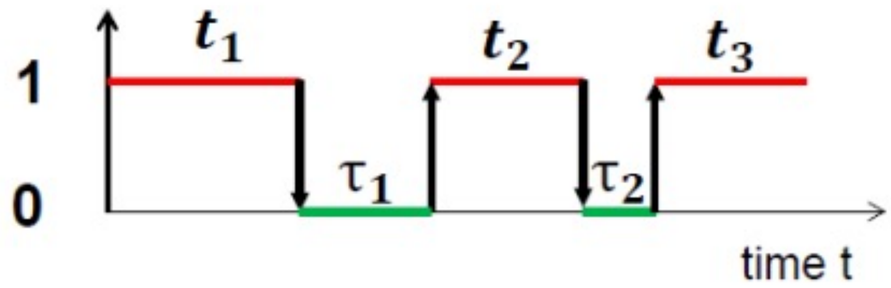
$$0 < u(t) < U, \qquad \frac{U}{1+I} > d \qquad \dot{z}(t) = u(z(t)) - d$$

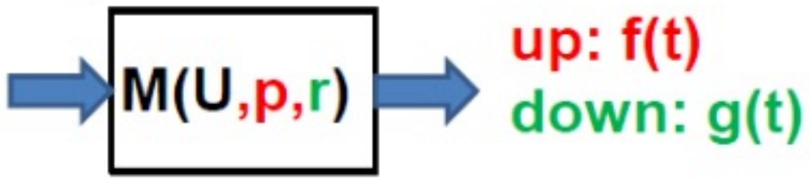
Safety Stock as Hedging Point Problem

Prone-to-failure machine

$$\int_0^\infty t f(t)dt \cong \frac{1}{n}\sum_{i=1}^{n} t_i = \frac{1}{\boldsymbol{p}}$$

$$\int_0^\infty t g(t)dt \cong \frac{1}{m}\sum_{i=1}^{m} \tau_i = \frac{1}{\boldsymbol{r}}$$





$$\frac{cum.\,production}{needed\ time} \cong \boldsymbol{U}$$ (measured during an operational run)

Safety Stock as Hedging Point Problem

Objective:

- **Choosing the production rate** in order to minimize the mean long-term inventory costs.

Optimal scheduling policies are often difficult to compute even for single product manufacturing systems.

We treat a situation a situation where the optimal scheduling policy is a **hedging point policy**.

Safety Stock as Hedging Point Problem

Hedging Point Policy:

- If the downstream inventory of finished goods is **below** a certain value $z*$: **produce at full rate**
- If the downstream inventory of finished goods is exactly **at** $z*$: **produce according to the demand**
- If the downstream inventory of finished goods is **above** $z*$: **stay idle**

Safety Stock as Hedging Point Problem

Determining an optimal policy:

1. Show that the hedging point **policy is optimal**
2. **Calculate the hedging point**

Nota bene: when the hedgint point ***z****** **is equal to 0**, then it follows the ***Just-In-Time*** philosophy.

Safety Stock as Hedging Point Problem

Single machine manufacturing facility:

- **Constant demand** with demand rate d
- **Occasional breakdowns** during which no production occurs
- Time between breakdowns and repair times are **exponentially distributed** with respective rates p and r
- **Mean-time-to-failure** $MTTF = 1/p$
- **Mean-time-to-repair** $MTTR = 1/r$
- **Indisposability** is given $I = MTTR/MTTF = p/r$

Safety Stock as Hedging Point Problem

Modelling:

- $\chi(t)$: **random process** describing the working status of the machine

$$\chi(t) = \begin{cases} 0 & \text{if the system is down at time } t \\ 1 & \text{if the system is up at time } t \end{cases}$$

- $z(t)$: level of the downstream inventory of finished goods at time t

- We consider unfulfilled demand to be **backlogged** as negative inventory

Safety Stock as Hedging Point Problem

Modelling (continued):

- When working, the production facility can produce at any rate between 0 and a maximum production rate $U > d$.
  - **Controllable production rate *u(t)*** limited in a range $[0, U]$
  - Assumption: production objective is feasible for the demand rate: $$\frac{U}{1+I} > d$$
- **Costs:**
  - **Holding costs c^+** > 0 per item kept in the downstream inventory and per time unit
  - **Shortage costs c^-** > 0 for each unit of backlogged demand per time unit

Safety Stock as Hedging Point Problem

Control Policy:

- **Objective**: Control the production rate $u(t) = u(z(t),\chi(t))$ as a function of:
  - The current inventory $z(t)$ of finished goods
  - The current working status $\chi(t)$ of the production facility

- **Constraints**:
  - On-times: $0 \leq u(z,\chi) \leq U \text{ if } \chi = 1$
  - Off-times: $u(z,\chi) = 0 \text{ if } \chi = 0.$

Safety Stock as Hedging Point Problem

Definitions:

- Inventory level: $z(t) = z^+(t) - z^-(t)$
- Negative part: $z^+(t) = \max\{0, z(t)\} \geq 0$
- Positive part: $z^-(t) = -\min\{0, z(t)\} \geq 0$
- $c^+ z^+(t)$: instantaneous costs due to positive inventory
- $c^- z^-(t)$: instantaneous costs due to shortage

Dynamics:

- Dynamical content of the downstream inventory:

$$\dot{z}(t) = u\big(z(t), \chi(t)\big) - d$$

Safety Stock as Hedging Point Problem

Minimization Problem:

$$J(z(0), \chi(0), t=0) := \min_u \left\{ \lim_{T\to\infty} \frac{1}{T} E\left(\int_0^T [c^+ z^+(s) + c^- z^-(s)] ds \right) \right\}$$

which satisfies the Bellmann equations:

$$\begin{aligned}
-\frac{\partial J}{\partial t}(z,0,t) &= \min_u \left\{ [c^+ z^+(t) + c^- z^-(t)] - J(z,0,t)r + J(z,1,t)r + \frac{\partial J}{\partial t}(z,0,t)(u-d) \right\} \\
-\frac{\partial J}{\partial t}(z,1,t) &= \min_u \left\{ [c^+ z^+(t) + c^- z^-(t)] + J(z,0,t)p - J(z,1,t)p + \frac{\partial J}{\partial t}(z,1,t)(u-d) \right\}
\end{aligned}$$

Safety Stock as Hedging Point Problem

Solution:

Minimum is realized by following dynamical production rate:

$$u(z(t),1) = \begin{cases} 0 & \text{if} \quad z(t) > z^* \\ d & \text{if} \quad z(t) = z^* \\ U & \text{if} \quad z(t) < z^* \end{cases}$$

With z^* equal to:

$$z^* = \begin{cases} \frac{1}{b} \ln\left(Kb(1+\frac{c^-}{c^+})\right) & \text{if } b < \frac{c^+}{c^+ + c^-} \\ 0 & \text{if } b \geq \frac{c^+}{c^+ + c^-} \end{cases}$$

Where: $b = \frac{r}{d} - \frac{p}{U-d}$ and $K = \frac{Up}{b(r+p)(U-d)}$

Safety Stock as Hedging Point Problem

Exercise 12 A student prepares sandwiches for a specific coffee-shop (during the opening hours, from 11am to 3pm). S/he produces sandwiches at a rate of $\frac{1}{3}$ sandwiches per minute. However, s/he is interrupted every 10 minutes (on average) for 30 seconds (on average). Suppose that every 20 minutes a sandwich is demanded by a client and that shortage costs equal the price of a sandwich, i.e. $c^- = 5\text{CHF}/20\text{min}$ (i.e., 5CHF per sandwich). Assuming $c^+ = 0.1\text{CHF}/\text{min}$, calculate the optimal safety stock policy. ⊙

Exercise 13 Suppose $c^- = c^+$. Explain in your own words the "Just-in-Time" boundary $b = 1/2$ in this case. *Hint:* interpret r/d and $p/(U-d)$.

⊙