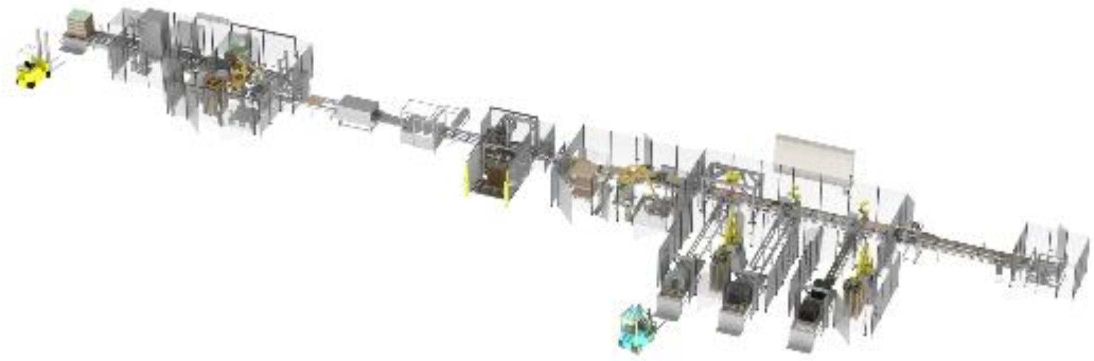


MANUFACTURING SYSTEMS AND SUPPLY CHAIN DYNAMICS

Chapter 12: Maintenance

EPFL, Master MT

Roger Filliger (BFH), Olivier Gallay (UniL)

Course Content

Predictive Maintenance

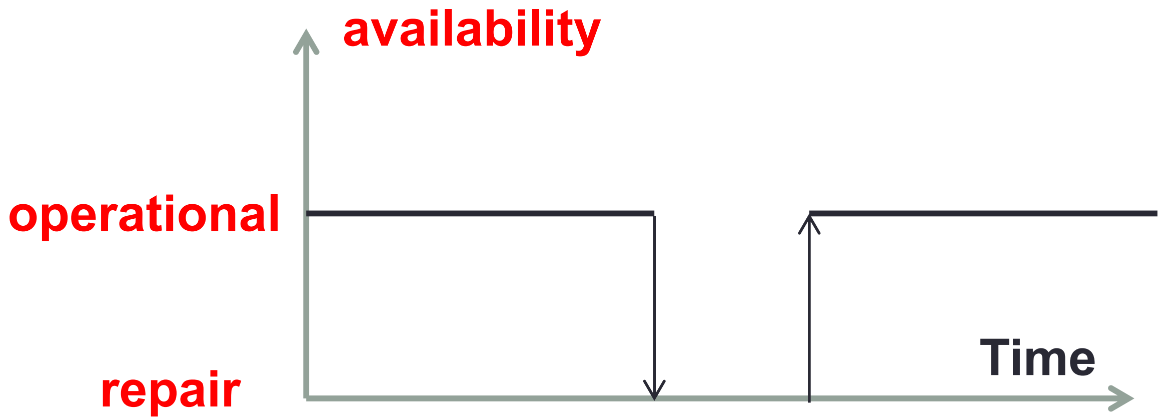
knowledge

know-how

Run to failure maintenance

Maintenance action

Run to failure maintenance



Maintenance action

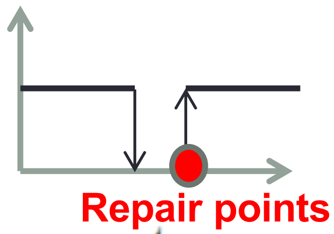
Event Based Policies

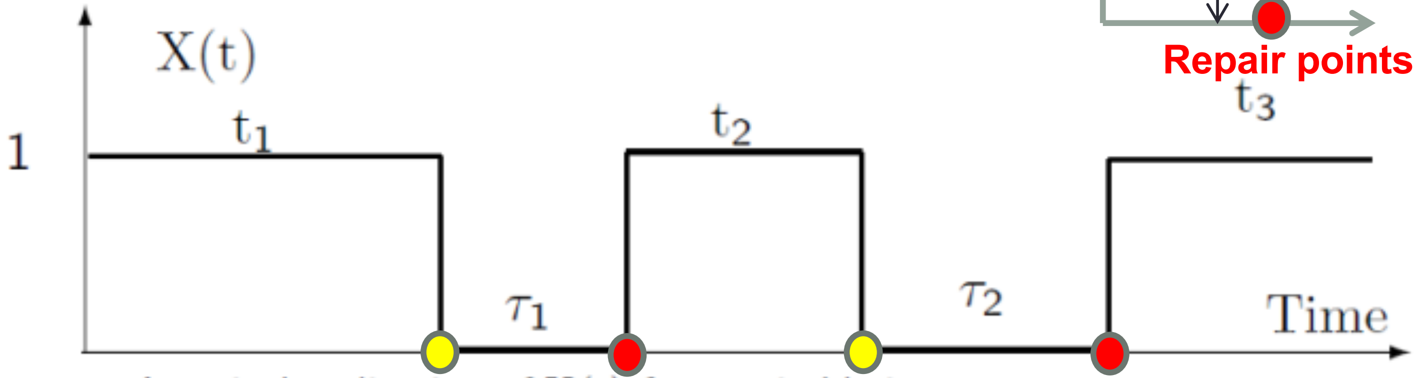
Easy decision making

Complex Planning

Run to failure maintenance

$$X_t = \begin{cases} 1 & \text{if the item is available at time t} \\ 0 & \text{otherwise} \end{cases}$$





A typical realization of X(t) for repairable items.

$$\text{mean time to failure} = \text{MTTF} = \frac{1}{n}\sum_{i=1}^{n} t_i$$

$$\text{mean time to repair} = \text{MTTR} = \frac{1}{n}\sum_{i=1}^{n} \tau_i$$

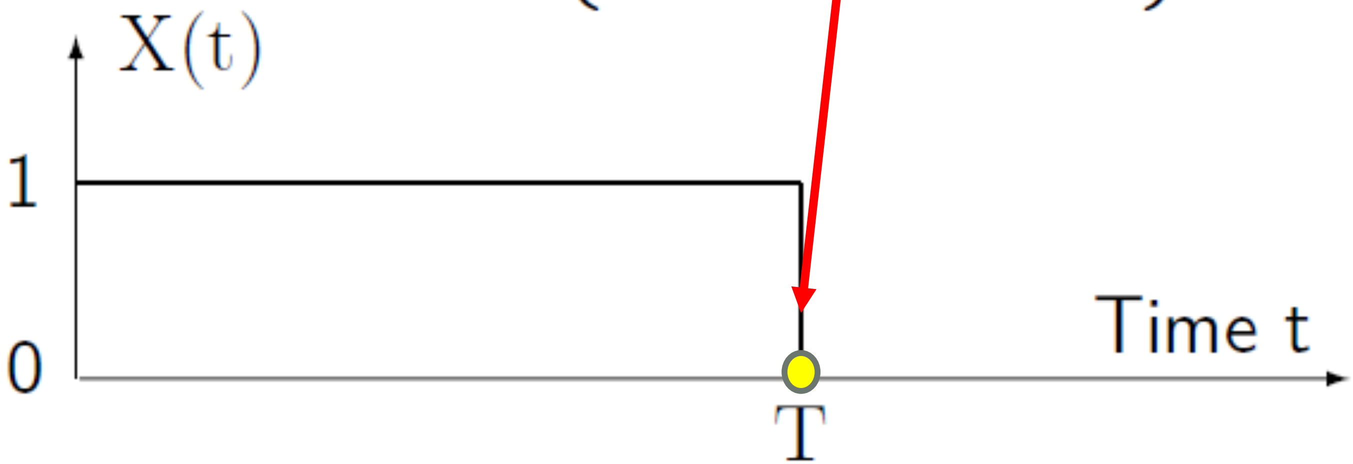
$$\text{inavailability factor} = I = \frac{\text{MTTR}}{\text{MTTF}}$$

Preventive maintenance

= Maintenance before T:

The time to failure T is the first time the item fails:

$$T = \min\left\{t \geq 0 \mid X(t) = 0\right\}$$

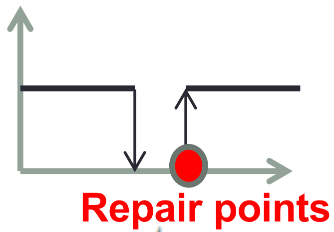


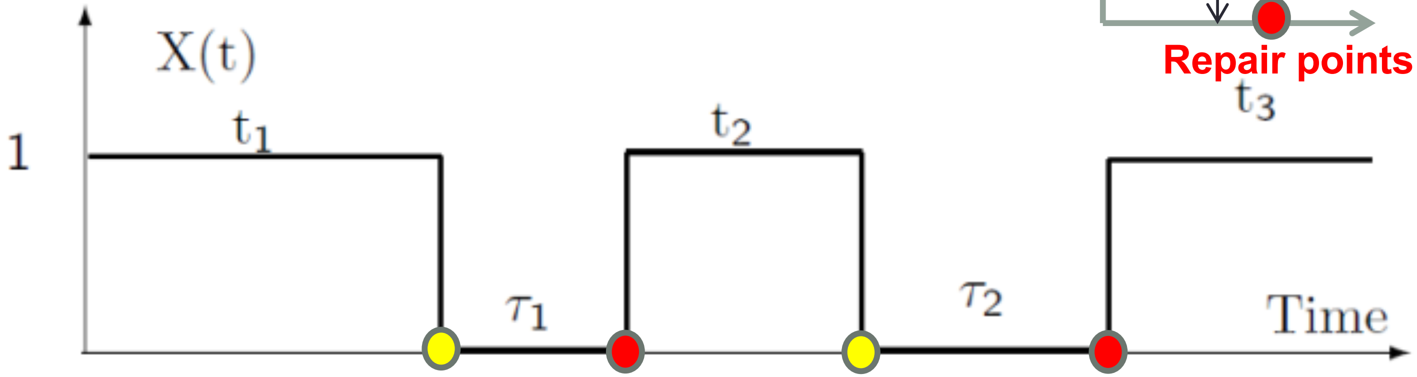
Run to failure maintenance

Maintenance action

Number of repair points in [0,t]

$$X_t = \begin{cases} 1 & \text{if the item is available at time t} \\ 0 & \text{otherwise} \end{cases}$$





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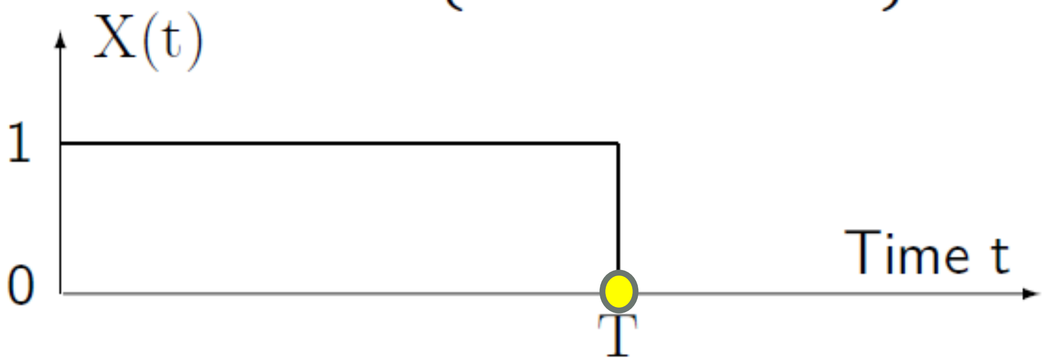
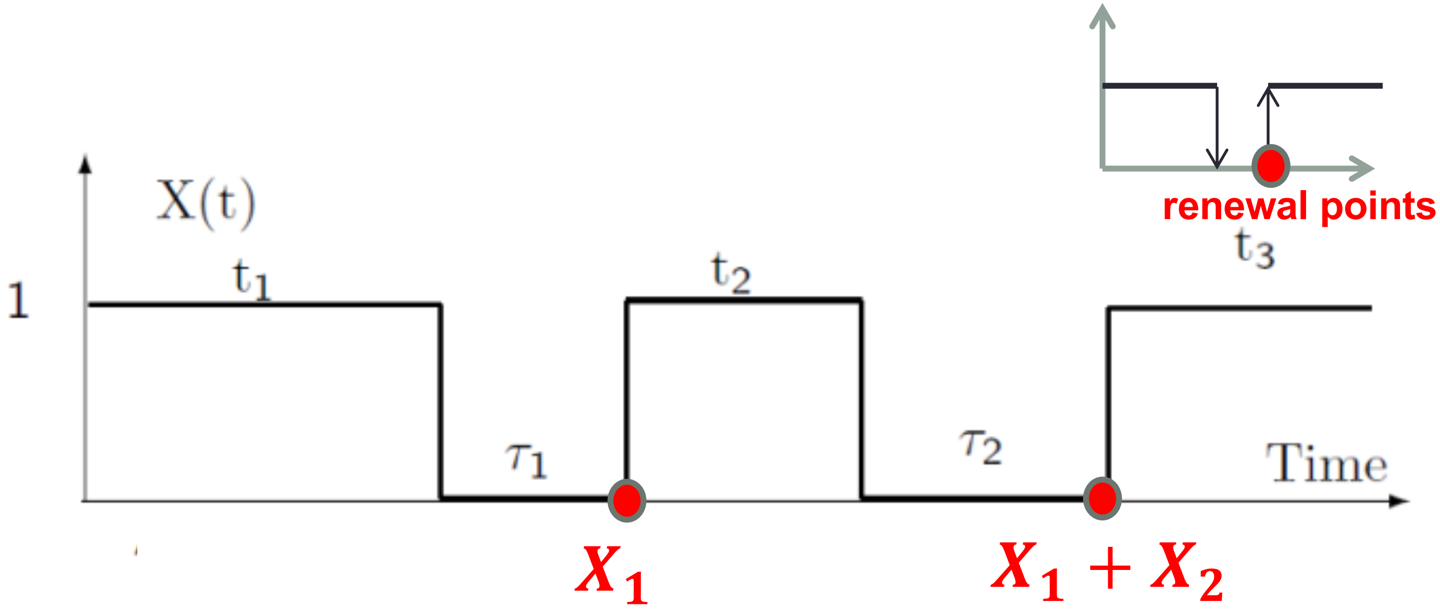


Number of repair points in [0,t]



$S_k = X_1 + X_2 + \cdots X_k,$ **time of k'th renewal**

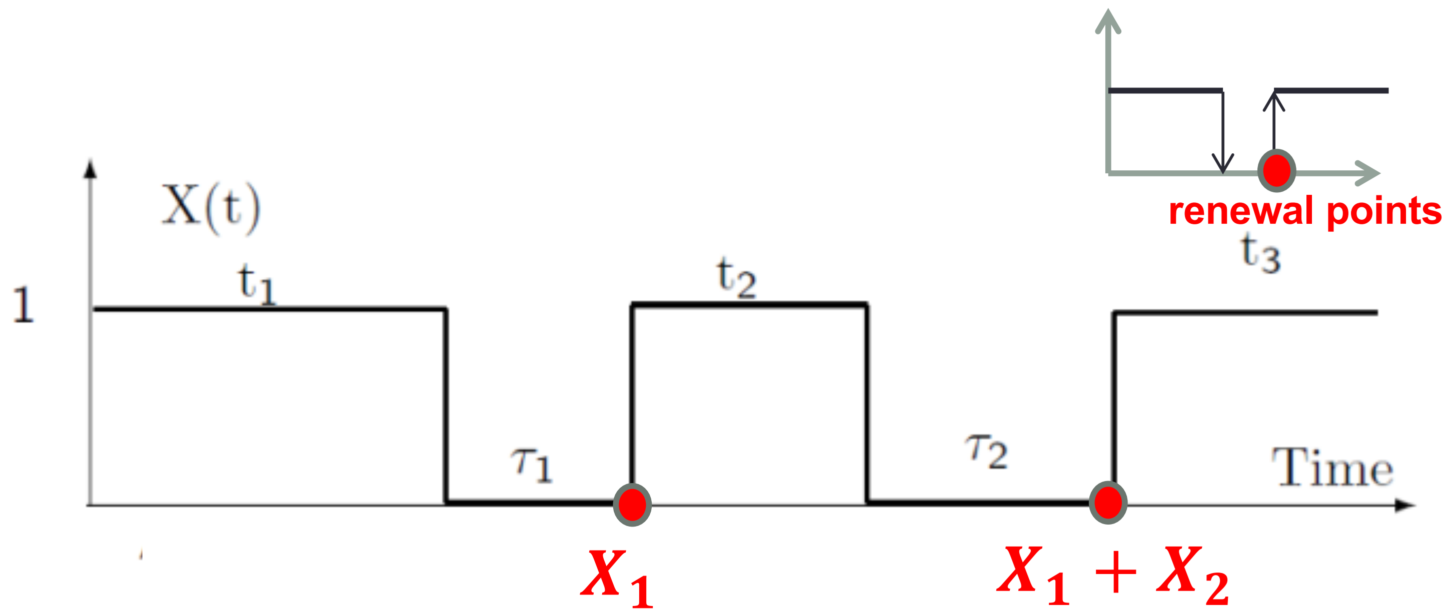
$F(x) = P(X_1 \leq x)$, $F^{(k)}$ k-fold convolution of F

$$P\Big(N(t) = n\Big) = P\Big(S_n \leq t \text{ and } S_{n+1} > t\Big)$$
$$= F^{(n)}(t) - F^{(n+1)}(t)$$

Renewal process

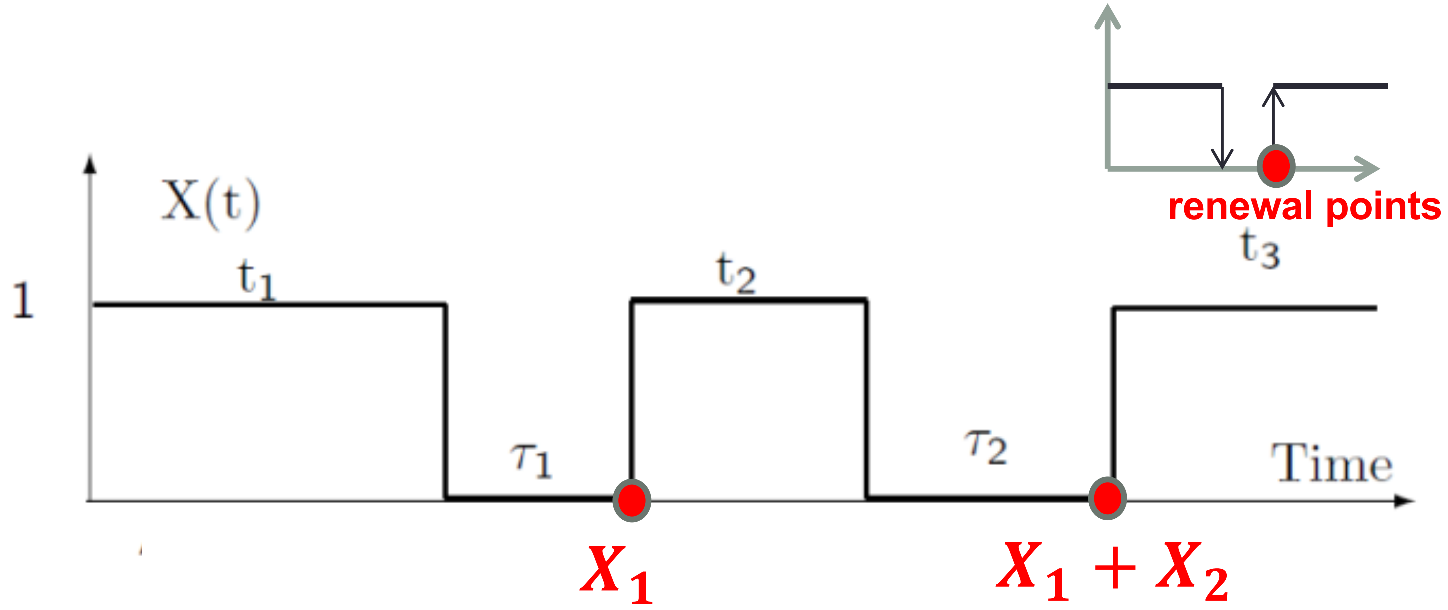
Number of renewal in [0,t]

Number of repair points in [0,t]



Example 23 : The Poisson Process. If $F(t) = 1 - e^{\lambda t}$, *i.e.*, the X_i's are exponentially distributed with parameter λ, $N(t)$ is the Poisson process, and $P\Big(N(t) \geq n\Big) = F^{(n)}(t) = 1 - e^{-t\lambda} \sum_{i=0}^{n-1} \frac{(t\lambda)^i}{i!}$. ◊

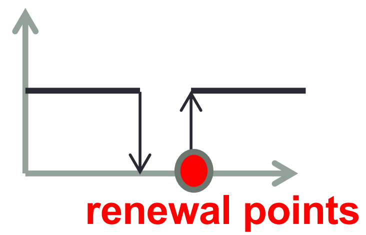
Number of repair points in [0,t]



Example 24 : IFR Renewal Process. If the underlying failure distribution $F(t)$ is IFR with mean μ, we have for $t \le \mu$ (using the bounds expressed in Figure 11.5):

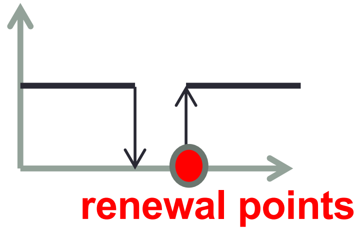
$$P\Big(N(t) \ge n\Big) = F^{(n)}(t) \le 1 - e^{-t/\mu} \sum_{i=0}^{n-1} \frac{(t/\mu)^i}{i!}.$$

Number of repair points in [0,t]



Exercise 60 Consider the situation when a machine breakdown occurs, in the mean, once in 1200 working hours. Estimate the probability for 2 or more breakdowns to happen within 600 working hours. ⊙

Number of repair points in [0,t]



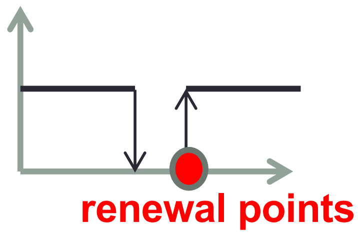
The renewal function is defined as the expected number of renewals happening in $[0, t]$, that is:

$$M(t) = E[N(t)].$$

Theorem: Under IFR assumption and for large t, the renewal function $M(t)$ satisfies:

$$\frac{t}{\mu} - 1 \leq \frac{t}{\int_0^t R(x)dx} - 1 \leq M(t) \leq \frac{tF(t)}{\int_0^t R(x)dx} \leq \frac{t}{\mu}.$$

Number of repair points in [0,t]



Example:
We observe the following renewal points:

$$x_1 = 8850h$$
$$x_2 = 3215h$$
$$x_3 = 9460h$$
$$x_4 = 6650h$$
$$x_5 = 7056h$$
$$x_6 = 7604h$$

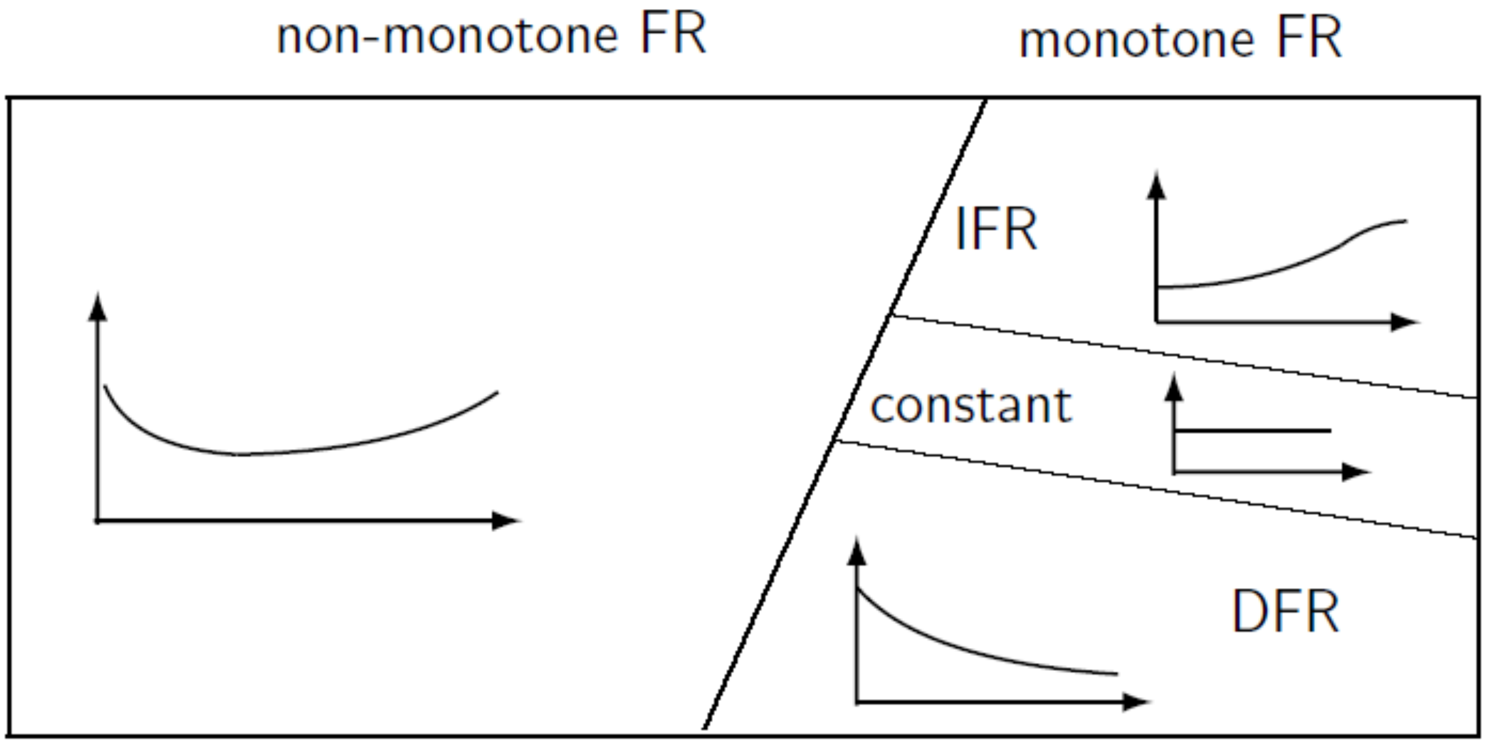
With mean $\mu = 7139$

and suppose an aging process behind the renewal points (->IFR).

Therefore, for t $\geqslant \mu$: $\mathbf{M}(\boldsymbol{t}) \in \left[\frac{\boldsymbol{t}}{\boldsymbol{\mu}} - \mathbf{1}, \frac{\boldsymbol{t}}{\boldsymbol{\mu}}\right]$

Preventive maintenance

**Maintenance planning
is based on T respectively on R
or equivalently on z:**

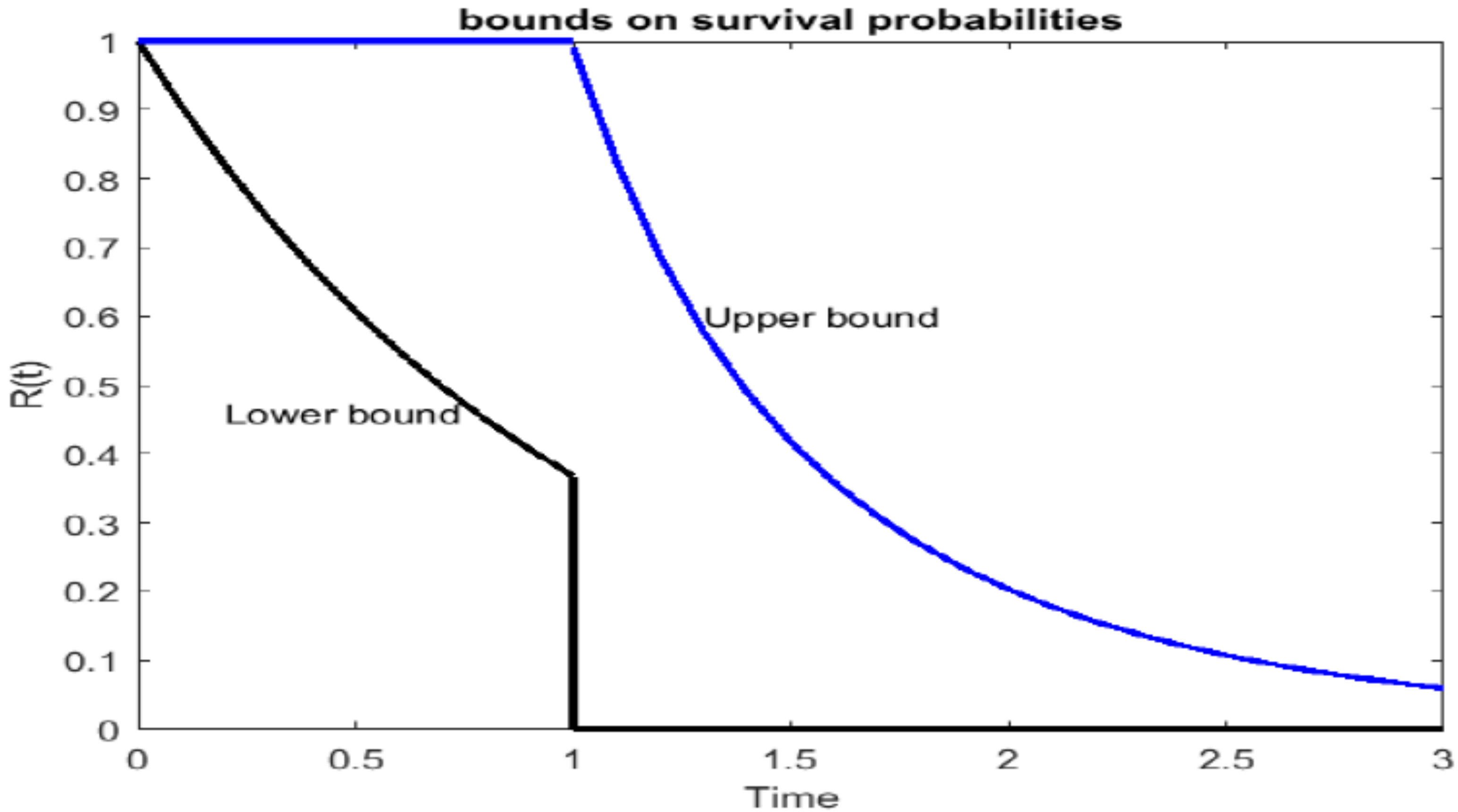


time to failure is **IFR** if $z(t)$ is increasing in t
time to failure is **DFR** if $z(t)$ is decreasing in t

$$\mathrm{IFR} \cap \mathrm{DFR} = \{\mathrm{const\ FR}\} = \{\mathrm{T} \sim \mathrm{Exp}\}$$

Preventive maintenance

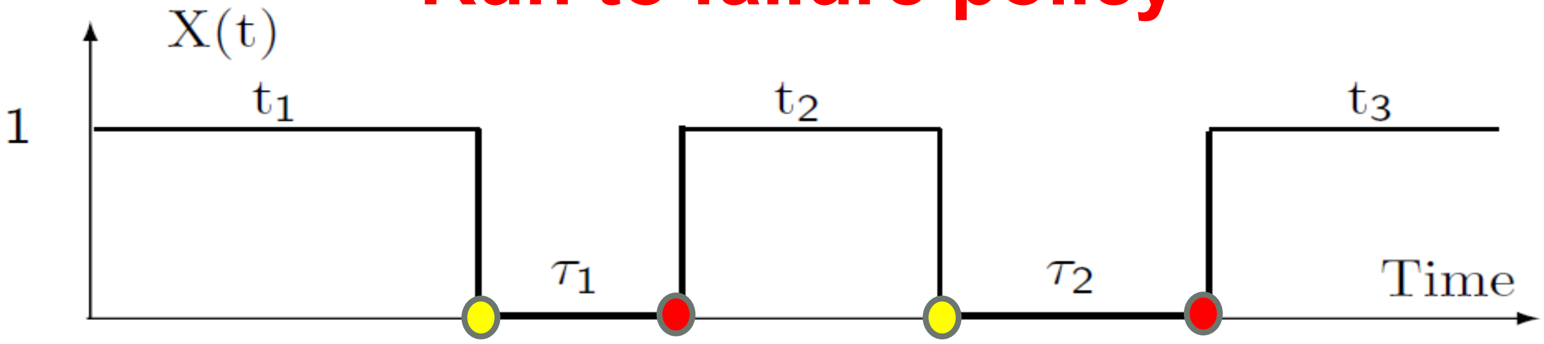
Maintenance planning is based on T respectively on R or equivalently on z:

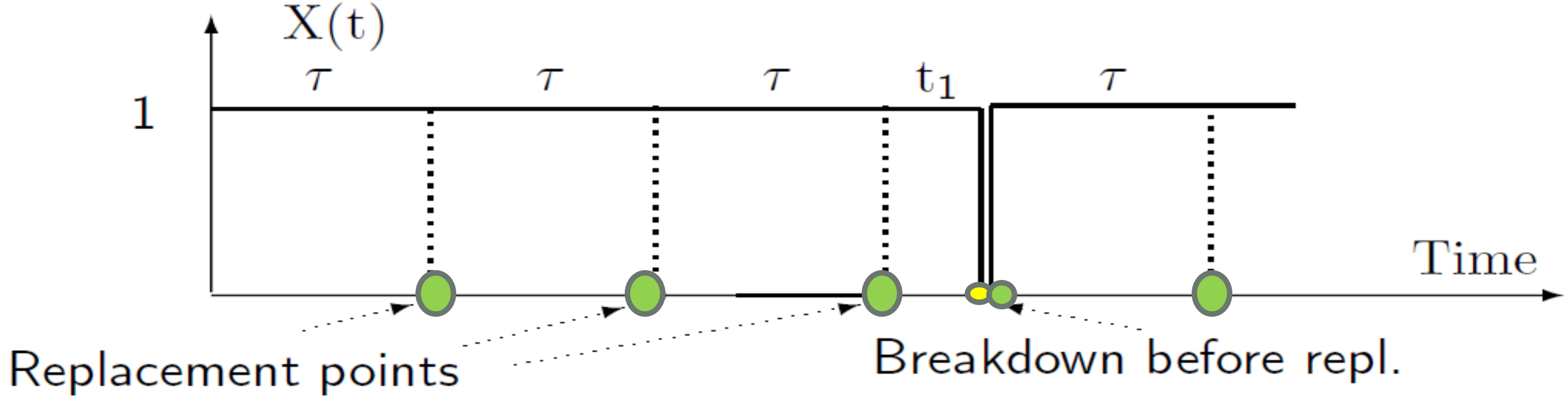


Bounds on reliability for IFR distributions (here, $\mu = 1$).

Preventive maintenance

Run to failure policy



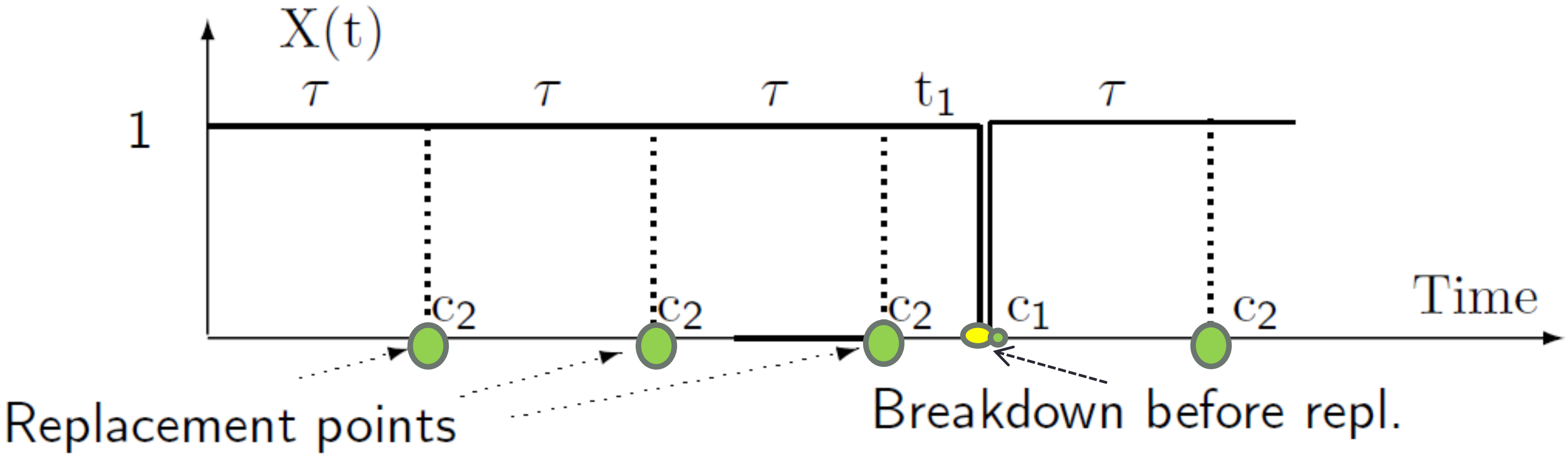


An age replacement policy replaces a deteriorating unit τ hours after its installation or at failure, whichever occurs first.

Age replacement policy

Preventive maintenance

Observation :

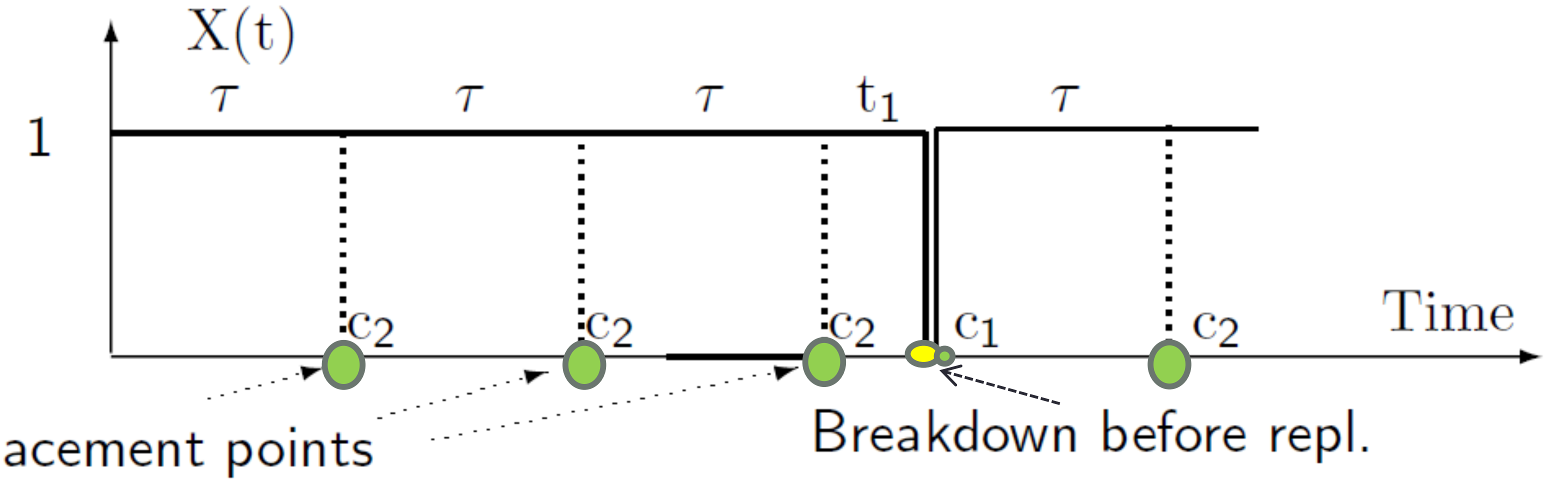


We fix $\tau > 0$ and denote by

$\bar{S}_\tau(t) = 1 - S_\tau(t)$ the **probability of no repair** during $[0, t]$

Preventive maintenance

Observation :



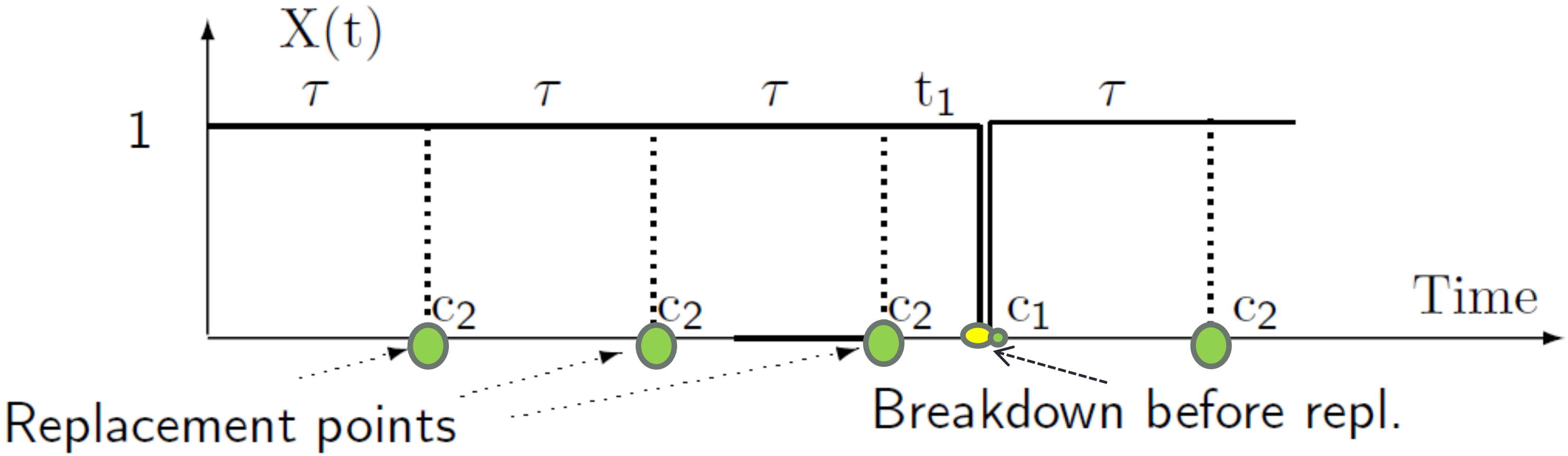
We fix $\tau > 0$ and denote by

$\bar{S}_\tau(t) = 1 - S_\tau(t)$ the **probability of no repair** during $[0, t]$

We have $\bar{S}_\tau(t) = R(\tau)^n R(t - n\tau), \quad n\tau \le t \le (n+1)\tau$

Preventive maintenance

Observation **: increasing τ increases the risk of run to failure!**



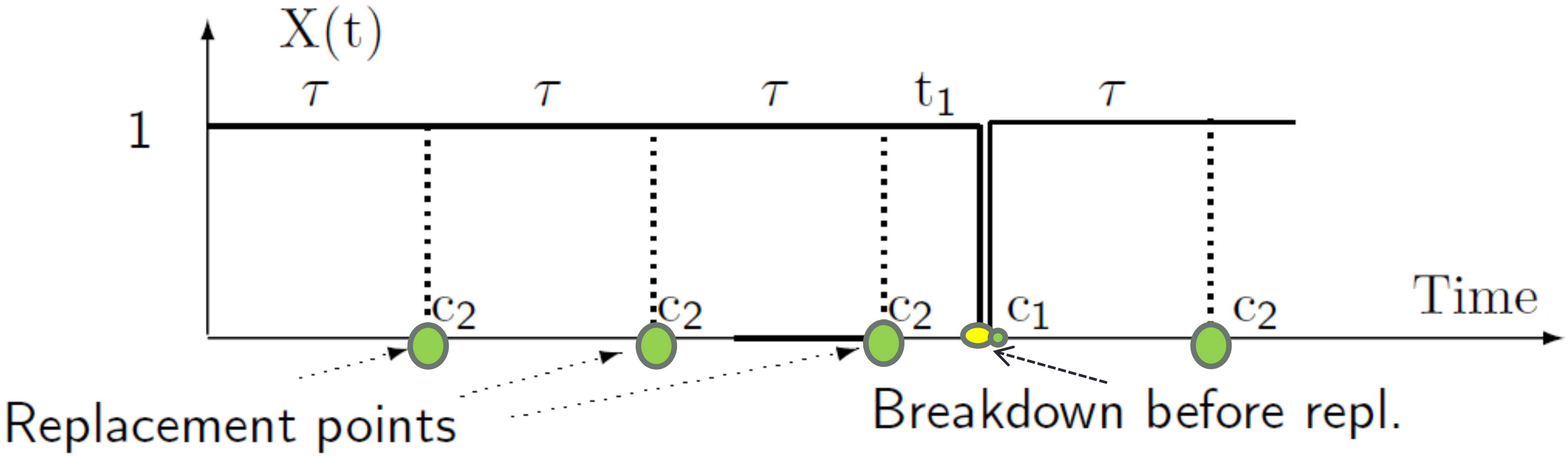
We fix $\tau > 0$ and denote by

$\bar{S}_\tau(t) = 1 - S_\tau(t)$ the **probability of no repair** during $[0, t]$

and $\bar{S}_{\tau_1}(t) \geq \bar{S}_{\tau_2}(t)$, for all $\tau_1 \leq \tau_2$ if and only if T is IFR.

Preventive maintenance

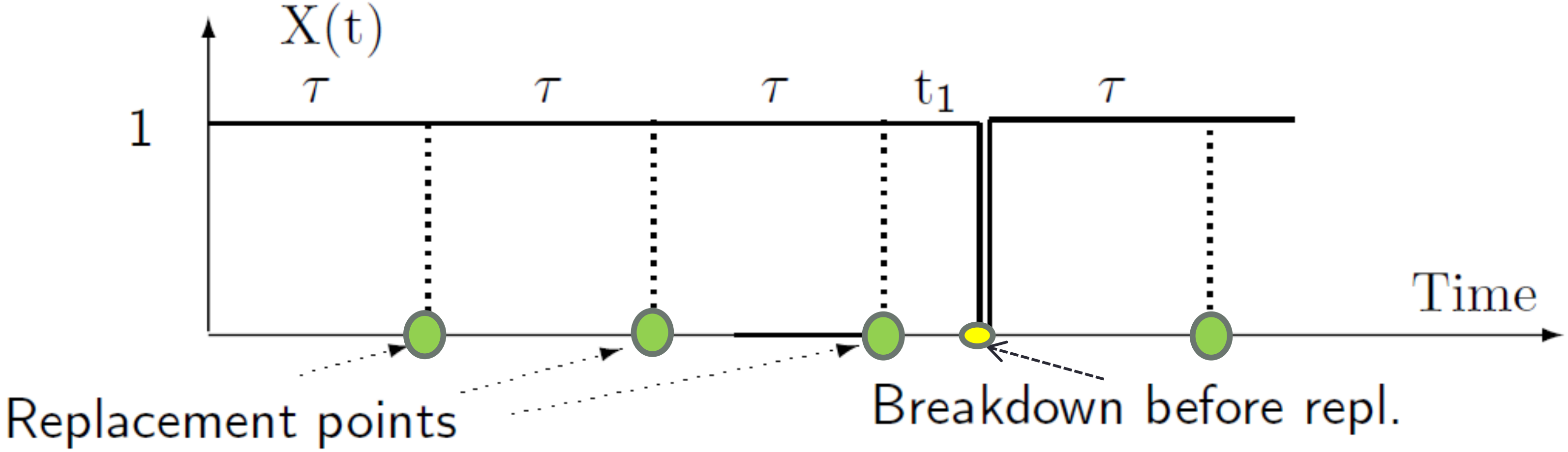
age replacement after τ:



Suppose a cost c_1 is suffered for each failed item which is replaced and...
...a cost $c_2 < c_1$ is suffered for each nonfailed item which is exchanged.

Preventive maintenance

Optimal age replacement policy:



Letting

$N_1(t)$ denote the number of failures during $[0, t]$ and

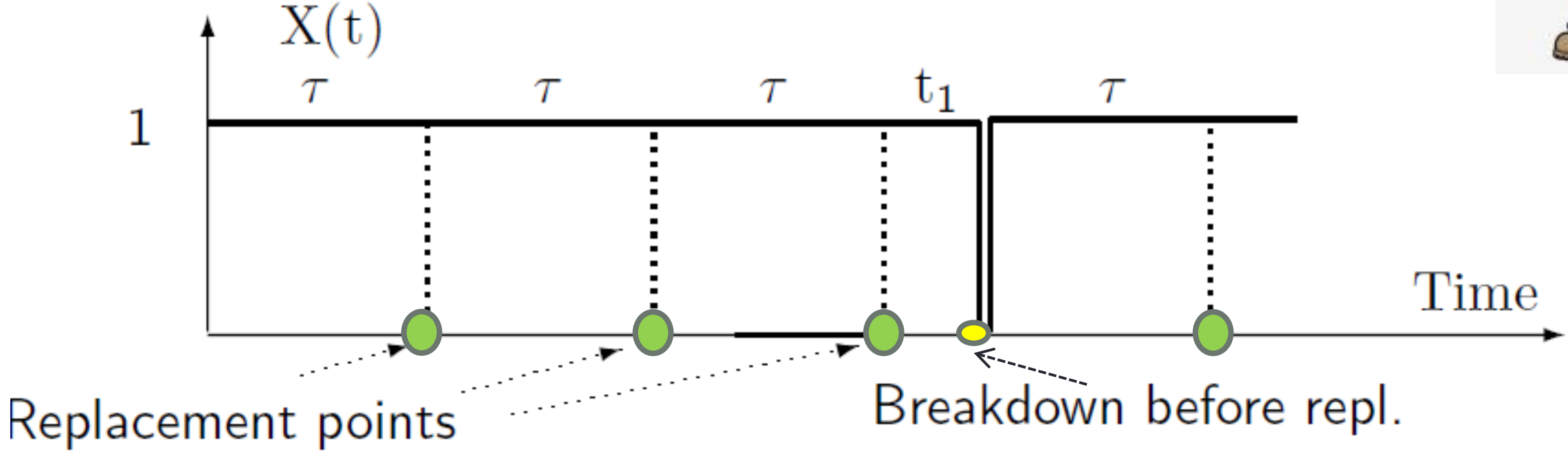
$N_2(t)$ denote the number of exchanges of nonfailed items during $[0, t]$,

and we seek **the age policy minimizing** $C(t)$

$$C(t) = c_1 E\big(N_1(t)\big) + c_2 E\big(N_2(t)\big)$$

Preventive maintenance

Optimal age replacement policy:



The solution $\tau = t$ to the equation

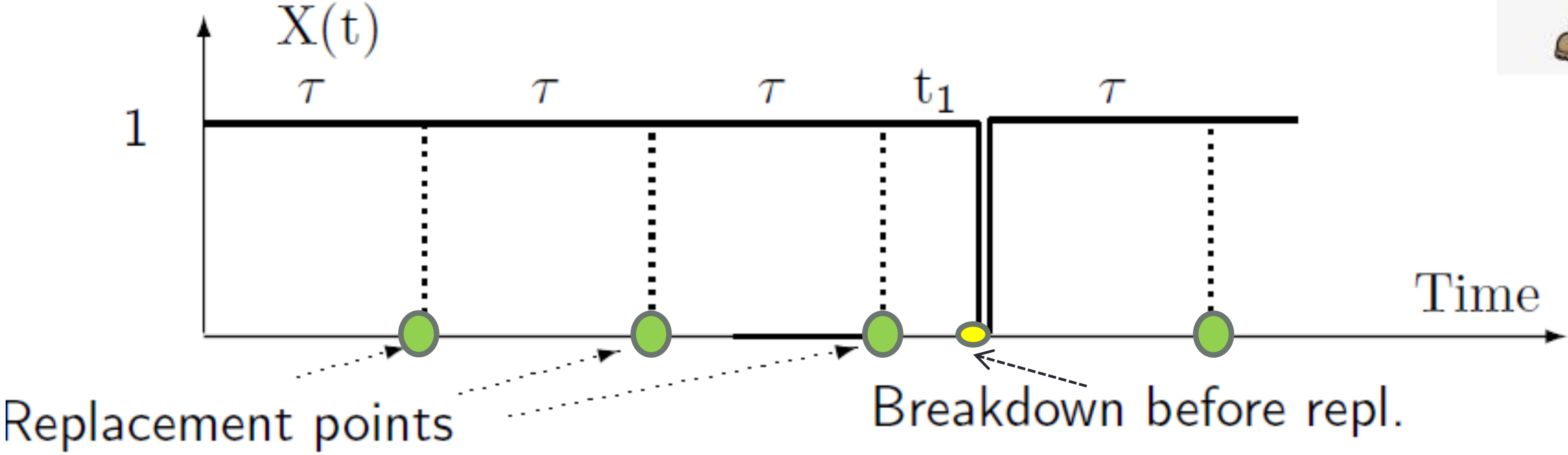
$$z(t)\int_0^t R(x)dx - F(t) = \frac{c_2}{c_1 - c_2}. \qquad (1)$$

defines the optimal replacement strategy. Moreover, if F is IFR, we have:

$$\tau > \frac{c_2}{c_1}\text{MTTF}$$

Preventive maintenance

Exercise: work out the details for a Weibull distribution with $\alpha=2$, $\beta=1$.



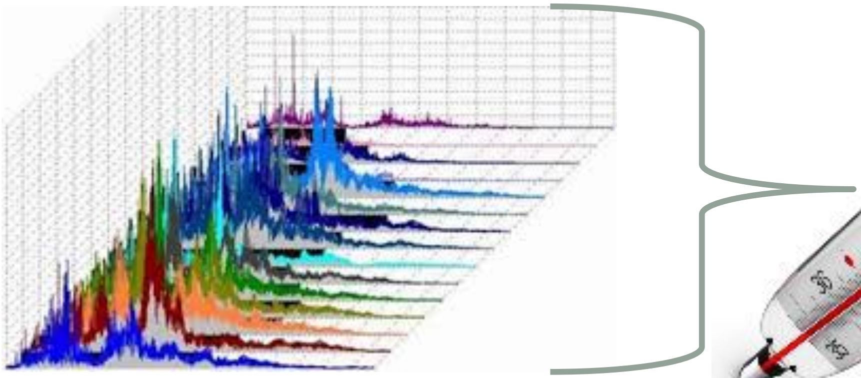
The solution $\tau = \mathrm{t}$ to the equation

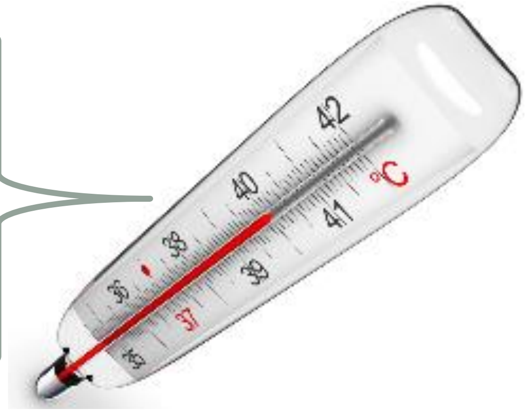
$$z(t)\int_0^t R(x)dx - F(t) = \frac{c_2}{c_1 - c_2}. \qquad (1)$$

defines the optimal replacement strategy. Moreover, if F is IFR, we have:

$$\tau > \frac{c_2}{c_1}\mathrm{MTTF}$$

Predictive maintenance





Condition monitoring
State prediction

Decision making

Maintenance action

Online estimation

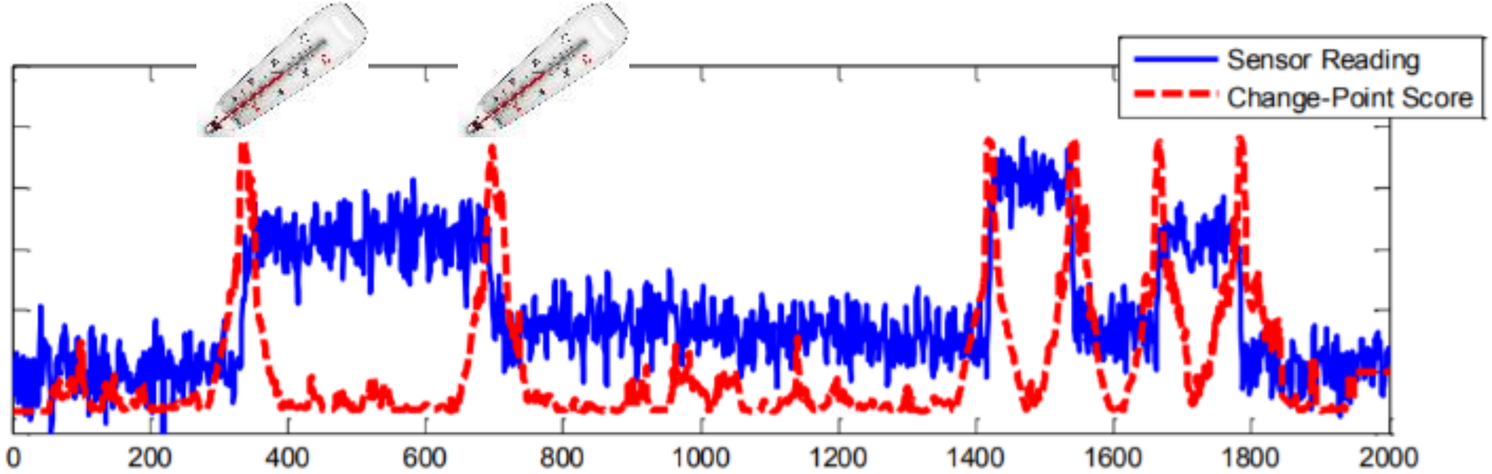
Index Policies

Maintenance when required

Theoretical formulation:

Predictive maintenance relies on

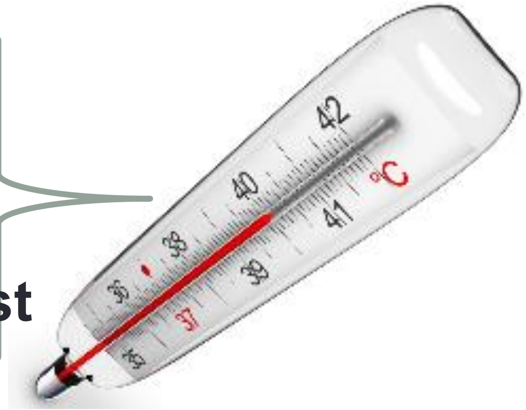
Distributional Change Detection in Time-series



- **Objective**: Detecting abrupt changes lying among time-series data

Predictive maintenance

Pioneering work
in 1940 by
Abraham Wald:
Sequential hypothesis test



Hypothesis test

H_0: Maintenance not needed

H_1: Maintenance needed

Condition monitoring
State prediction

Decision making

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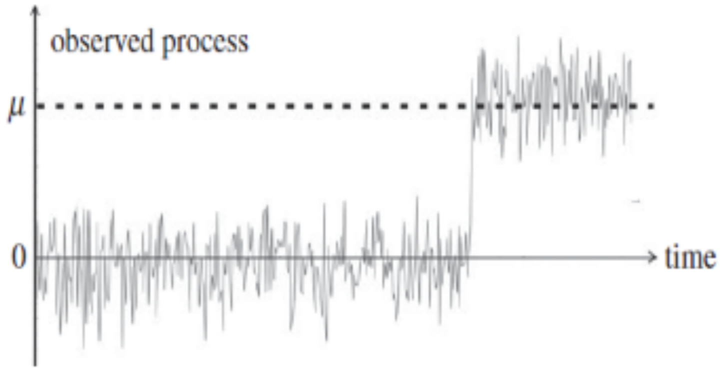
Maintenance when required

Predictive maintenance



Condition monitoring

Minimize Probability of false alarm (accept H1 when H0 is true)

Trade off

Minimize Probability of false serenity (accept H0 when H1 is true)

Hypothesis test

H_0: Maintenance not needed

H_1: Maintenance needed

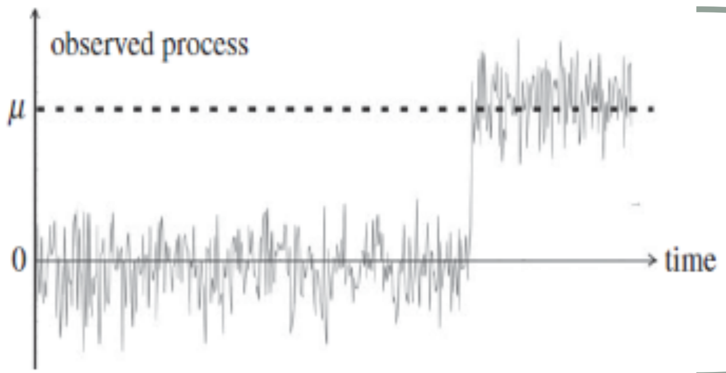
$$H_0: \mathbf{x}[i] = \sigma \mathbf{w}[i]$$
$$H_1: \mathbf{x}[i] = \sigma \mathbf{w}[i] + \mu$$

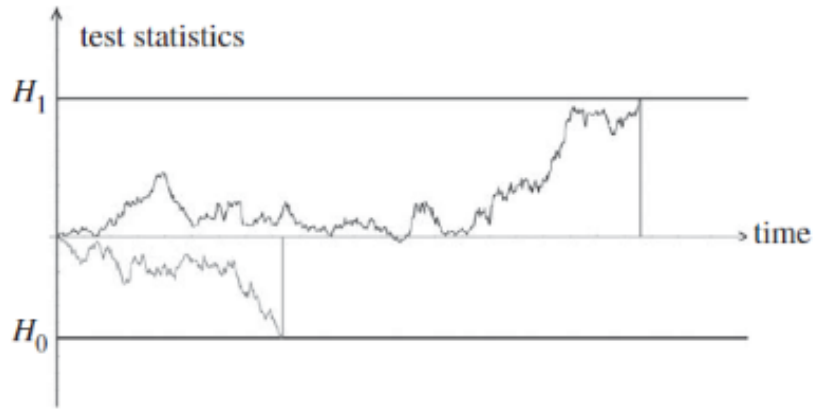
Model based

Typical Index Policy:
If likelihood ≥ K : we accept H0
If likelihood < K: we accept H1 → maintenance

Maintenance when required

Predictive maintenance





Condition monitoring

Decision making

Maintenance action

Sequential probability ratio testing

$$H_0: \mathbf{x}[i] = \sigma \mathbf{w}[i]$$

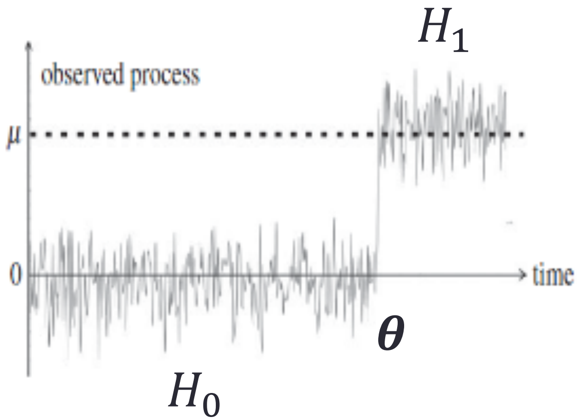
$$H_1: \mathbf{x}[i] = \sigma \mathbf{w}[i] + \mu$$

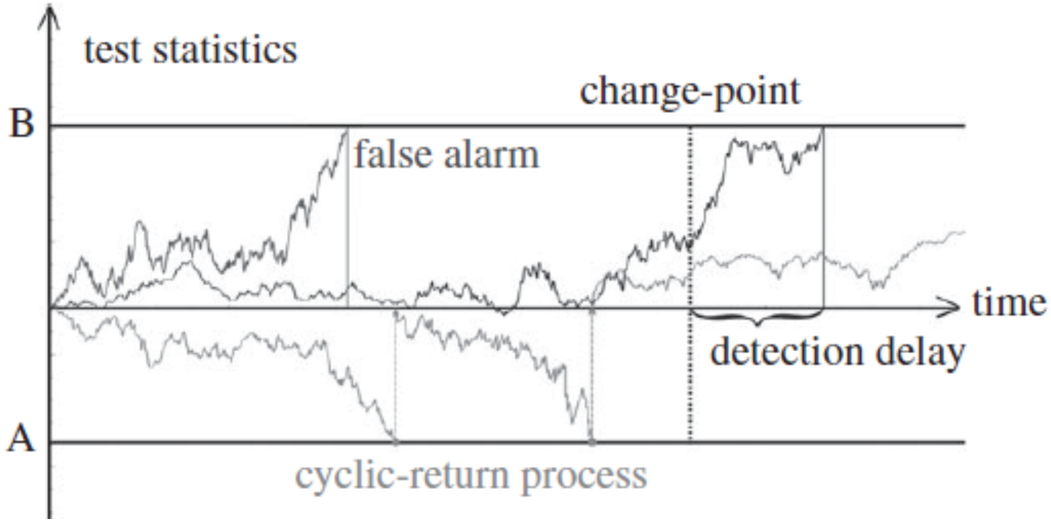
D. Siegmund, 2013, Sequential analysis, N.Y. Springer

Maintenance when required

Model based

Predictive maintenance





Change-point detection

$$H_0: \mathbf{x}[i] = \sigma \mathbf{w}[i]$$

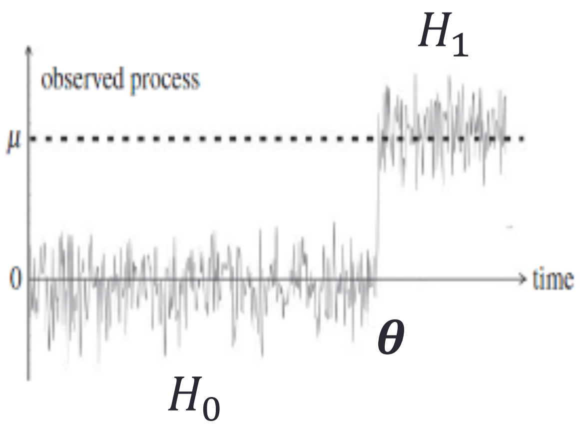
$$H_1: \mathbf{x}[i] = \sigma \mathbf{w}[i] + \mu$$

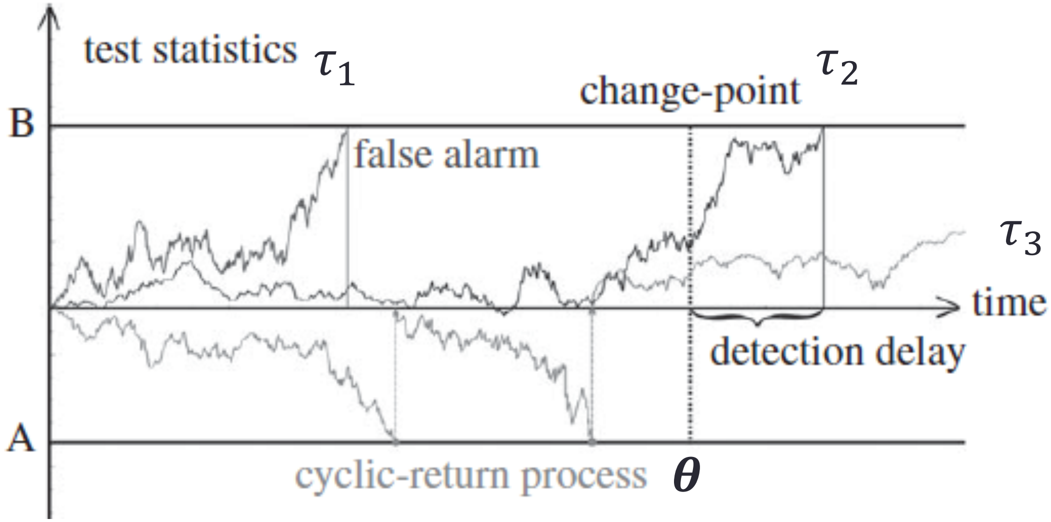
Minimize average **delay** and respect constraints on **false alarm**

Maintenance when required

Model based

Predictive maintenance





Change-point detection

$$H_0: \mathbf{x}[i] = \sigma \mathbf{w}[i]$$

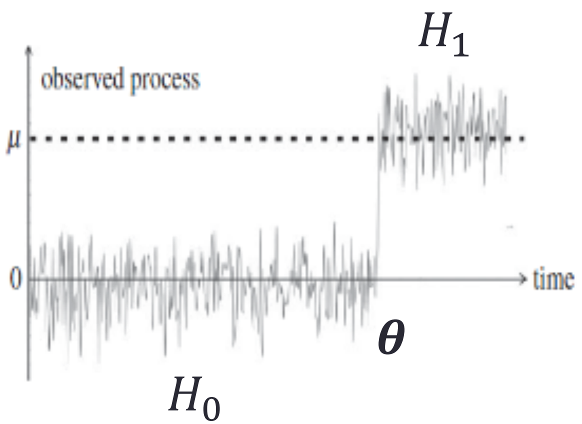
$$H_1: \mathbf{x}[i] = \sigma \mathbf{w}[i] + \mu$$

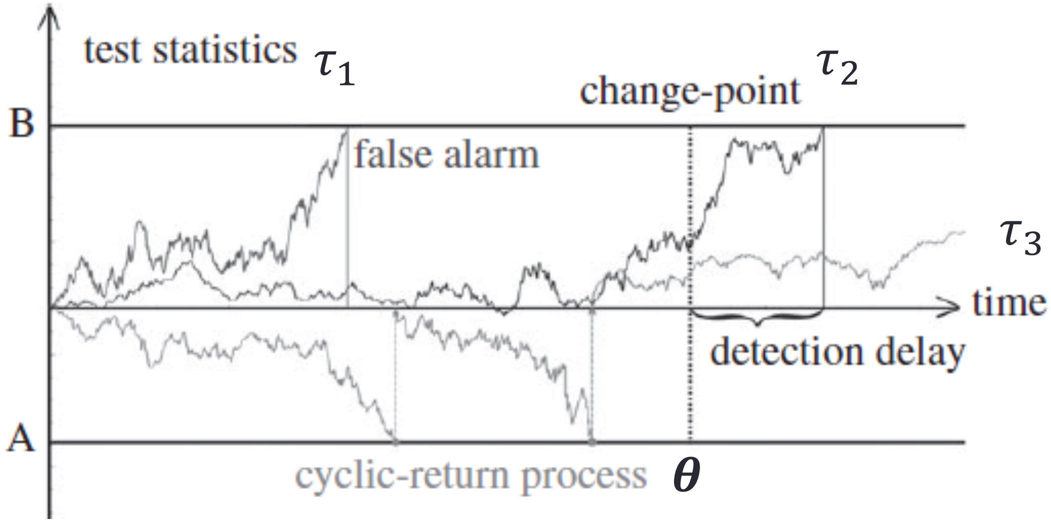
Model based

Bayesian problem: find stopping time τ^* s.t.

$$\boldsymbol{inf_\tau[P(\tau \leq \theta) + cE(\tau - \theta)^+] = P(\tau^* \leq \theta) + cE(\tau^* - \theta)^+}$$

Predictive maintenance





Change-point detection

$$H_0: \mathbf{x}[i] = \sigma \mathbf{w}[i]$$

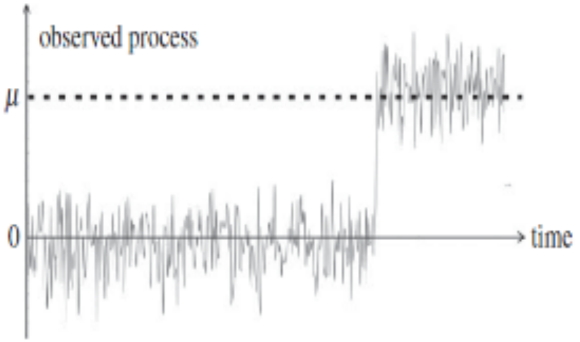
$$H_1: \mathbf{x}[i] = \sigma \mathbf{w}[i] + \mu$$

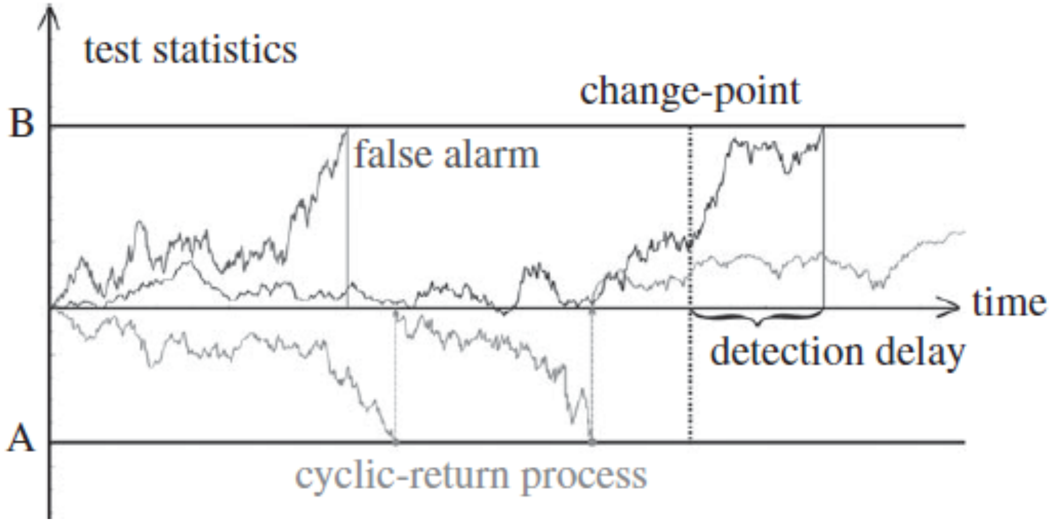
Model based

Conditional extremal problem: for $\boldsymbol{P(\tau^* \leq \theta)} \leq \alpha$ find stopping time τ^* s.t.

$$\boldsymbol{inf_{\tau}[E(\tau - \theta | \tau \geq \theta)] = E(\tau^* - \theta | \tau^* \geq \theta)}$$

Predictive maintenance





Change-point detection

$$H_0: \mathbf{x}[i] = \sigma \mathbf{w}[i]$$

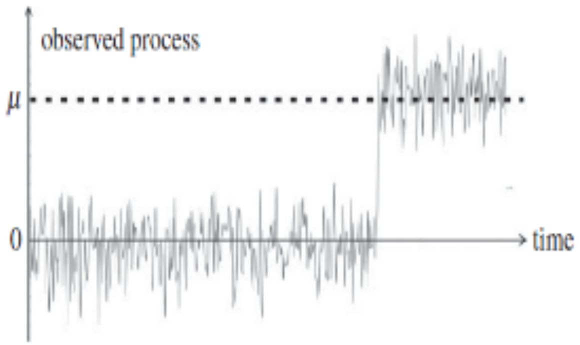
$$H_1: \mathbf{x}[i] = \sigma \mathbf{w}[i] + \mu$$

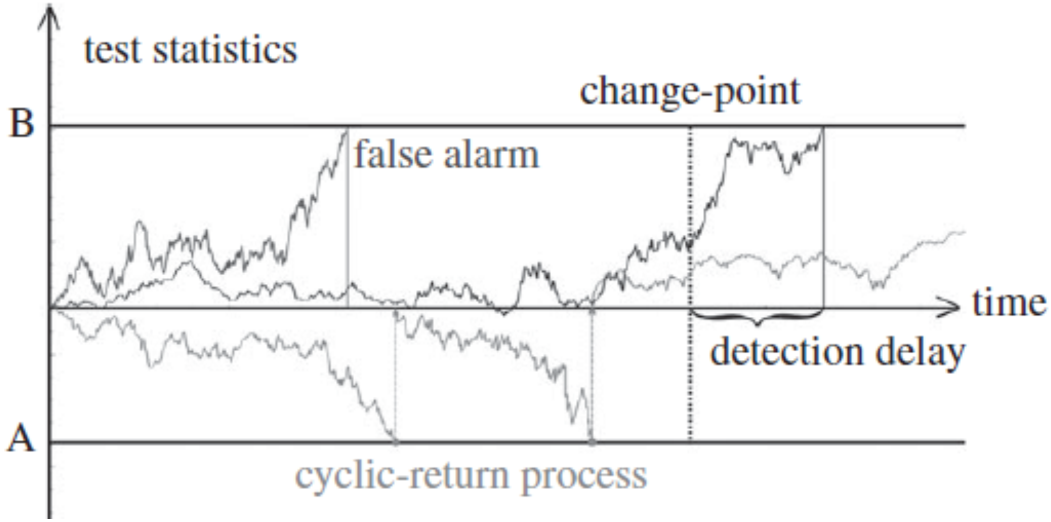
Model based

A. Shiryaev, **1963**
On detecting of disorders in industrial processes.
Ann. Stat. **36**, 787–807.

Maintenance when required

Predictive maintenance





Change-point detection

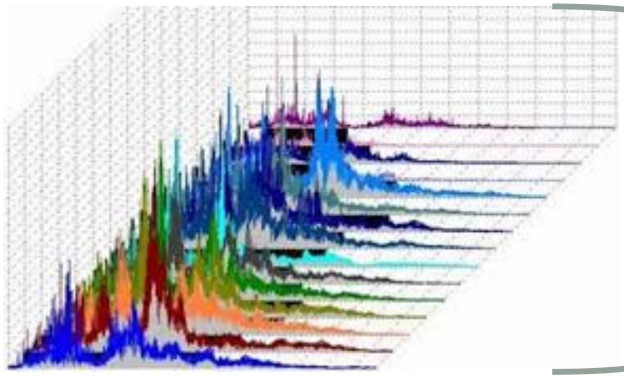
$$H_0: \mathbf{x}[i] = \sigma \mathbf{w}[i]$$

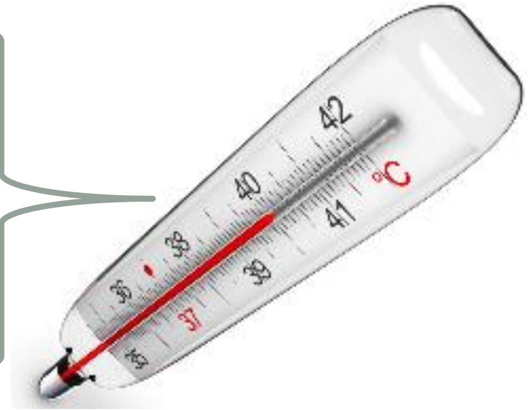
$$H_1: \mathbf{x}[i] = \sigma \mathbf{w}[i] + \mu$$

Model based

A. Shiryaev, and P. Y. Zryumov, (**2009**).
On the Linear and Nonlinear Generalized
Bayesian Disorder Problem.
Optimality and Risk, Modern
Trends in Mathematical Finance.

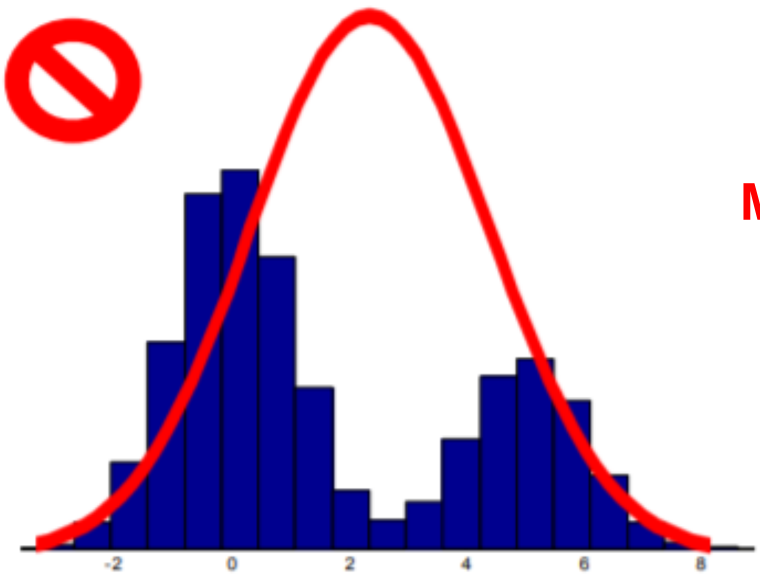
Predictive maintenance





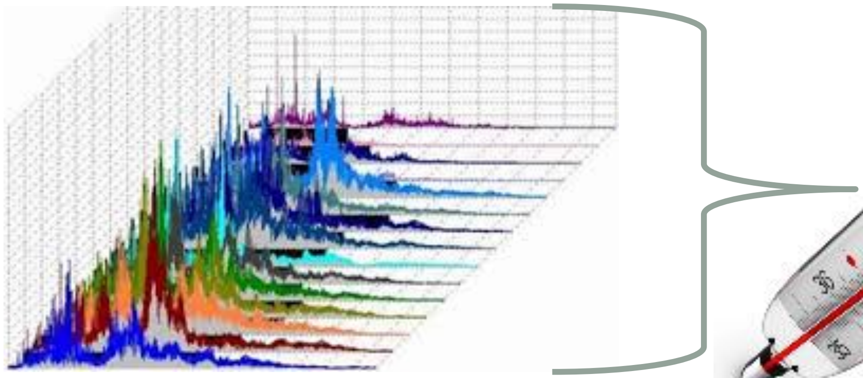
Condition monitoring

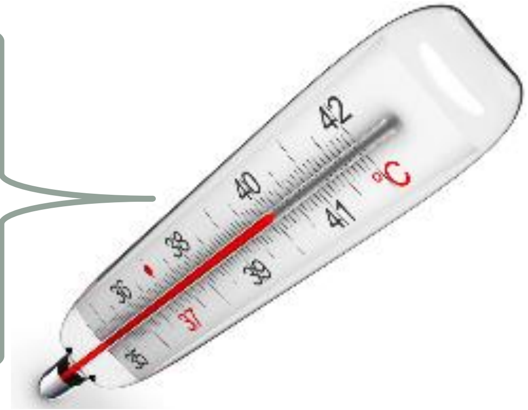
Decision making



Model mismatch!

Predictive maintenance

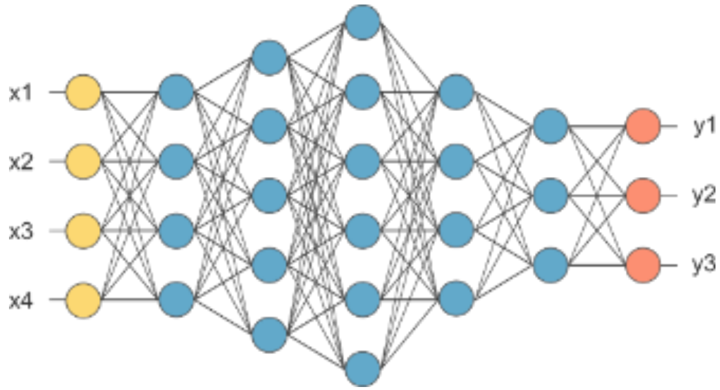




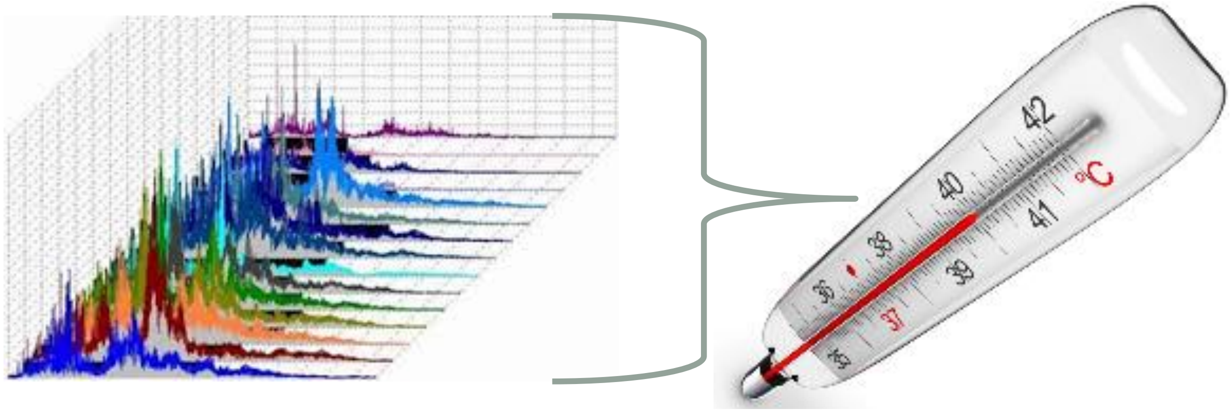
Condition monitoring

Decision making

Data based (model free) estimation



Predictive maintenance



Condition monitoring

Decision making

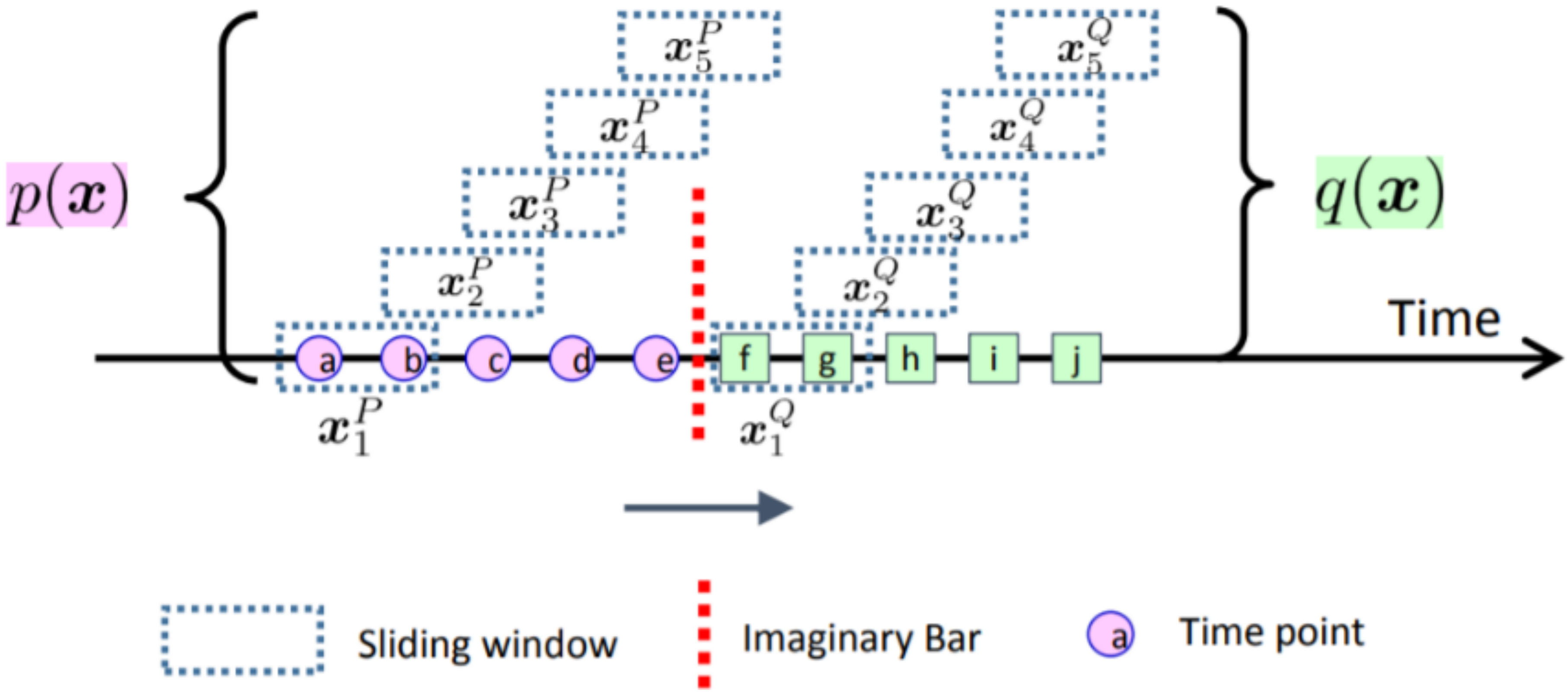
Get relevant, clean and synchronized data

- **Support Vector Machine:** Desorby et al. 2005
- **Likelyhood based methods:** Kawahara & Sugiyama, 2009

Predictive maintenance

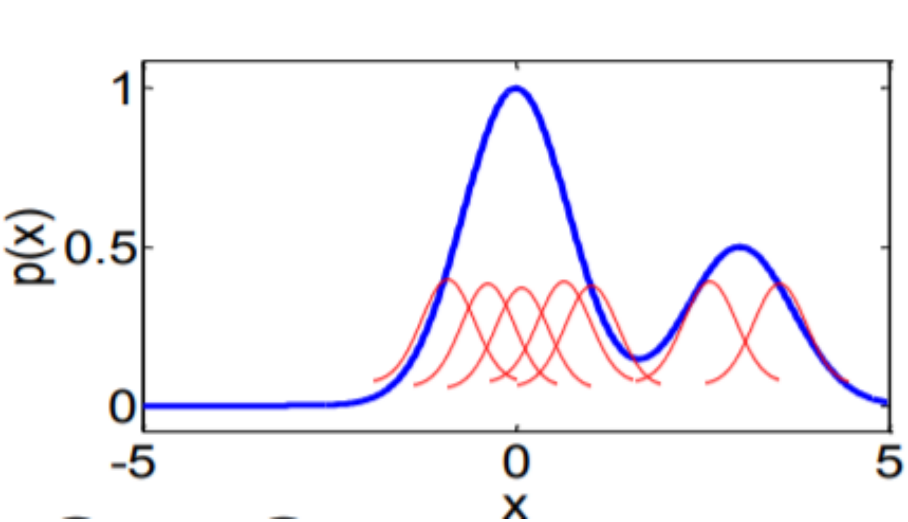
Theoretical problem: Estimate and compare densities

- Construct samples by using sliding window.
- An imaginary bar in the middle divides samples into two groups.
- Assume two groups of samples are from p and q.



Predictive maintenance

Kernel density estimation for p and q:

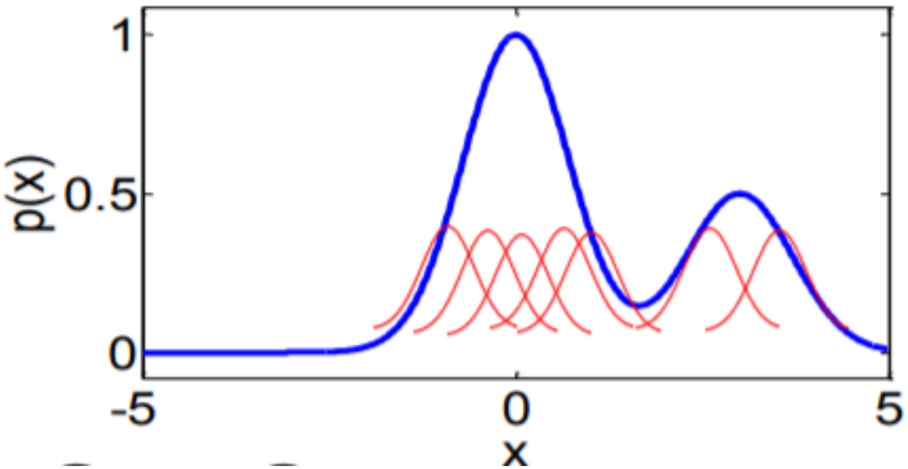


$$\hat{p}\ (\boldsymbol{x};\sigma) = \frac{1}{n}\sum_{i=1}^{n} K_\sigma(x, x_i)$$

$$K_\sigma(\boldsymbol{x},\boldsymbol{x}') = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(\frac{\|\boldsymbol{x}-\boldsymbol{x}'\|^2}{-2\sigma^2}\right)$$

Predictive maintenance

Direct ratio estimation for r=p/q:



$$\hat{p}\,(\boldsymbol{x};\sigma) = \frac{1}{n}\sum_{i=1}^{n} K_\sigma(x, x_i)$$

$$K_\sigma(\boldsymbol{x}, \boldsymbol{x}') = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(\frac{\|\boldsymbol{x}-\boldsymbol{x}'\|^2}{-2\sigma^2}\right)$$

Direct Density Ratio Estimation (Sugiyama et al., 2012).

$$\frac{p(\boldsymbol{x})}{q(\boldsymbol{x})} \approx \hat{r}(\boldsymbol{x};\boldsymbol{\theta}) = \sum_{i}^{n} \theta_i K_\sigma(\boldsymbol{x}, \boldsymbol{x}_i)$$

Predictive maintenance

Direct ratio estimation for r=p/q:

$$\widehat{\mathrm{KL}} := \frac{1}{n}\sum_{i=1}^{n} \log \hat{r}(\boldsymbol{x}_i)$$

Kulback Leibler

$$\widehat{\mathrm{PE}} := -\frac{1}{2n}\sum_{i=1}^{n} \hat{r}(\boldsymbol{x}'_i)^2 + \frac{1}{n}\sum_{i=1}^{n} \hat{r}(\boldsymbol{x}_i) - \frac{1}{2}$$

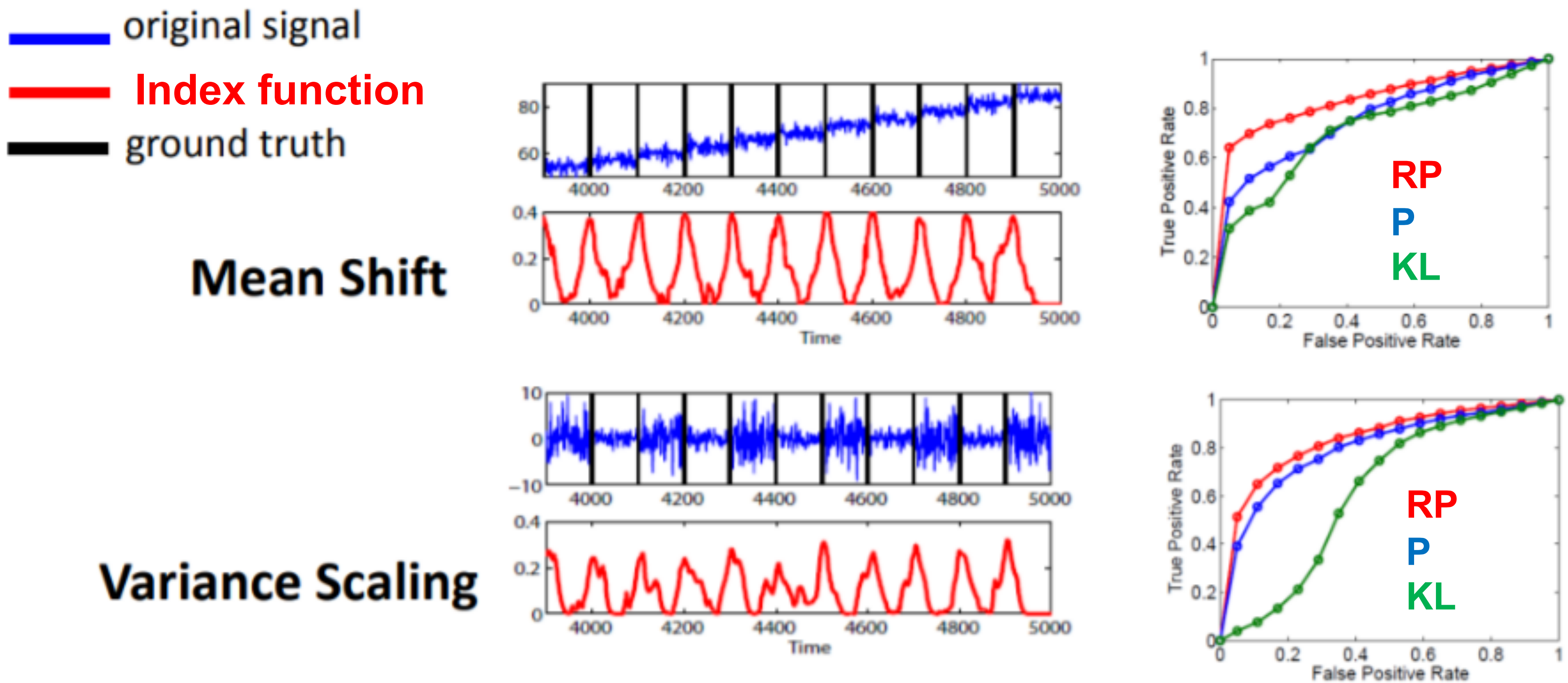
Pearson

Relative Pearson

$$\widehat{\mathrm{PE}}_\alpha = -\frac{\alpha}{2n}\sum_{i=1}^{n} \hat{r}(\boldsymbol{x}_i)^2 - \frac{1-\alpha}{2n}\sum_{i=1}^{n} \hat{r}(\boldsymbol{x}'_i)^2 + \frac{1}{n}\sum_{i=1}^{n} \hat{r}(\boldsymbol{x}_i) - \frac{1}{2}$$

Predictive maintenance

Experiment with synthetic data



Example: train track maintenance

On Board Monitoring System for Tilting Trains

- Measurement of vehicle reactions
- Monitoring of the track condition

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