PAB

# Exercice 9.1: Base commune Alice-Bob-Charlie

Base commune: $\left|h_{ijk}\right\rangle \equiv \left|h_{p}\right\rangle = \left|e_{i}\right\rangle \otimes \left|f_{j}\right\rangle \otimes \left|g_{k}\right\rangle$ $\qquad p = 0, \ldots ?$

Exemple: spin Up/Down

Base «Alice»: $\left|U\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \left|D\right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Base «Bob»: $\left|u\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \left|d\right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Base «Charlie»: $\left|\uparrow\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \left|\downarrow\right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Base commune:

?

Etat produit

$$\left|\psi_{i,j,k}\right\rangle = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} \otimes \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} \oplus \begin{pmatrix} \gamma_0 \\ \gamma_1 \end{pmatrix} = \begin{pmatrix} ? \end{pmatrix}$$

# Exercice 9.1: Base commune Alice-Bob-Charlie

Base commune: $\left|h_{ijk}\right\rangle \equiv \left|h_p\right\rangle = \left|e_i\right\rangle \otimes \left|f_j\right\rangle \otimes \left|g_k\right\rangle$ $\qquad p = 0, \ldots (n^3 - 1)$

Base commune spin Up/Down:

$$\begin{pmatrix} Uu\uparrow \\ Uu\downarrow \\ Ud\uparrow \\ Ud\downarrow \\ Du\uparrow \\ Du\downarrow \\ Dd\uparrow \\ Dd\downarrow \end{pmatrix}$$

Etat produit spin Up/Down

$$\left|\psi_{i,j,k}\right\rangle = \left|A\right\rangle \otimes \left|B\right\rangle \otimes \left|C\right\rangle = \begin{pmatrix} \alpha_0 \begin{pmatrix} \beta_0 \begin{pmatrix} \gamma_0 \\ \gamma_1 \end{pmatrix} \\ \beta_1 \begin{pmatrix} \gamma_0 \\ \gamma_1 \end{pmatrix} \end{pmatrix} \\ \alpha_1 \begin{pmatrix} \beta_0 \begin{pmatrix} \gamma_0 \\ \gamma_1 \end{pmatrix} \\ \beta_1 \begin{pmatrix} \gamma_0 \\ \gamma_1 \end{pmatrix} \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \alpha_0 \cdot \beta_0 \cdot \gamma_0 \\ \alpha_0 \cdot \beta_0 \cdot \gamma_1 \\ \alpha_0 \cdot \beta_1 \cdot \gamma_0 \\ \alpha_0 \cdot \beta_1 \cdot \gamma_1 \\ \alpha_1 \cdot \beta_0 \cdot \gamma_0 \\ \alpha_1 \cdot \beta_0 \cdot \gamma_1 \\ \alpha_1 \cdot \beta_1 \cdot \gamma_0 \\ \alpha_1 \cdot \beta_1 \cdot \gamma_1 \end{pmatrix}$$

# Exercice 9.2: opérateur produit entre Alice, Bob et Charlie

$$O = M \otimes N \otimes L$$

**Alice**

$$M = \begin{pmatrix} M_{00} & M_{01} \\ M_{10} & M_{11} \end{pmatrix}$$

**Bob**

$$N = \begin{pmatrix} N_{00} & N_{01} \\ N_{10} & N_{11} \end{pmatrix}$$

**Charlie**

$$L = \begin{pmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{pmatrix}$$

$O =$ ?

# Exercice 9.2:
# opérateur produit entre Alice, Bob et Charlie

Alice Bob Charlie

$$O = M \otimes N \otimes L$$

$$O = \begin{pmatrix} M_{00} \begin{pmatrix} N_{00} \begin{pmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{pmatrix} & N_{01} \begin{pmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{pmatrix} \\ N_{10} \begin{pmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{pmatrix} & N_{11} \begin{pmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{pmatrix} \end{pmatrix} & M_{01} \begin{pmatrix} N_{00} \begin{pmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{pmatrix} & N_{01} \begin{pmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{pmatrix} \\ N_{10} \begin{pmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{pmatrix} & N_{11} \begin{pmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{pmatrix} \end{pmatrix} \\ M_{10} \begin{pmatrix} N_{00} \begin{pmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{pmatrix} & N_{01} \begin{pmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{pmatrix} \\ N_{10} \begin{pmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{pmatrix} & N_{11} \begin{pmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{pmatrix} \end{pmatrix} & M_{11} \begin{pmatrix} N_{00} \begin{pmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{pmatrix} & N_{01} \begin{pmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{pmatrix} \\ N_{10} \begin{pmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{pmatrix} & N_{11} \begin{pmatrix} L_{00} & L_{01} \\ L_{10} & L_{11} \end{pmatrix} \end{pmatrix} \end{pmatrix}$$

# Exemple

**Alice**

$$M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

**Bob**

$$N = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

**Charlie**

$$L = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\Rightarrow \quad O = M \otimes N \otimes L = \ ?$$

**Etat produit**

$$|A\rangle = \tfrac{1}{5} \cdot \begin{pmatrix} 3 \\ 4 \end{pmatrix} \qquad |B\rangle = \tfrac{1}{5} \cdot \begin{pmatrix} 4 \\ -3 \end{pmatrix} \qquad |C\rangle = \tfrac{1}{5} \cdot \begin{pmatrix} 4 \\ 3 \cdot i \end{pmatrix}$$

$$\langle M \rangle = \langle A | M | A \rangle = \ ?$$

$$\langle N \rangle = \langle B | N | B \rangle = \ ?$$

$$\langle L \rangle = \langle C | L | C \rangle = \ ?$$

$$|\psi\rangle = |A\rangle \otimes |B\rangle \otimes |C\rangle = \ ?$$

$$\langle O \rangle = \langle \psi | O | \psi \rangle = \ ?$$

# Exemple

**Alice**

$$M=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$\sigma_Z$

**Bob**

$$N=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

$\sigma_X$

**Charlie**

$$L=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$

$\sigma_Y$

$$\Rightarrow \quad O=\begin{pmatrix}0&0&0&-i&0&0&0&0\\0&0&i&0&0&0&0&0\\0&-i&0&0&0&0&0&0\\i&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&i\\0&0&0&0&0&0&-i&0\\0&0&0&0&0&i&0&0\\0&0&0&0&-i&0&0&0\end{pmatrix}$$

**Etat produit**

$$|A\rangle=\tfrac{1}{5}\cdot\begin{pmatrix}3\\4\end{pmatrix}\qquad |B\rangle=\tfrac{1}{5}\cdot\begin{pmatrix}4\\-3\end{pmatrix}\qquad |C\rangle=\tfrac{1}{5}\cdot\begin{pmatrix}4\\3\cdot i\end{pmatrix}$$

$$\langle M\rangle=\langle A|M|A\rangle=-0.28$$

$$\langle N\rangle=\langle B|N|B\rangle=-0.96$$

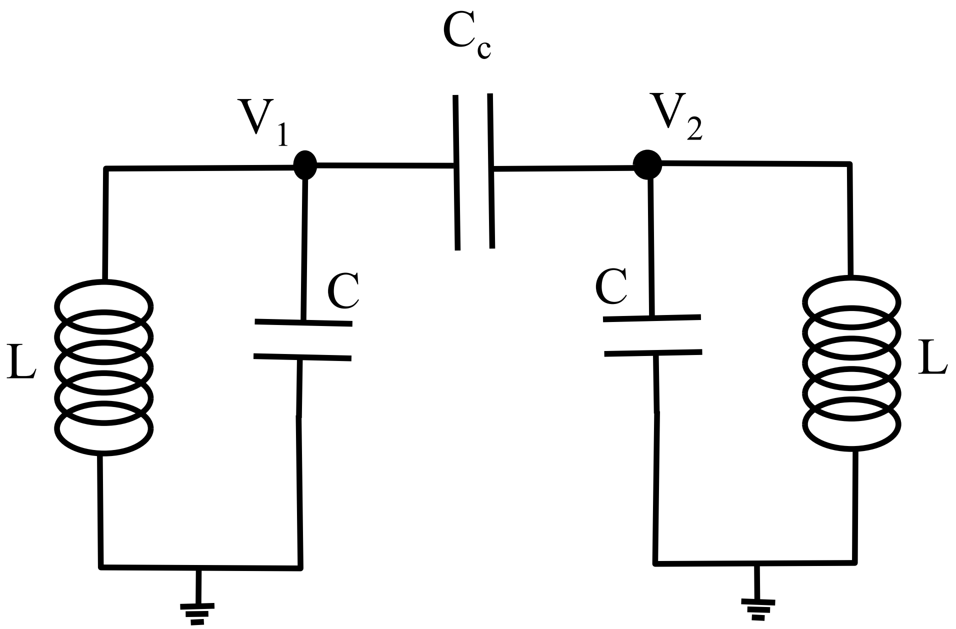
$$\langle L\rangle=\langle C|L|C\rangle=0.96$$

$$|\psi\rangle=|A\rangle\otimes|B\rangle\otimes|C\rangle=\tfrac{1}{25}\cdot\begin{pmatrix}12\\-9\\16\\-12\end{pmatrix}\otimes|C\rangle=\tfrac{1}{125}\begin{pmatrix}48\\36\cdot i\\-36\\-27\cdot i\\64\\48\cdot i\\-48\\-36\cdot i\end{pmatrix}$$

$$\langle O\rangle=\langle\psi|O|\psi\rangle=0.258=\langle M\rangle\cdot\langle N\rangle\cdot\langle L\rangle$$

# Exercice 9.3:
# Hamiltonien de couplage iSWAP

$$H_c \approx -T \cdot \left( a_{1+} a_{2-} + a_{1-} a_{2+} \right)$$




Reprenons l'exercice 8.3:
Considérez seulement deux modes dans chaque résonateur (2 qubits)

**Exprimez l'opérateur $H_c$ sous forme matricielle**

# Hamiltonien de couplage

$$a_{+} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \Rightarrow \qquad a_{1+}a_{2-} = \begin{array}{cc} \begin{matrix} \mathrm{Gg} & \mathrm{Ge} & \mathrm{Eg} & \mathrm{Ee} \end{matrix} & \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{matrix} \mathrm{Gg} \\ \mathrm{Ge} \\ \mathrm{Eg} \\ \mathrm{Ee} \end{matrix} \end{array}$$

$$\boxed{H_c \approx -T \cdot \left( a_{1+}a_{2-} + a_{1-}a_{2+} \right)}$$

$$\Downarrow$$

$$a_{-} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \Rightarrow \qquad a_{1-}a_{2+} = \begin{array}{cc} \begin{matrix} \mathrm{Gg} & \mathrm{Ge} & \mathrm{Eg} & \mathrm{Ee} \end{matrix} & \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{matrix} \mathrm{Gg} \\ \mathrm{Ge} \\ \mathrm{Eg} \\ \mathrm{Ee} \end{matrix} \end{array}$$

$$\boxed{H_c \approx -T \cdot \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}$$

# Exercice 9.4: singulet / triplet

Supposons une énergie d'interaction de la forme «spin-spin» entre deux qubits:

$$H \approx \vec{\sigma}_1 \otimes \vec{\sigma}_2 \approx \sigma_{1x} \otimes \sigma_{2x} + \sigma_{1y} \otimes \sigma_{2y} + \sigma_{1z} \otimes \sigma_{2z}$$

**1) Exprimez H sous la forme d'une matrice 4x4**

**2) Montrez que les états de Bell sont les modes propres de cet Hamiltonien.**
**Quelles sont les valeurs propres correspondantes ?**

# Singulet / triplet

**Energie d'interaction spin/spin**

$$H \approx \vec{\sigma}_1 \otimes \vec{\sigma}_2 \approx \sigma_{1x} \otimes \sigma_{2x} + \sigma_{1y} \otimes \sigma_{2y} + \sigma_{1z} \otimes \sigma_{2z}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_{1x} \otimes \sigma_{2x} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \qquad \sigma_{1y} \otimes \sigma_{2y} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \qquad \sigma_{1z} \otimes \sigma_{2z} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# Singulet / triplet

$$H \approx \sigma_{1x} \otimes \sigma_{2x} + \sigma_{1y} \otimes \sigma_{2y} + \sigma_{1z} \otimes \sigma_{2z} \quad \Rightarrow \quad H \approx \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

**Singulet:**

$$\left|\varphi_s\right\rangle \approx \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \qquad \lambda_s = -3$$

**Triplet:**

$$\left|\varphi_{t1}\right\rangle \approx \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \qquad \left|\varphi_{t2}\right\rangle \approx \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} \qquad \left|\varphi_{t3}\right\rangle \approx \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad \lambda_t = 1$$

**Etats de Bell, intrication maximale**

# Exercice 9.5:
# Etats de Bell

**Montrez que les états de Bell ne sont pas des états produits**

$$\left|\varphi_{t1}\right\rangle \approx \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \neq A \otimes B \qquad \left|\varphi_{t2}\right\rangle \approx \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} \neq A \otimes B \qquad \left|\varphi_{t3}\right\rangle \approx \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \neq A \otimes B \qquad \left|\varphi_{s}\right\rangle \approx \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \neq A \otimes B$$

# Etats de Bell

$$|\varphi_{t1}\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \qquad |\varphi_{t2}\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

$$\left.\begin{aligned} \alpha_0\beta_0 &= 1 \\ \alpha_0\beta_1 &= 0 \end{aligned}\right\} \quad \beta_1 = 0$$

$$\left.\alpha_1\beta_1 = \pm 1\right\} \quad \beta_1 \neq 0$$

$$\boxed{|\varphi_{t1}\rangle \neq A \otimes B}$$

$$\boxed{|\varphi_{t2}\rangle \neq A \otimes B}$$

$$|\varphi_{t3}\rangle = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \qquad |\varphi_{s}\rangle = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$$

$$\left.\begin{aligned} \alpha_0\beta_1 &= 1 \\ \alpha_0\beta_0 &= 0 \end{aligned}\right\} \quad \beta_0 = 0$$

$$\left.\alpha_1\beta_0 = \pm 1\right\} \quad \beta_0 \neq 0$$

$$\boxed{|\varphi_{t3}\rangle \neq A \otimes B}$$

$$\boxed{|\varphi_{s}\rangle \neq A \otimes B}$$

# Exercice 9.6

Montrez que ces opérateurs ne sont pas des opérateurs «produit»

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \neq M \otimes N$$

$$H_C = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \neq M \otimes N$$

$$\vec{\sigma}_1 \otimes \vec{\sigma}_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \neq M \otimes N$$

# Exercice 9.6

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\left.\begin{array}{l} M_{00}N_{00} = 1 \\ M_{00}N_{01} = 0 \end{array}\right\} N_{01} = 0$$

$$\left.M_{11}N_{01} = 1\right\} N_{11} \neq 0$$

$$\boxed{CNOT \neq M \otimes N}$$

$$H_C = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\left.\begin{array}{l} M_{01}N_{10} = 1 \\ M_{01}N_{01} = 0 \end{array}\right\} N_{01} = 0$$

$$\left.M_{10}N_{01} = 1\right\} N_{01} \neq 0$$

$$\boxed{\vec{\sigma}_1 \otimes \vec{\sigma}_2 \neq M \otimes N}$$

$$\vec{\sigma}_1 \otimes \vec{\sigma}_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\left.\begin{array}{l} M_{01}N_{10} = 2 \\ M_{01}N_{01} = 0 \end{array}\right\} N_{01} = 0$$

$$\left.M_{10}N_{01} = 2\right\} N_{01} \neq 0$$

$$\boxed{H_C \neq M \otimes N}$$