# The International System of Units

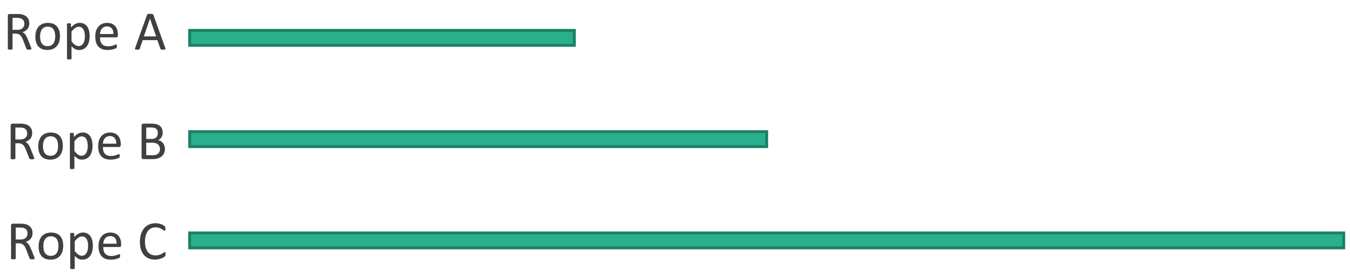
# Why do we need units?

Let’s assume we have two pieces of rope: A & B


We can compare these two pieces by saying Rope B is 50% longer than rope A

# Why do we need units?

Now let’s assume we have three pieces of rope: A,B & C




We can compare the previous pieces with the new piece C

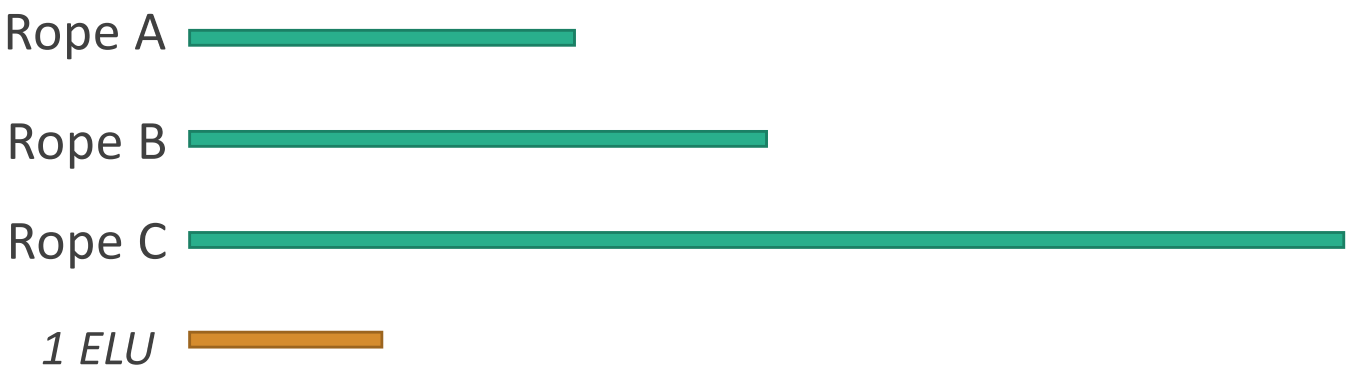
Rope C is twice as long as rope B: C=2*B
Rope C is three times as long as rope A:C=3*A

For every new piece of rope we could do such a comparison …

…would get quite messy!

# Why do we need units?

OR: we could agree on a standard length unit: Let’s call it the *EPFL Length Unit (ELU)*




We can then say:
Rope A = 2 *ELU*
Rope B = 3 *ELU*
Rope C = 6 *ELU*

Every new piece of rope we can now measure in *ELU* and therefore that piece is automatically compared to all previously measured ropes!

# How many units do we need

- Do we want to have a unit for every quantity we can measure? We could define:
  - EPFL Length Unit: *ELU*
  - EPFL Area Unit : *EAU*
  - EPFL Volume Unit: *EVU*
  - …

- *We would end up with a large number of units that are related, because we know Area (A) is defined in terms of Length (L) as:*

$$A=L*L$$

*Therefore:*

$$1\ EAU=1\ ELU^2$$

In the same way, many other physical quantities are interdependent with or are defined by other physical quantities. What we need is….

# Coherent System of Units

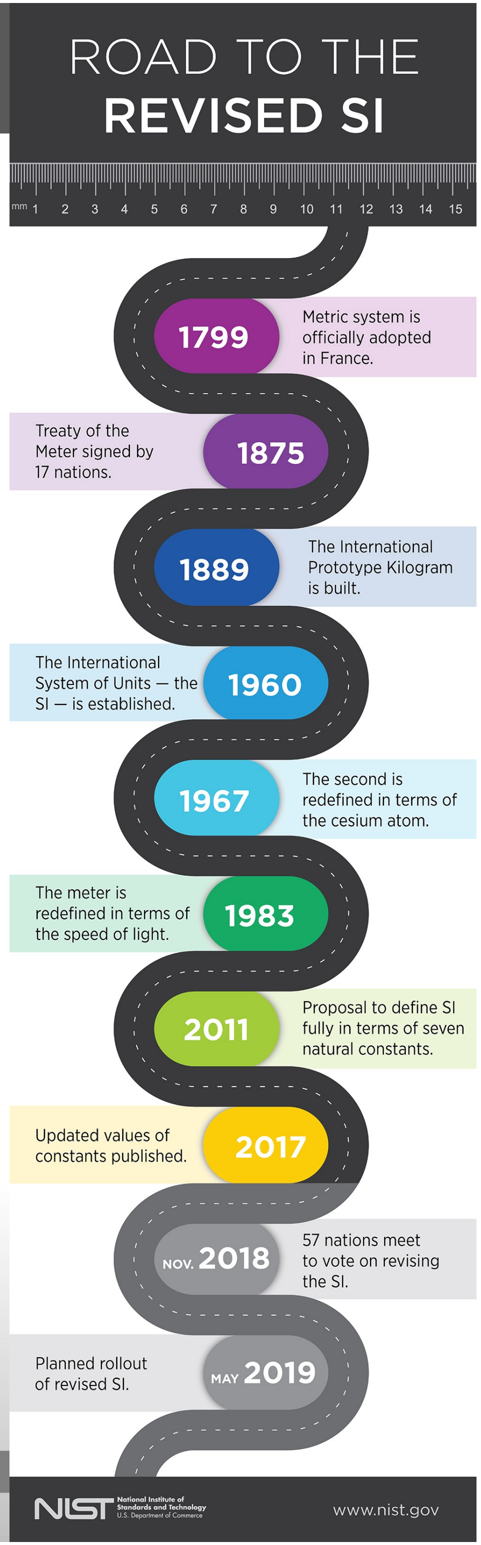
A *coherent system of units* is defined such that equations between units have the same form as equations of the physical quantities or numerical values expressed in the units. Any numerical factors must be the same in both representations

There could be many possibilities for a coherent system of units.

We use the ***International System of Units*** (*Système international (d'unités)*)

# International System of Units

- The SI is the modern form of the metric system
- The metric system was developed from 1791 onwards by a committee of the French Academy of Sciences
- In 1799 *mètre des Archives* and *kilogramme des Archives* were deposited in the French National Archives.
- 1812, Napoleon ”reversed” this new development
- It took until 1875 for a “second attempt for unification of units”: the <u>meter convention</u>
- 1899  International prototype metre and International prototype kilogram replace the *mètre des Archives* and *kilogramme des Archives*
- 1960: Definition of the SI
- 1967: Definition of second based on  Cs-133 hyperfine structure
- 1983: definition of meter in terms of vacuum speed of light
- 2019: The NEW SI based on definition of exact universal physical constants.



# International System of Units

- The SI is the modern form of the metric system
- It is maintained by the General Conference on Weights and Measures *(CGPM, Conférence générale des poids et mesures)*
- It is based on seven **base units** with their corresponding **physical quantities:**

| Base unit | Phys. Quantity |
|---|---|
| Ampere (A) | electrical current |
| Kelvin (K) | temperature |
| second (s) | time |
| meter (m) | distance |
| kilogram (kg) | mass |
| candela (cd) | luminous intensity |
| mole (mol) | amount of substance |

# International System of Units

- It has 20 pre-fixes to depict multiples and fractions of the base units:

**SI prefixes**

| Prefix | | Base 1000 | Base 10 | Decimal | English word | |
|---|---|---|---|---|---|---|
| Name | Symbol | | | | Short scale | Long scale |
| yotta | Y | $1000^{8}$ | $10^{24}$ | 1 000 000 000 000 000 000 000 000 | septillion | quadrillion |
| zetta | Z | $1000^{7}$ | $10^{21}$ | 1 000 000 000 000 000 000 000 | sextillion | trilliard |
| exa | E | $1000^{6}$ | $10^{18}$ | 1 000 000 000 000 000 000 | quintillion | trillion |
| peta | P | $1000^{5}$ | $10^{15}$ | 1 000 000 000 000 000 | quadrillion | billiard (Proposed) |
| tera | T | $1000^{4}$ | $10^{12}$ | 1 000 000 000 000 | trillion | billion |
| giga | G | $1000^{3}$ | $10^{9}$ | 1 000 000 000 | billion | milliard |
| mega | M | $1000^{2}$ | $10^{6}$ | 1 000 000 | million | |
| kilo | k | $1000^{1}$ | $10^{3}$ | 1 000 | thousand | |
| hecto | h | $1000^{2/3}$ | $10^{2}$ | 100 | hundred | |
| deca | da | $1000^{1/3}$ | $10^{1}$ | 10 | ten | |
| | | $1000^{0}$ | $10^{0}$ | 1 | one | |
| deci | d | $1000^{-1/3}$ | $10^{-1}$ | 0.1 | tenth | |
| centi | c | $1000^{-2/3}$ | $10^{-2}$ | 0.01 | hundredth | |
| milli | m | $1000^{-1}$ | $10^{-3}$ | 0.001 | thousandth | |
| micro | μ | $1000^{-2}$ | $10^{-6}$ | 0.000 001 | millionth | |
| nano | n | $1000^{-3}$ | $10^{-9}$ | 0.000 000 001 | billionth | milliardth |
| pico | p | $1000^{-4}$ | $10^{-12}$ | 0.000 000 000 001 | trillionth | billionth |
| femto | f | $1000^{-5}$ | $10^{-15}$ | 0.000 000 000 000 001 | quadrillionth | billiardth (Proposed) |
| atto | a | $1000^{-6}$ | $10^{-18}$ | 0.000 000 000 000 000 001 | quintillionth | trillionth |
| zepto | z | $1000^{-7}$ | $10^{-21}$ | 0.000 000 000 000 000 000 001 | sextillionth | trilliardth |
| yocto | y | $1000^{-8}$ | $10^{-24}$ | 0.000 000 000 000 000 000 000 001 | septillionth | quadrillionth |

Source: Wikipedia: https://en.wikipedia.org/wiki/International_System_of_Units#Base_units

# International System of Units

- It has 22 derived units:

| Name | Symbol | Quantity | Expressed in terms of other SI units | Expressed in terms of SI base units |
|---|---|---|---|---|
| radian | rad | angle | | $m \cdot m^{-1}$ |
| steradian | sr | solid angle | | $m^2 \cdot m^{-2}$ |
| hertz | Hz | frequency | | $s^{-1}$ |
| newton | N | force, weight | | $kg \cdot m \cdot s^{-2}$ |
| pascal | Pa | pressure, stress | $N/m^2$ | $kg \cdot m^{-1} \cdot s^{-2}$ |
| joule | J | energy, work, heat | $N \cdot m$ | $kg \cdot m^2 \cdot s^{-2}$ |
| watt | W | power, radiant flux | J/s | $kg \cdot m^2 \cdot s^{-3}$ |
| coulomb | C | electric charge or quantity of electricity | | $s \cdot A$ |
| volt | V | voltage (electrical potential difference), electromotive force | W/A | $kg \cdot m^2 \cdot s^{-3} \cdot A^{-1}$ |
| farad | F | electric capacitance | C/V | $kg^{-1} \cdot m^{-2} \cdot s^4 \cdot A^2$ |
| ohm | Ω | electric resistance, impedance, reactance | V/A | $kg \cdot m^2 \cdot s^{-3} \cdot A^{-2}$ |
| siemens | S | electrical conductance | A/V | $kg^{-1} \cdot m^{-2} \cdot s^3 \cdot A^2$ |
| weber | Wb | magnetic flux | $V \cdot s$ | $kg \cdot m^2 \cdot s^{-2} \cdot A^{-1}$ |
| tesla | T | magnetic field strength | $Wb/m^2$ | $kg \cdot s^{-2} \cdot A^{-1}$ |
| henry | H | inductance | Wb/A | $kg \cdot m^2 \cdot s^{-2} \cdot A^{-2}$ |
| degree Celsius | °C | temperature relative to 273.15 K | | K |
| lumen | lm | luminous flux | $cd \cdot sr$ | cd |
| lux | lx | illuminance | $lm/m^2$ | $m^{-2} \cdot cd$ |
| becquerel | Bq | radioactivity (decays per unit time) | | $s^{-1}$ |
| gray | Gy | absorbed dose (of ionizing radiation) | J/kg | $m^2 \cdot s^{-2}$ |
| sievert | Sv | equivalent dose (of ionizing radiation) | J/kg | $m^2 \cdot s^{-2}$ |
| katal | kat | catalytic activity | | $s^{-1} \cdot mol$ |

Source: http://getmetricized.blogspot.com/2015/04/introducing-si-base-units-and-derived_7.html

# Defining units based on physical phenomena

In order to define a unit, we all have to agree on some things:

1. We could use a physical phenomenon what we assume to be constant and that is measurable

*Example 1:* We could define time as a fraction of a day:

1s = 1/86 400 of a day

Problem: The physical phenomenon needs to be constant and must be clearly defined!

How do we define a day?

- The time it takes for the earth to do a full rotation (stellar day)
- The time it takes between two times that the sun is at its highest point (solar day)

These two definitions are NOT the same! The solar day is also not constsnt! It depends on the time of year!

# Defining units based on physical phenomena

*Example 2:* We could define 1 degree centigrade:

1°C = 1/100 of the difference in temperature between the freezing and boiling point of water

Problem: The freezing and boiling points of water depend on the atmospheric pressure, and hence the altitude and weather!

*Example 3:* 1m = $10^{-7}$ * distance between north pole and equator when passing through Paris

Problem: This definition is neither very practical or measurable.

# Defining units based on physical artefacts

2. Another way is to agree on one physical manifestation of the unit to which all others are compared. Such physical manifestations are called artefacts (or sometimes prototypes).

*Example 4:* The prototype *kg* and *m* were made in 1889 out of 90% platinum and 10% iridium. Multiple copies were made to which all other objects could be compared.

<u>Problem:</u>

- There are a limited number of copies. Making more copies from copies can cause systematic errors.
- Artefacts can change over time. The prototype *kg* <u>looses ca 0.5 ug/year</u>

# Defining units based on universally true constants

3. A better approach would be to define units based on universally true constants and "proven" physical laws that link the constants to quantities that have specific units.

*Example 5:* The second (s) is defined based on the hyperfine structure of Cs-133. It is taking the fixed numerical value of the cesium frequency:

$$\Delta\nu_{cs} = \Delta\nu({}^{133}Cs)_{HFS} = 9'192'631'770\ Hz$$

with 1Hz=1s$^{-1}$

Based on quantum mechanics we assume that this transition is constant and universal all over the world when (measured under the same conditions).

We can then derive the definition of other units based on physical laws and physical constants with <u>fixed</u> numerical values.

# Defining units based on universally true constants

Let's look at the relationship between distance and time:

$$\Delta x = v \cdot \Delta t$$

- Let's first use the s based on the CS-133 method, and the meter based on the prototype meter artefact. We can then calculate for any object the speed that it travels at:

$$v = \frac{\Delta x}{\Delta t}$$

- We can also measure this speed for a photon travelling in vacuum:

$$v_{photon} = c = 299'792'458{,}0 \ldots \frac{m}{s}$$

  The precision of *c* depends on the precision of the prototype meter we used.

But we know from the theory of relativity that *c* is constant!

- Instead of using the prototype meter, we could **define:**

$$v_{photon} = c = 299'792'458 \frac{m}{s} \quad \text{EXACT!}$$

From that we can define that the meter is equal to the distance light travels in vacuum in $299'792'458$s.

If *c* is universal, then this defines the *meter* universally!

EPFL

# Defining units based on universally true constants

In a similar way we can define all other base units by agreeing on exact numerical values for specific physical constants.

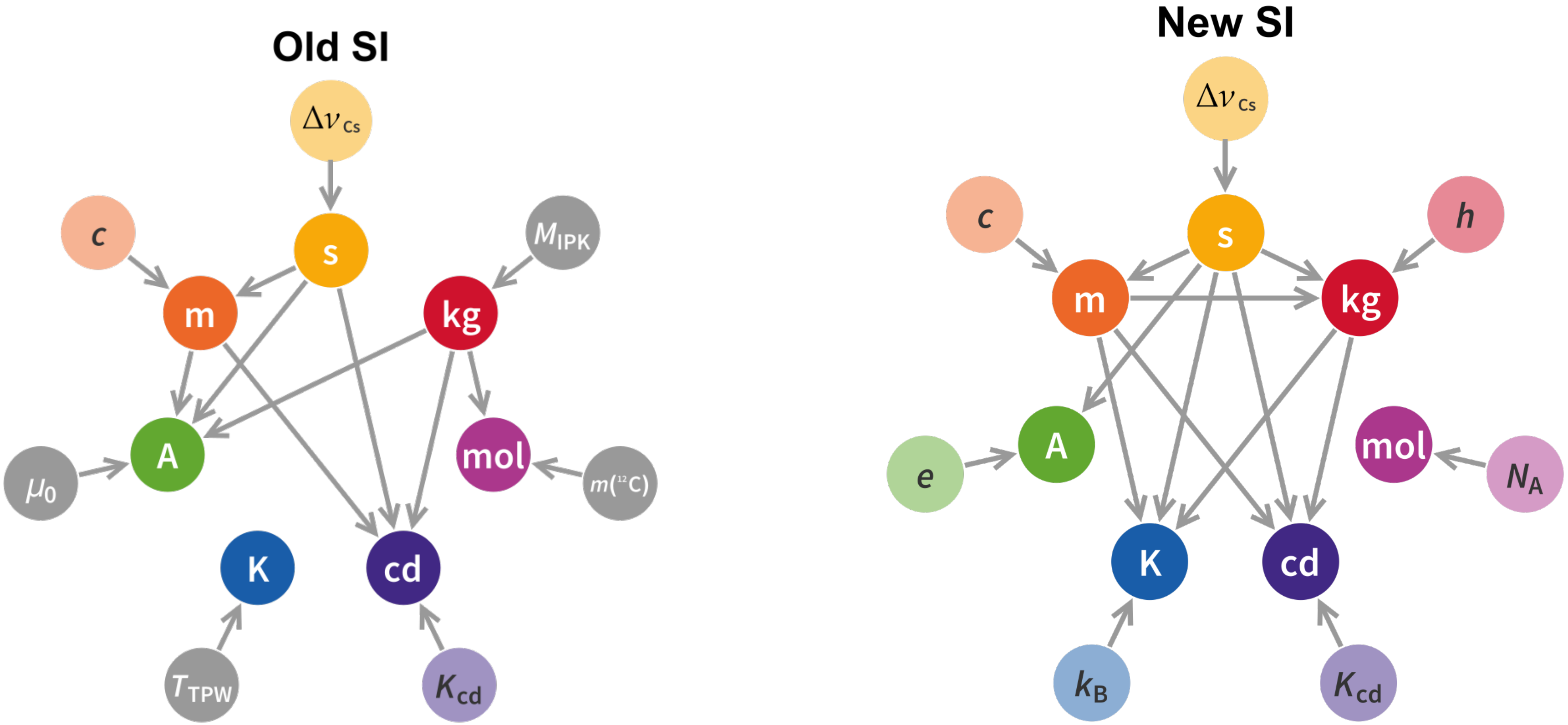
*This is the BIGH CHANGE in the International System of Units from May 20th 2019!*

*All base units were re-defined based on physical constants and no more physical artefacts are in use.*

The constants with exact numerical values are:

| | |
|---|---|
| h = 6.62607015×10−34 kg·m2·s−1 | Planck's constant |
| $e = 1.602176634\times10^{-19}$ A·s | elementary charge |
| $k = 1.380649\times10^{-23}$ kg·m$^2$·K$^{-1}$·s$^{-2}$ | Boltzmann constant |
| $N_A = 6.02214076\times10^{23}$ mol$^{-1}$ | Avogadro's number |
| $c = 299792458$ m·s$^{-1}$ | Vacuum speed of light |
| $\Delta\nu_{Cs} = \Delta\nu(^{133}Cs)_{hfs} = 9192631770$ s$^{-1}$ | Hyperfine structure transition frequency of the caesium-133 atom |
| $K_{cd} = 683$ cd·sr·s$^3$·kg$^{-1}$·m$^{-2}$ | Luminous efficacy |

# Fundamental differences between old and new SI



# Dimensional Analysis

- The SI is a coherent system of units so that equations between units have the same form as the equations expressed in physical quantities or numerical values.

- This is very convenient, because it allows us to use dimensional analysis to find relationships that we might not know yet.
- Let's look at a pendulum: what is the formula for a pendulum period?
- What quantities could influence the time the pendulum takes for one period?

# The universe by orders of magnitude

**Table 1-3 The Universe by Orders of Magnitude**

| Size or Distance | (m) | Mass | (kg) | Time Interval | (s) |
|---|---|---|---|---|---|
| Proton | $10^{-15}$ | Electron | $10^{-30}$ | Time for light to cross nucleus | $10^{-23}$ |
| Atom | $10^{-10}$ | Proton | $10^{-27}$ | Period of visible light radiation | $10^{-15}$ |
| Virus | $10^{-7}$ | Amino acid | $10^{-25}$ | Period of microwaves | $10^{-10}$ |
| Giant amoeba | $10^{-4}$ | Hemoglobin | $10^{-22}$ | Half-life of muon | $10^{-6}$ |
| Walnut | $10^{-2}$ | Flu virus | $10^{-19}$ | Period of highest audible sound | $10^{-4}$ |
| Human being | $10^{0}$ | Giant amoeba | $10^{-8}$ | Period of human heartbeat | $10^{0}$ |
| Highest mountain | $10^{4}$ | Raindrop | $10^{-6}$ | Half-life of free neutron | $10^{3}$ |
| Earth | $10^{7}$ | Ant | $10^{-4}$ | Period of Earth's rotation | $10^{3}$ |
| Sun | $10^{9}$ | Human being | $10^{2}$ | Period of Earth's revolution around the Sun | $10^{7}$ |
| Distance from Earth to the Sun | $10^{11}$ | Saturn V rocket | $10^{6}$ | | |
| | | Pyramid | $10^{10}$ | Lifetime of human being | $10^{9}$ |
| Solar system | $10^{13}$ | Earth | $10^{24}$ | Half-life of plutonium-239 | $10^{12}$ |
| Distance to nearest star | $10^{16}$ | Sun | $10^{30}$ | Lifetime of mountain range | $10^{15}$ |
| Milky Way galaxy | $10^{21}$ | Milky Way galaxy | $10^{41}$ | Age of Earth | $10^{17}$ |
| Visible universe | $10^{26}$ | Universe | $10^{52}$ | Age of universe | $10^{18}$ |

# Exercise problems 1:

Redefinition of the *kg* based on Planck's constant:

- Since May 20th 2019, the kg is no longer defined based on the prototype kg but based on the exact definition of Planck's constant.
- How is the base unit kg defined when having *s* defined using the Cs-133 definition and *m* defined through *c*? Prove that the dimensions are correct.
- *Hint: think about the energy of a photon.*