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Measuring the frequency of light with mode-locked lasers [1]

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Abstract

We have stabilized the modes of a comb of optical frequencies emitted by a mode-locked femtosecond-laser and used it as a ruler to measure differences of up to 45.2 THz between laser frequencies in a new type of frequency chain. Directly converting optical to radio frequencies, we have used it for an absolute frequency measurement of the 1S–2S transition in the hydrogen atom. Here, an intuitive model of the comb's properties is given and essential techniques for its stabilization and efficient detection of beat signals are presented.

Keywords: Frequency comb; Femtosecond laser; Frequency chain; Optical frequency measurement

In recent years, we have witnessed dramatic progress in the development of optical frequency standards based on trapped ions [1] and narrow spectral lines of neutral atoms such as the hydrogen 1S–2S transition [2]. Having the potential to reach accuracies far beyond the current state of the art cesium atomic clocks [3], the missing part of future atomic clocks still remains a precise, reliable 'clock-work', converting frequencies in the optical range of several hundred Terahertz to radio frequencies. In the past, phase coherent comparisons between optical and radio frequencies have been performed with the few harmonic frequency chains that reach all the way up to the visible or the UV region [2,4,5]. A promising approach for a new type of frequency chain is the determination of an optical frequency f by measuring a frequency interval in the optical region. For example, the interval $\Delta f = 2f - f = f$ between f and its second harmonic $2f$ could be measured. Optical frequency interval dividers [6,7] that allow the phase coherent bisection of arbitrarily large frequency differences can reduce that interval. These dividers generate the sum frequency of the two input laser frequencies in a nonlinear crystal and phase-lock the second harmonic of a third laser to it. This forces the fundamental frequency of the third laser to oscillate at the precise center of the interval. If n such dividers are cascaded the initial frequency gap is reduced by a factor 2^n. The simplicity and reliability of such a new frequency chain depends critically on the capability to directly measure large optical frequency differences, avoiding the need for a large number of frequency interval dividers.

While we have demonstrated a 5 stage frequency interval divider chain about two years ago [2], in recent experiments we have used the comb of modes emitted by a mode-locked femtosecond-laser to

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[1] We dedicate this article to Marlan O. Scully on occasion of his 60th birthday.

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bridge large frequency intervals in the optical region. Although the application of mode-locked picosecond lasers for the measurement of frequency intervals of about 1 GHz had been demonstrated in elementary form already 20 years ago [8], not much work had been done on that subject since then. Using pulses of about 70 fs length, we have recently bridged frequency differences of up to 20 THz in a phase coherent measurement of the absolute frequency of the Cs D_1 line [9] and in experiments to verify the mode spacing constancy [10]. After further spectral broadening of the comb by self phase modulation in an optical fiber [11], we could even bridge an interval of 45.2 THz. With this system we were able to directly compare the optical frequency of the 1S–2S two photon resonance in atomic hydrogen ($\lambda = 243$ nm) with the radio frequency of our local cesium atomic clock [12], demonstrating a prototype of a new type of frequency chain. As the spectral width of the frequency comb scales with the inverse of the pulse length, even larger frequency gaps may be measured with shorter pulses. To be able to make use of femtosecond-lasers as a tool for high precision metrology, we apply new techniques to stabilize the mode comb, to efficiently detect beat signals with continuous wave (CW) lasers and to count the number of modes in a frequency interval. In this article, these techniques are presented on the basis of an intuitive model of the properties of the frequency comb and we discuss applications of mode-locked femtosecond-lasers for absolute frequency measurements.

Kerr-lens mode-locked lasers are able to generate pulse trains with pulse-lengths as short as 5.4 fs [13]. The resonator of these lasers is designed in a way that it experiences lower losses if a Kerr-lens is created by the high peak intensity of a short pulse [14]. The group velocity dispersion $\partial^2 k/\partial\omega^2$ in the cavity, that would lead to a rapid pulse broadening, is compensated with so called chirped mirrors [15] or a pair of prisms [16]. If started appropriately, a short stationary pulse with high peak intensity will continuously circulate in the cavity. In the frequency domain, Kerr-lens mode-locking can be viewed as an intensity dependent mechanism that locks the relative phases of a large number of active longitudinal laser modes in such a way that their superposition builds up this circulating pulse. The strong amplitude modulation can be thought of as imposing sidebands on the active laser modes that injection lock their neighboring modes. To make this work the mode separation has to be sufficiently constant over the entire optical bandwidth participating in the mode-locking process. In a passive resonator of length L the mode separation is determined by demanding that $2Lk(\omega)$ is a multiple of 2π. Dispersion is accounted for by the frequency dependence of the wave vector $k(\omega)$ which can be expanded about some center frequency ω_0:

$$k(\omega) = k(\omega_0) + \left.\frac{\partial k}{\partial\omega}\right|_{\omega_0}(\omega-\omega_0) + \frac{1}{2}\left.\frac{\partial^2 k}{\partial\omega^2}\right|_{\omega_0} \times(\omega-\omega_0)^2 + \dots \quad (1)$$

To obtain a constant mode separation all but the first two terms in this series have to vanish. As already pointed out with a time domain argument the third term, which is proportional to the group velocity dispersion, is compensated in a Kerr-lens mode-locked laser. The remaining higher order deviations from a constant mode spacing are removed by mode pulling of the active laser modes, i.e. injection locking of adjacent laser modes. The mode separation is calculated from the second term and can be expressed with the group velocity $v_g = \partial\omega_0/\partial k$ to be $v_g/2L$ which is the inverse round trip time of the circulating pulse. The mode spacing of passive resonators is usually given as the free spectral range (FSR) calculated from the phase velocity FSR = $c/n2L$, where n is the refractive index. In this expression, dispersion is neglected completely. Therefore it is not suitable for accurate metrological applications. However, the corresponding expression for mode-locked lasers, the pulse repetition rate $f_r = v_g/2L$, is easily measured with radio frequency accuracy. As it is the modulation frequency which governs the mode locking process and precisely fixes the mode separation, it will be used in the further discussion as the appropriate expression to describe the mode spacing.

The electric field at the output coupler of the mode-locked laser can be expressed as $E(t) = A(t)e^{-2\pi i f_c t} + \text{c.c.}$, with a (possibly complex) periodic envelope-function $A(t)$, amplitude- and phase-modulating an underlying light field function oscillating with some optical carrier frequency f_c. If the envelope function $A(t)$ is strictly periodic, it can be expressed as a Fourier series in f_r: $A(t)=\sum_q A_q e^{-2\pi i q f_r t}$. The electric field is then

$$E(t)=\sum_{q=-\infty}^{+\infty} A_q e^{-2\pi i(f_c+qf_r)t}+\text{c.c.} \tag{2}$$

The spectrum of Eq. (2) represents a comb of laser frequencies precisely spaced by the pulse repetition rate f_r, where the coefficients A_q, that contain the spectral intensity and the relative phases of the modes, do not depend on time. For example, a pulse train made up of a purely amplitude-modulated optical carrier frequency f_c would correspond to a symmetrical comb of modes, centered around f_c (see Fig. 1). The uniform spacing of the modes by precisely f_r is a result of the periodicity of the pulse envelope function. We have verified this property within the limits of our experiment with an uncertainty of a few mHz for a 20 THz frequency interval [10].

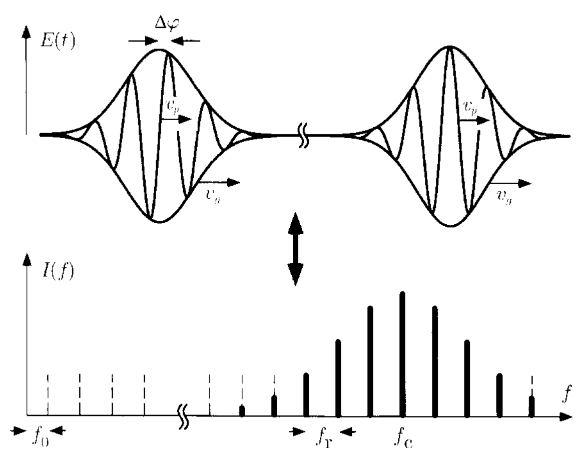

Fig. 1. Two consecutive pulses of the pulse train emitted by a mode-locked laser and intensity spectrum of the train. Within the cavity, the envelope is traveling with the group velocity v_g which, in general, differs from the phase velocity of the carrier v_p. The carrier phase relative to the envelope changes from pulse to pulse by $\Delta\varphi$. The modes are offset from being integer multiples of the pulse repetition rate f_r by $f_0=(\Delta\varphi/2\pi)f_r$.

In general, the carrier frequency f_c and therefore the frequency of each mode f_n is *not* an integer multiple of the pulse repetition rate. The reason for this is obvious, if an ideal pulse train consisting of a pure amplitude modulated carrier frequency is assumed: The envelope of the circulating pulse travels with the group velocity, while the carrier phase advances with its phase velocity. Thus, the carrier phase will be shifted after each round trip with respect to the pulse envelope by say $\Delta\varphi$ (see Fig. 1). This phase shift can be calculated from the difference of the group velocity of the pulse and the phase velocity of the carrier and can be chosen to obey $0\le\Delta\varphi\le 2\pi$. Clearly, the electric field emitted by the laser is in general not periodic with the pulse repetition time $T=f_r^{-1}$ so that it can not be expressed by a Fourier series in f_r. The comb of laser frequencies is therefore shifted by an offset frequency f_0 from the integer multiples of f_r. This shift is given by

$$f_0=(\Delta\varphi/2\pi)T^{-1}=(\Delta\varphi/2\pi)f_r\,. \tag{3}$$

Since $0\le\Delta\varphi\le 2\pi$, the offset frequency f_0 is smaller than f_r and by comparison with Eq. (2) the frequency of the nth cavity mode is given by

$$f_n=nf_r+f_0=(n+\Delta\varphi/2\pi)f_r\,. \tag{4}$$

where we set $f_c=n_c f_r+f_0$ with some large number n_c.

In a real laser the carrier frequency is usually not constant across the pulse. This so called frequency chirp can be described by a complex envelope function $A(t)=|A(t)|e^{-i\alpha(t)}$ in Eq. (2). The phase variation across the pulse $\alpha(t)$ is the same from pulse to pulse i.e. $\alpha(t)=\alpha(t-T)$, as confirmed by our experiment proving the mode spacing constancy [10]. Independent of the particular shape of $\alpha(t)$ the pulse is collecting a phase factor $e^{-2\pi i f_c T}$ during the round trip time T, which equals $e^{-2\pi i(n_c+f_0T)}=e^{-i\Delta\varphi}$. The relation $f_0=(\Delta\varphi/2\pi)f_r$ still holds even though $\Delta\varphi$ does not have the clear interpretation sketched in Fig. 1. The mode spacing constancy is thus maintained even for a chirped pulse and Eq. (4) remains valid. In our recent experiments we were able to control both f_r and f_0 independently, which supports the intuitive picture presented here.

The spectral width of the frequency comb can be broadened while maintaining the mode spacing by self phase modulation in a nonlinear optical material. Additional modes outside the original spectrum are created from the existing modes by this four wave mixing process: The frequency of one mode is subtracted from the sum frequency of two other modes. This $\chi^{(3)}$-process is most efficient if all possible combinations of original modes, that lead to a specific new mode, add up coherently. This is the case if the phases of the original modes are adjusted to give the shortest pulse, i.e. the highest peak power. The strong focusing of the pulses over an extended length in a single mode optical fiber has been used for this purpose [11]. Due to group velocity dispersion and self phase modulation the pulse is broadened and the peak intensity is reduced as it travels along the fiber so that the effective broadening comes to an end. The useful fiber length is limited to a value proportional to the initial pulse length squared. If the initial pulse has a negative chirp and the fiber a positive group velocity dispersion the pulse will reach its shortest duration in the fiber enhancing the process of self phase modulation. The group velocity dispersion of the fiber imposes a large frequency chirp on the pulses. As long as this chirp stays the same from pulse to pulse the mode spacing remains constant. To circumvent systematic uncertainties caused by slow drifts of the ambient fiber temperature the beat signals for phase-locking, as described below, are measured after the fiber passage. Fig. 2 shows the observed broadening of the spectrum of pulses obtained from a commercial Ti:Sapphire Kerr-lens mode-locked laser (Coherent model Mira 900 pumped by a 5 W frequency-doubled single-frequency $Nd:YVO_4$ laser, Coherent model Verdi). This set-up was used for an absolute frequency measurement of the hydrogen 1S–2S transition [12].

Like f_r, the frequency offset f_0 lies in the radio frequency domain. Eq. (4) would therefore allow a direct determination of the optical frequency f_n. While the pulse repetition rate is easily measured the frequency offset could be measured in principle by cross correlation of subsequent pulses [17], but not with radio frequency accuracy. As shown in Fig. 1 the peak electric field should depend on the relative phase φ of the carrier to the envelope if the pulses are sufficiently short. Highly nonlinear processes like high harmonic generation [18] and above threshold ionization [19] could then be sensitive to φ and should therefore be modulated with $2f_0$. This would allow a radio frequency measurement of f_0. Assuming a pulse shape in the time domain [2] of $E(t) = E_0 e^{-t^2/2\tau^2}\cos(2\pi f_c t + \varphi)$, like in Fig. 1, the detec-

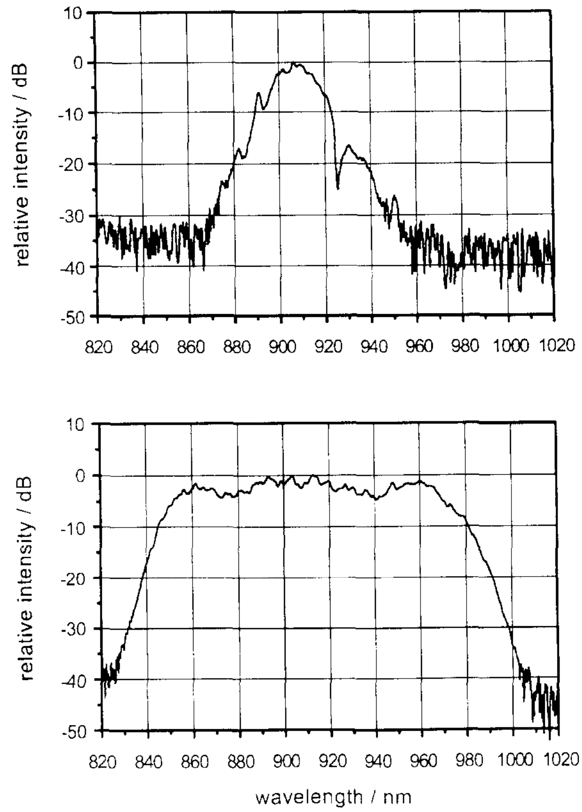

Fig. 2. Observed spectral broadening of 73 fs pulses (f_r = 76 MHz, 1.5 times the Fourier limited time-bandwidth product) after passage through a 40 cm long single mode optical fiber (Newport FS-F). Top: 8 mW average power from a Ti:Sapphire Kerr-lens mode-locked laser is coupled through the fiber. Bottom: The same with 225 mW average power. With our current set-up it is possible to phase-lock a laser diode to the modes that are more than 10 dB less intense than the peak of the spectrum.

[2] The intensity pulse width (FWHM) is given $2\tau\sqrt{\ln(2)} = 1.67\tau$.

tion after a process that depends on the Nth power of the electric field will yield a signal proportional to

$$\int_{-\infty}^{+\infty}\exp\left(\frac{-Nt^2}{2\tau^2}\right)\cos^N(2\pi f_c t+\varphi)\,dt$$
$$=\frac{1}{2^N}\int_{-\infty}^{+\infty}\exp\left(\frac{-Nt^2}{2\tau^2}\right)\sum_{j=0}^{N}\binom{N}{j}$$
$$\times\cos([N-2j](2\pi f_c t+\varphi))\,dt$$
$$=\frac{1}{2^N}\sum_{j=0}^{N}\binom{N}{j}\sqrt{\frac{2\pi}{N}}\,\tau$$
$$\times\exp\left(-\frac{[N-2j]^2}{2N}(2\pi f_c\tau)^2\right)$$
$$\times\cos([N-2j]\varphi)\,. \qquad (5)$$

If N is an even number, which is usually the case if the intensity of the nonlinear yield is measured (e.g. $N=4$ for the second harmonic), then the largest contribution ($N-2j=\pm 2$) to the signal is proportional to

$$1+\frac{2N}{N+2}\,e^{-2(2\pi f_c\tau)^2/N}\cos(2\varphi)\,. \qquad (6)$$

The second term, which is modulated with $2f_0$, is exceedingly small compared to 1 unless a very short pulse length and/or a high harmonic of the electric field is used. For constant pulse-to-pulse energy the result should be divided by this energy term ($N=2$), but the relative correction to the second term in Eq. (6) is small. The creation of high harmonics of femtosecond-pulses usually demands high pulse powers. These can be created with the help of regenerative amplifiers which, however, reduce the repetition rate down to a few 100 kHz or less. A low repetition rate, i.e. a dense frequency comb, is difficult to use for frequency metrology, as the number of unwanted beat notes is drastically increased. Additionally, assuming unchanged average output power, the available power per mode is reduced with the repetition rate.

Another way to measure the offset frequency becomes possible if the frequency comb contains at the blue side the second harmonic of modes from the red side. In this case two modes with frequencies $f=n_1 f_r+f_0$ and $n_2 f_r+f_0$ with $n_2=2n_1$ should oscillate at the same time. By phase-locking a laser diode to f which is then frequency doubled the beat note $2f-(n_2 f_r+f_0)=f_0$ can be observed. A white light continuum covering the visible range could for example serve this purpose if it can be created with a sufficiently high repetition rate. If the width of the frequency comb is not wide enough a number of optical frequency interval dividers [2] may be employed reducing the necessary span by a factor of two for each installed divider.

This method of measuring an absolute optical frequency f as an interval between f and its second harmonic $2f$ can be viewed as a special case of the general principle. Other combinations of nonlinear processes can be used to convert an absolute optical frequency into a frequency gap whose measurement with mode-locked lasers relies solely on the mode spacing constancy. A prototype of such a new type

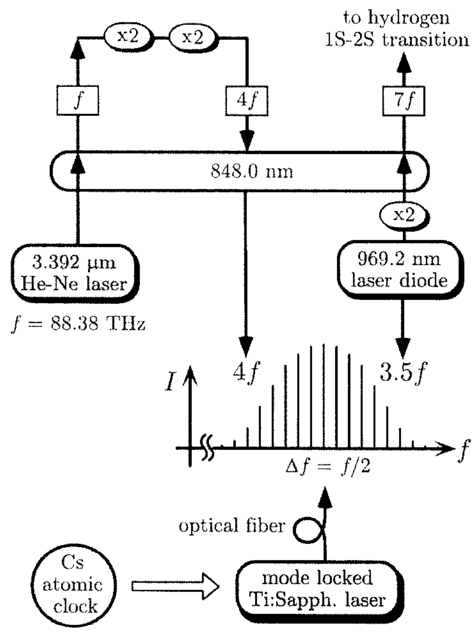

Fig. 3. Example for a new type of frequency chain (simplified scheme) as used in our recent experiment to determine the hydrogen 1S–2S transition frequency at 121 nm ($4\times 7f$). The optical frequency interval divider (big oval symbol) fixes the ratio of its frequencies to f:$4f$:$7f$. Measuring the frequency of a stabilized He–Ne laser as an interval of $f/2$ at 900 nm determines all other frequencies in this system setup.

of frequency chain, that has already been demonstrated in our laboratory in an experiment to measure the 1S–2S transition frequency in atomic hydrogen [12], is shown in Fig. 3.

To measure large frequency differences with high accuracy, the beat signal of a mode of the comb with a CW laser must be detected in presence of a large number of other modes that only contribute to noise. A low noise beat signal can be achieved by mode preselection with the help of a grating as shown in Fig. 4. In this case the noise due to the unused modes is reduced by preventing them from reaching the detector. The shot-noise limited signal to noise ratio of the beat signal with the nth mode on a detector with a quantum efficiency η and a detection bandwidth B_w is calculated from [20]

$$S/N = \frac{\eta}{h\nu B_w} \frac{tP_n(1-t)P_{LD}}{t\sum_k P_k + (1-t)P_{LD}}, \tag{7}$$

where P_{LD} and P_n are the power of the laser diode and the nth mode respectively and $h\nu$ is the energy of a single photon. The transmission of the adjustable beam splitter used to match the beams is given by t. The summation Σ_k extends over all the modes that reach the detector, this might be approximated by NP_n if the resolution Nf_r of the grating in combination with the detector aperture is such that the power is approximately constant within the sum:

$$S/N \approx \frac{\eta}{h\nu B_w} \frac{tP_n(1-t)P_{LD}}{NtP_n + (1-t)P_{LD}}. \tag{8}$$

If we assume that t is always adjusted to give the optimum signal to noise ratio $t_{opt} = \sqrt{P_{LD}}/(\sqrt{NP_n} + \sqrt{P_{LD}})$ we find that for $N \ll P_{LD}/P_n$ the detection is limited only by the shot noise of the weak signal P_n:

$$(S/N)_{opt} = \frac{\eta P_n}{h\nu B_w}. \tag{9}$$

As a rule, the grating must therefore remove enough modes so that the total power of the remaining modes is much smaller than the power of the laser diode. In the wings of the frequency comb a rather poor resolution is usually sufficient to achieve a signal to noise ratio limited by the shot noise of the weak signal P_n.

To create the beat notes necessary for the determination of the Cs D_1 line and the hydrogen 1S–2S

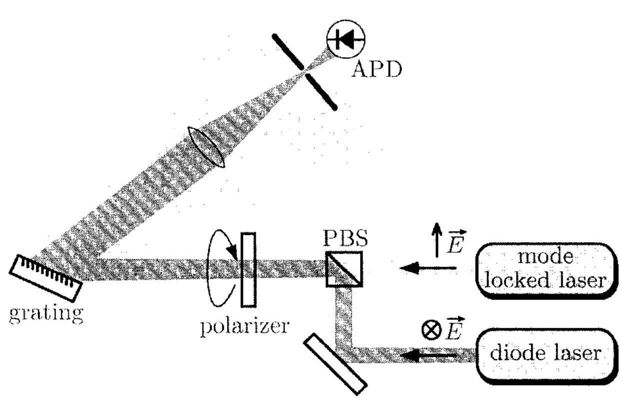

Fig. 4. Detection of a low noise beat signal with the mode-locked laser on a Si avalanche photo diode (APD). The relative intensity of the matched beams is adjusted by rotating the polarizer that is placed after the polarizing beam splitter (PBS). To avoid possible beam pointing changes and to provide the polarization for the optimal refraction efficiency of the grating the rotatable beam splitter may be replaced by a fixed beam splitter and a half wave plate. In most cases, the width of the aperture shown is not critical and could be the area of the photo diode itself.

frequency we used a 15×10 mm^2 grating with 1800 lines/mm and a $f = 15$ mm lens at a distance of 50 cm from the grating. The aperture of the lens (diameter of 5 mm) was sufficient for selecting one of the modes in vicinity of the laser diode frequency and reliably phase-locking the diode to it [21].

To use the mode-locked laser as a ruler to measure large optical frequency differences, the mode spacing, i.e. the pulse repetition rate of the laser, must either be measured very accurately or set to a predefined value by phase-locking it to a radio frequency reference. In principle, this can be done by controlling the laser cavity length with a piezo driven mirror. However, for most applications, it is desirable to phase-lock the pulse repetition rate *and* the beat note of one of the modes with another laser simultaneously. For this purpose it is necessary to control the phase velocity of that particular mode and the group velocity of the pulse independently. A piezo driven folding mirror changes both the pulse round trip time and the mode wavelength. Changing the cavity length in this way, no extra dispersion is added leaving the pulse to pulse phase shift $\Delta\varphi$ constant. According to Eq. (4), the optical frequencies are shifted by $\Delta f_n/f_n = \Delta f_r/f_r = -\Delta L/L$.

A mode-locked laser that uses two intra cavity prisms to compensate the intra cavity group velocity dispersion provides us with an elegant means to independently control the pulse repetition rate. As shown in Fig. 5, we use a second piezo-transducer to tilt the cavity end mirror near one of the prisms. The reflection thus introduces an additional phase delay proportional to the frequency distance $f - f_n$ for all frequency components f of the pulse. This displaces the pulse in time and changes the effective cavity round trip time. The frequency f_n corresponds to the cavity mode that coincides with the tilting axis on the surface of the cavity end mirror. A similar scheme has been used before in a rapid scanning optical delay line [22].

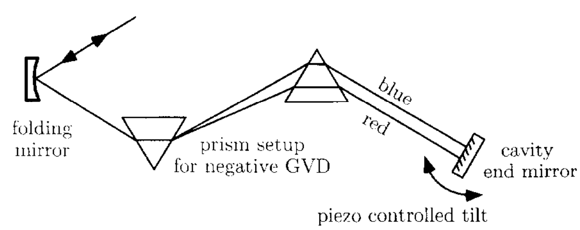

Fig. 5. The usual prism set-up at one of the end mirrors used to compensate the group velocity dispersion can be used to change the group delay time per round trip. This is done by introducing an additional round trip phase delay that is a linear function of the optical frequency.

Arguing in the frequency domain, the insertion of an additional length proportional to $f_m - f_n$ for each laser mode f_m simply changes the mode separation. Ideally the absolute frequency f_n is the one that is phase-locked with the folding mirror. In this case the tilt angle will change every frequency in the comb except f_n so that the control of the f_r decouples from the control of f_n. However, the phase-lock of f_n with the folding mirror is not decoupled from the control of the pulse repetition rate. Using this method [3] with our commercial femtosecond system we found that it operates well if the bandwidth that controls the cavity length is as high as possible (typically 10 kHz) while the bandwidth that controls the cavity end mirror tilt (f_r) is low in order to exploit the short term stability provided by the laser cavity. The position of the tilt axis was not critical and was situated even beyond the mirror. The misalignment of the cavity due to the necessary tilting of the end mirror was negligible. To reduce the necessary servo bandwidth for phase-locking one of the modes to a laser diode with a piezo transducer controlling the cavity length, we used a prescaler that divides the phase-locked beat frequency by 128. In addition we used a large range ($\pm 32\pi$) digital phase detector [21]. In principle, a fast phase-lock by an intra-cavity electro-optic modulator seems to be feasible as well, if the dispersion compensation in the resonator is designed appropriately.

When phase-locking the pulse repetition rate to a signal provided by a synthesizer one faces the problem of phase noise multiplication. It is well known that the integrated phase noise intensity grows as N^2 when a radio frequency is multiplied by a factor of N [23]. By the detection and phase locking process of the pulse repetition rate additional phase noise is

[3] Patent pending.

introduced. We reduced this noise contribution by a factor of 100^2 since we did not phase-lock the fundamental pulse repetition rate but its 100th harmonic. Further, we used a low servo bandwidth to exploit the intrinsic stability of the laser resonator. The small servo bandwidth and the intrinsic stability of the laser resonator that is filtering the modes helps to prevent the high frequency noise components from propagating through the frequency comb [24]. In fact, using an externally referenced low noise DRO (dielectric resonator oscillator, CTI Communication Techniques model XPDRO-6313) instead of a synthesizer (Hewlett–Packard model 8360) for this purpose did not improve the performance in any noticeable way. Unlike the synthesizer the DRO showed a small but significant frequency drift attributed to drifts or instabilities of the DRO's internal phase locked loops to the reference. The observed frequency drift relative to its reference frequency was as high as several 100 mHz per second at the measured frequency interval of 4 THz.

To unambiguously measure large optical frequency differences with a mode-locked laser, it is necessary to determine the number of modes between the two CW laser frequencies that form the interval. If the pulse repetition rate is sufficiently high a wavemeter could identify the modes, provided it has a resolution better than f_r. However, this is not the case with a commonly used commercial wavemeter (e.g. Burleigh WA-20 Series model) in connection with a pulse repetition rate around 75 MHz commonly employed in commercial femtosecond systems. The observation of a frequency shift $n \times \delta f_r$ of one of the modes after changing f_r by δf_r may be difficult because it demands a resolution of δf_r to distinguish between the mode number n and $n \pm 1$. The shift $n \times \delta f_r$ could not be chosen too large if one has to track it with a phase-locked diode laser. This scheme becomes particularly difficult when using large mode numbers which is of course the main interest.

When applied to a determination of some optical reference frequency the mode number may be unambiguous if a previous measurement with sufficient accuracy exists. If this is not possible another solution of the problem is to use an external cavity that is adjusted to have a free spectral range of some multiple N of f_r. This cavity then, when operated on resonance, would transmit only every Nth mode of the frequency comb and is therefore increasing the mode spacing by a factor of N. In the time domain, one could argue that the pulse is bouncing N times in the cavity until the next pulse arrives. The pulse repetition rate is thereby multiplied by N. To make

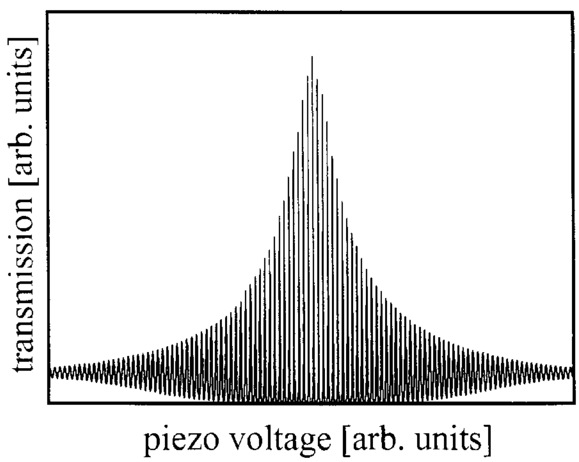

Fig. 6. The transmitted light from a mode-locked laser through a cavity of finesse F as a function of the piezo supply voltage that controls the mirror separation. Far detuned the cavity will transmit one out of $\pi/2F \approx 0.005$ modes giving rise to the constant background. At the central peak the ratio of the pulse repetition rate and the free spectral range of the cavity is exactly 1:20.

sure that the free spectral range of the cavity is indeed N times the pulse repetition rate the transmission through the cavity is observed. If the fraction of the free spectral range and the pulse repetition rate is some arbitrary number, the cavity will transmit on average one out of $2F/\pi$ modes from the frequency comb where F is the finesse of the external cavity. Taking into account the large number of modes in the frequency comb the transmission is not changed if the cavity length is scanned with a piezo mounted mirror. If however the fraction of the free spectral range and the pulse repetition rate is a small integer, resonant fringes appear as shown in Fig. 6. The transmitted intensity has a peak if the piezo controlled mirror spacing allows the transmission of every Nth mode and it vanishes if these modes are blocked [9,25]. Higher order ratios produce fringes with smaller contrast. Although all peaks shown in Fig. 6 correspond to a transmission of every 20th laser mode for a certain spectral range, the matching of the cavity resonance spacing to the comb of laser modes is fulfilled best for the central peak. In this case, dispersion in the external cavity limits the spectral width of the transmission. The cavity used by us allowed the simultaneous transmission of modes 18 THz apart as necessary in our experiment. Applying this mode number counting method to much larger intervals would demand a cavity with flat dispersion over a larger spectral range. To validate this scheme we have used 26,500 modes of our mode-locked laser and checked the resulting frequency difference [10] by comparison with a frequency comb produced by a fast electro optic modulator [26].

If two frequency combs are available at the same time the mode number can be determined most conveniently by operating them at slightly different repetition frequencies say f_{r1} and f_{r2}. If the difference is say 1 Hz then the beat frequency $nf_{r1} - nf_{r2}$ yields immediately the mode number.

In conclusion, we have provided and applied the techniques necessary to operate a mode-locked femtosecond laser as a ruler to measure large optical frequency differences. Full frequency stabilization of the comb of modes has been achieved, while the mode spacing constancy had been confirmed on a level of 3 parts in 10^{17} for a 20 THz interval before. This new approach to measuring optical frequencies can be understood in an intuitive picture. The reported techniques have made it possible to construct a new type of frequency chain which has been applied for an absolute frequency measurement of the 1S–2S transition in hydrogen. The next step is the implementation of these ideas with all solid state and diode lasers that will allow a continuous operation of such a chain as a reliable optical clockwork.

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