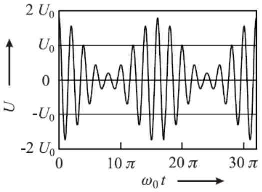
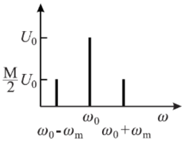
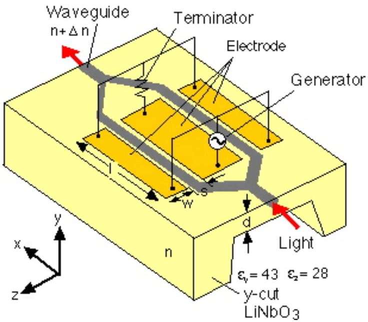
Modules of the 2024 Course

Topics covered	No	Lecture/Date
Introductory presentation; Basic of laser operation I: dispersion theory, atoms	1	11. 09. 2024
Basic of laser operation II: dispersion theory, atoms	2	18. 09. 2024
Laser systems I: 3 and 4 level lasers, gas lasers, solid state lasers, applications	3	25. 09. 2024
Laser systems II: semi-conductor lasers, external cavity lasers, applications	4	02. 10. 2024
Noise characteristics of lasers: linewidth, coherence, phase and amplitude noise, OSA (1)	5	09. 10. 2024
Noise characteristics of lasers: linewidth, coherence, phase and amplitude noise, OSA (2)	6	16. 10. 2024
Optical detection	7	30. 10. 2024
Optical fibers: light propagation in fibers, specialty fibers and dispersion (GVD)	8	06. 11. 2024
Ultrafast lasers I.: Passive mode locking and ultrafast lasers	9	13. 11. 2024
Ultrafast lasers II: mode locking, optical frequency combs / frequency metrology	10	20. 11. 2024
Ultrafast lasers III: pulse characterization, applications	11	27. 11. 2024
Nonlinear frequency conversion I: theory, frequency doubling, applications	12	04. 12. 2024
Nonlinear frequency conversion II: optical parametric amplification (OPA)	13	11. 12. 2024
Laboratory visits (lasers demo)	14	20. 12. 2024

Amplitude Modulation







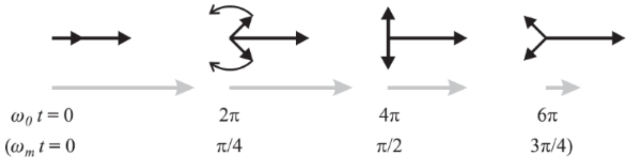
Amplitude modulation:

$$U_{\text{AM}}(t) = \left(U_0 + \Delta U_0 \cos(\omega_m t)\right) \cos \omega_0 t$$

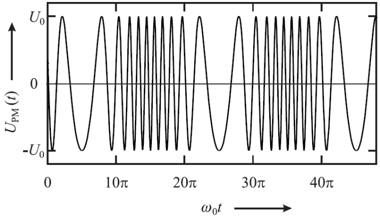
$$P_{\text{AM}} = U_0 \left[e^{i\omega_0 t} + \frac{M}{2} e^{i(\omega_0+\omega_m)t} + \frac{M}{2} e^{i(\omega_0-\omega_m)t} \right]$$

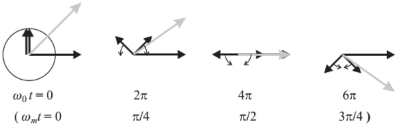
$$M \equiv \frac{\Delta U_0}{U_0}$$

$$P_{\text{AM}} \propto |U_0|^2 \left[1 + \frac{M^2}{2} \right]$$



Phase Modulation





Phase modulation

$U_{\mathrm{PM}}(t) = U_0 \cos\varphi = U_0 \cos\left[\omega_0 t + \delta\cos(\omega_m t)\right]$

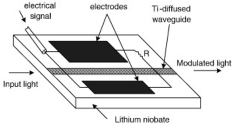
$\omega(t) = \omega_0 + \omega_m \delta \sin(\omega_m t) \equiv \omega_0 - \Delta\omega \sin(\omega_m t)$

$\Delta\omega \equiv \omega_m \delta$

Sideband picture

$$U_{PM}(t) = U_0 \sum_{n=-\infty}^{\infty} \Re\left\{ i^n J_n(\delta) e^{\left[(i\omega_0 + n\omega_m)t\right]} \right\}$$

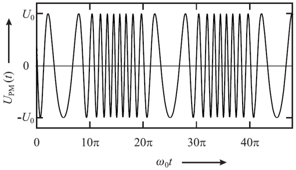
Application note: Principle of phase modulator



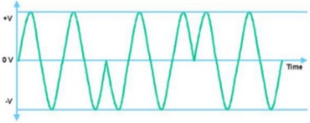
$$\exp\left[i\delta\cos(\omega_m t)\right] = J_0(\delta) + 2iJ_1(\delta)\cos(\omega_m t)$$

$$J_1(\delta) = \frac{\delta}{2}$$

Phase Modulation



Application note: Quadrature Amplitude Modulation (QAM) is a combination of two modulation techniques: Amplitude Modulation (AM) and Phase Modulation (PM). To explain QAM we will start with a brief explanation of AM and PM.



Phase modulation

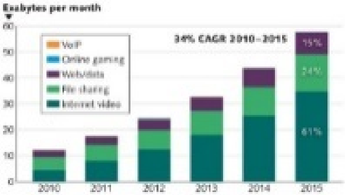
$$U_{\mathrm{PM}}(t) = U_0 \cos\varphi = U_0 \cos\left[\omega_0 t + \delta\cos(\omega_m t)\right]$$
$$\omega(t) = \omega_0 + \omega_m \delta \sin(\omega_m t) \equiv \omega_0 - \Delta\omega \sin(\omega_m t)$$
$$\Delta\omega \equiv \omega_m \delta$$

Sideband picture

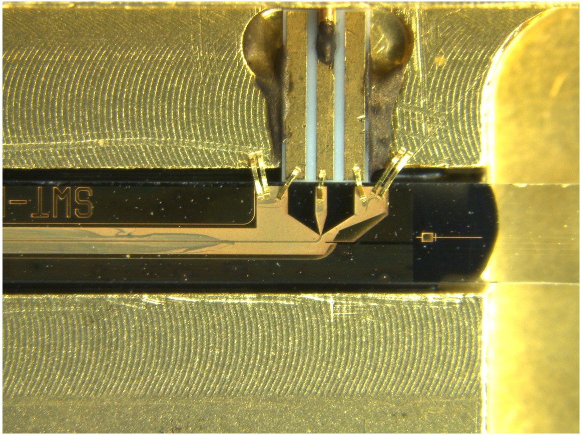
$$U_{PM}(t) = U_0 \sum_{n=-\infty}^{\infty} \Re\left\{ i^n J_n(\delta) e^{\left[(i\omega_0 + n\omega_m)t\right]} \right\}$$

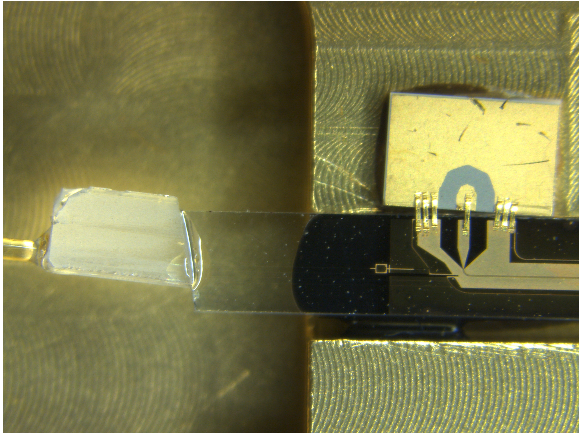
$$\exp\left[i\delta\cos(\omega_m t)\right] = J_0(\delta) + 2iJ_1(\delta)\cos(\omega_m t)$$

$$J_1(\delta) = \frac{\delta}{2}$$

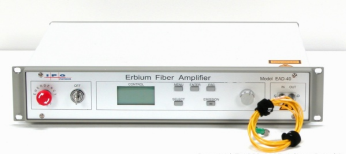


Phase Modulator. Zoom in





Amplification, noise figure and detectors



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Learning outcomes Fiber amplifiers

1. Understand the *Gain* of an Amplifier
2. Understand the *Noise figure* concept (NF)
3. Understand *where the Noise originates* from
4. Understand the *fundamental limit* of the noise figure

Learning outcomes Detectors

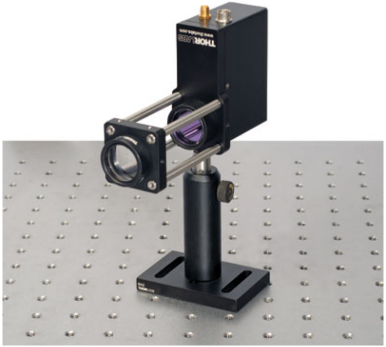
1. Understand the different detector types
2. How to detect a signal? Understand the noise (*shot noise*, *thermal*)

High Power EDFA Series

Model Number	AMP-ST-18	AMP-ST-22	AMP-ST-30	AMP-ST-37
Output Power (dBm)*	18	22	30	37
Input Power Range (dBm)	-25 ~ +10	-25 ~ +10	0 ~ 3	0 ~ 3
Wavelength (nm)**	1530 ~ 1565	1530 ~ 1565	1540 ~ 1565	1540 ~ 1565
Typical Noise Figure (dB)	5.5	5.5	6.0	6.0
Operating Temp (°C)	0 ~ 50			
Operating Voltage (VAC)	85 ~ 264			
Dimension (cm)	34(w) x 42(d) x 9(h)		48(w) x 44(d) x 14(h)	

* *Other output powers are available.*
** *L-band is available.*

Detectors



1. Photomultiplier
2. Photoconductive detector
3. Photodiode (Avalanche Photodiode)
4. The Eye

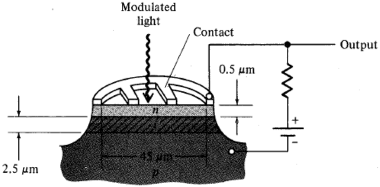
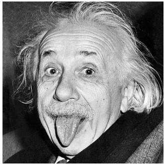


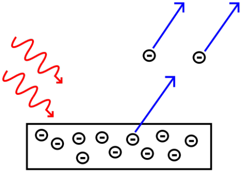
Figure 11-14 A *p-i-n* photodiode. (After Reference [13].)

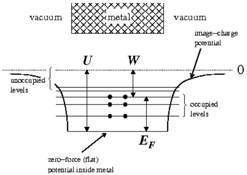
Photoelectric effect

Review: photoelectric effect

Historically, the photoelectric effect is associated with Albert Einstein, who relied upon the phenomenon to establish the fundamental principle of quantum mechanics, in 1905,[3] an accomplishment for which Einstein received the 1921 Nobel Prize.







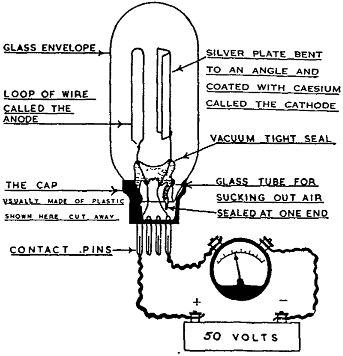
First observed by Heinrich Hertz in 1887,[2] the phenomenon is also known as the "Hertz effect",[3][4], although the latter term has fallen out of general use.

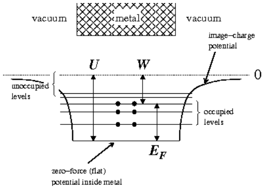
In the free electron model, non-interacting electrons bounce around inside a potential well of depth U. The Fermi Level is the highest energy level that is occupied by electrons. Here E_F is defined relative to the bottom of the potential well, and the work function W is the energy required to eject the electron in the Fermi Level.

Detectors: Phototube

Simplest possible detector is a phototube that utilizes the photoelectric effect.

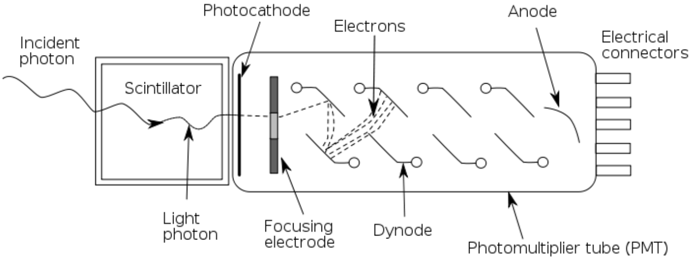
1. Photomultiplier
2. Photoconductive detector
3. Photodiode (Avalanche Photodiode)
4. The Eye





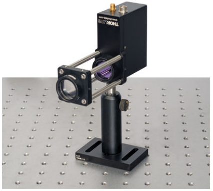
Photomultiplier tube (PMT)

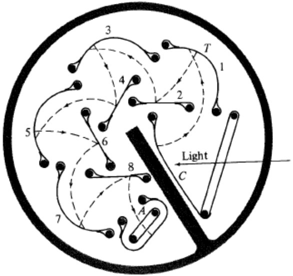
Photomultiplier tubes (photomultipliers or PMTs for short), members of the class of vacuum tubes, and more specifically phototubes, are extremely sensitive detectors of light in the ultraviolet, visible, and near-infrared ranges of the electromagnetic spectrum. *These detectors multiply the current produced by incident light by as much as 100 million times* (i.e., 160 dB), in multiple dynode stages, enabling (for example) individual photons to be detected when the incident flux of light is very low.



Photomultiplier tube

Item #	PMM01	PMM02
Photocathode Type	Bialkali	Multialkali (S20)
Photocathode Geometry	Head-On	
Dynode Chain Orientation	Circular	
Photocathode Active Diameter	22 mm	
Wavelength Range	280 - 630 nm	280 - 850 nm
Gain (Max)	7.1×10^6	3.1×10^6
Peak Responsivity (Max)	86 mA/W	67 mA/W
Quantum Efficiency at Peak (Typ.)	28% at 400 nm	21% at 420 nm
Transimpediance Gain	Hi-Z: 1×10^6 V/A 50 Ω: 5×10^5 V/A	
Dark Current (@ 20°C)	0.3-3 nA	0.5-5 nA
Dark Count Rate (@ 20°C)	100 s^{-1}	3000 s^{-1}
Bandwidth (6 dB)*	0-20 kHz	
Amplifier Noise (Typ.)	2 mV RMS	
Output Rise and Fall Times	15 µs	
Output Impedance	50 Ω	
Output Signal**	0-10 V (unterminated) 0-5 V (terminated into 50 Ω)	
Power Input	+12 V (+12 to +15): 40 mA -12 V (-12 to -15): 10 mA	
Anode Current (Max)	100 µA	
Tube Voltage (Anode to Cathode)†	0 to -1500 V	0 to -1800 V
Tube Voltage Control‡	0 to 1.5 V	0 to 1.8 V
HV Control Sensitivity	-1000 V/V	
HV Control Volts (Max)	1.8 V	
Warm Up Time	<10 s	
Output Connector	SMA	
General		
Module Dimensions	3.66" x 1.6" x 2.46" (92.9 mm x 40.6 mm x 62.5 mm)	
Operating Temperature	5 to 55°C	
Storage Temperature	-40 to 55°C	
Mounting Holes	8-32 (M4 on -EC version)	
Weight (Power Supply)	1.1 kg (2.42 lbs)	
Weight (PMT)	0.2 kg (0.5 lbs)	
Window Characteristics		
Material	Borosilicate	
Type	Plano-Concave	
Refractive Index	1.49	
Potassium (K)	300 ppm	
Thorium (Th)	250 ppm	
Uranium (U)	100 ppm	





Photoconductive detectors

Element	eV	Element	eV	Element	eV	Element	eV	Element	eV
Ag:	4.52-4.74	Al:	4.06-4.26	As:	3.75	Au:	5.1-5.47	B:	~4.45
Ba:	2.52-2.7	Be:	4.98	Bi:	4.34	C:	~5	Ca:	2.87
Cd:	4.08	Ce:	2.9	Co:	5	Cr:	4.5	Cs:	2.14
Cu:	4.53-5.10	Eu:	2.5	Fe:	4.67-4.81	Ga:	4.32	Gd:	2.90
Hf:	3.9	Hg:	4.475	In:	4.09	Ir:	5.00-5.67	K:	2.29
La:	4	Li:	2.93	Lu:	~3.3	Mg:	3.66	Mn:	4.1
Mo:	4.36-4.95	Na:	2.36	Nb:	3.95-4.87	Nd:	3.2	Ni:	5.04-5.35
Os:	5.93	Pb:	4.25	Pd:	5.22-5.6	Pt:	5.12-5.93	Rb:	2.261
Re:	4.72	Rh:	4.98	Ru:	4.71	Sb:	4.55-4.7	Sc:	3.5
Se:	5.9	Si:	4.60-4.85	Sm:	2.7	Sn:	4.42	Sr:	~2.59
Ta:	4.00-4.80	Tb:	3.00	Te:	4.95	Th:	3.4	Ti:	4.33
Tl:	~3.84	U:	3.63-3.90	V:	4.3	W:	4.32-5.22	Y:	3.1
Yb:	2.60 [2]	Zn:	3.63-4.9	Zr:	4.05				

As evident from the chart most materials have a work function that is so high, that it requires light of visible or UV to achieve photoelectron emission.

Photoconductive detector

In a photoconductive detector, an optical photon increases the conductivity of a doped semiconductor, leading to a drop in resistance.

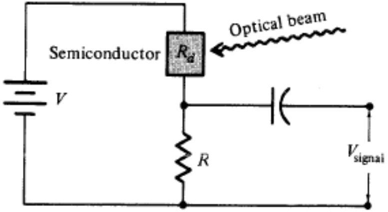


Figure 11-7 Typical biasing circuit of a photoconductive detector.

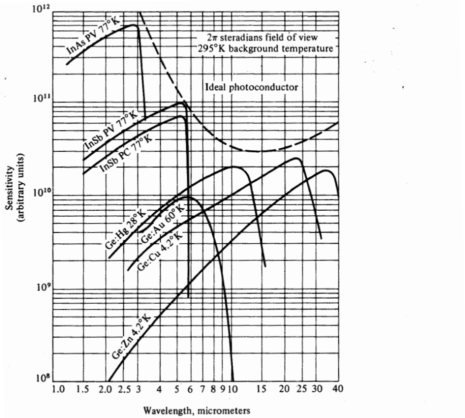
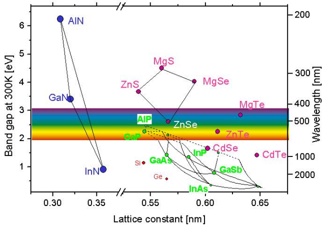


Figure 11-9 Relative sensitivity of a number of commercial photoconductors. (Courtesy Santa Barbara Research Corp.)

Photodiode

Detectors converts a photon (i.e. a quanta of electromagnetic energy into one or more electrons). Common detectors are:



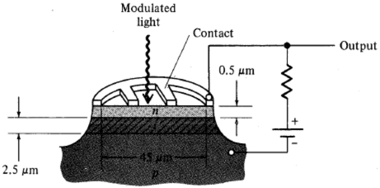
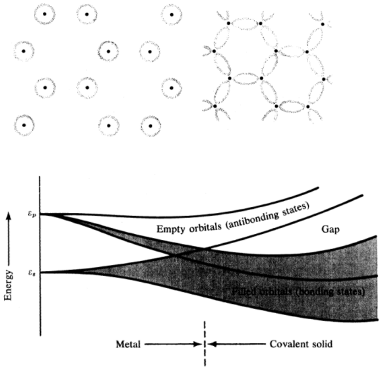


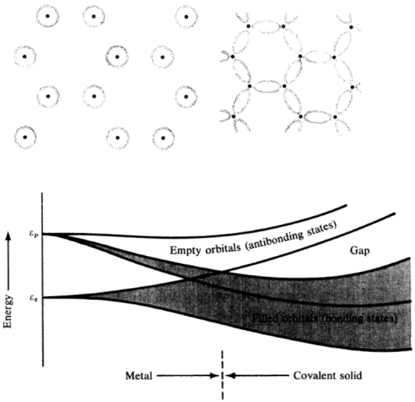
Figure 11-14 A *p-i-n* photodiode. (After Reference [13].)



(from Harrison)

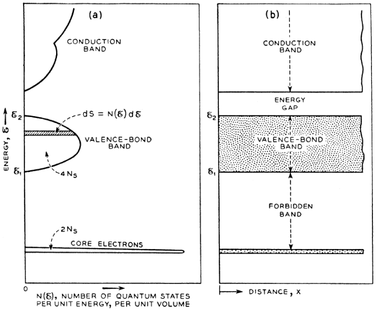
Photodiodes

Detector converts a photon (i.e. a quanta of electromagnetic energy into one or more electrons). Common detectors are:



(from Harrison)

Energy band structure

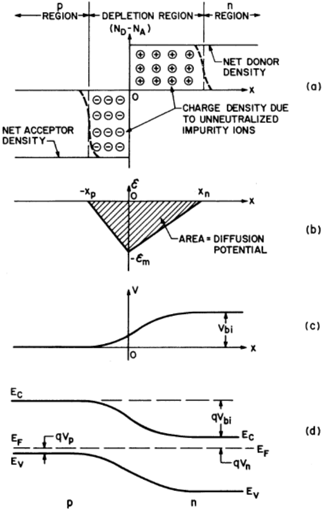


(from Schockley)

Typical band gaps (valence – conduction band)

Ge	0.7 eV	GaAs	1.4 eV
Si	1.1 eV	Diamond	5.5 eV

Photodiode and Avalanche Photodiode (APD)



(from Sze, *Physics of Semiconductor Devices*)

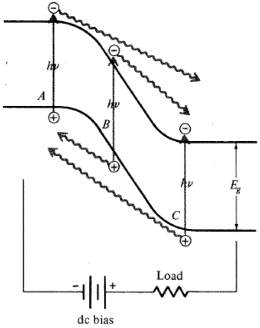


Figure 11-13 The three types of electron–hole pair creation by absorbed photons that contribute to current flow in a *p-n* photodiode.

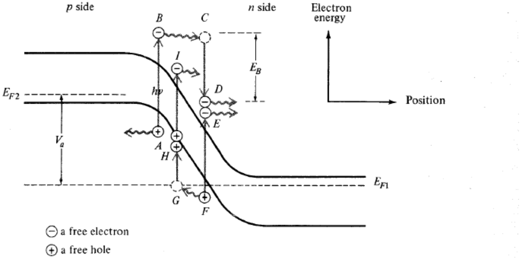
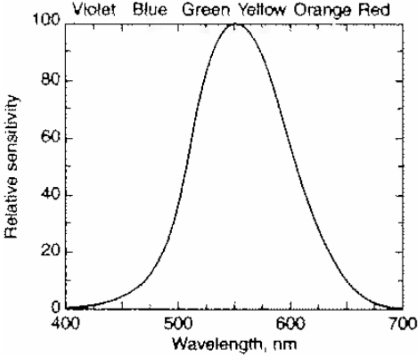
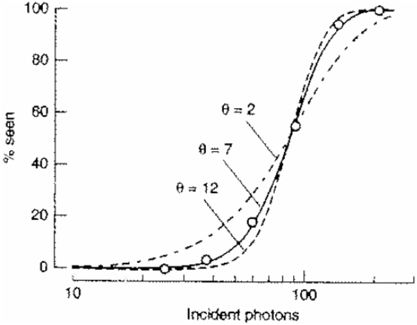


Figure 11-22 Energy-position diagram showing the carrier multiplication following a photon absorption in a reverse-biased avalanche photodiode.

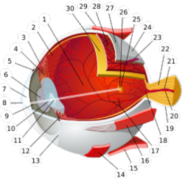
Human eye





Cones: 560 nm peaks sensitivity
Rods: 510 nm peak sensitivitity

The experiment by Pirenne revealed that a rod can detect a single photon (!).

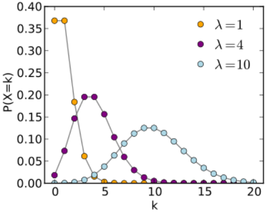


Review: Poisson distribution

Variance of the Poisson distribution:

$$\Delta n = \sqrt{\langle n \rangle} \quad p(n) = \frac{\langle n \rangle^n e^{-n}}{n!}$$

In probability theory and statistics, the **Poisson distribution** (or **Poisson law of small numbers**) is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time and/or space if these events occur with a known average rate and independently of the time since the last event. (The Poisson distribution can also be used for the number of events in other specified intervals such as distance, area or volume.)



The **Poisson** distribution approaches a **Gaussian** Distribution for large mean values.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The Poisson distribution (Background)

Proof of the Poisson distribution

$$P(t)\Delta t = \alpha I(t)\Delta t$$

Let $P_n(t)$ be the probability of counting n photons during a time $t(0 \leq t \leq T)$

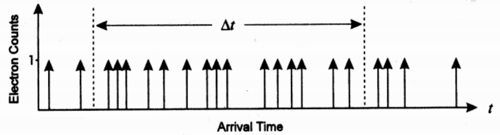


Figure A.4 Random arrival time of photogenerated electrons.

$$P_{n-1}(t)p(t)\Delta t = (\text{probability of } n-1 \text{ photons in time } t) \times (\text{probability of } 1 \text{ photon in } \Delta t)$$

$$P_n(t)[1 - p(t)\Delta t] = (\text{probability of } n \text{ photons in time } t) \times (\text{probability of no photon in } \Delta t)$$

$$P_n(t+\Delta t) = P_{n-1}(t)p(t)\Delta t + P_n(t)[1 - p(t)\Delta t]$$

$$\frac{P_n(t+\Delta t) - P_n(t)}{\Delta t} = \left[P_{n-1}(t) - P_n(t)\right] p(t)$$

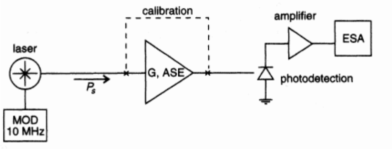
$$\frac{dP_n}{dt} = \alpha I(t)\left[P_{n-1}(t) - P_n(t)\right]$$

$$P_n(T) = \frac{(\bar{n})^n}{n!} e^{-\bar{n}}$$

$$p(n) = \frac{\langle n \rangle^n e^{-n}}{n!}$$

$$\left\langle \Delta n(T)^2 \right\rangle = \bar{n}$$

Brief review shot noise in optical detection



Shot noise can be viewed in three (equivalent) ways.

1. As a consequence of the particle nature of light
2. As a consequence of the fact that current is composed of individual electrons
3. As a consequence of the interference of the signal beam with the vacuum fields of other electromagnetic modes

A quick and simple derivation of shot noise:

$$SNR = \frac{P_{\text{Signal}}}{P_{\text{Noise}}} = \left(\frac{N}{\sqrt{N}}\right)^2 = \left(\sqrt{N}\right)^2$$

$$P_{\text{Signal}} = R(eN/T)^2$$

$$P_{\text{Noise}} = R\left\langle \Delta I^2 \right\rangle = R\left(\frac{e^2}{T^2}\left\langle \Delta N^2 \right\rangle\right) = R \cdot \frac{e^2}{T^2} N$$

Number of detected photons:
$N = P/2\hbar\omega B$

Bandwidth: $B = 1/2T$

$$SNR = \frac{P_{\text{Signal}}}{P_{\text{Noise}}} = \frac{P}{2\hbar\omega B}$$

Noise figure (NF)

$$NF[dB] = 10\log_{10}\left(\frac{SNR_{in}}{SNR_{out}}\right)$$

Johnson or Nyquist Noise

THERMAL AGITATION OF ELECTRICITY IN CONDUCTORS

By J. B. Johnson

ABSTRACT

Statistical fluctuation of electric charge exists in all conductors, producing random variation of potential between the ends of the conductor. The effect of these fluctuations has been measured by a vacuum tube amplifier and thermocouple, and can be expressed by the formula $\bar{I}^2=(2kT/\pi)\int_0^\infty R(\omega)|Y(\omega)|^2 d\omega$. I is the observed current in the thermocouple, k is Boltzmann's gas constant, T is the absolute temperature of the conductor, $R(\omega)$ is the *real* component of impedance of the conductor, $Y(\omega)$ is the transfer impedance of the amplifier, and $\omega/2\pi=f$ represents frequency. *The value of Boltzmann's constant* obtained from the measurements lie near the accepted value of this constant. *The technical aspects of the disturbance* are discussed. In an amplifier having a range of 5000 cycles and the input resistance R the power equivalent of the effect is $\overline{V^2}/R=0.8\times10^{-16}$ watt, with corresponding power for other ranges of frequency. The least contribution of *tube noise* is equivalent to that of a resistance $R_c=1.5\times10^5 i_p/\mu$, where i_p is the space current in milliamperes and μ is the effective amplification of the tube.

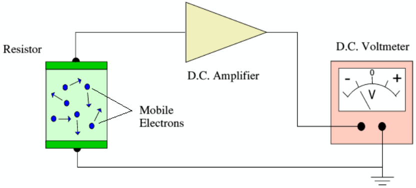


Figure 3.1 Fluctuating voltage produced by random movements of mobile electrons.

In 1927, J. B. Johnson observed random fluctuations in the voltages across electrical resistors. A year later H. Nyquist published a theoretical analysis of this noise which is thermal in origin. Hence this type of noise is variously called *Johnson* noise, *Nyquist* noise, or *thermal* noise.

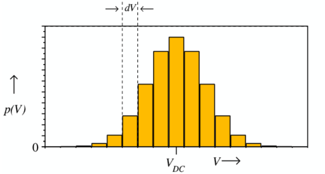


Figure 3.2 Histogram of some noise voltage measurements.

$$p(V)dV \propto \mathrm{Exp}\left[\frac{-2V^2}{\sigma^2}\right]$$

Johnson or Nyquist Noise

$$\overline{v_n^2} = 4k_B T R$$

The voltage fluctuations on a resistor are directly linked to the thermal Brownian motion of the electrons.

$$\overline{v_x^2} = k_B T/m$$

$$i_{th} = \sqrt{\frac{4kT\Delta f}{R}}[A]$$

THERMAL AGITATION OF ELECTRICITY IN CONDUCTORS

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Abstract

Statistical fluctuation of electric charge exists in all conductors, producing random variation of potential between the ends of the conductor. The effect of these fluctuations has been measured by a vacuum tube amplifier and thermocouple, and can be expressed by the formula $\bar{I}^2 = (2kT/\pi)\int_0^\infty R(\omega)|Y(\omega)|^2 d\omega$. I is the observed current in the thermocouple, k is Boltzmann's gas constant, T is the absolute temperature of the conductor, $R(\omega)$ is the *real* component of impedance of the conductor, $Y(\omega)$ is the transfer impedance of the amplifier, and $\omega/2\pi = f$ represents frequency. *The value of Boltzmann's constant* obtained from the measurements lie near the accepted value of this constant. *The technical aspects of the disturbance* are discussed. In an amplifier having a range of 5000 cycles and the input resistance R the power equivalent of the effect is $\overline{V}^2/R = 0.8 \times 10^{-16}$ watt, with corresponding power for other ranges of frequency. The least contribution of *tube noise* is equivalent to that of a resistance $R_c = 1.5 \times 10^5 i_p/\mu$, where i_p is the space current in milliamperes and μ is the effective amplification of the tube.

$$R = \frac{d}{\sigma A} = \frac{md}{Ne^2\tau_0 A}$$

Table A.1 Thermally generated current noise for various resistance values.

R	$\hat{i}_{tn}$ *(pA/√Hz)*
50 Ω	18
100 Ω	13
1 KΩ	3.9
10 KΩ	1.3
100 KΩ	0.39
1 MΩ	0.13
10 MΩ	0.04

$$\frac{1}{2}m\bar{v}^2 = \frac{3}{2}kT$$

Electronic shot noise / quantum noise

Spectral density of current fluctuations

$$i_{sn} = \sqrt{2qI_{dc}\Delta f} \quad [A]$$

$$\Delta N_{\rm rms} = \sqrt{N} \qquad i_{sn} = \frac{q\Delta N_{\rm rms}}{\Delta t}$$

$$I_{dc} = \frac{q\bar{N}}{\Delta t} \qquad i_{sn} = \sqrt{\frac{qI_{dc}}{\Delta t}}$$

Random intensity noise (RIN)

$$\mathrm{RIN} = \frac{\left\langle \Delta i^2 \right\rangle}{I^2} \left[\mathrm{Hz}^{-1}\right]$$

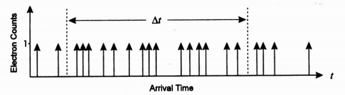


Figure A.4 Random arrival time of photogenerated electrons.

Introducing the band-with:
$\Delta f = \frac{1}{2\Delta t} \quad [Hz]$

Application note: electronic shot noise

Table A.2 Representative shot-noise values.

I_{dc}	I_{sn} (pA/$\sqrt{Hz}$)	RIN_{sn} (dB/Hz)
100 nA	0.18	−115
1 μA	0.57	−125
10 μA	1.8	−135
100 μA	5.7	−145
1 mA	18	−155

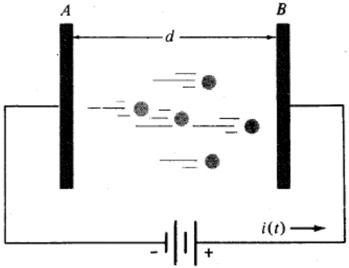
The concept of shot noise was first introduced in 1918 by Walter Schottky who studied fluctuations of current in vacuum tubes.[1]

3. *Über spontane Stromschwankungen in verschiedenen Elektrizitätsleitern; von W. Schottky.*

Durch Hintereinanderschalten von Glühkathodenverstärkern ist es in den letzten Jahren gelungen, Wechselströme von äußerst geringer Amplitude wahrnehmbar und meßbar zu machen. Viele technische Probleme haben dadurch eine ruckweise Förderung erfahren, aber auch dem Forscher scheint sich ein neues Gebiet zu erschließen; die Verstärkerschaltungen haben für elektrische Untersuchungen sicher dieselbe Bedeutung wie in der Optik das Mikroskop. Da sich bisher noch keine deutliche Grenze für die erreichbare Verstärkung gezeigt hat, konnte man hoffen, durch genügenden Schutz störungsfreie Aufstellung usw. hier sozusagen bis zum unendlich Kleinen vorzudringen; der Traum vom „Gras wachsen hören“ stellte sich wieder einmal recht greifbar der Menschheit dar.

Absicht der folgenden Zeilen ist, gewisse unüberschreitbare Grenzen für die Verstärkung mit Glühkathoden- und Gasentladungsröhren nachzuweisen. Das erste unüberwindliche Hindernis ist merkwürdigerweise durch die Größe des Elementarquantums der Elektrizität gegeben. Die Wärmebewegung der

Electronic shot noise



Spectral density of current fluctuations

$$\overline{i_N^2}(\nu) \equiv S(\nu)\Delta\nu = 2e\bar{I}\Delta\nu$$

Electronic shot noise and the definition of the spectral density

$$\overline{i_N^2}(\nu) \equiv S(\nu)\Delta\nu = 2e\bar{I}\Delta\nu$$

Detector noise

$$i_{\text{total}} = \sqrt{\frac{4kT\Delta f}{R} + 2qI_{dc}\Delta f + I_{dc}^2\,\mathrm{RIN}\,\Delta f}\ [A]$$

(thermal) (shot) (intensity)

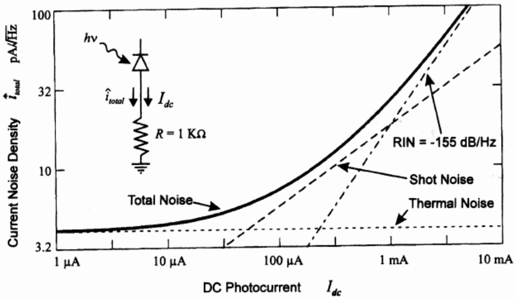
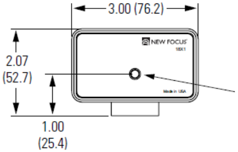


Figure A.7 Total rms photocurrent noise normalized to a 1 Hz bandwidth, caused by the combined effects of thermal, shot, and intensity noise.

Noise equivalent power

Photodetector Specifications

Model #	1801	1811
Wavelength Range	300–1050 nm	900–1700 nm
Coupling	DC or AC	DC or AC
3-dB Bandwidth (DC versions)	DC–125 MHz (typical)	DC–125 MHz (typical)
3-dB Bandwidth (AC versions)	25 kHz–125 MHz (typ.)	25 kHz–125 MHz (typ.)
DC Bias Monitor Bandwidth (AC versions only)	DC–50 kHz (typical)	DC–50 kHz (typical)
Risetime	3 ns (typical)	3 ns (typical)
Transimpedance Gain (AC-coupled version)	40 V/mA (AC) 1 V/mA (DC)	40 V/mA (AC) 1 V/mA (DC)
Transimpedance Gain (DC-coupled version)	40 V/mA	40 V/mA
Output Impedance	50 Ω	50 Ω
Minimum NEP*	3.3 pW/$\sqrt{\text{Hz}}$	2.5 pW/$\sqrt{\text{Hz}}$
CW Saturation Power	120 μW @ 950 nm	120 μW @ 950 nm
Maximum Pulse Power	5 mW	5 mW
Detector Material/Type	Silicon/PIN	InGaAs/PIN
Detector Diameter	0.8 mm	0.3 mm (FS) 0.1 mm (FC)
Power Requirements	±15 V DC; 250 mA	±15 V DC; 250 mA
Optical Input	FC or free space (FS)	FC or free space (FS)
RF Output	SMA	SMA
DC Bias Monitor output (AC-coupled units only)	SMB	SMB



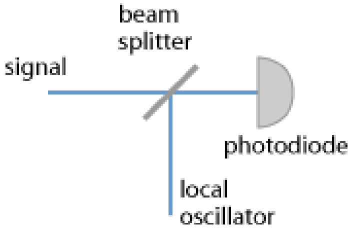
Noise-equivalent power (NEP) is a measure of the sensitivity of a photodetector or detector system. It is defined as the signal power that gives a signal-to-noise ratio of one in a one hertz output bandwidth

The units of NEP are watts per square root hertz.

For example, a detector with an NEP of 1 pW can detect a signal power of one picowatt with a signal-to-noise ratio (SNR) of one after one half second of averaging

$$P_{\min} = NEP(\lambda) \cdot \sqrt{B}$$

Detecting weak signals: Heterodyne detection



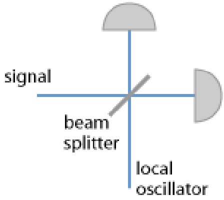
Heterodyne detection (also called coherent detection) is a detection method which was originally developed in the field of radio waves and microwaves. There, a weak input signal is mixed with some strong "local oscillator". The frequency of the mixing product is the sum or the difference of the frequencies of the signal and the local oscillator.

$$\frac{S}{N} = \frac{\left\langle I^2 \right\rangle_{\text{signal}}}{\left\langle I^2 \right\rangle_{\text{noise}}} = \frac{\eta_m \left(\frac{\eta_q e}{hv}\right)^2 2 P_s P_{\text{lo}}}{2e\frac{\eta_q e}{hv} P_{lo} B} = \frac{\frac{\eta_q}{hv} P_s}{B}$$

A remaining problem of heterodyne detection is that excess noise of the local oscillator wave directly affects the signal. This is avoided with a *balanced* heterodyne setup.

Detecting the phase: balanced homodyne detection

In a balanced homodyne detector, the beam splitter must have a reflectivity of precisely 50%. With a simple electronic circuit, one can obtain the difference of the two photocurrents. A key advantage is *that difference is to first order not influenced by noise of the local oscillator*.

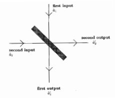


Beamsplitter

$$E_{\text{out1}} = (E_{LO} + E_s)/\sqrt{2}$$
$$E_{\text{out2}} = (E_{LO} - E_s)/\sqrt{2}$$

Difference photocurrent

$$|E_{\text{out2}}|^2 - |E_{\text{out1}}|^2 = (E_{LO} + E_S)^2/2 - (E_{LO} - E_S)/2 = 2E_{LO}E_S$$



Transmission matrix

$$A = \begin{pmatrix} t & -r \\ r & t \end{pmatrix}$$

Energy conservation

$$t^2 + r^2 = 1$$

50:50 beamsplitter

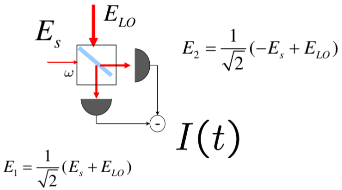
$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

Homodyne measurement (1/2)

Detected photocurrent

$$E_s = E_s e^{i\omega t + \delta\theta(t)},$$
$$\Delta I(t) \propto |E_1|^2 + |E_2|^2 = \frac{1}{2}\left(iE_s + iE_{L0}\right)\left(E_s + E_{L0}\right) - \frac{1}{2}\left(iE_s - iE_{LO}\right)\left(E_s - E_{LO}\right) = E_s^* E_{LO} + E_s E_{LO}^*$$

Homodyne detector



Photocurrent for 90 degree shift between LO and signal

$$E_{LO} = E_{LO} e^{i\Phi}$$
$$\Delta I(t) \propto |E_{LO}| \left(E_s e^{-i\phi} - E_s^* e^{+i\phi}\right)$$

Photocurrent for 90 degree shift between LO and signal

$$\Delta I(t) \propto |E_{LO}| \left(E_s - E_s^*\right)$$
$$\Delta I(t) \propto \delta\theta$$

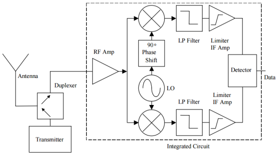
Homodyne measurement (2/2)

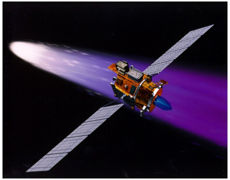
Transmission matrix 50:50 beamsplitter

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

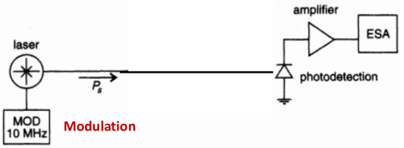
Application Note: Homodyne receivers are used in mobile phones and deep space missions

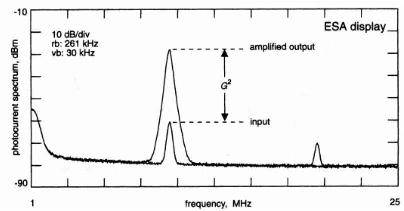
Figure 1. Homodyne (Zero-IF) Receiver





The signal to noise ratio for the input field

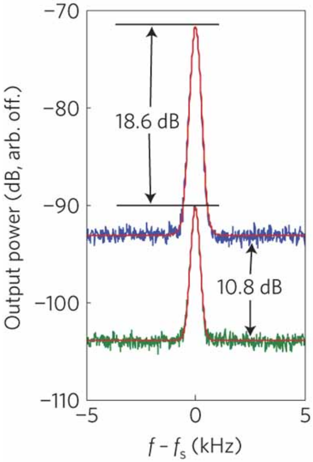




Amplifier noise figure

$$NF = \log_{10}\left(\frac{SNR_{\text{in}}}{SNR_{\text{out}}}\right)$$

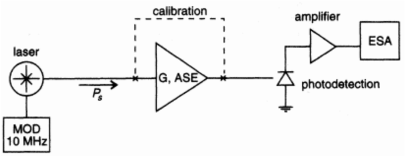
What is the signal to noise of the input of the amplifier?



Amplified signal modulation
Signal to Noise: SHOT NOISE and AMPILFIER ADDED NOISE

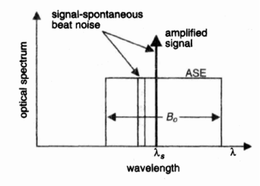
Input signal modulation
Signal to noise SHOT NOISE

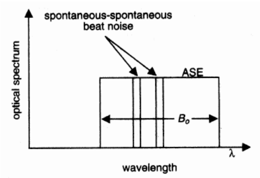
ASE Noise in a Fiber Amplifier



The ASE gives rise to:

- Electronic shot noise
- Signal spontaneous beat noise (SSE)
- Spontaneous-Spontaneous Noise (SSN)



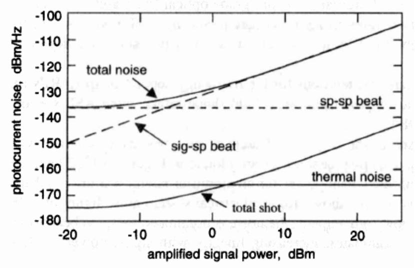


The Noise in an Fiber Amplifier originates from the amplified spontaneous emission noise (ASE)

$$P_{\text{ASE}} = 2n_{sp}h\nu(G-1)B_0$$

$$n_{sp} = \frac{\sigma_e N_2}{\sigma_e N_2 - \sigma_a N_1}$$

Photocurrent noise of versus the power of an amplified signal:



Amplified spontaneous emission noise

Where does the ASE Noise originate from?

$$dP = \gamma P dz$$

Optical gain

$$\gamma(v) = (N_2 - N_1)\frac{c^2 g(v)}{8\pi n^2 v^2 t_{\text{spont}}}$$

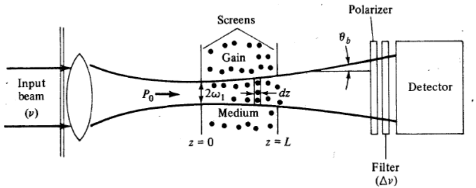
Spontaneous emission noise

$$P_N = \frac{N_2 h v A dz}{t_{\text{spont}}}$$

Total noise in the volume and in interval $V \ldots V + \Delta V$

$(dP)' = \frac{1}{2} \cdot \frac{N_2 h v g(v) \Delta v A}{t_{\text{spont}}} \frac{d\Omega}{4\pi} dz$ *(Rewrite expression with optical gain)*

$$(d\Omega)_{\min} = (d\Omega)_b = \pi \vartheta_b^2 = \frac{\lambda^2}{n^2 A}$$



Diffraction angle (Gaussian ebeam optics)

$$\vartheta_b = \frac{\lambda}{\pi^2 \omega_1}$$

The spontaneous emission noise yields the expression

$$(dP)' = \frac{N_2 \gamma h v}{N_2 - N_1} \Delta v \cdot dz$$

Spontaneous emission noise (1/2)

$$dP = \gamma P dz$$

$$\gamma(v) = (N_2 - N_1)\frac{c^2 g(v)}{8\pi n^2 v^2 t_{\text{spont}}}$$

$$(dP)' = \frac{N_2 \gamma h v}{N_2 - N_1}\Delta v dz$$

Inversion factor

$$\mu = \frac{N_2}{N_2 - N_1} = n_{sp}$$

Bandwith

$$2B_0 = \Delta v$$

Gain

$$G = e^{\gamma z}$$

Spontaneous emission noise (2/2)

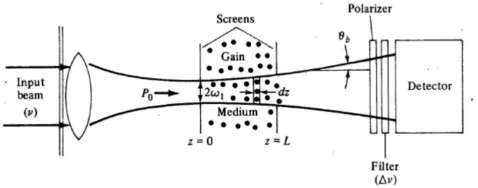
Both signal and noise are being amplified

$$\frac{dP}{dz} = \gamma P + \frac{N_2}{N_2 - N_1}\gamma h v \Delta v,$$
$$P(z) = P_0 e^{\gamma z} + \mu h v \Delta v \left(e^{\gamma z} - 1\right) =$$
$$= \text{ampl. sign.} + \text{ampl. noise}$$

- The pre-factor has an important consequence: minimum noise is expected if all ions are inverted! Never run an amplifier with low pump current.
- Also: never run an amplifier without input signal.

ESA Noise of the Amplifier

$$P_{\text{Noise}} = \mu 2 h v B (G - 1)$$



Signal to noise at the output of the amplifier

$$\left(\frac{S}{N}\right)_{\text{output}} = \frac{P_0}{\mu h v 2B}\frac{G}{G-1}$$

$$NF = \left(\frac{S}{N}\right)_{\text{in}} / \left(\frac{S}{N}\right)_{\text{out}} = \frac{P_0}{hvB} / \frac{P_0}{\mu h v 2B}\frac{G}{G-1} = 2\mu\frac{G-1}{G}$$

Spontaneous emission noise

A more accurate calculation yields (valid also for low gain)

$$NF = 2\mu\frac{(G-1)}{G} + \frac{1}{G}$$

Consequently, it is very important that the amplifier is operated in a mode,

- where all ions are invert (and thus the amplifier gain is high),
- where the amplified input signal is large compared to the ASE.

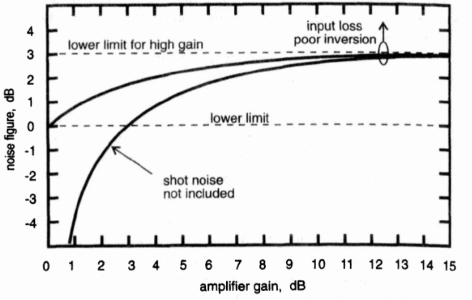


Figure 13.21 Noise figure dependence on optical amplifier gain with and without the shot-noise contribution.

Supplementary Info: spontaneous emission noise

The total spontaneous emission in a bandwidth B is given by

$$P_{\text{ASE}} = 2n_{sp}hv(G-1)B_0.$$

Consequently, there is a beatnote between spontaneous emission and the signal (si-sp noise):

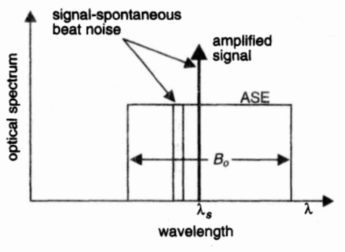
$$S_{i_{sg-sp}}(f) = 4\,\text{Re}^2\,GP_s\rho_{\text{ASE}} \quad \left[\text{A}^2/Hz\right].$$

A second contribution of noise on a photodetector is that of the shot noise of the signal:

$$S_p(f)\Big|_{\text{shot}} = \frac{2hvi_{dc}}{\text{Re}} = 2hv\langle P\rangle \quad \left[\text{W}^2/Hz\right].$$

The total noise (SNR in versus out) figure is thus:

$$NF = \frac{2\rho_{\text{ASE}}}{GhV} + \frac{1}{G}, \quad NF = 2n_{sp}\frac{(G-1)}{G} + \frac{1}{G}.$$



Supplementary: Fundamental limit on amplifiers (1/2)

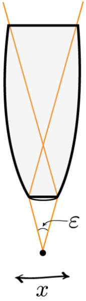
Position momentum uncertainty

$$\Delta p \cdot \Delta x \geq \hbar/2$$

"If one wants to be clear about what is meant by 'position of an object,' for example of an electron..., then one has to specify definite experiments by which the 'position of an electron' can be measured; otherwise this term has no meaning at all." – Heisenberg, in uncertainty paper, 1927

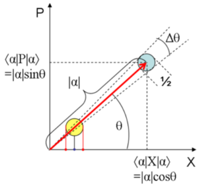
Let the cone of light rays leaving the microscope lens and focusing on the electron make an angle with the electron. Then, according to the laws of classical optics

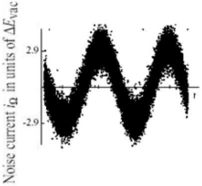
$$\Delta x = \frac{\lambda}{\sin\left(\frac{\varepsilon}{2}\right)}$$



Photon-number phase uncertainty

$$\Delta \phi \cdot \Delta n \geq 1/2$$





Supplementary: Fundamental limit on amplifiers (2/2)

When a photon strikes an electron, the latter has a Compton recoil with momentum proportional to

$$\Delta p_x \approx \frac{h}{\lambda}\sin\left(\frac{\varepsilon}{2}\right)$$

$$\Delta x \Delta p_x \approx \frac{\lambda}{\sin\left(\frac{\varepsilon}{2}\right)} \cdot \frac{h}{\lambda}\sin\left(\frac{\varepsilon}{2}\right) = h$$

Supplementary info: Number-phase uncertainty

1604 PROCEEDINGS OF THE IRE July

The Fundamental Noise Limit of Linear Amplifiers*

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Input noise (minimum uncertainty state)

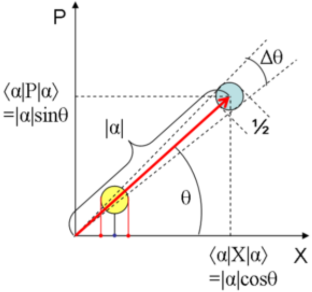
$$\Delta n_1 \Delta\Phi_1 = \frac{1}{2}$$

Amplification process (Gain G)

$$(n_2 \pm \Delta n_2) = G\,(n_1 \pm \Delta n_1)$$
$$(\Phi_1 \pm \Delta\Phi_1) = (\Phi_2 \pm \Delta\Phi_2)$$

Uncertainty in the output fluctuations:

$$\Delta n_2 \Delta\Phi_2 = (G\Delta n_1)\,\Delta\Phi_1$$
$$\Delta n_1 \Delta\Phi_1 = \frac{1}{G}\cdot\frac{1}{2}$$



This violates the photon number uncertainty relation!

Any linear amplifier must therefore add noise.

Photon-number phase uncertainty

$$\Delta\phi \cdot \Delta n \geq 1/2$$

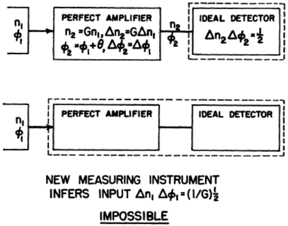


Fig. 1—Thought experiment to show the nonexistence of a perfect amplifier.