Waveguides

Konstantinos Makris

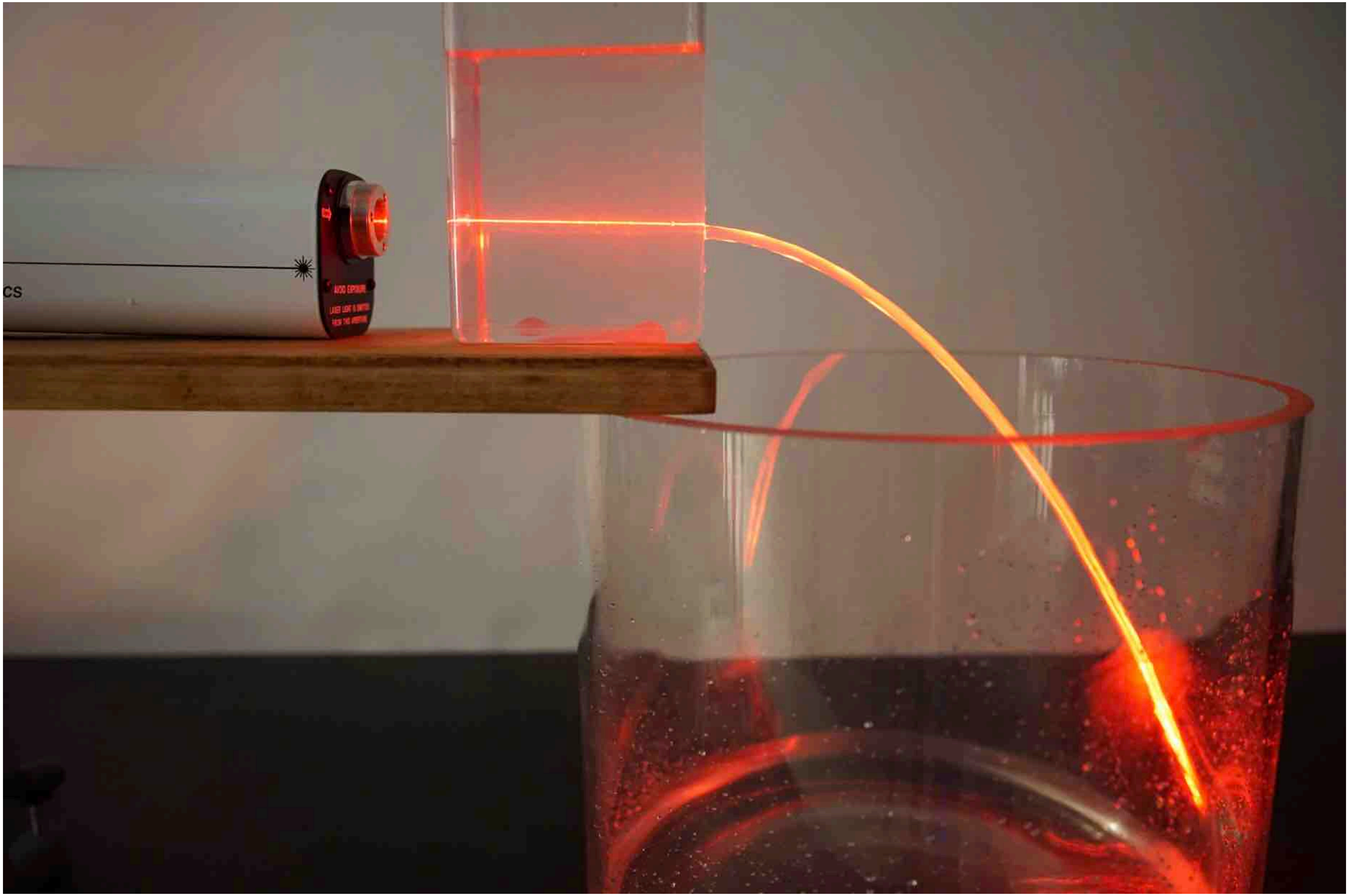
University of Crete, Greece

Demetri Psaltis

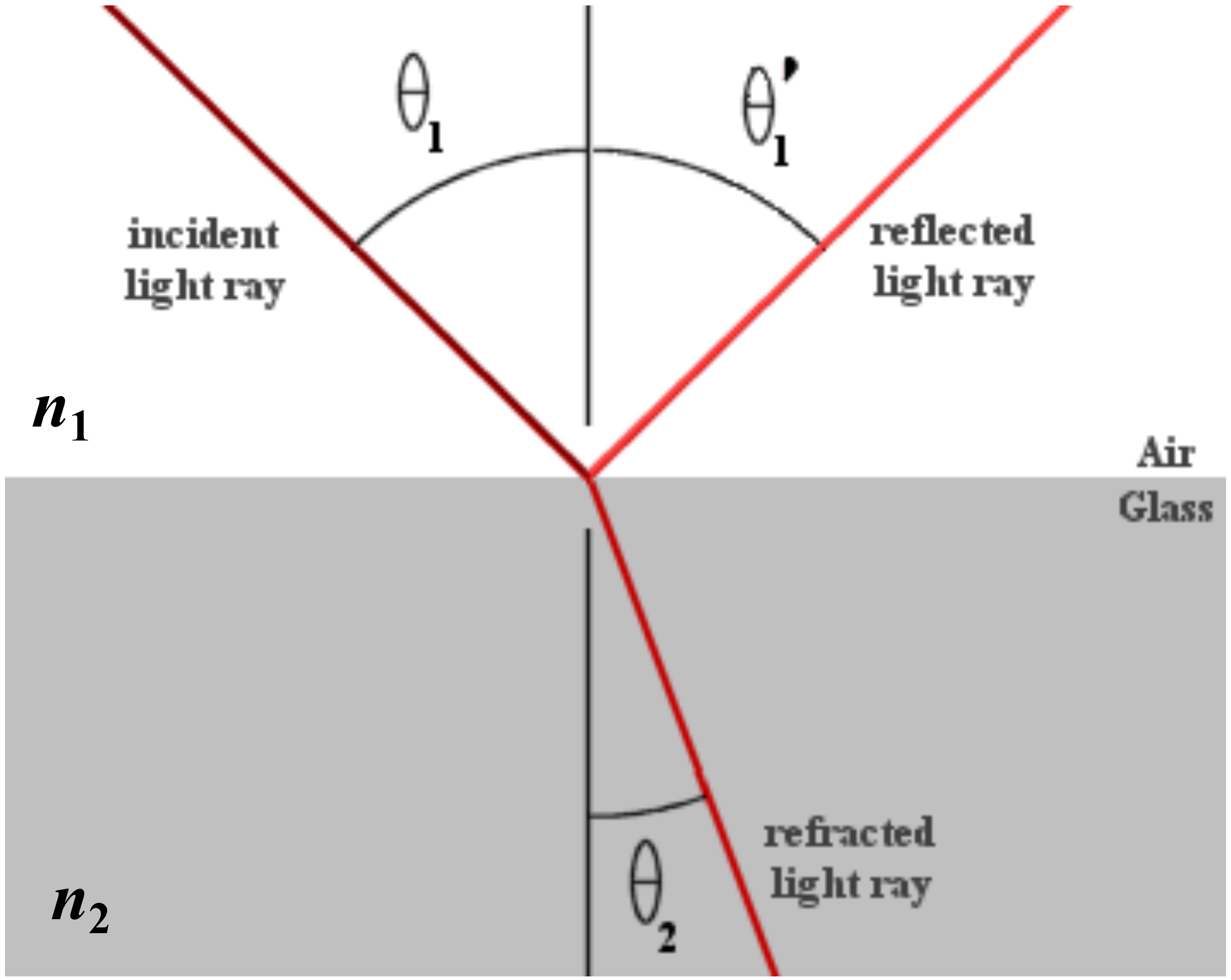
Optics Lab-EPFL, Switzerland

TRAPPING OF LIGHT - WAVEGUIDES

Trapping of Light



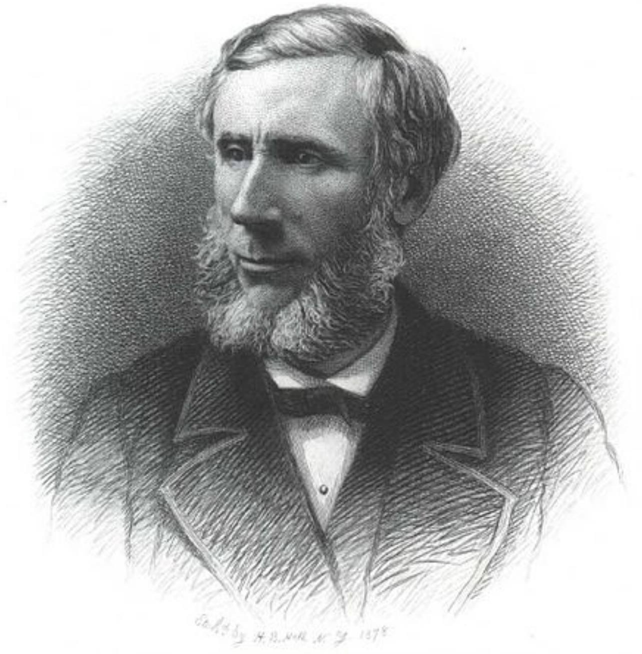
Snell's Law



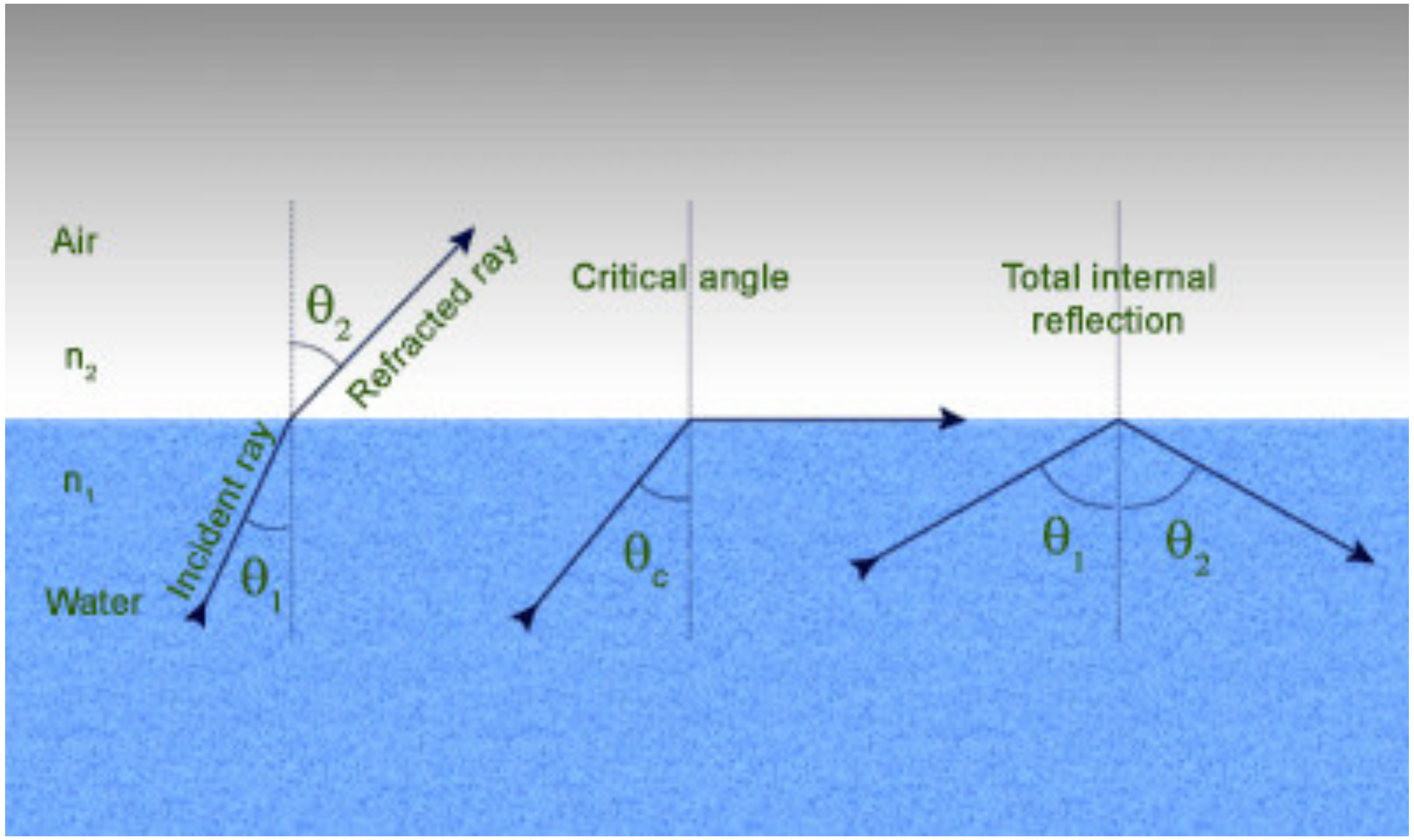
$$\boxed{\theta_1 = \theta_1'}$$ **Reflection angles**

$$\boxed{n_1 sin\theta_1 = n_2 sin\theta_2}$$ **Snell's Refraction Law**

Total Internal Reflection (TIR)

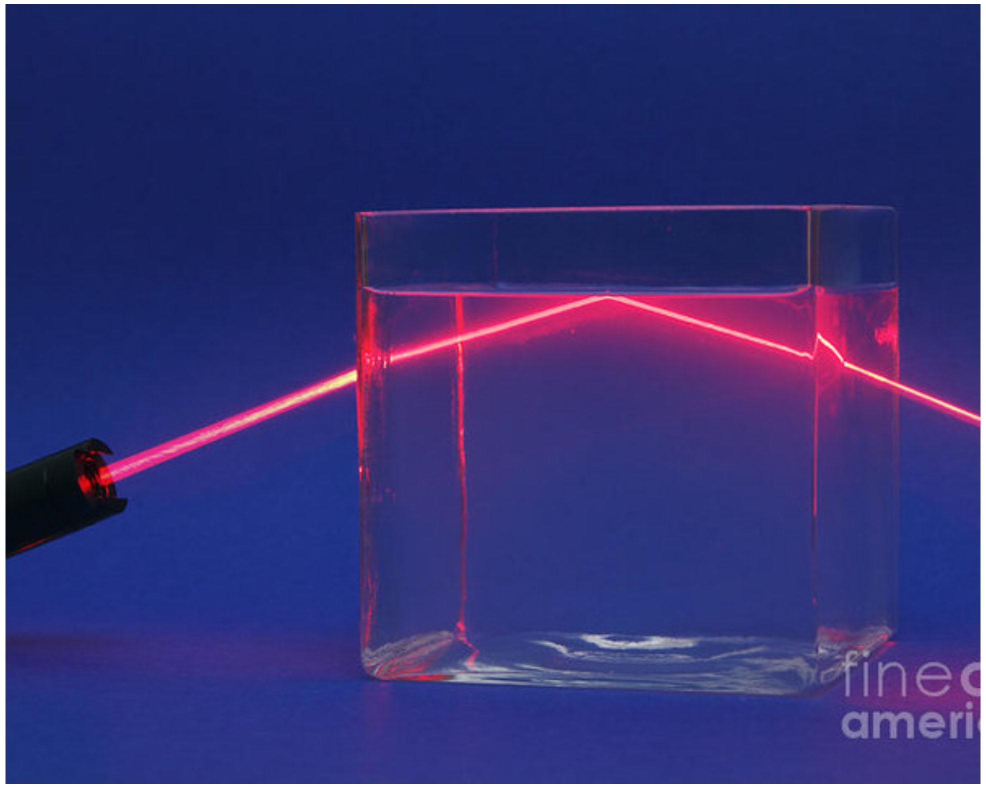


John Tyndall
1820-1893

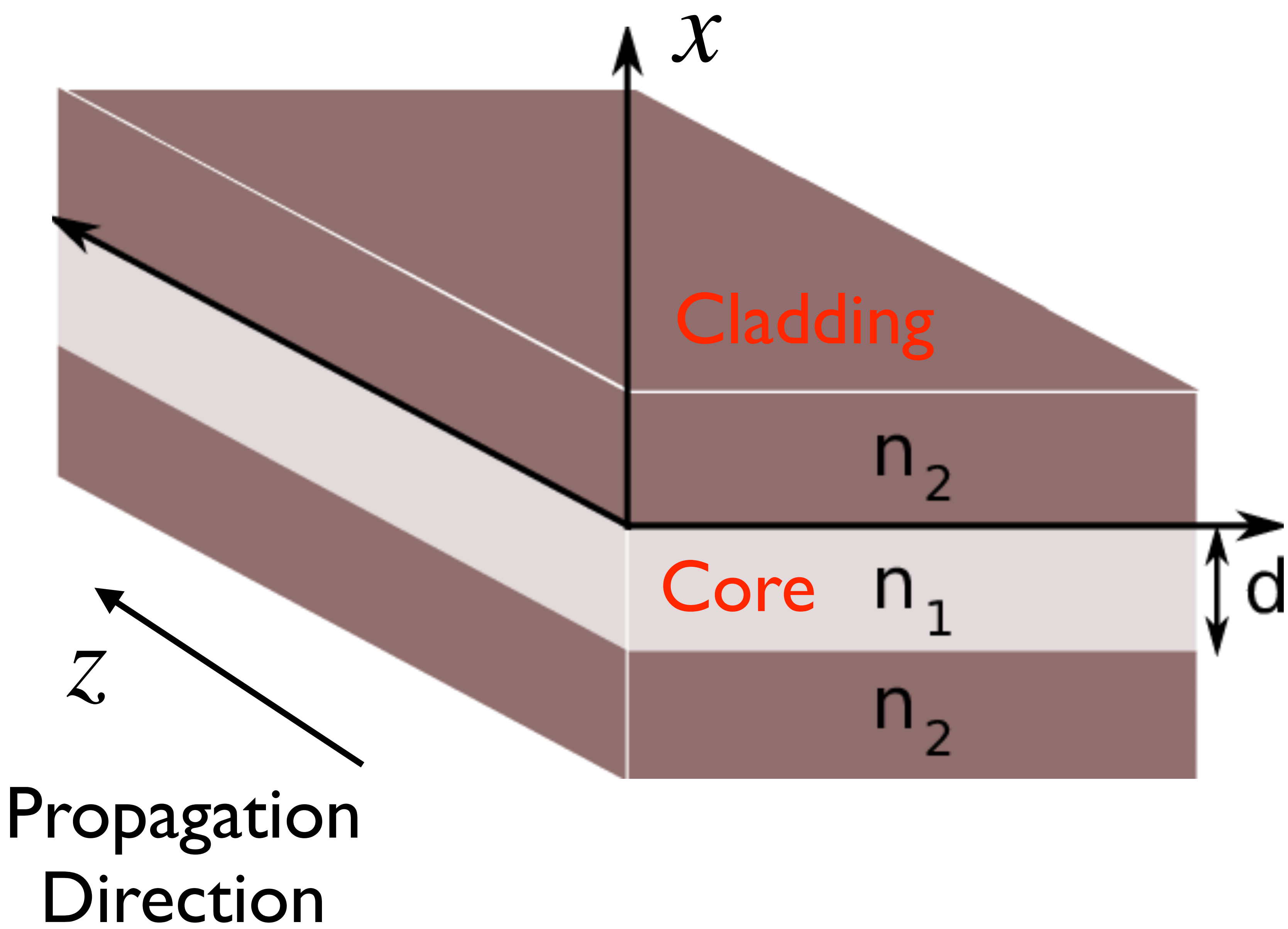


Critical Angle for TIR

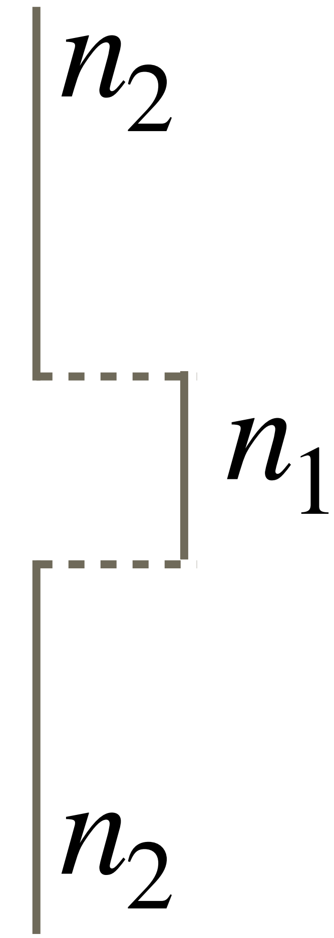
$$n_1 sin\theta_c = n_2 \Rightarrow \boxed{\theta_c = arcsin(\frac{n_2}{n_1})}$$



Slab Waveguide

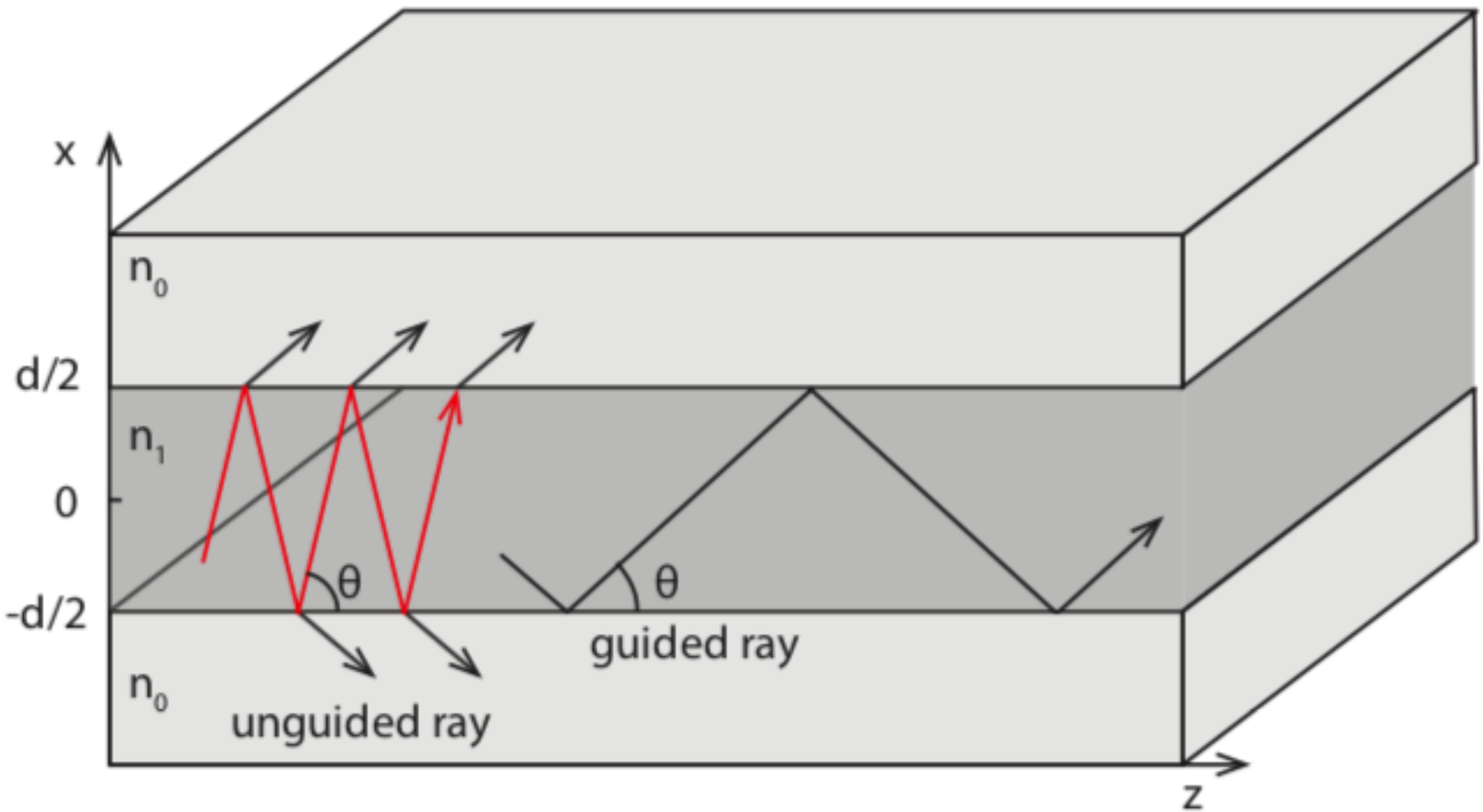


Step index profile

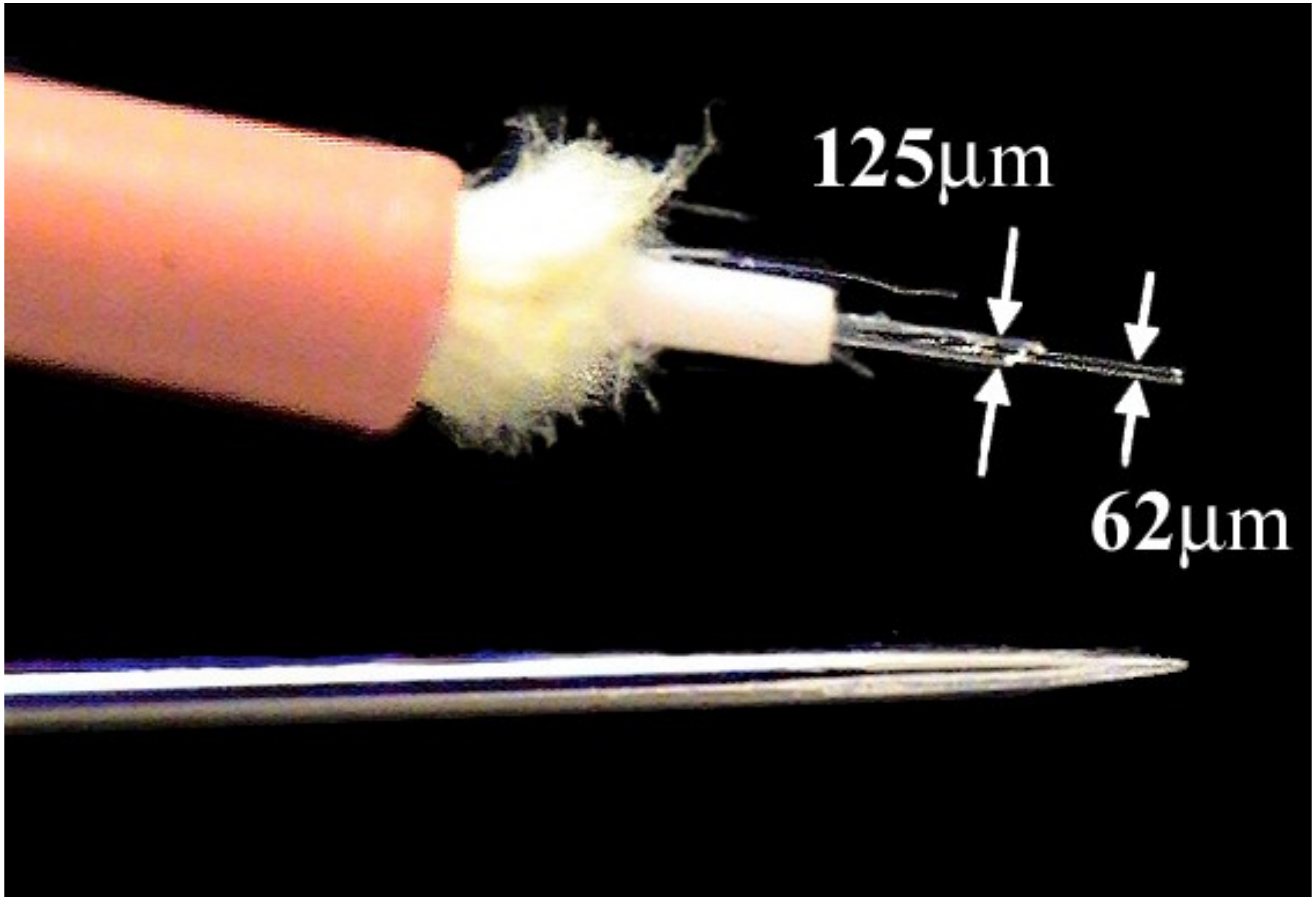


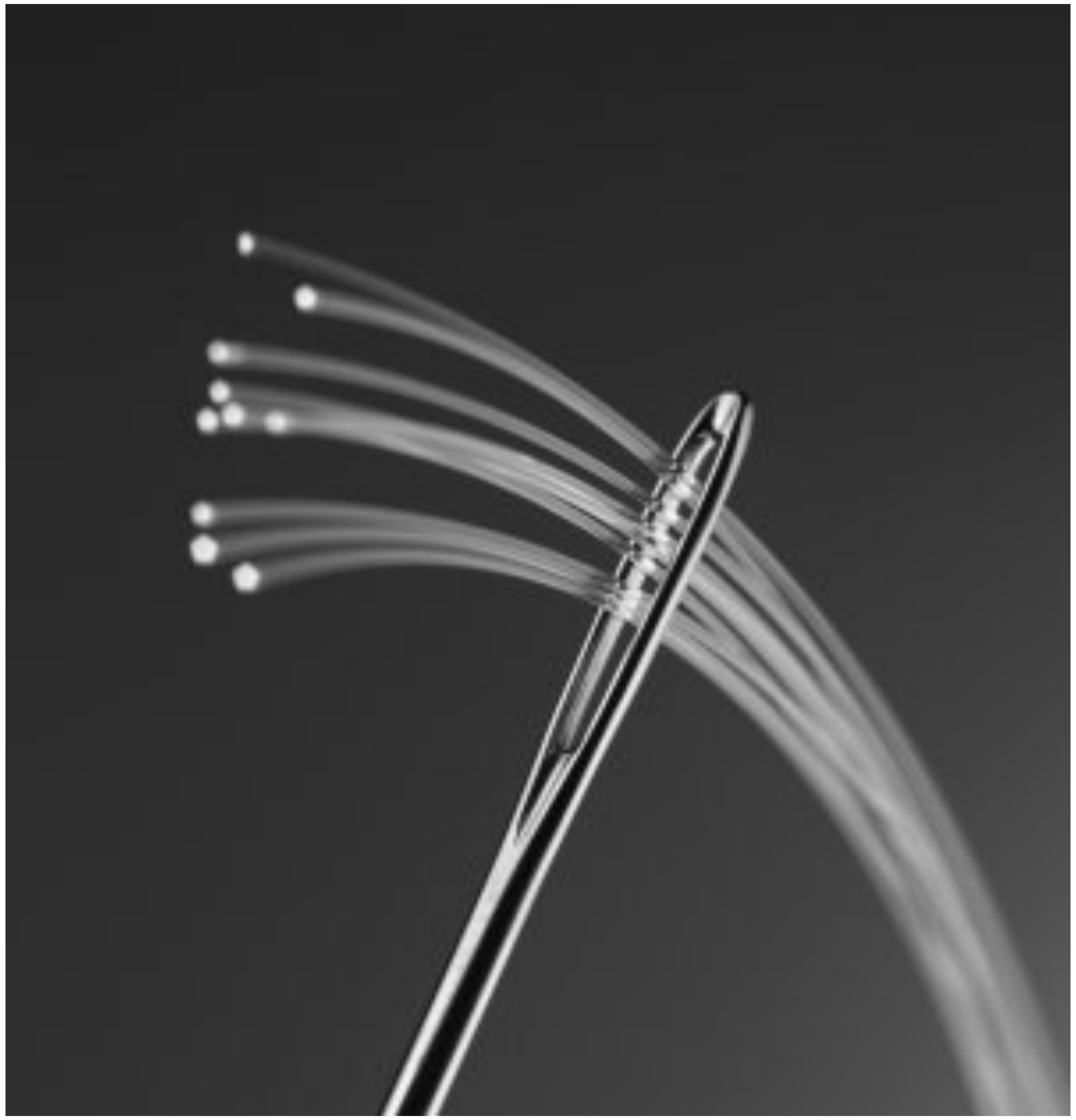
$$\boxed{n_1 = n_{core} > n_{cladding} = n_2}$$

Propagation in a Slab Waveguide



Optical Fiber





Wavelength ~1.55 µm
Optical fiber length ~ m-Km

Thickness of human hair fiber ~ 50-70 µm

Optical Fibers

The Nobel Prize in Physics 2009

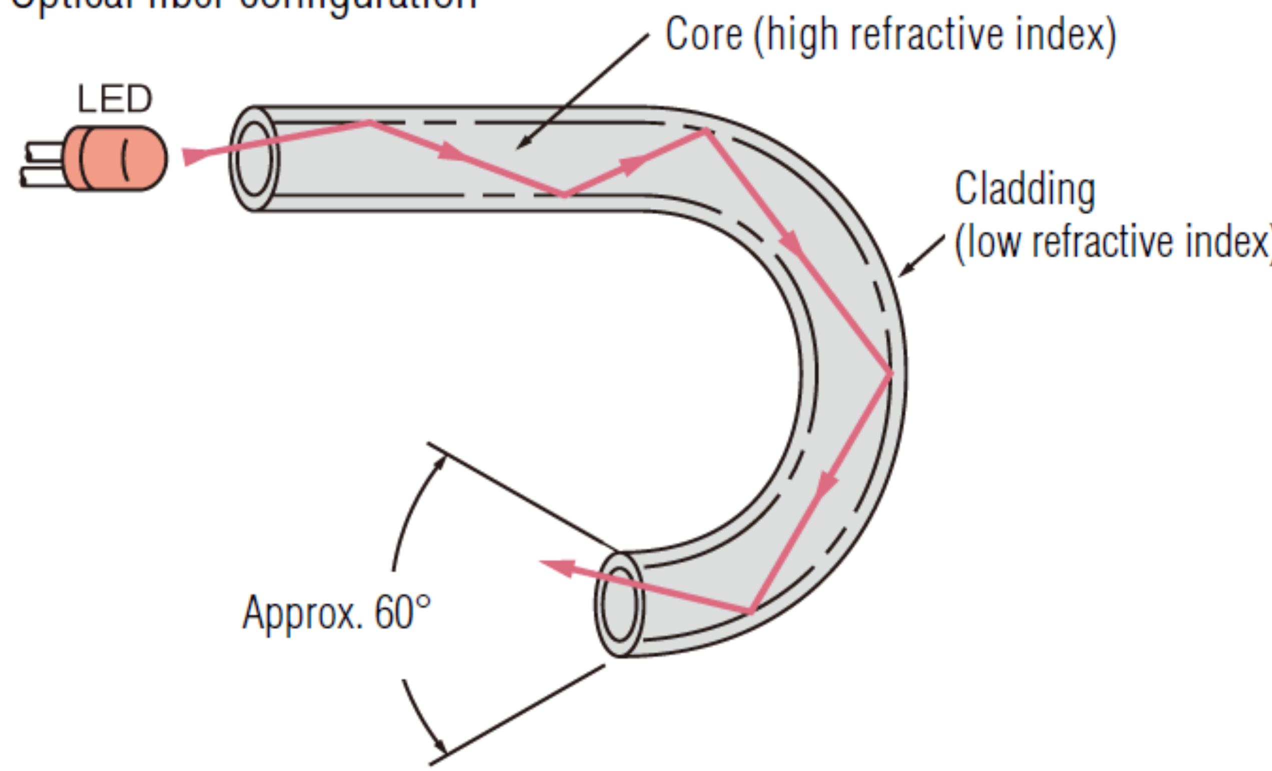
Photo: U. Montan
Charles Kuen Kao
Prize share: 1/2

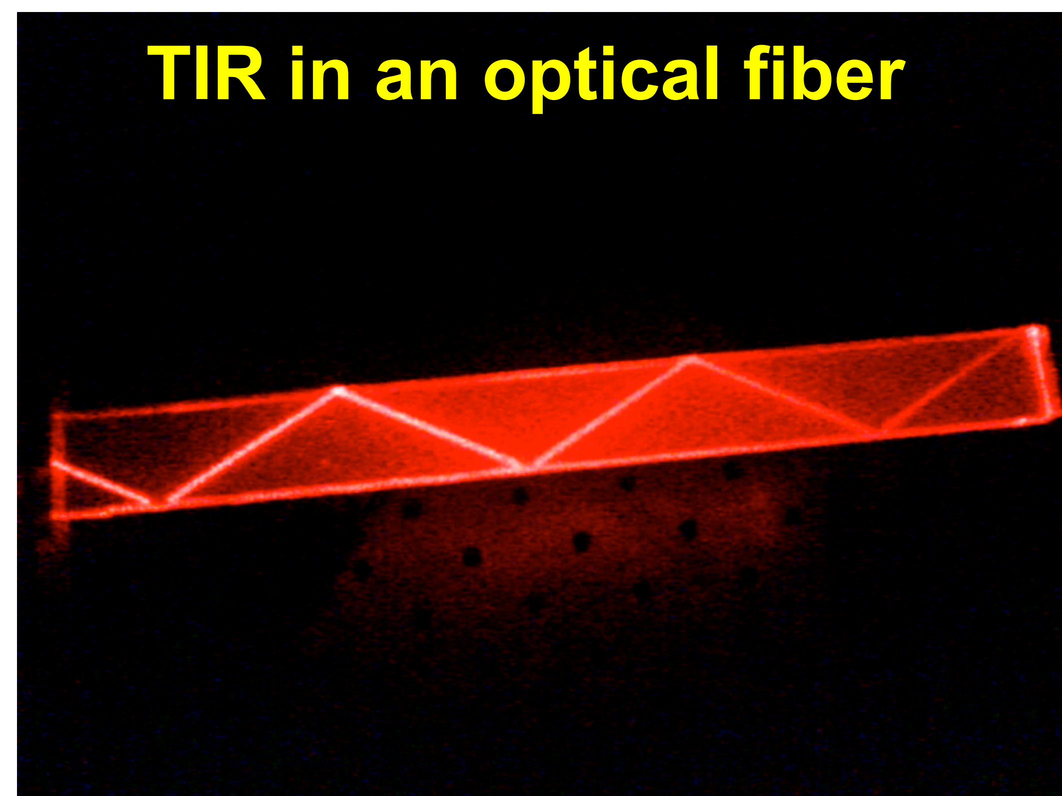
Photo: U. Montan
Willard S. Boyle
Prize share: 1/4

Photo: U. Montan
George E. Smith
Prize share: 1/4

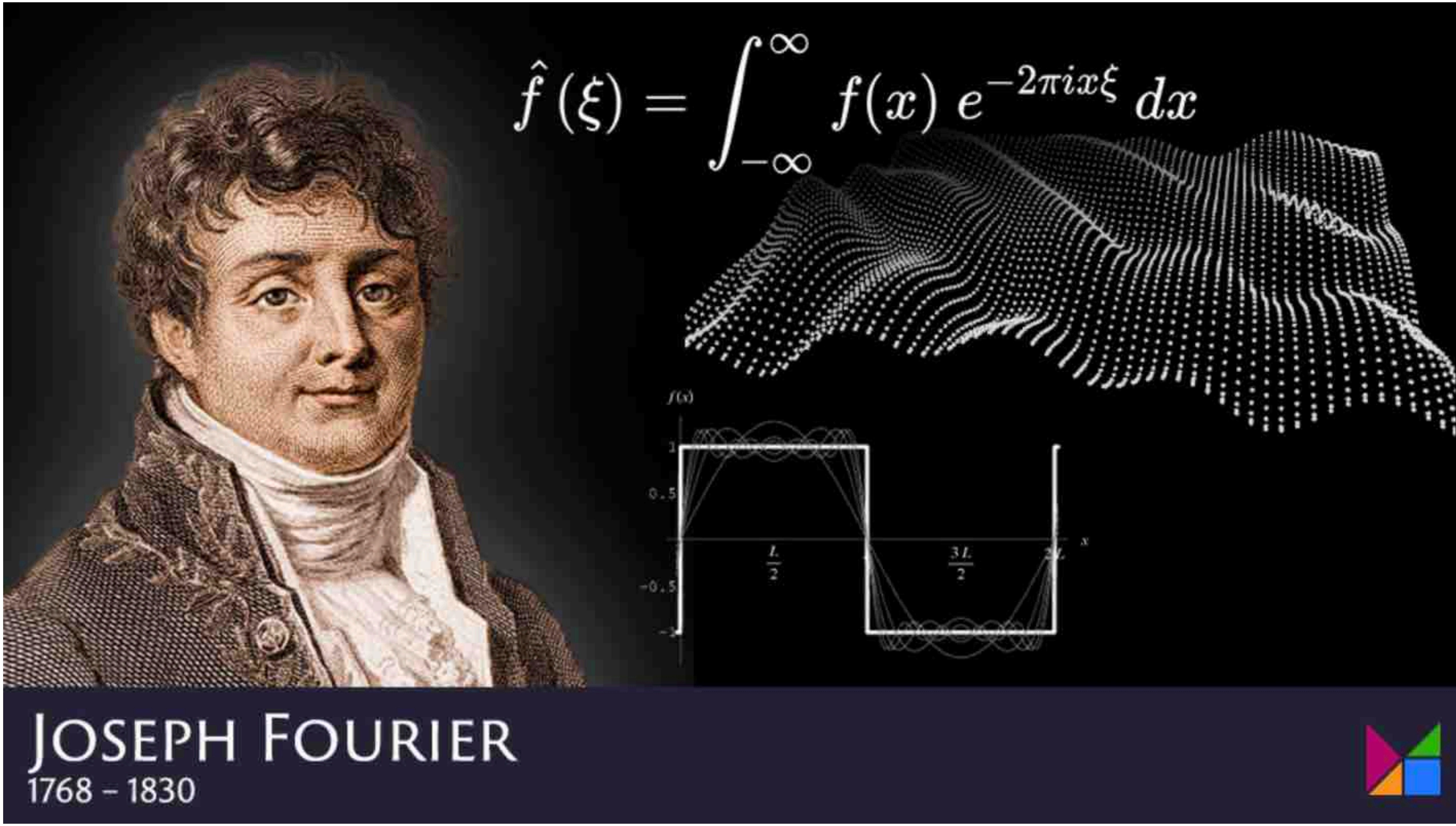
The Nobel Prize in Physics 2009 was divided, one half awarded to Charles Kuen Kao *"for groundbreaking achievements concerning the transmission of light in fibers for optical communication"*, the other half jointly to Willard S. Boyle and George E. Smith *"for the invention of an imaging semiconductor circuit - the CCD sensor"*.

Optical fiber configuration





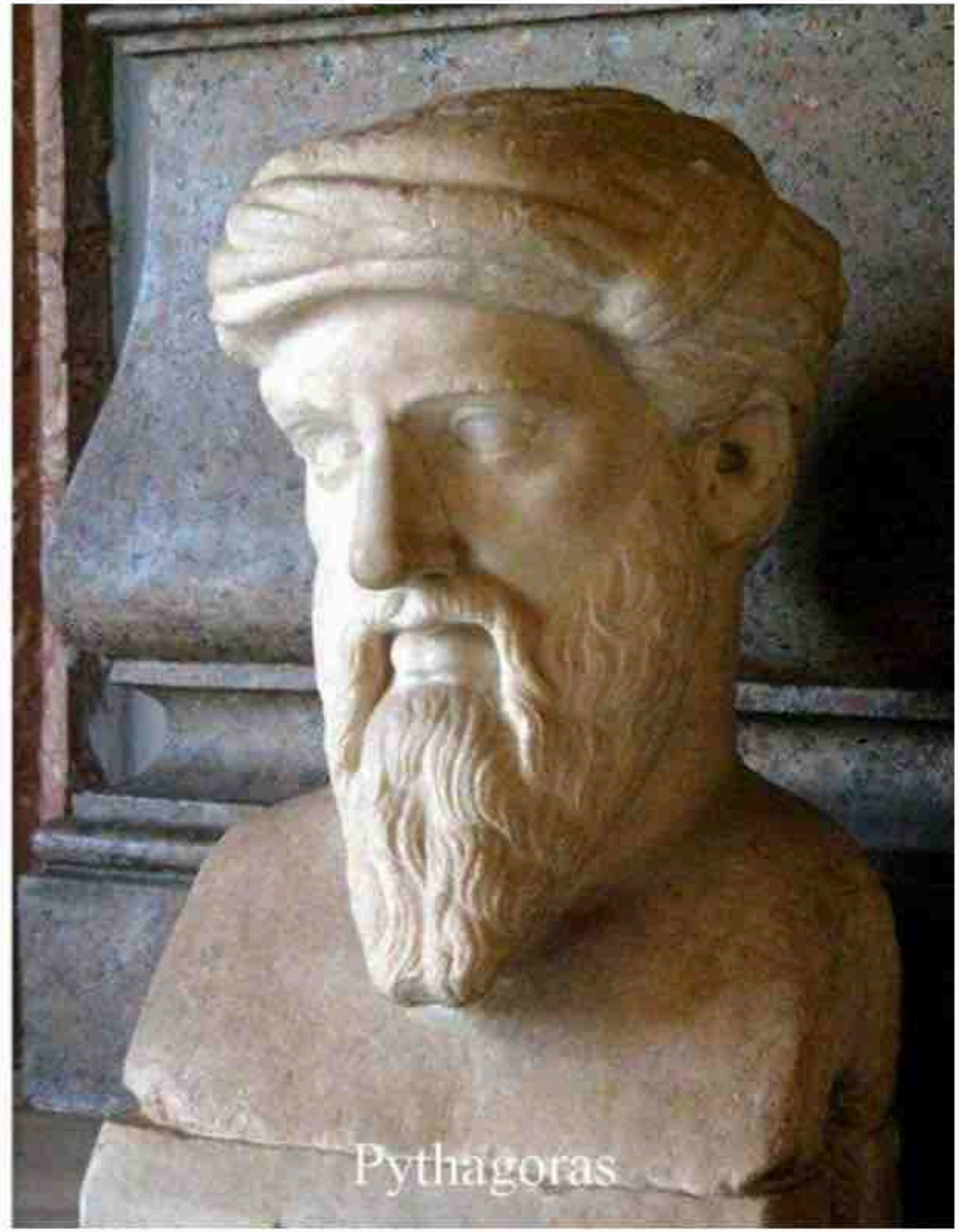
OPTICAL EIGENMODES

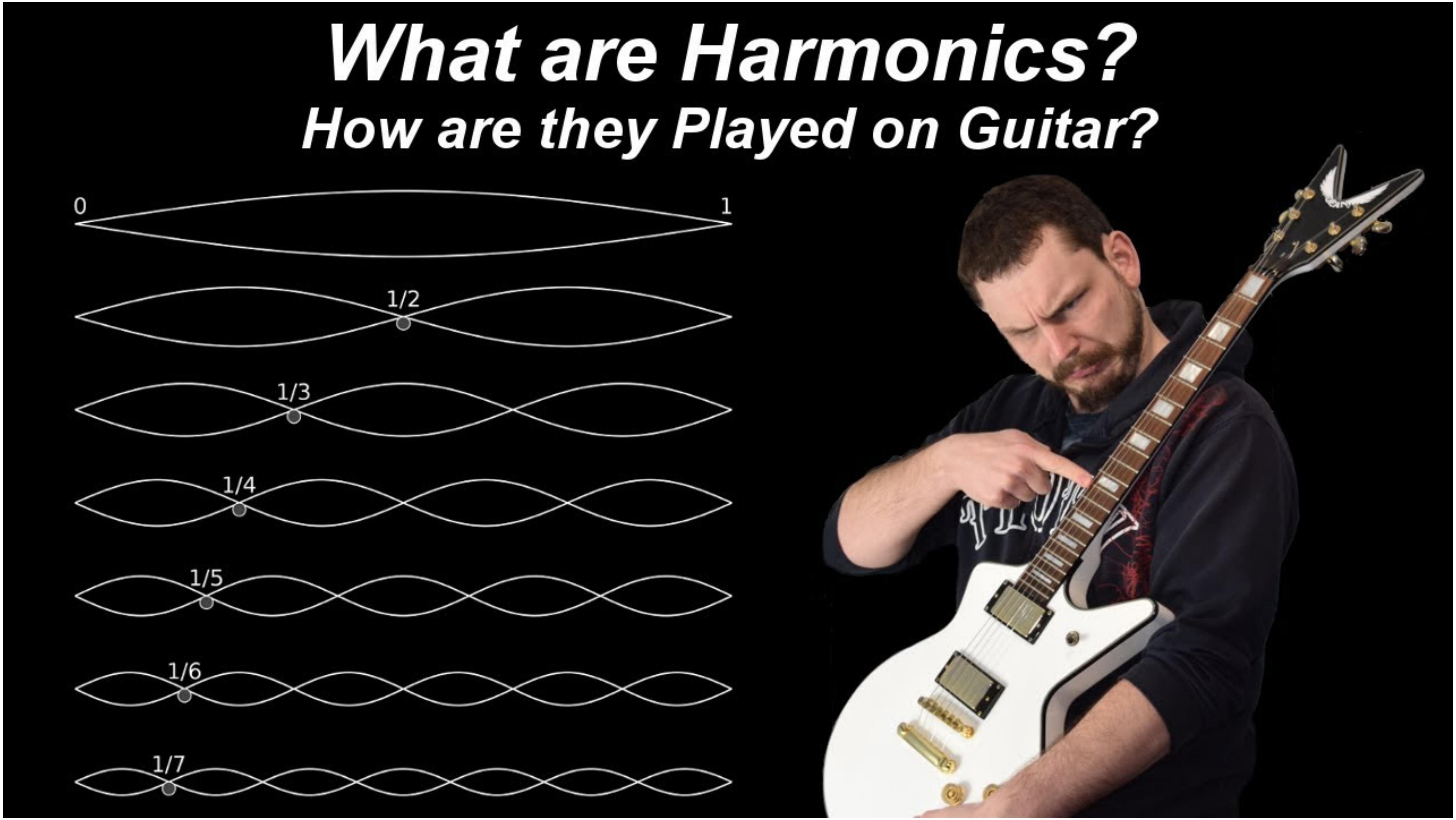


Plane waves as eigenmodes of free space

Pythagorian Harmonics

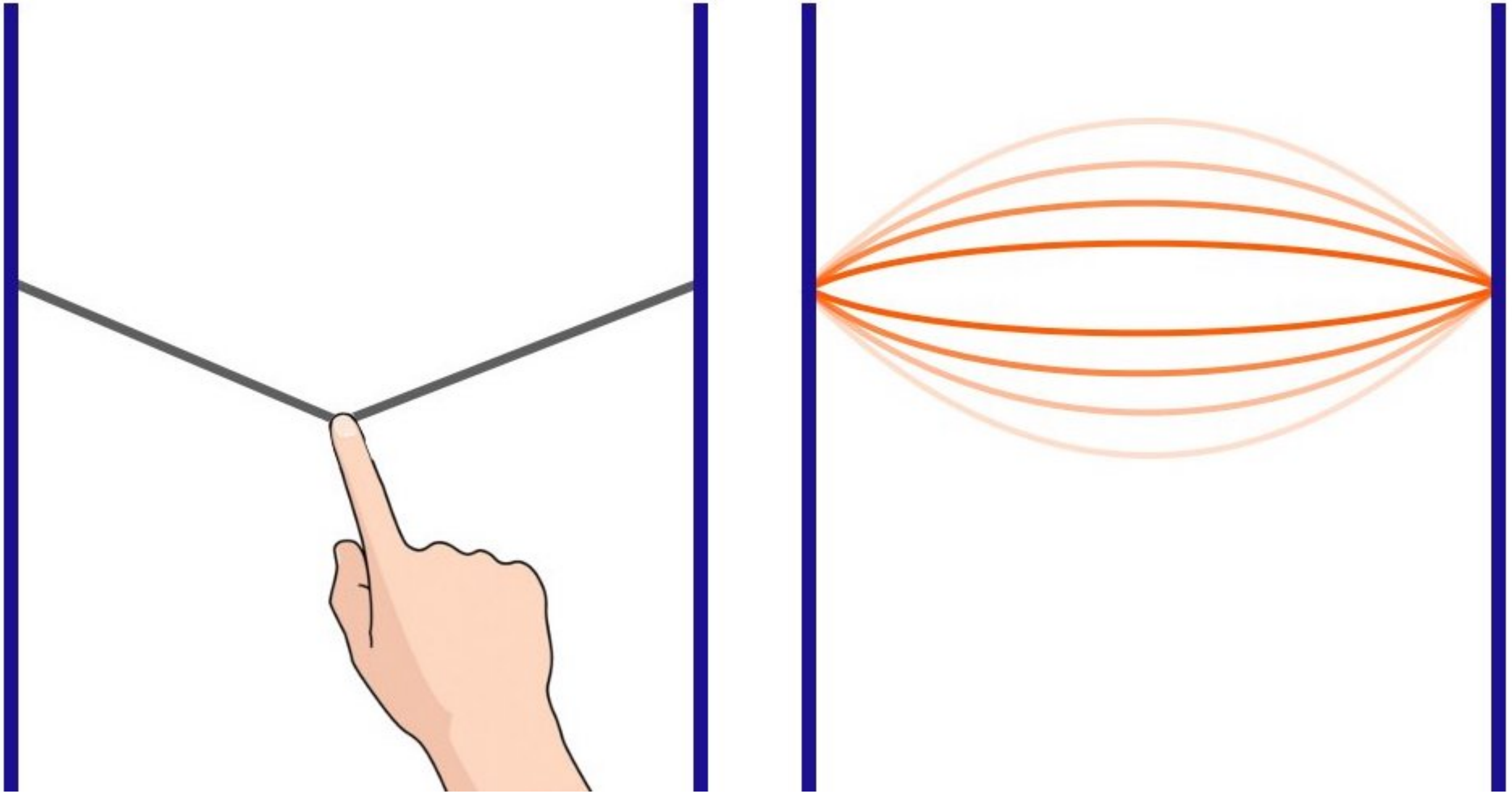
“There is geometry in the humming of the strings, there is music in the spacing of the spheres.”




What are Harmonics?
How are they Played on Guitar?
0
1
1/2
1/3
1/4
1/5
1/6
1/7

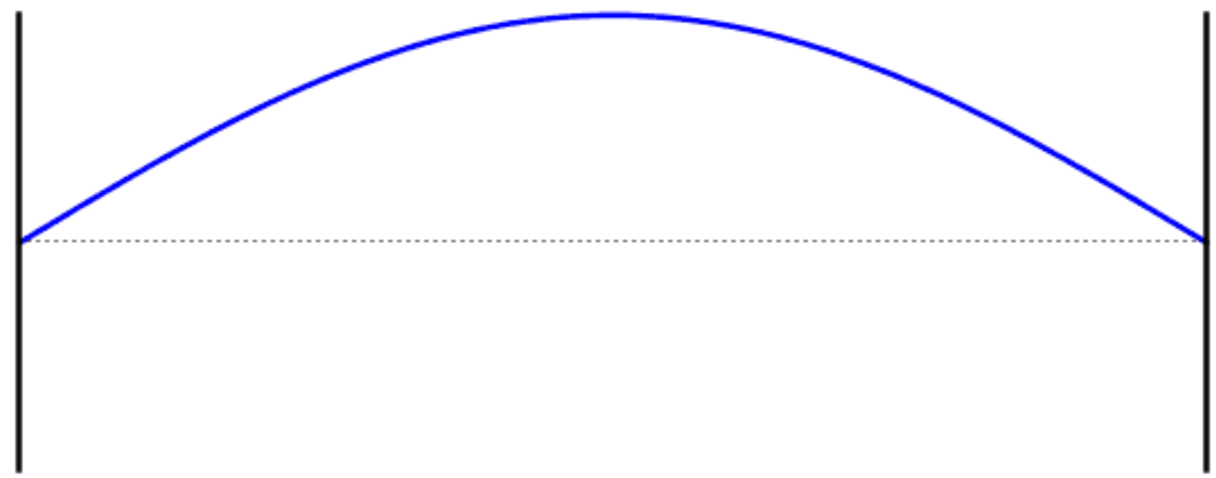
Wave motion of a string

$t = 0$ $\qquad$ $t > 0$

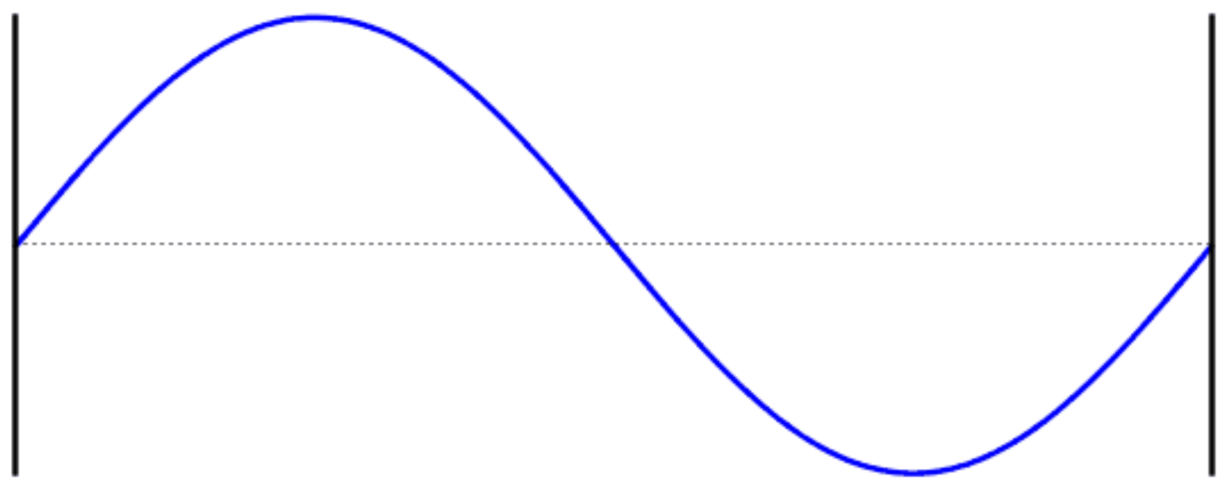


Vibrating string

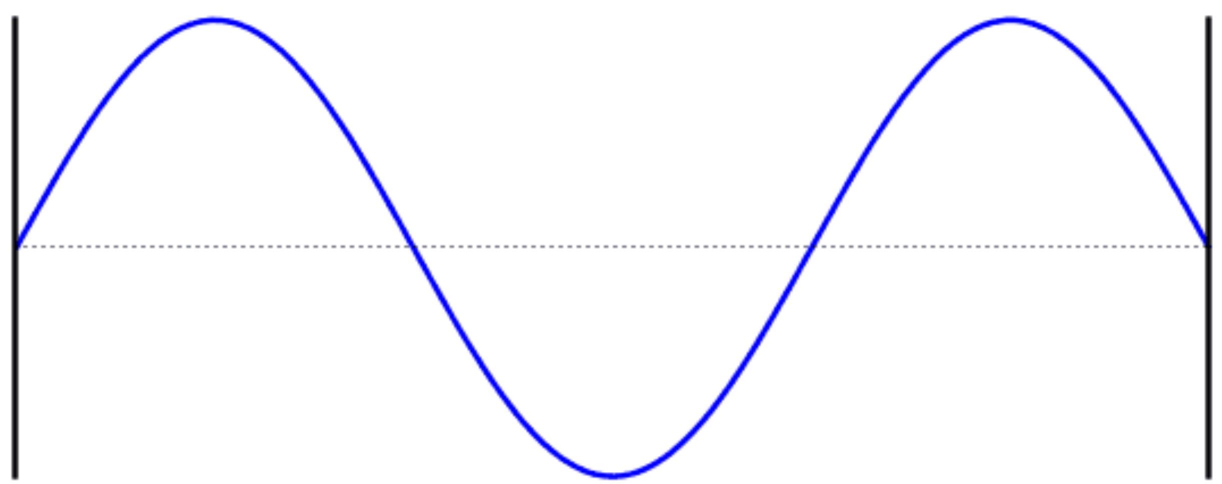
1st harmonic



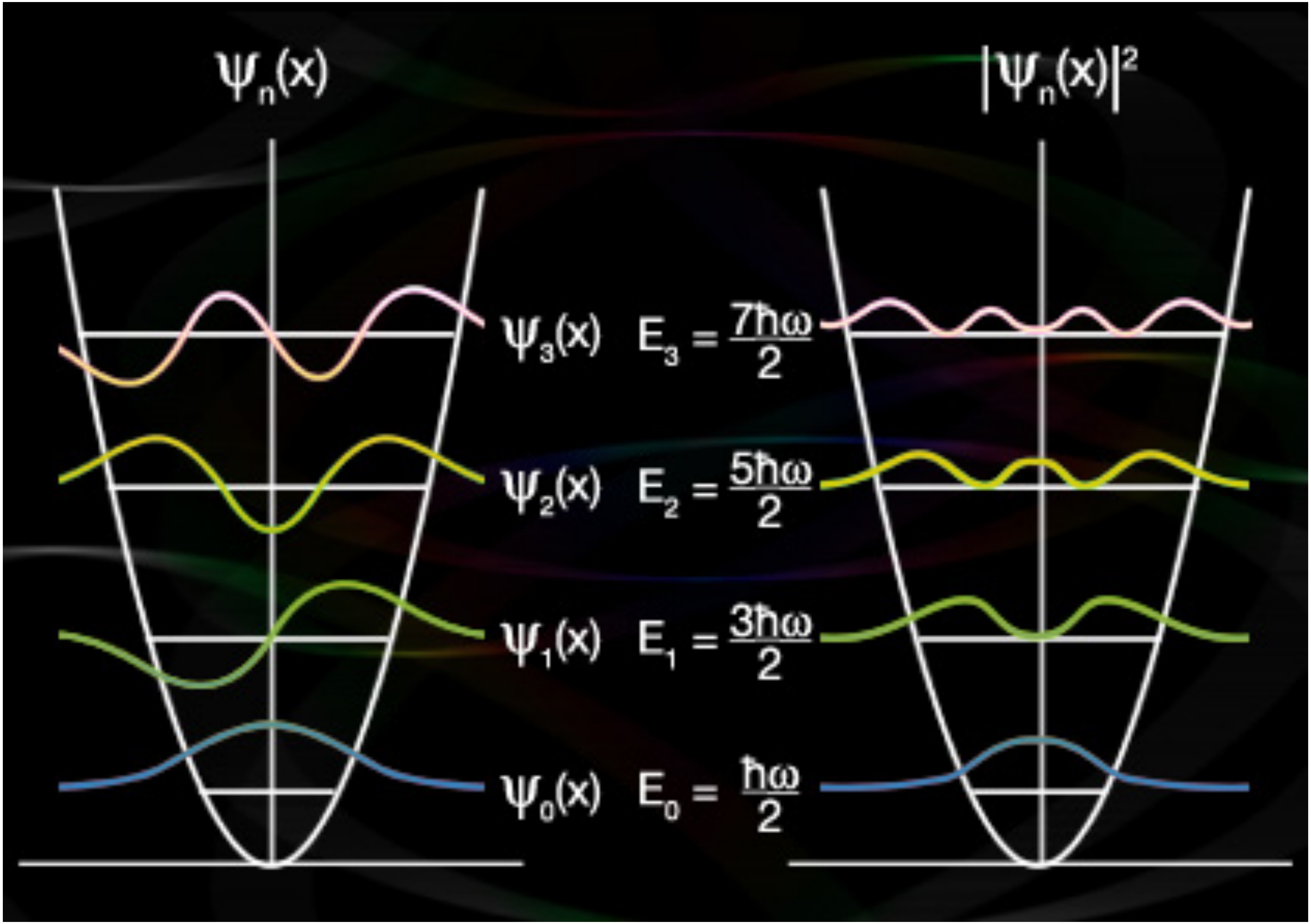
2nd harmonic



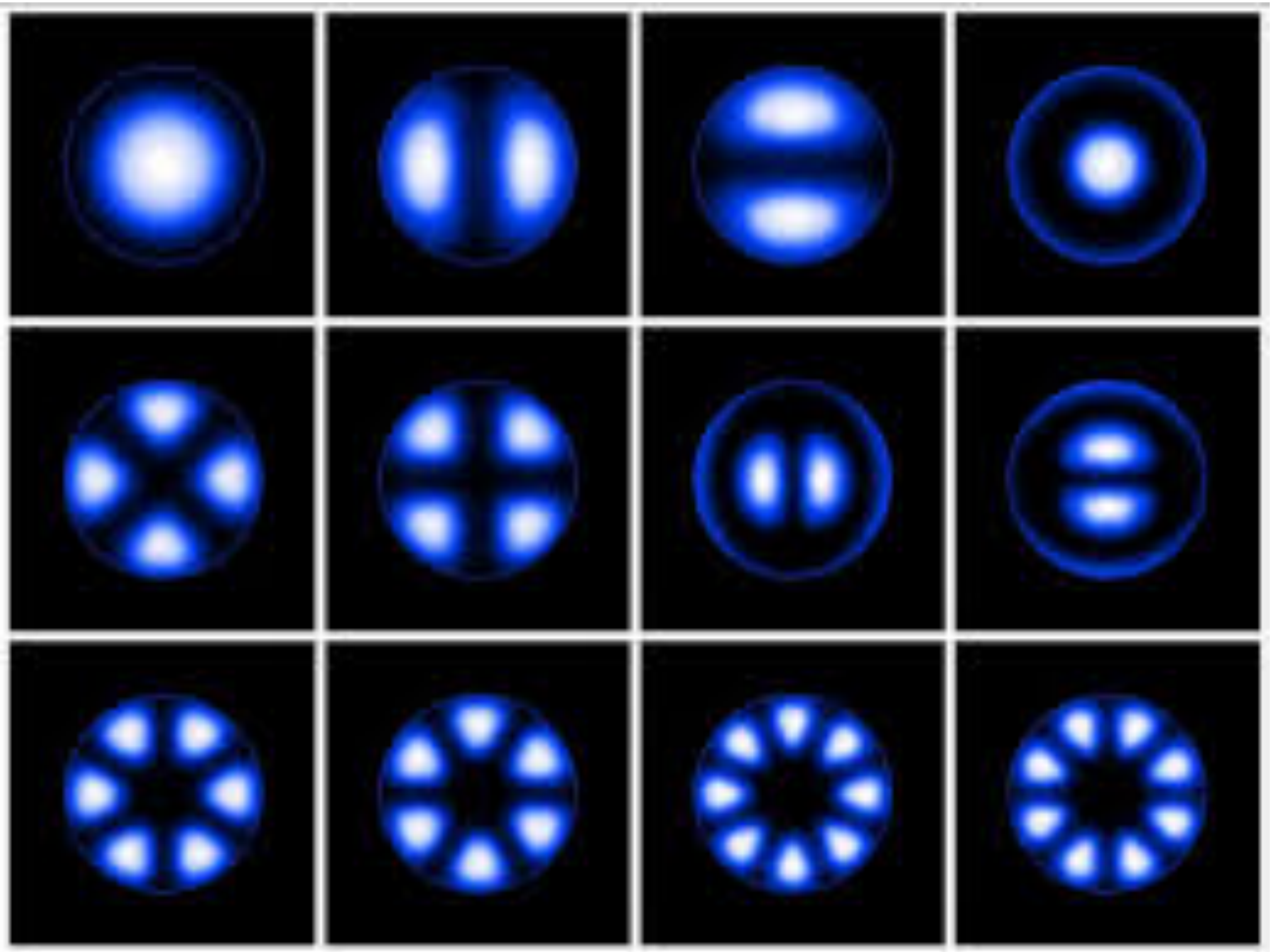
3rd harmonic



Eigenmodes of Harmonic oscillator



Eigenmodes of an optical fiber



Concept of an Eigenstate

Stationary Ansatz

$$\boxed{\psi_n(x,z) = u_n(x)\, e^{i\beta_n z}}$$

Eigenstate Profile

Eigenvalue

Eigenvalue Problem

$$\hat{H} u_n = \beta_n u_n$$

$$\boxed{|\psi_n(x,z)|^2 = |u_n(x)|^2}$$

The intensity of an eigenmode is invariant with respect to z

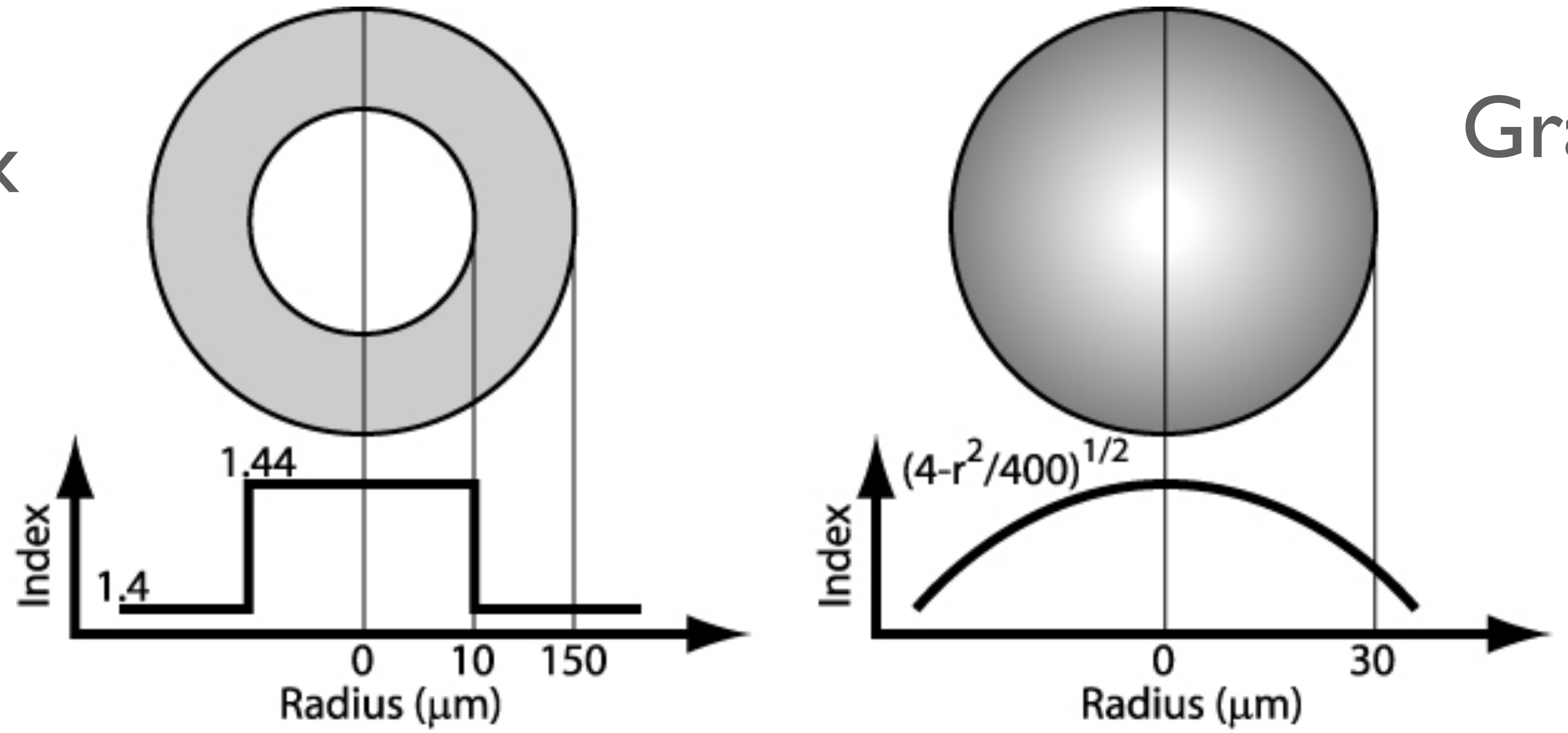
Paraxial Diffraction Equation for Waveguides

$$i\frac{\partial A}{\partial z} + \frac{1}{2k}\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}\right) + k_0 \delta n(x,y) A = 0$$

Optical potential term

$$n(x,y) = n_{cl} + \delta n(x,y)$$

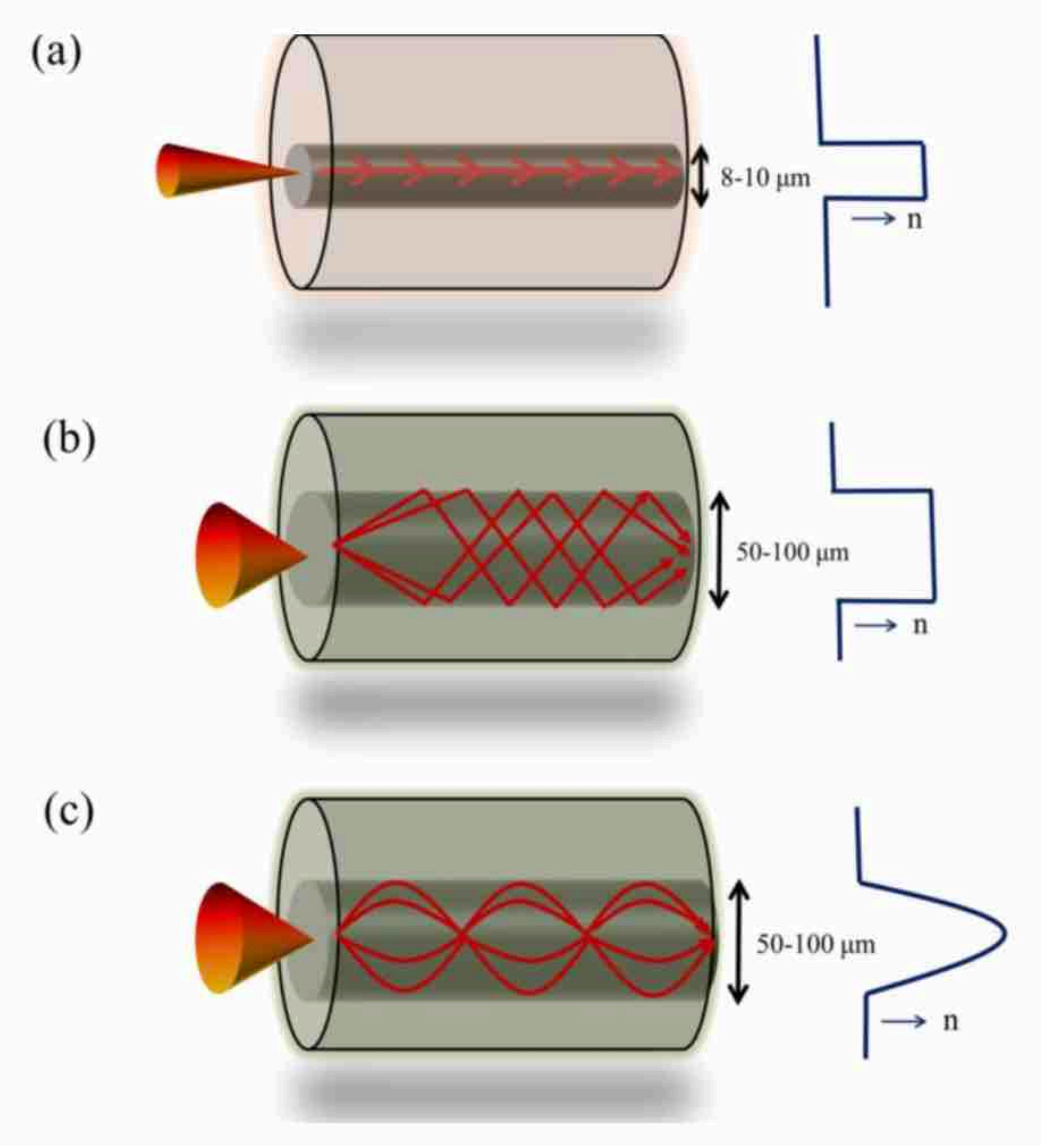
Step index Fiber



Graded index Fiber

Weakly guiding Approximation: $\delta n \ll n_{cl}$

GRIN Fibers – Harmonic Oscillator



Eigenmodes of a Waveguide

$$i\frac{\partial A}{\partial z} + \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + V(x, y)A = 0$$

We are looking for eigenstate solutions of the form:

$$\boxed{A(x, y, z) = u_n(x, y)e^{i\beta_n z}}$$

Eigenmode Ansatz

$$\boxed{\frac{\partial^2 u_n}{\partial x^2} + \frac{\partial^2 u_n}{\partial y^2} + V(x, y)u_n = \beta_n u_n}$$

Eigenvalue problem

Finite number of guided modes - Bound eigenstates

$$\boxed{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u_m^*(x, y)u_n(x, y)dxdy = \delta_{nm}}$$

Orthogonality condition

Spectrum of a Waveguide

Superposition Principle

$$\boxed{A(x,y,z) = \sum_{n=1}^{N} c_n u_n(x,y) e^{i\beta_n z} + \int_k c(k) u_k(x,y) e^{i\beta(k)z}}$$

Guided modes

Radiation modes

We can project any input field to our basis:

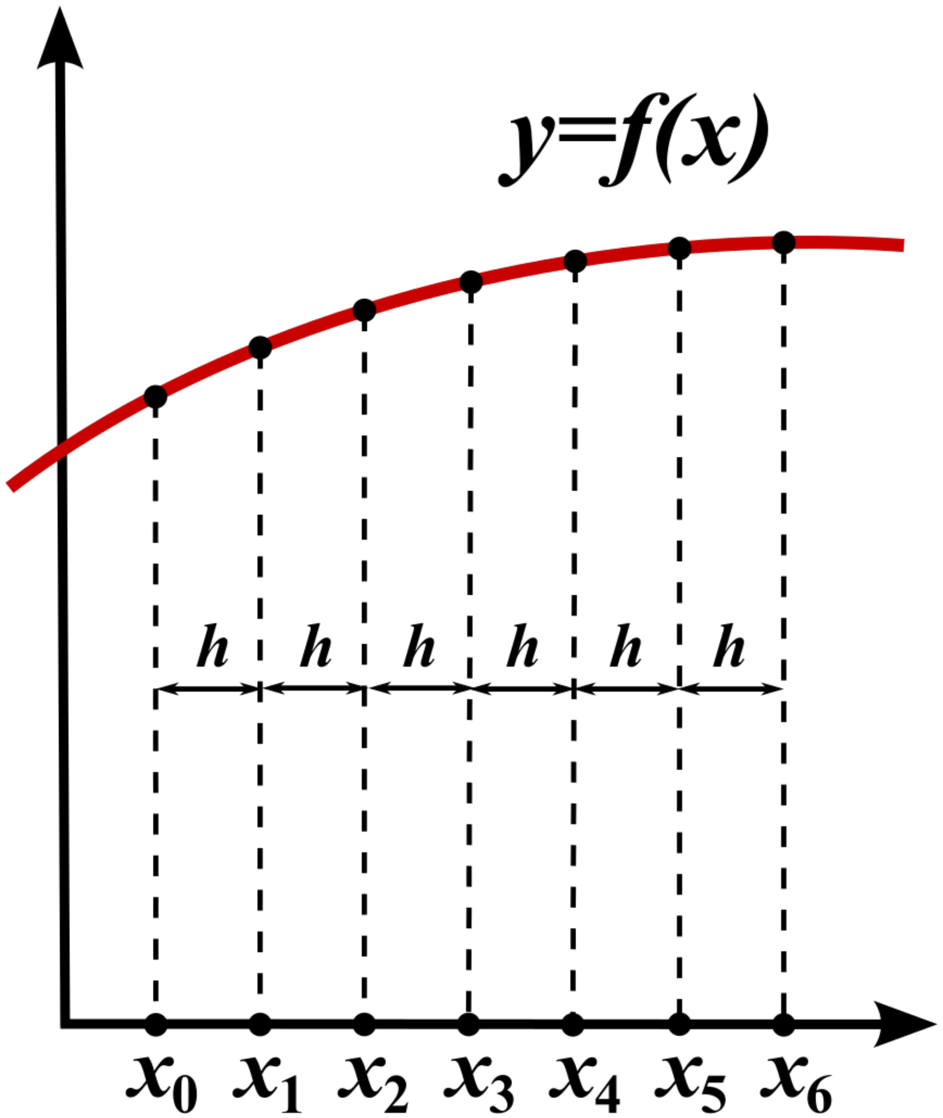
$$\boxed{c_n = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u_n^*(x,y) A(x,y,0)\,dx\,dy}$$

Projection Coefficients

Initial Condition

FINITE DIFFERENCE METHOD

Finite Differences



$$x \to [x_1 \quad x_2 \ldots \quad x_n \ldots \quad x_N]$$ x-grid

$$y(x) \to [y_1 \quad y_2 \ldots \quad y_n \ldots \quad y_N]$$

$$n = 1,2,\ldots,N$$

$$x \in [a, b]$$

$$h \equiv \Delta x$$

$$\boxed{\frac{d^2y}{dx^2} \approx \frac{y_{n+1} - 2y_n + y_{n-1}}{(\Delta x)^2}}$$

Finite difference Derivative

Discretization the Eigenvalue problem

Continuous
Eigenvalue problem

$$\boxed{\frac{d^2u}{dx^2} + V(x)u = \beta u}$$

$$x \to [x_1 \quad x_2 \ldots \quad x_n \ldots \quad x_N]$$ x-grid

$$u(x) \to [u_1 \quad u_2 \ldots \quad u_n \ldots \quad u_N]$$

$$V(x) \to [V_1 \quad V_2 \ldots \quad V_n \ldots \quad V_N]$$

$$\frac{d^2u}{dx^2} \approx \frac{u_{n+1} - 2u_n + u_{n-1}}{(\Delta x)^2}$$

$$n = 1,2,\ldots,N \qquad x \in [a, b]$$

$$\Rightarrow$$

$$\frac{u_{n+1} - 2u_n + u_{n-1}}{(\Delta x)^2} + V_n u_n = \beta u_n \Rightarrow \frac{u_{n+1} + u_{n-1}}{(\Delta x)^2} + [V_n - \frac{2}{(\Delta x)^2}]u_n = \beta u_n \Rightarrow$$

Discrete
Eigenvalue problem

$$\boxed{B u_{n+1} + C_n u_n + B u_{n-1} = \beta u_n}$$

$$C_n = V_n - 2(\Delta x)^{-2}$$
$$B = (\Delta x)^{-2}$$

Discretization the Eigenvalue problem

$$\frac{d^2u_n}{dx^2} + V(x)u_n = \beta_n u_n$$

Continuous Eigenvalue problem

Finite difference Discretization

$$\boxed{\overrightarrow{M} \cdot \overrightarrow{u}_n = \beta_n \overrightarrow{u}_n}$$

$$\boxed{\begin{bmatrix} C_1 & B & 0 \dots & 0 \\ B & C_2 & B \dots & 0 \\ & & & \\ 0 & 0 \dots & B & C_N \end{bmatrix} \begin{bmatrix} u_{n1} \\ u_{n2} \\ \vdots \\ u_{nN} \end{bmatrix} = \beta_n \begin{bmatrix} u_{n1} \\ u_{n2} \\ \vdots \\ u_{nN} \end{bmatrix}}$$

Eigenvalue

$$\overrightarrow{u}_n = |u_n> = \begin{bmatrix} u_{n1} \\ u_{n2} \\ \vdots \\ u_{nN} \end{bmatrix}$$

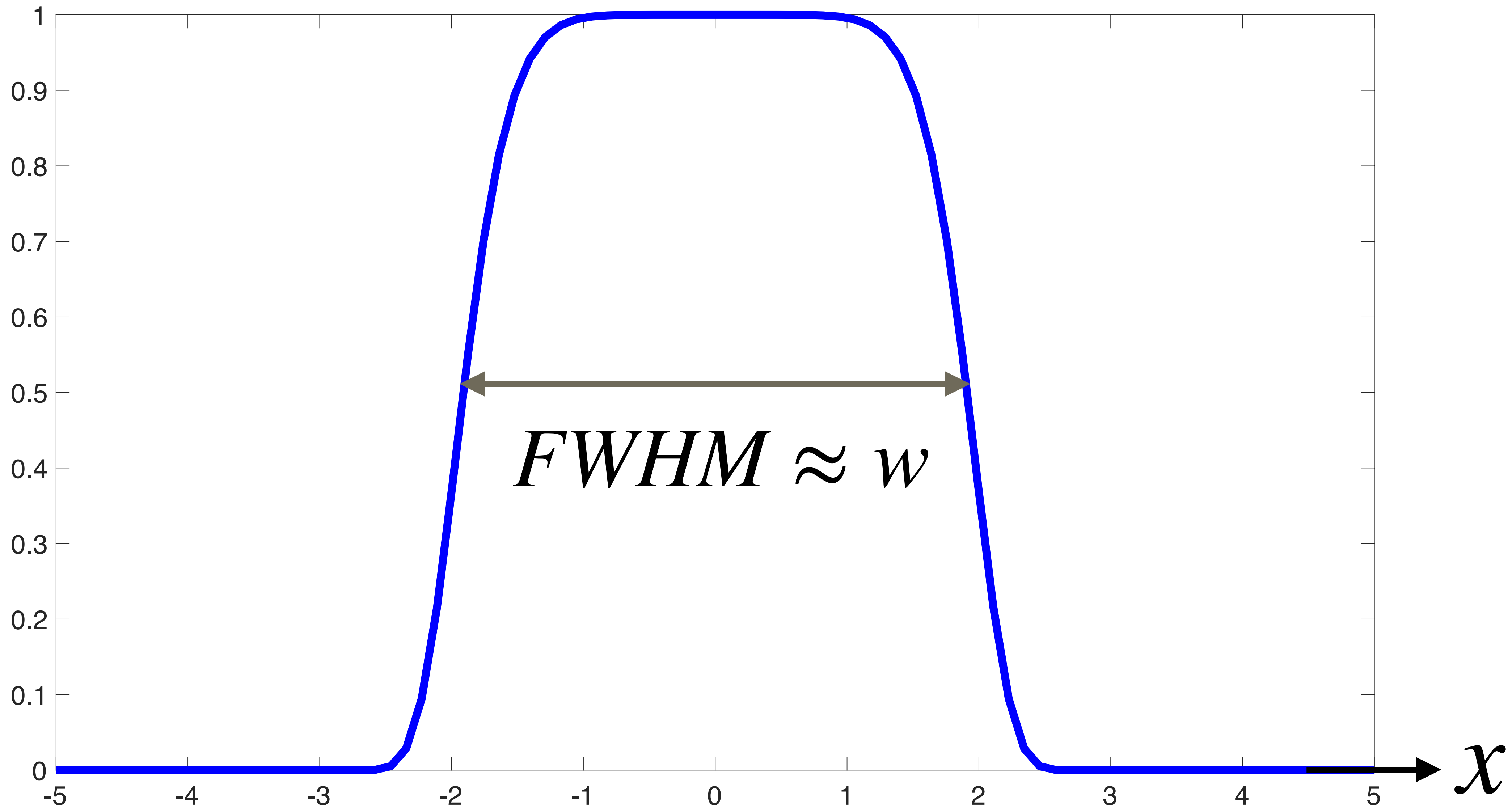
Eigenmode

Slab 1D–waveguide

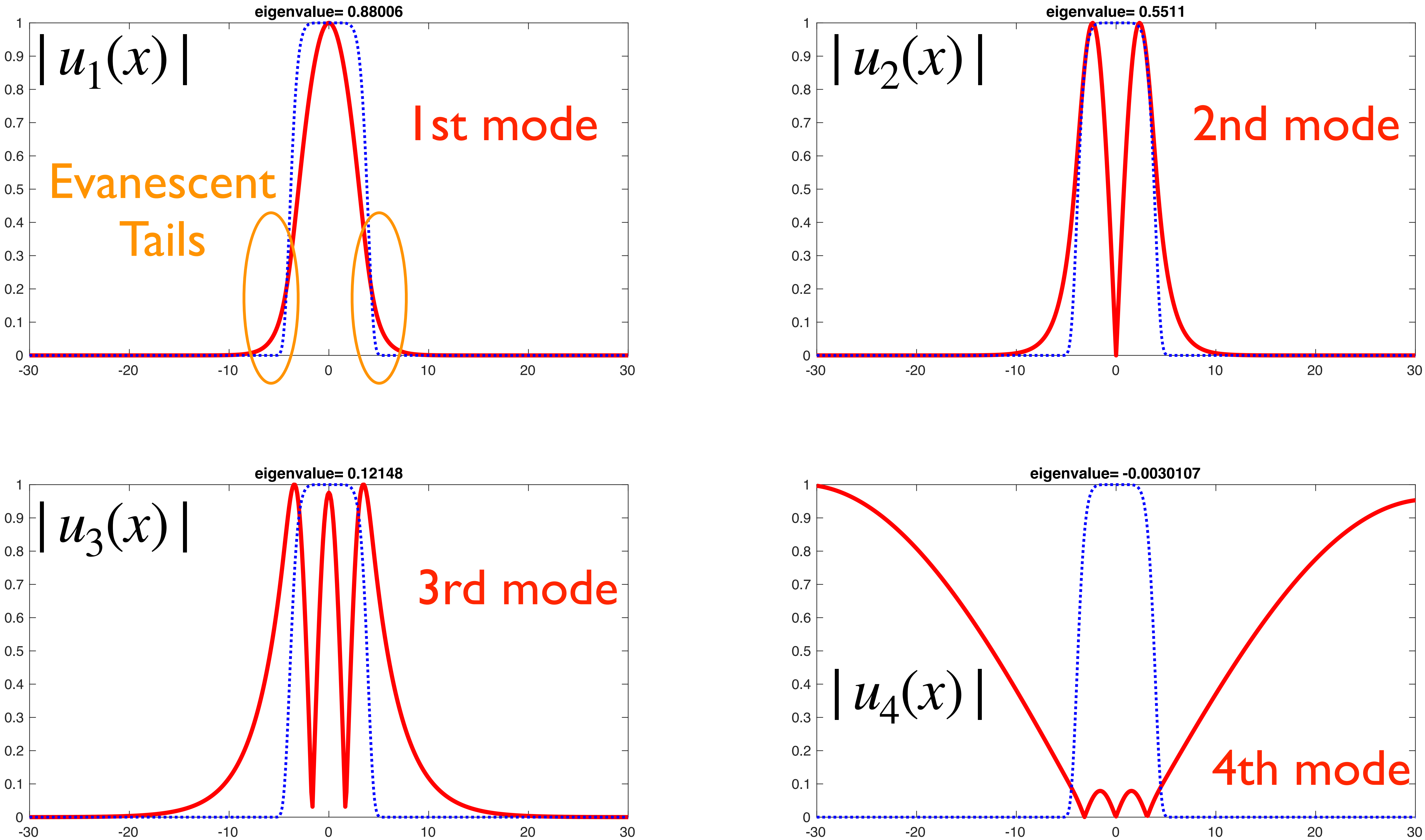
$$\boxed{V(x) = V_0 e^{-(x/w)^8}}$$

Supergaussian profile

Smooth function ideal for fft calculations based on BPM



Eigenmodes of a slab waveguide



The particular waveguide supports only 3 guided modes

SPLIT STEP FOURIER METHOD

Beam Propagation Method

$$x_n = n \cdot \Delta x \to (L/N)[0,\ldots,N-1] \qquad (n,m = 0,1,\ldots,N-1)$$

$$k_m = 2\pi \cdot p_m = 2\pi \cdot m \cdot \Delta p \to (2\pi / L)[0,\ldots,N/2-1,-N/2,\ldots,-1]$$

x-grid

k-grid

$$A_1\left(n\Delta x, z_0 + \frac{\Delta z}{2}\right) = IFFT\left\{FFT\left[A\left(n\Delta x, z_0\right)\right]\exp\left(-ik_m^2 \frac{\Delta z}{2}\right)\right\}$$

Half step diffraction

$$A_2\left(n\Delta x, z_0 + \frac{\Delta z}{2}\right) = \exp\left[i\Delta z \cdot V\left(n\Delta x\right)\right] A_1\left(n\Delta x, z_0 + \frac{\Delta z}{2}\right)$$

Full step Phase screen

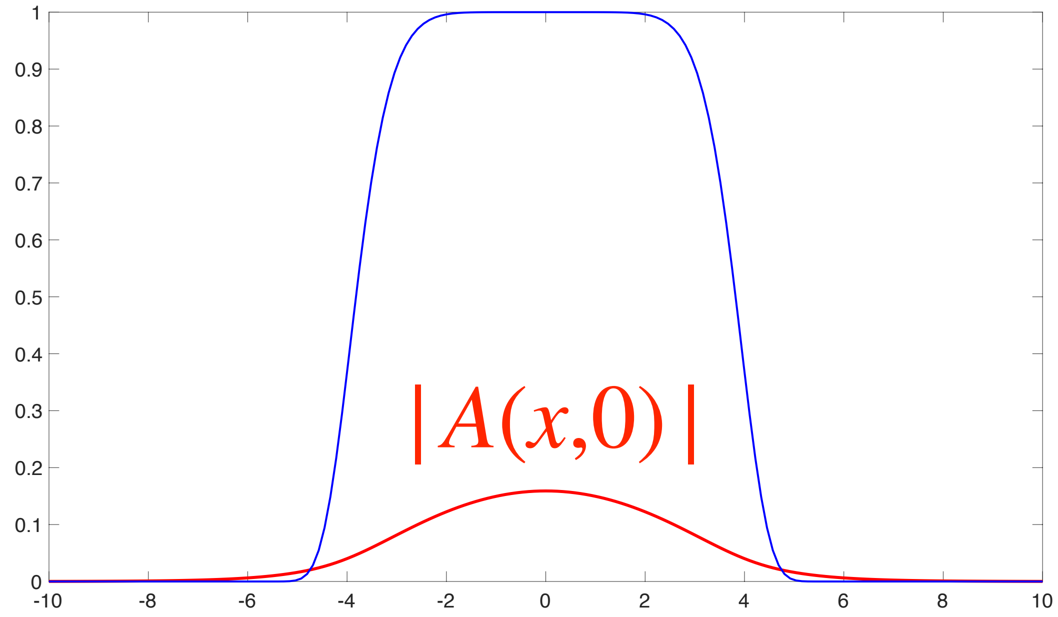
$$A_3\left(n\Delta x, z_0 + \Delta z\right) = IFFT\left\{FFT\left[A_2\left(n\Delta x, z_0 + \frac{\Delta z}{2}\right)\right]\exp\left(-ik_m^2 \frac{\Delta z}{2}\right)\right\}$$

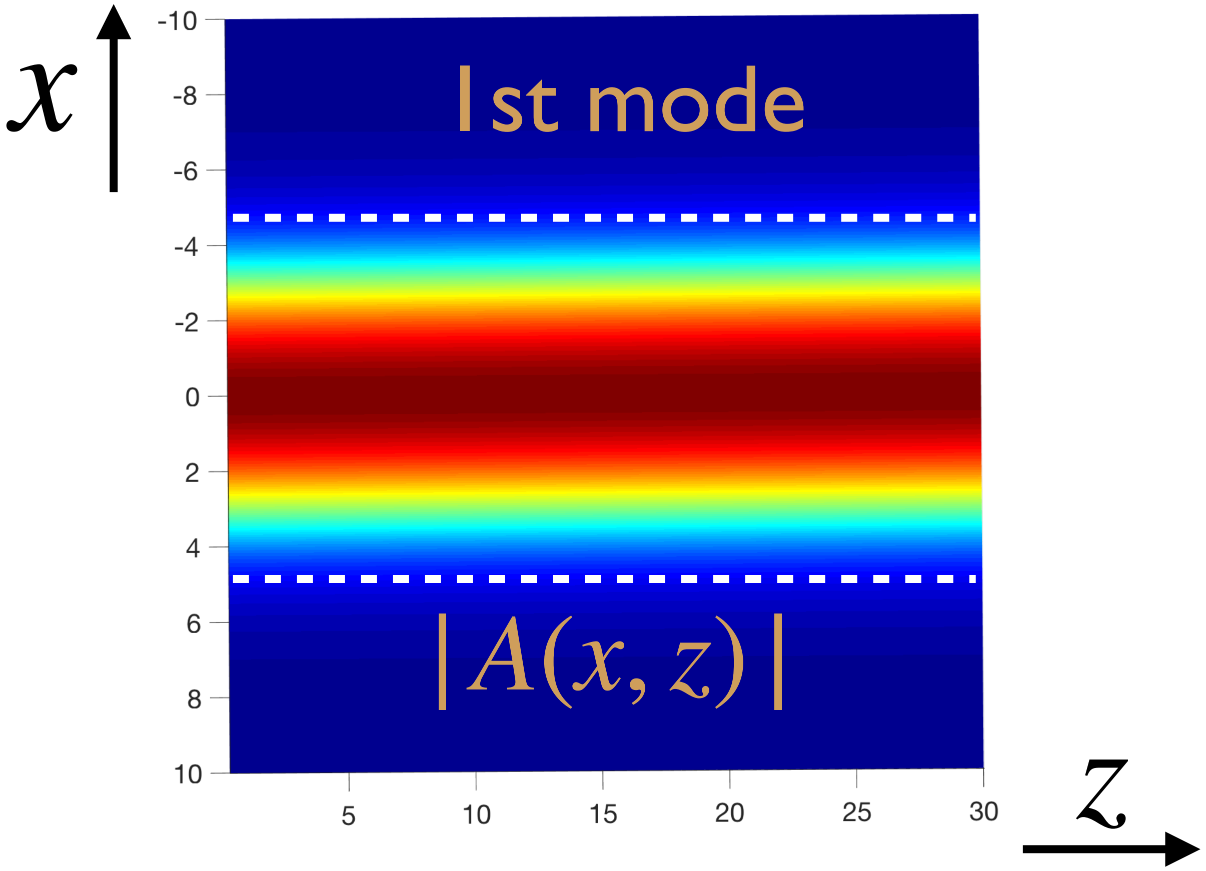
Half step diffraction

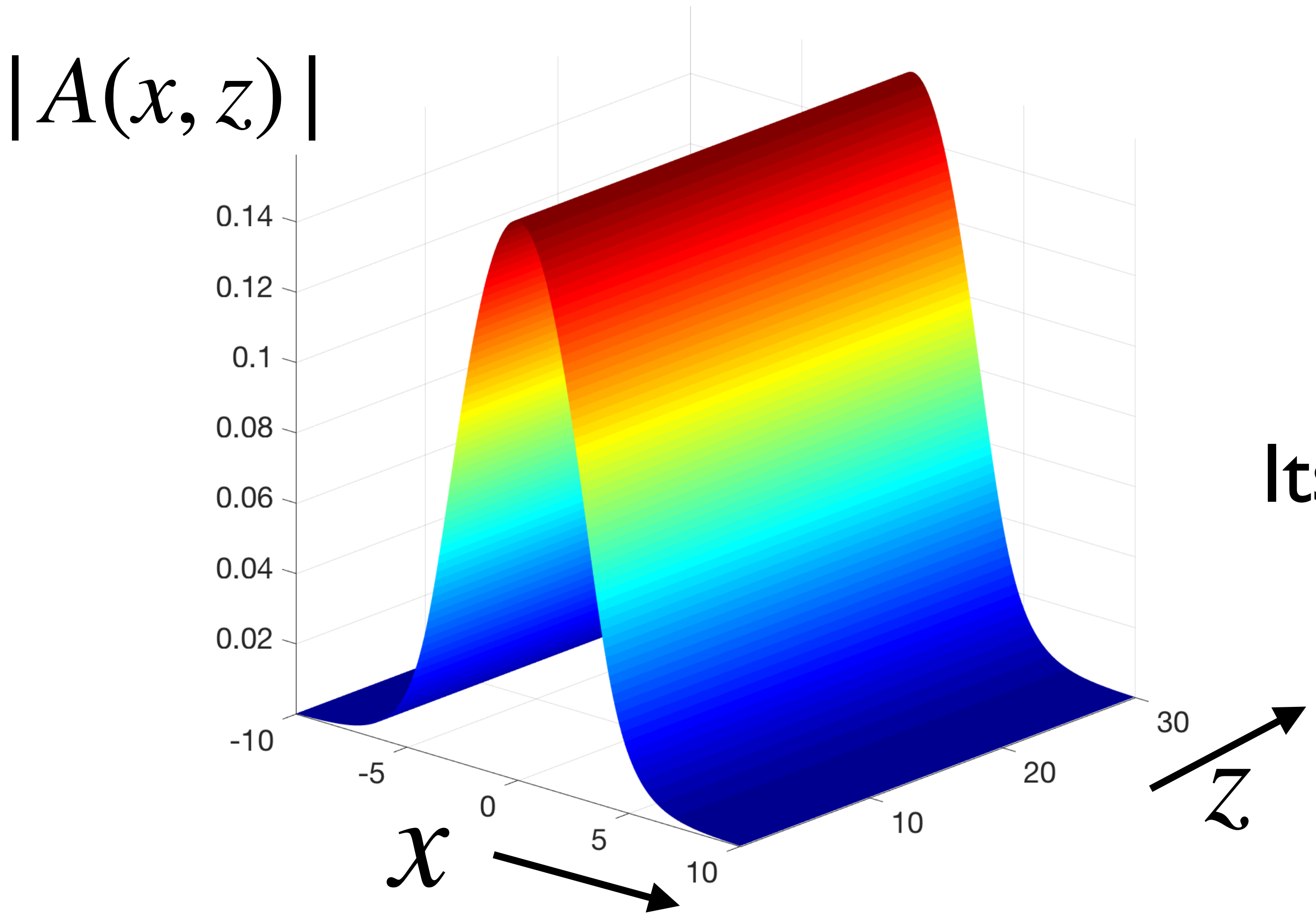
Symmetrized BPM for paraxial waveguides

Fundamental mode – Ground State

Excitation condition

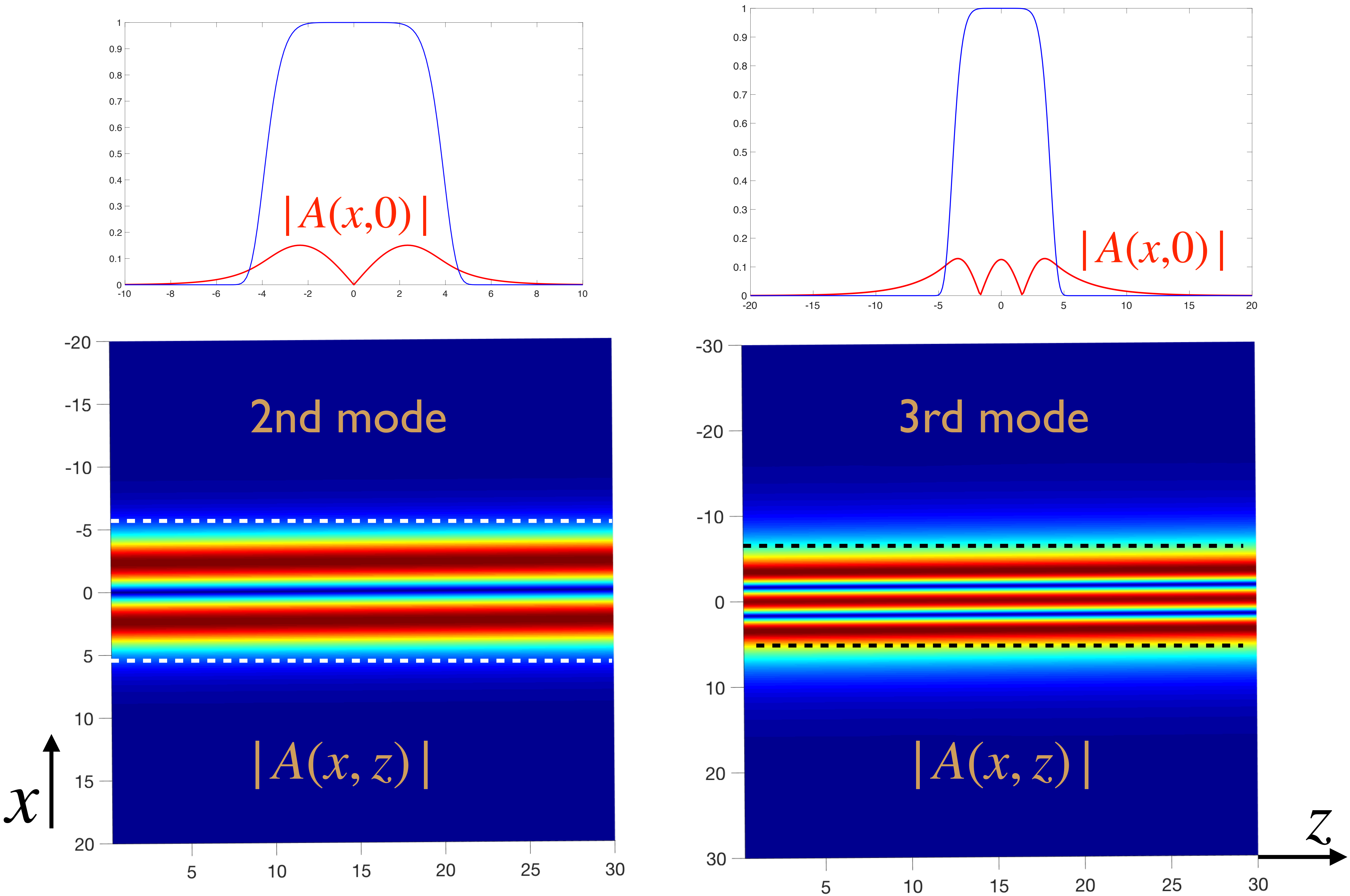






Indeed is an eigenmode
Its intensity remains invariant
Upon propagation

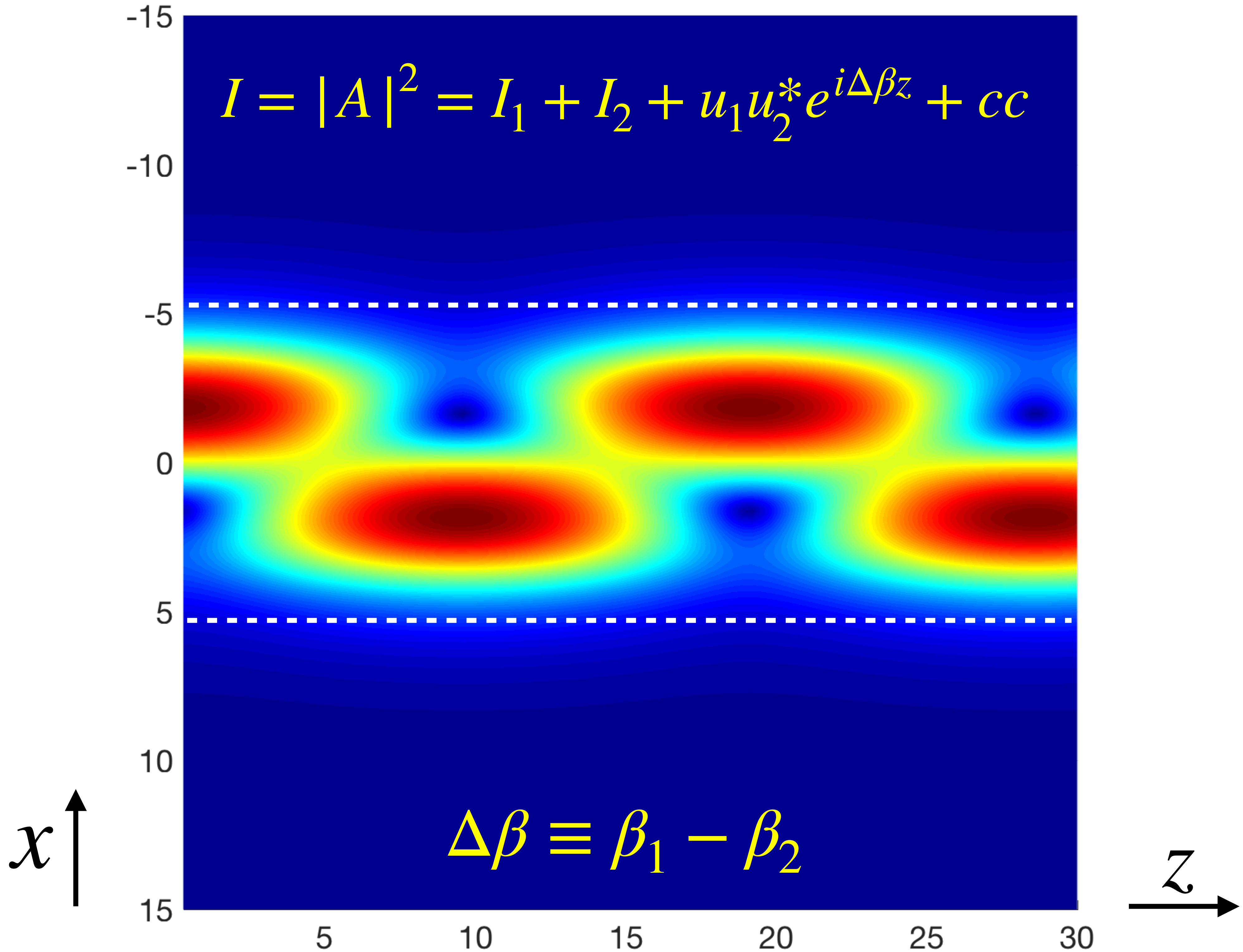
Excitation of higher order modes



Modal Interference

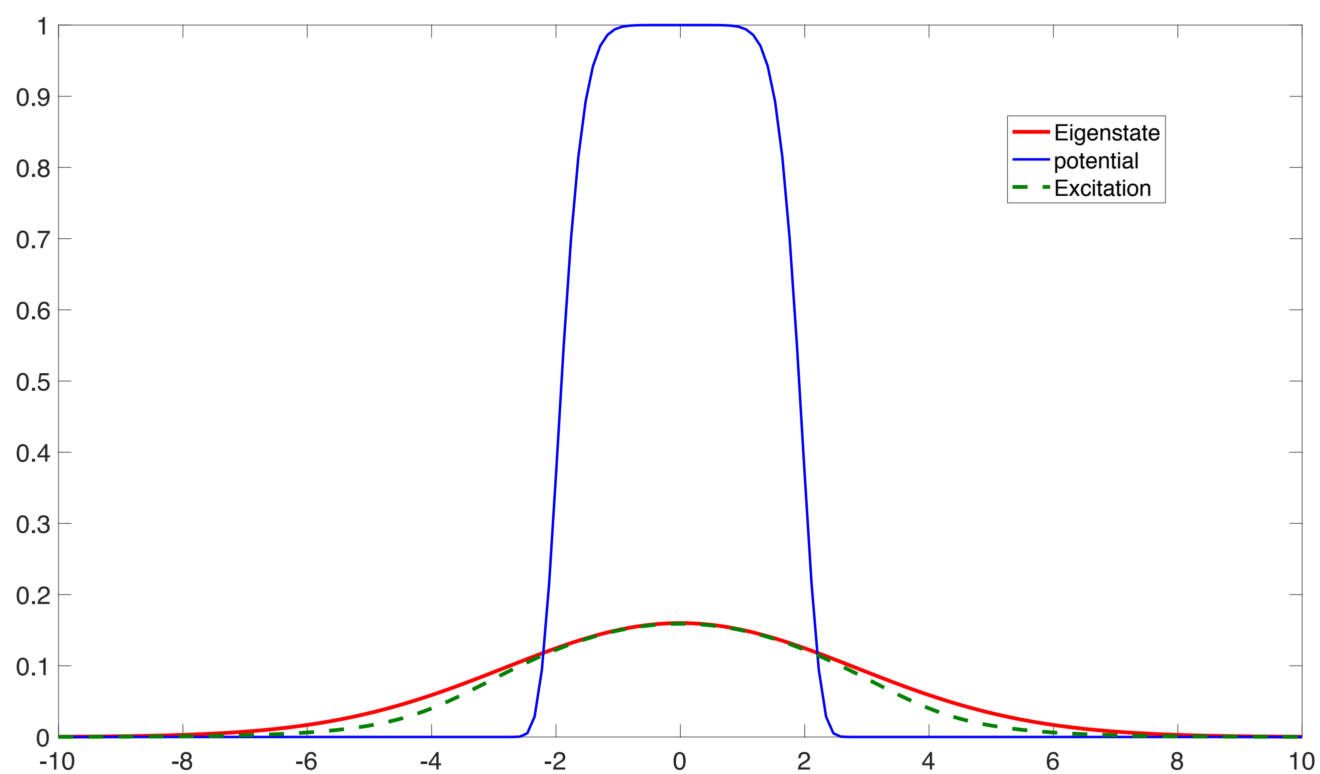
$$\boxed{A(x,y,z) = u_1(x,y)e^{i\beta_1 z} + u_2(x,y)e^{i\beta_2 z}}$$

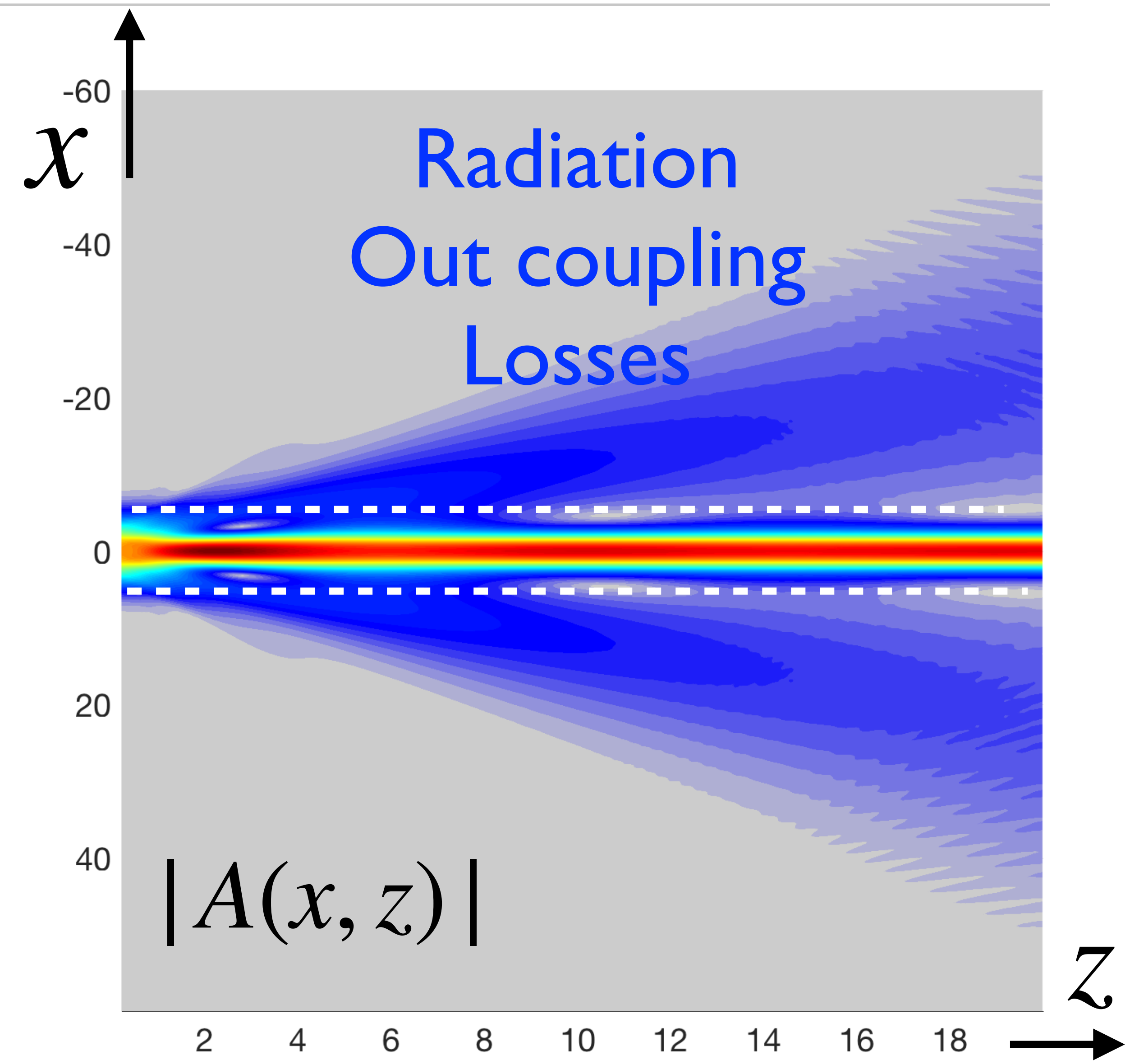
Superposition of 2 modes

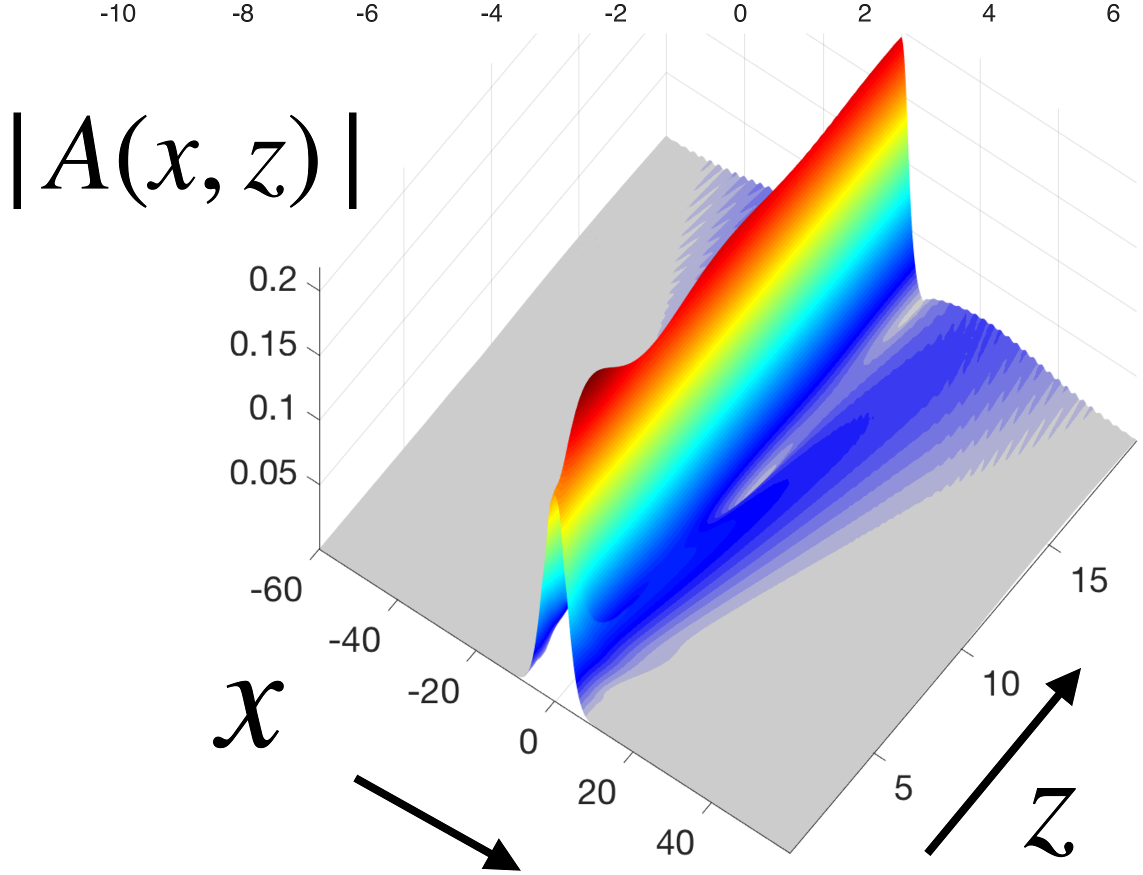


Coupling on a waveguide

Excitation condition







We lost some energy on the coupling but we have mostly excited the first order mode