Computational Optical Imaging

Lecture 4

4F imaging system

oultine

- Imaging with a single lens
- Resolution $\Delta x = \frac{\lambda}{2\,NA}$
- Exercise 1
- 4F system
- Intensity detection
- Dark field imaging
- Exercise 2

3 ways to simulate free space propagation
BPM, ASM, Fresnel integral

BPM

$$\frac{\partial A}{\partial z} \simeq -\frac{j}{2k}\left(\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}\right)$$

FT

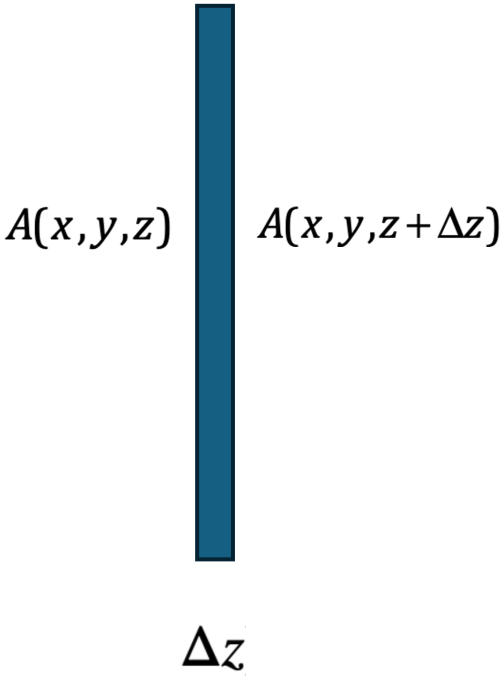
Angular Spectrum

$$\tilde{A}(k_x,k_y,z) = e^{j\left[\frac{k_x^2+k_y^2}{2k}\right]z}\tilde{A}(k_x,k_y,z=0)$$

FT

$$E_x\left(x',y',z,t\right) = A(x',y',z)e^{j\omega t}e^{-jkz} = \frac{1}{j\lambda z}e^{j\omega t}e^{-jkz}\iint A(x,y,z=0)\,e^{-\frac{j\pi[(x-x')^2+(y-y')^2}{\lambda z}}dxdy$$

Fresnel Diffraction



Thin transparency

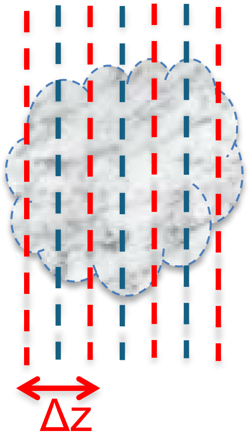
$$t(x,y)=\frac{A(x,y,z+\Delta z)}{A(x,y,z)}=e^{-\frac{j}{2k}\omega^2\mu\Delta\varepsilon(x,y,z)\Delta z}$$

Real part of $\Delta\varepsilon$ is index; imaginary part of $\Delta\varepsilon$ is absorption

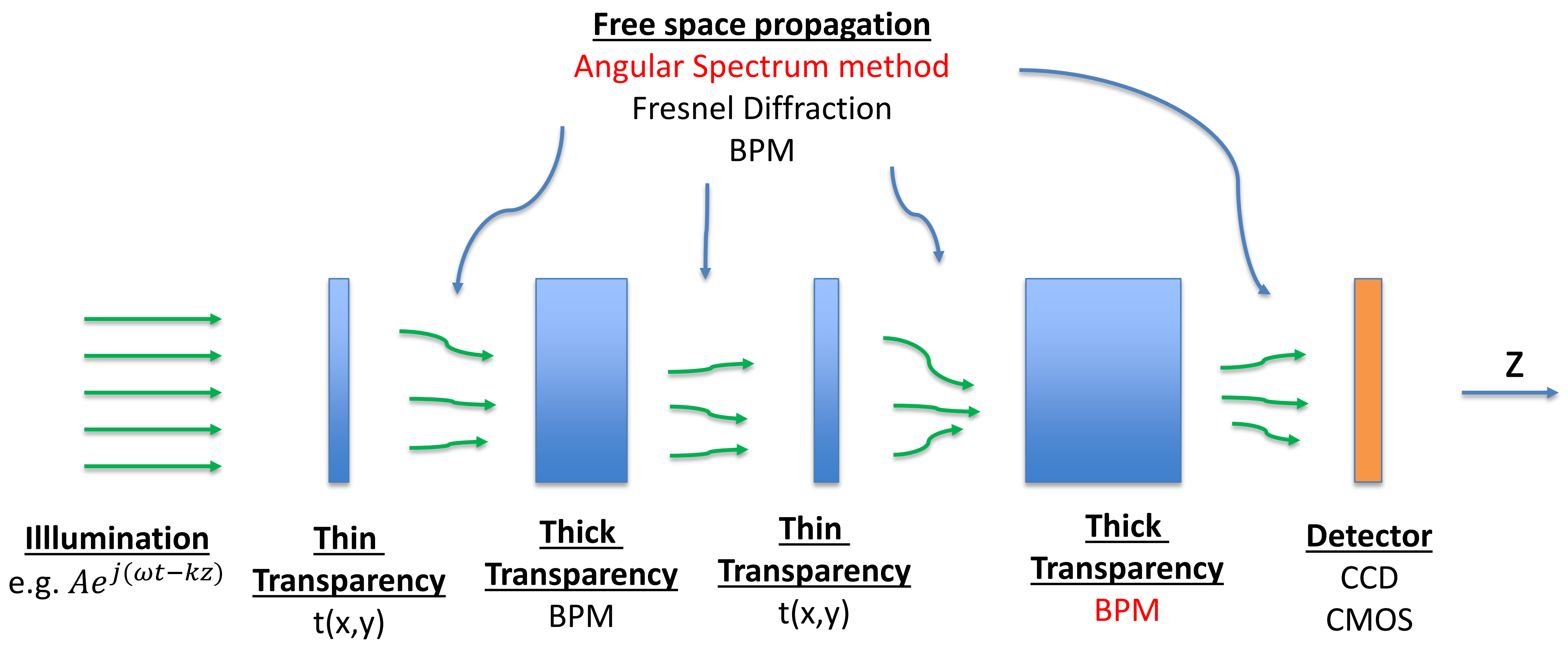
Thick transparencies or objects

$$\frac{\partial A}{\partial z}=-\frac{j}{2k}\left(\frac{\partial A^2}{\partial x^2}+\frac{\partial A^2}{\partial y^2}\right)-\frac{j}{2k}\omega^2\mu\Delta\epsilon(x,y,z)A$$

$$A(z+\Delta z)\approx A(z)+\left[-\frac{j}{2k}\left(\frac{\partial A^2}{\partial x^2}+\frac{\partial A^2}{\partial y^2}\right)-\frac{j}{2k}\omega^2\mu\Delta\epsilon(x,y,z)A\right]\Delta z$$

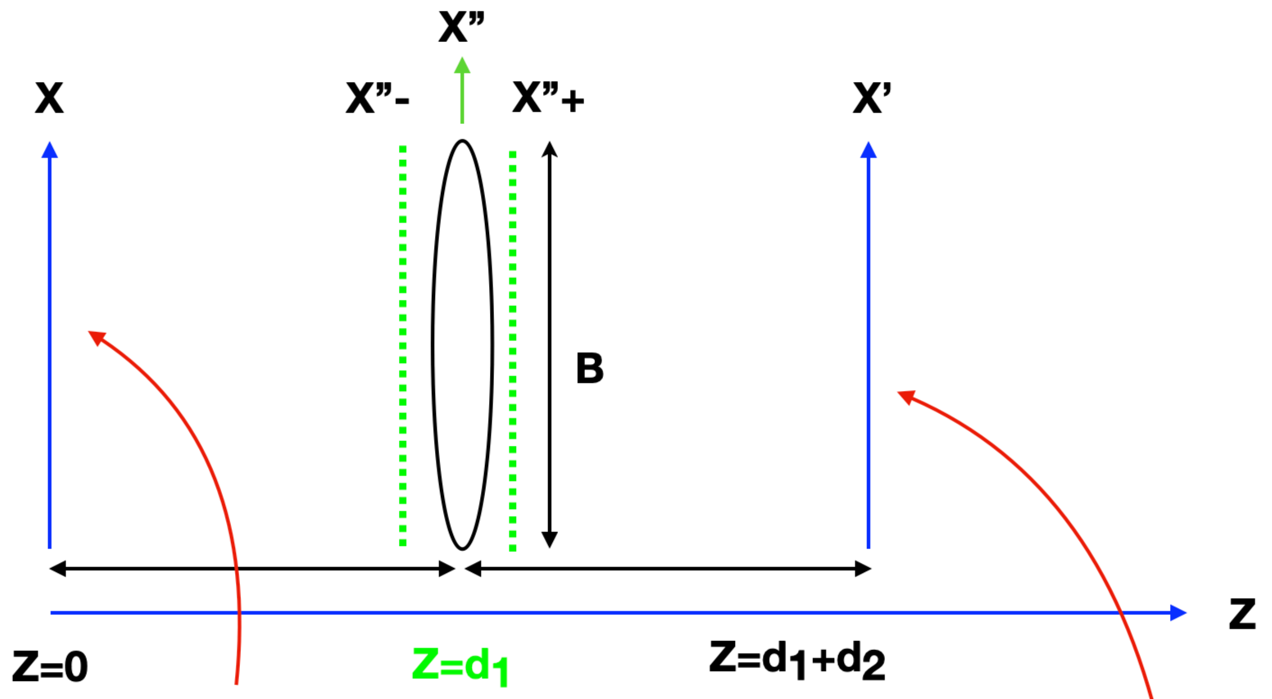


Toolkit



Free space propagation
Angular Spectrum method
Fresnel Diffraction
BPM
Z
Illlumination
e.g. $Ae^{j(\omega t-kz)}$
Thin Transparency
t(x,y)
Thick Transparency
BPM
Thin Transparency
t(x,y)
Thick Transparency
BPM
Detector
CCD
CMOS

Imaging lens



$$E(x,y,z=0,t) = A_0 f(x) e^{j(\omega t - kz)} \quad or \quad f(x)$$

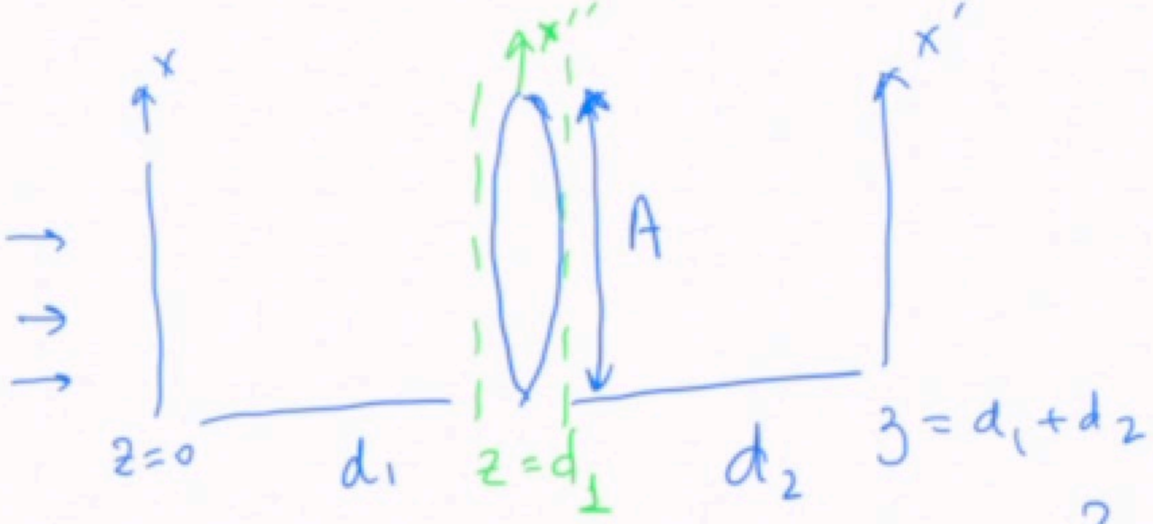
$$g(x') = ?$$

At x"-: $\int f(x) e^{-j\frac{\pi(x-x'')^2}{\lambda d_1}} dx$

$$\downarrow$$

At x"+: $h(z'') = (\quad) e^{+j\frac{\pi}{\lambda F}x''^2} rect(\frac{x''}{B})$

At x'+: $g(x') \approx e^{-j\pi x'^2/\lambda d_2}\ B \int f(x) sinc(\frac{B(x+x'/M)}{\lambda d_1}) dx \ where \ M = \frac{d_2}{d_1}|$



$f(x)$ $\qquad$ $g(x') = ?$

$$x''_- : \quad \underbrace{\int f(x)\, e^{-j\frac{\pi}{\lambda d_1}(x-x'')^2}\, dx}$$

Fresnel diffraction

$$x''_+ : \quad (\ \ )\, \underbrace{e^{+j\frac{\pi}{\lambda F}x''^2}\, \mathrm{rect}\left(\frac{x''}{A}\right)}_{\text{lens transmittance}} = h(x'')$$

$$x' : \quad g(x') = \int h(x'')\, e^{-j\frac{\pi}{\lambda d_2}(x''-x')^2}\, dx''$$

$$x' : \quad g(x') = \int h(x'')\, e^{-j\frac{\pi}{\lambda d_2}(x''-x')^2}\, dx''$$

$$= e^{-j\frac{\pi}{\lambda d_2}x'^2} \int f(x)\, e^{-j\frac{\pi}{\lambda d_1}x^2}\, (\Psi)\, dx$$

$$\Psi = \int_{-A/2}^{A/2} e^{-j\frac{\pi}{\lambda}\cancel{\left(\frac{x''^2}{d_1} + \frac{x''^2}{d_2} - \frac{x''^2}{F}\right)}}\, e^{+j\frac{2\pi}{\lambda d_1}\left(x x'' + x' x''/M\right)}\, dx''$$

Imaging condition: $\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{F}$

Magnification: $M = \frac{d_2}{d_1}$

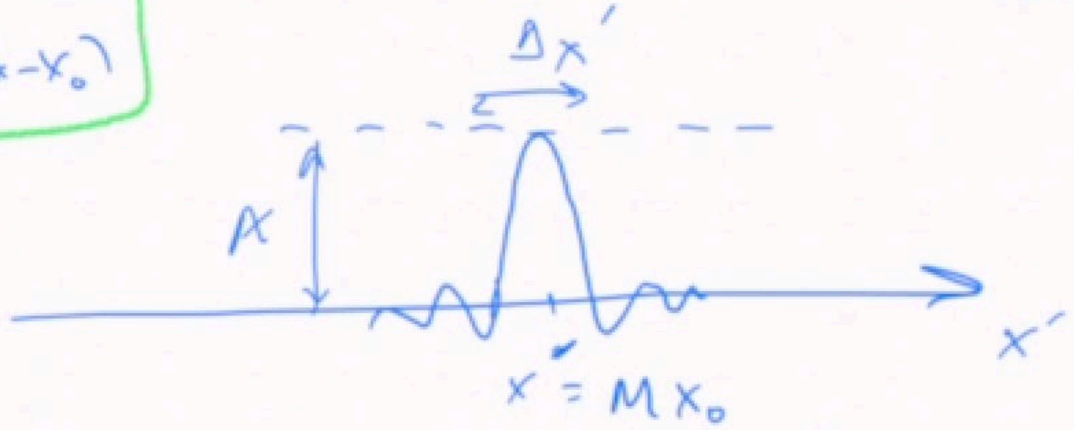
Reminder: FT of $\mathrm{rect}\left(\frac{x''}{A}\right)$ is $A\,\mathrm{sinc}(Au)$

$$\psi = \int_{-\infty}^{\infty} \mathrm{rect}\left(\frac{x''}{A}\right) e^{+j2\pi \overbrace{\frac{(x + x'/M)}{\lambda d_1}}^{=u} x''} \, dx''$$

$$= A \, \mathrm{sinc}\left[\frac{A(x + x'/M)}{\lambda d_1}\right]$$

POINT SPREAD FUNCTION PSF

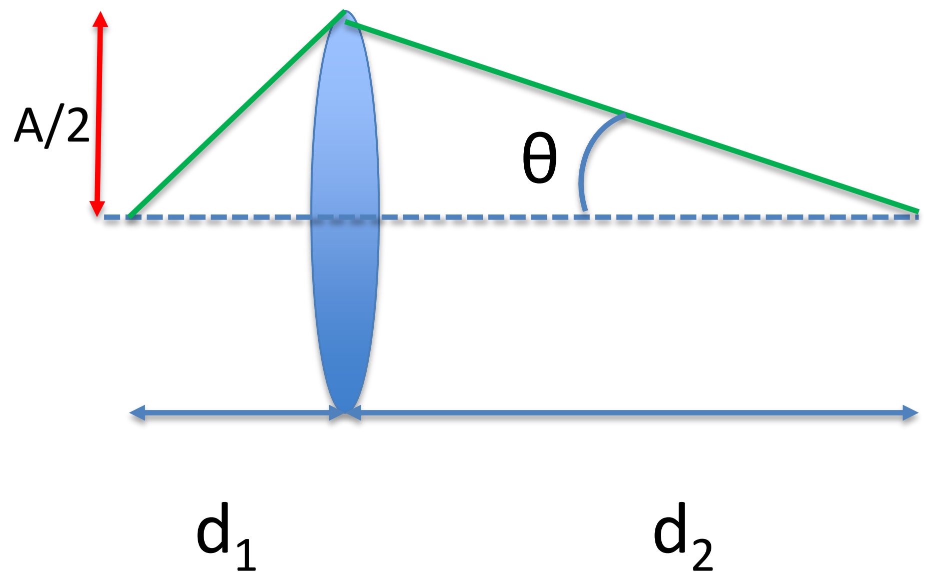
$$\boxed{f(x) = \delta(x - x_0)}$$



First zero at $\sin \pi \frac{A x'}{M \lambda d_1} = 0$

$$\Rightarrow \quad \Delta x' = \frac{\lambda d_1 M}{A} \quad \text{or} \quad \Delta x = \frac{\lambda d_1}{A}$$

Resolution



$$\boxed{\Delta x' = \frac{\lambda}{2\ NA}}\quad NA \quad is\ the\ numerial\ aperture$$

$$NA = nsin\theta \quad where \quad sin\theta \approx \frac{A}{2d_2}$$

$$\Delta x' = M\frac{\lambda d_1}{A} = \frac{\lambda d_2}{A} = \frac{\lambda}{2NA} \quad where\ the\ magnificaton \quad M = \frac{d_2}{d_1}$$

$$n = \frac{c}{v} \quad is \quad the\ index\ of\ refraction$$

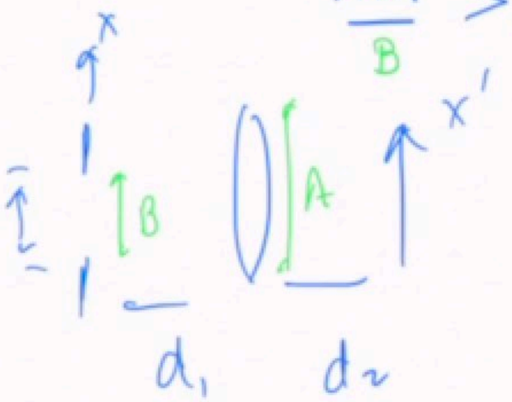
for an arbitrary input object $f(x)$:

$$g(x') = e^{-j\frac{\pi x'^2}{\lambda d_2}} A \int f(x)\, e^{-j\frac{\pi x^2}{\lambda d_1}} \operatorname{sinc}\left[\frac{A\,(x + x'/M)}{\lambda d_1}\right] dx$$

?

$$\frac{\lambda d_1}{B} \gg \frac{\lambda d_1}{A}$$

PARAXIAL APPROXIMATION



$$\Longrightarrow \boxed{g(x') \sim A \int f(x) \operatorname{sinc}\left[\frac{A\,(x + x'/M)}{\lambda d_1}\right] dx}$$

$$\Rightarrow \boxed{g(x') \sim A \int f(x)\, \mathrm{sinc}\left[\frac{A\,(x + x'/m)}{\lambda d_1}\right] dx}$$

Convolution theorem:

$$g(x') = \int f(x)\, h(x' - x)\, dx$$

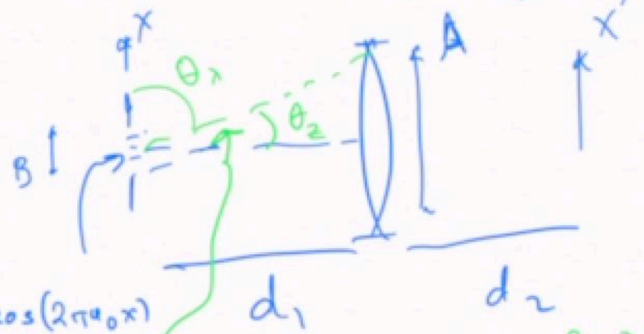
$$\Rightarrow \quad \underbrace{G(u)}_{\text{Angular Spectrum of OBJECT}} = \underbrace{F(u)}_{\text{ANGULAR SPECTRUM OF IMAGE}}\; H(\underbrace{u}_{\text{DUE TO LENS APERTURE}})$$

because

$$H(u) = \mathrm{rect}\left(\frac{u \lambda d_1}{A}\right) \qquad \mathrm{sinc}(\alpha x) \leftrightarrow \frac{1}{\alpha}\,\mathrm{rect}\left(\frac{u}{\alpha}\right)$$

because

$$H(u) = \mathrm{rect}\left(\frac{u \lambda d_1}{A}\right) \qquad \mathrm{Sinc}(\alpha x) \leftrightarrow \frac{1}{\alpha}\,\mathrm{rect}\left(\frac{u}{\alpha}\right)$$



$$\boxed{A \gg B}$$

$$\sin\theta \approx \frac{A}{2 d_1}$$

$f(x) = \cos(2\pi u_0 x)$

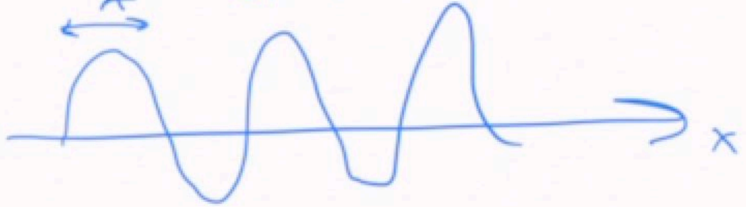
Plane wave $\sim e^{-j(k_x x + k_z z)}$

$$2\pi u_0 = k_x = \frac{2\pi}{\lambda}\cos\theta_x = \frac{2\pi}{\lambda}\sin\theta_z \qquad (\theta_x = 90 - \theta_z)$$

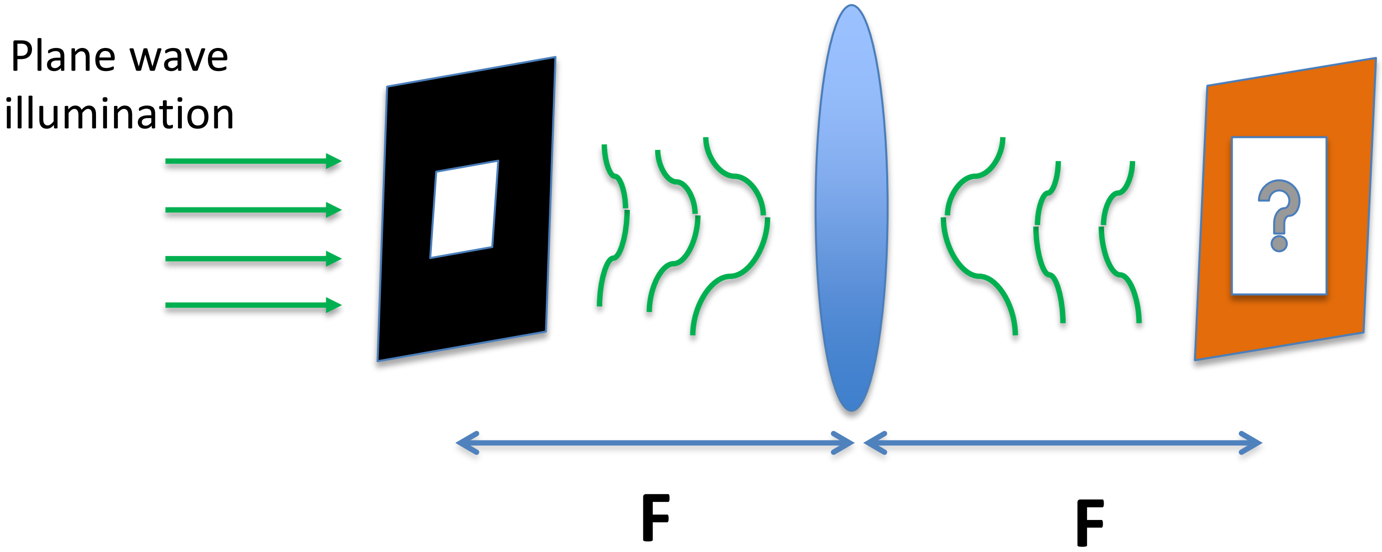
Maximum u_0 that will pass $\Big\}$ $\sin\theta_z \approx \frac{A/2}{d_1} \Rightarrow \frac{2\pi}{\lambda}\sin\theta_z = 2\pi u_0$

cut-off frequency $\boxed{u_0 < \frac{A}{2\lambda d_1}}$

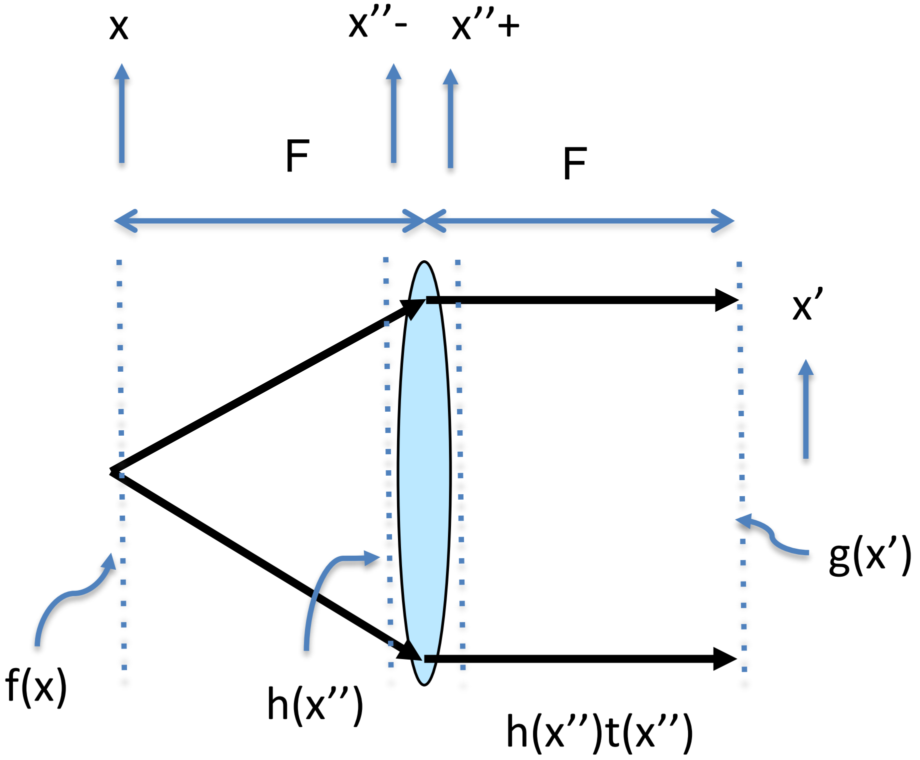
$\frac{\lambda d_1}{A} = \Delta x$! Resolution



Exercise 1



Fourier transforming lens



$$t(x'') = e^{+j\pi x''^2/\lambda F}$$

$$g(x') = \int f(x) e^{+j\pi x x'/\lambda F} dx$$

$$\mathbf{F}(u) = \int f(x) e^{-j\pi u x} dx \quad => \quad g(x') = \mathbf{F}\left(\frac{-x'}{\lambda F}\right)$$

$Step\ 1: \ h(x'') => g(x')$

$$g(x') = \int h(x'') e^{+j\pi x''^2/\lambda F} e^{-j\pi (x'-x'')^2/\lambda F} dx''$$

$$= \int h(x'') e^{+j\pi x''^2 \lambda F} e^{-j\pi x''^2 \lambda F} e^{+j2\pi x' x''/\lambda F} e^{-\pi x'^2/\lambda F} dx''$$

$$= e^{-\pi x'^2/\lambda F} \int h(x'') e^{+j2\pi x' x''/\lambda F} dx'' \quad =. \ e^{-\pi x'^2/\lambda F}\ \mathbf{H}(x'/\lambda F)$$

$Step\ 2: \ f(x) => h(x'') \qquad FT\ of\ f(x)\ is\ \mathbf{F}(u)$

$$ASM\ to\ x'': \quad \mathbf{H}(u) = \mathbf{F}(u) e^{+j\pi u^2 \lambda F} \quad with \quad u = \frac{-x'}{\lambda F}$$

$$f(x) = \mathbf{F}(u) e^{+j\pi u^2 \lambda F} e^{-j\pi u^2 \lambda F} = \mathbf{F}(u)$$

Reminder: Far Field

$$E_x\left(x',y',z,t\right) = A(x',y',z)e^{j\omega t}e^{-jkz}$$

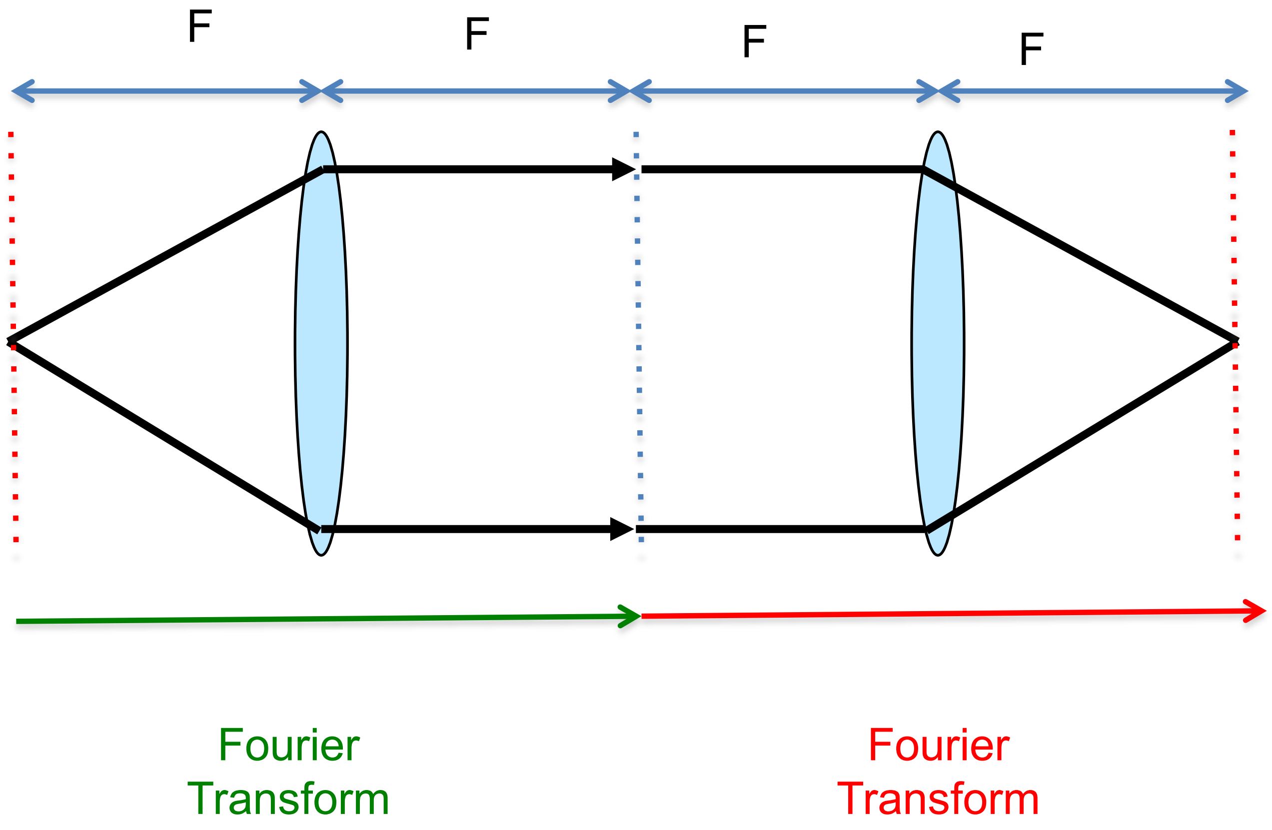
$$= e^{j\omega t}e^{-jkz}\iint A(x,y,z=0)e^{-\frac{j\pi[(x-x')^2+(y-y')^2}{\lambda z}}dxdy$$

$$= e^{j\omega t}e^{-jkz}e^{-\frac{j\pi(x'^2+y')^2}{\lambda z}}\iint A(x,y,z=0)e^{-\frac{j\pi(x^2+y)^2}{\lambda z}}e^{\frac{j2\pi(xx'+yy')}{\lambda z}}dxdy$$

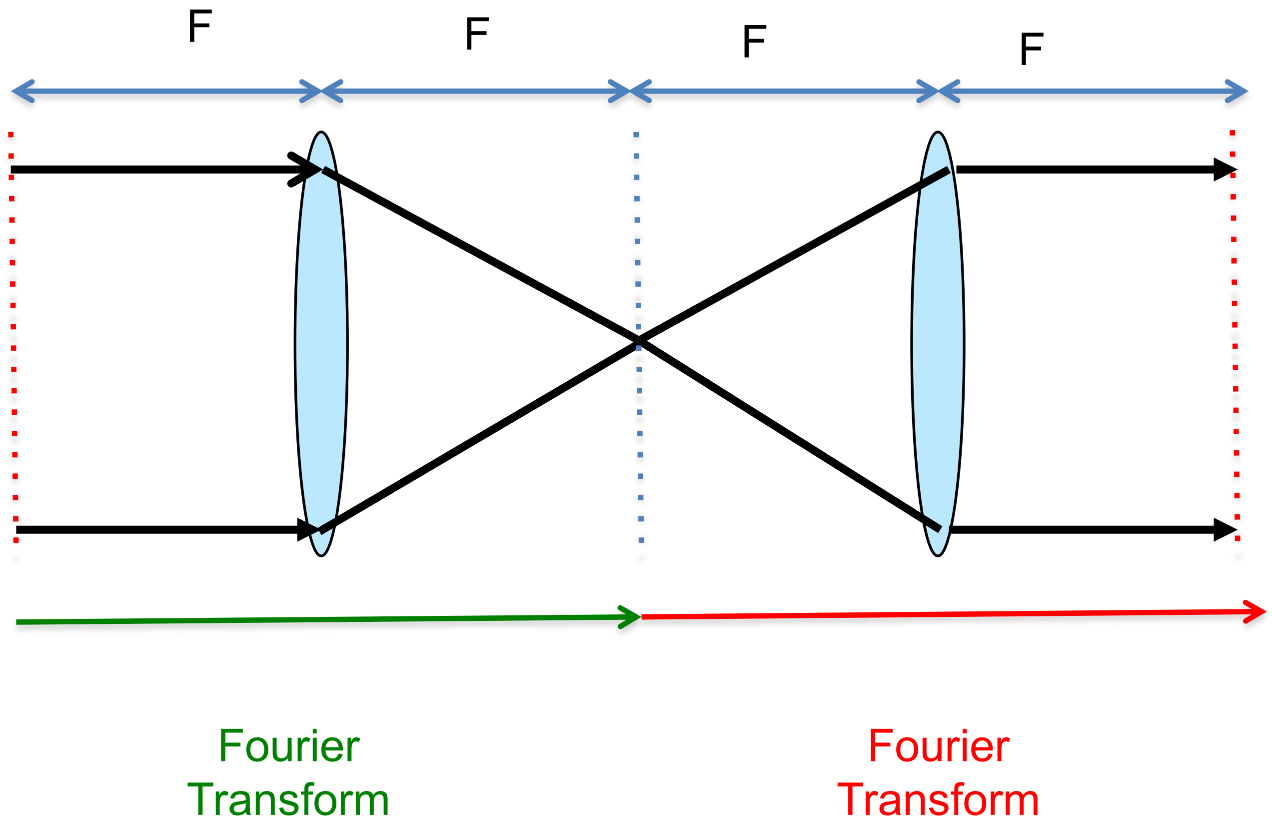
$$\text{if } e^{-\frac{j\pi(x^2+y)^2}{\lambda z}} \approx 1 \text{ then } A(x',y',z) \text{ is the FT of } A(x,y,z=0)$$

$$u = x'/\lambda z \qquad v = y'/\lambda z$$

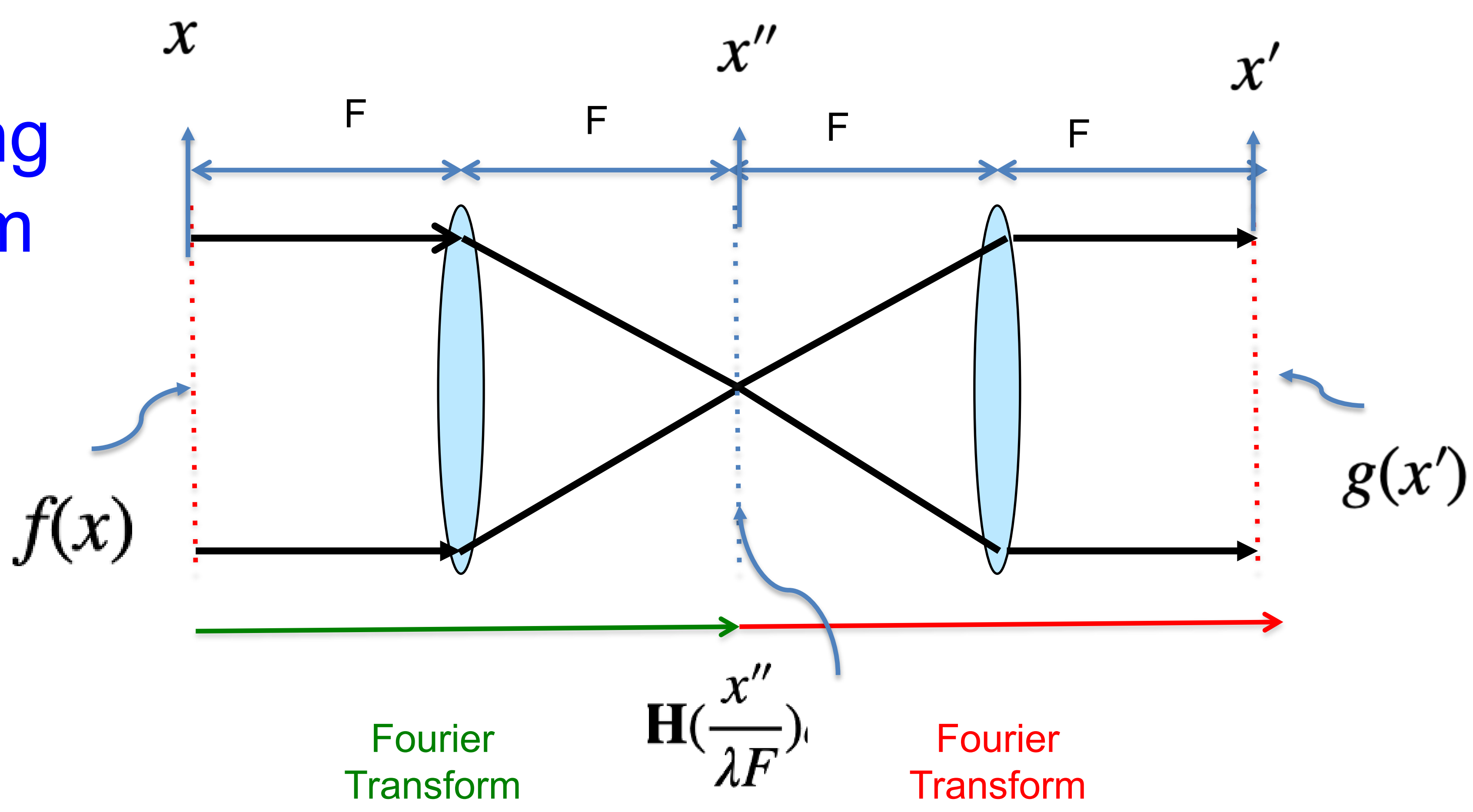
4F Imaging System



4F Imaging System



4F Imaging System

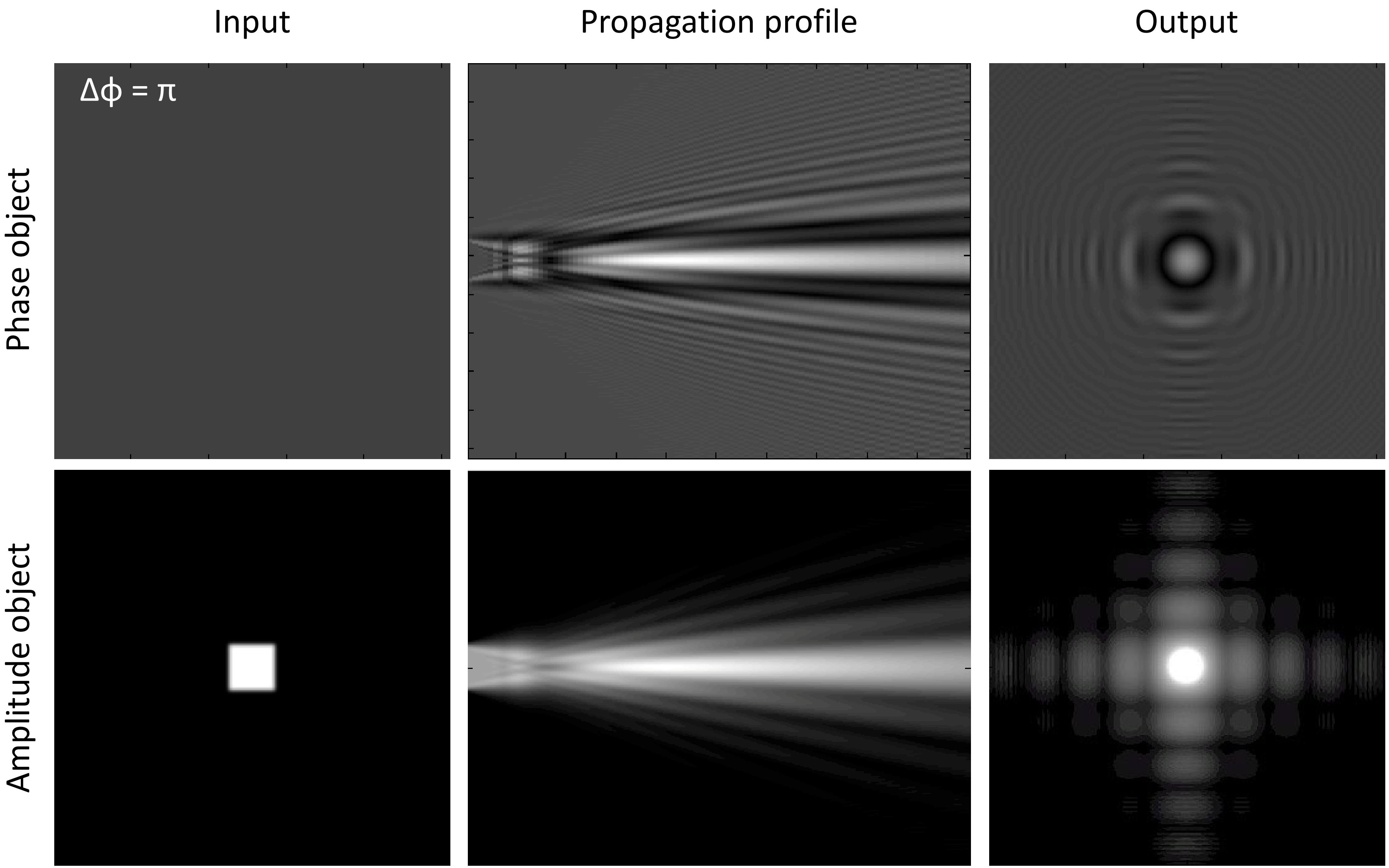


$$g(x') = \int \mathbf{F}(\frac{x''}{\lambda F})\mathbf{H}(\frac{x''}{\lambda F})e^{j2\pi(x''x'/\lambda F)}dx'' = \int f(x)h(x-x')dx$$

Phase objects

Phase vs amplitude object

λ = 800nm



D. Psaltis, *Optical Waves Propagation* - EPFL Photonics program doctoral course, Fall semester 2014

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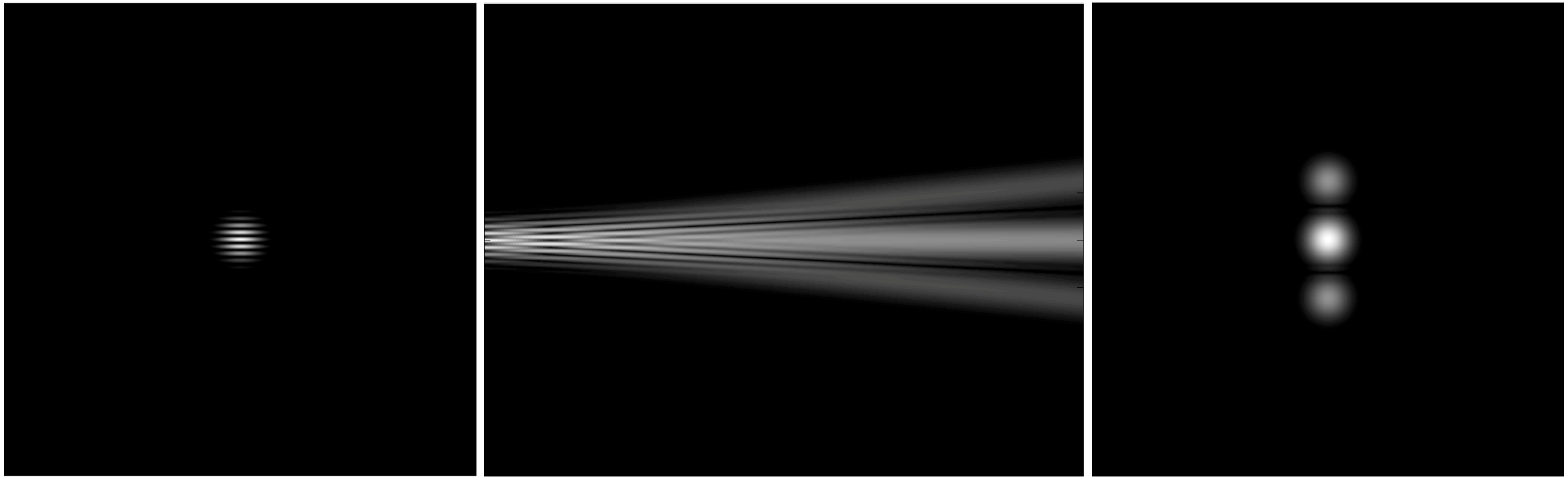
Amplitude Grating

$$A_{\mathrm{out}} = A_{\mathrm{in}} T(x) \qquad\qquad T(x) = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi}{\Lambda}x\right)$$

λ = 800nm

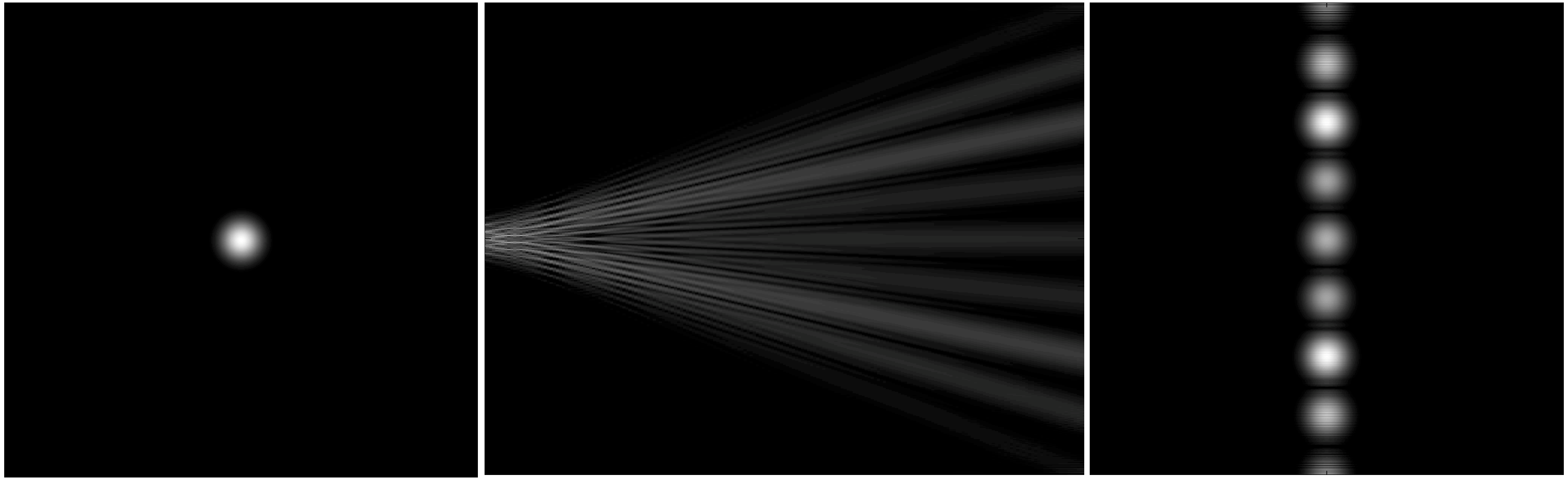


Thin Phase Grating

$$A_{\text{out}} = A_{\text{in}} \exp[j\phi(x)] \qquad \phi(x) = \phi_0 \left[\frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi}{\Lambda}x\right)\right]$$

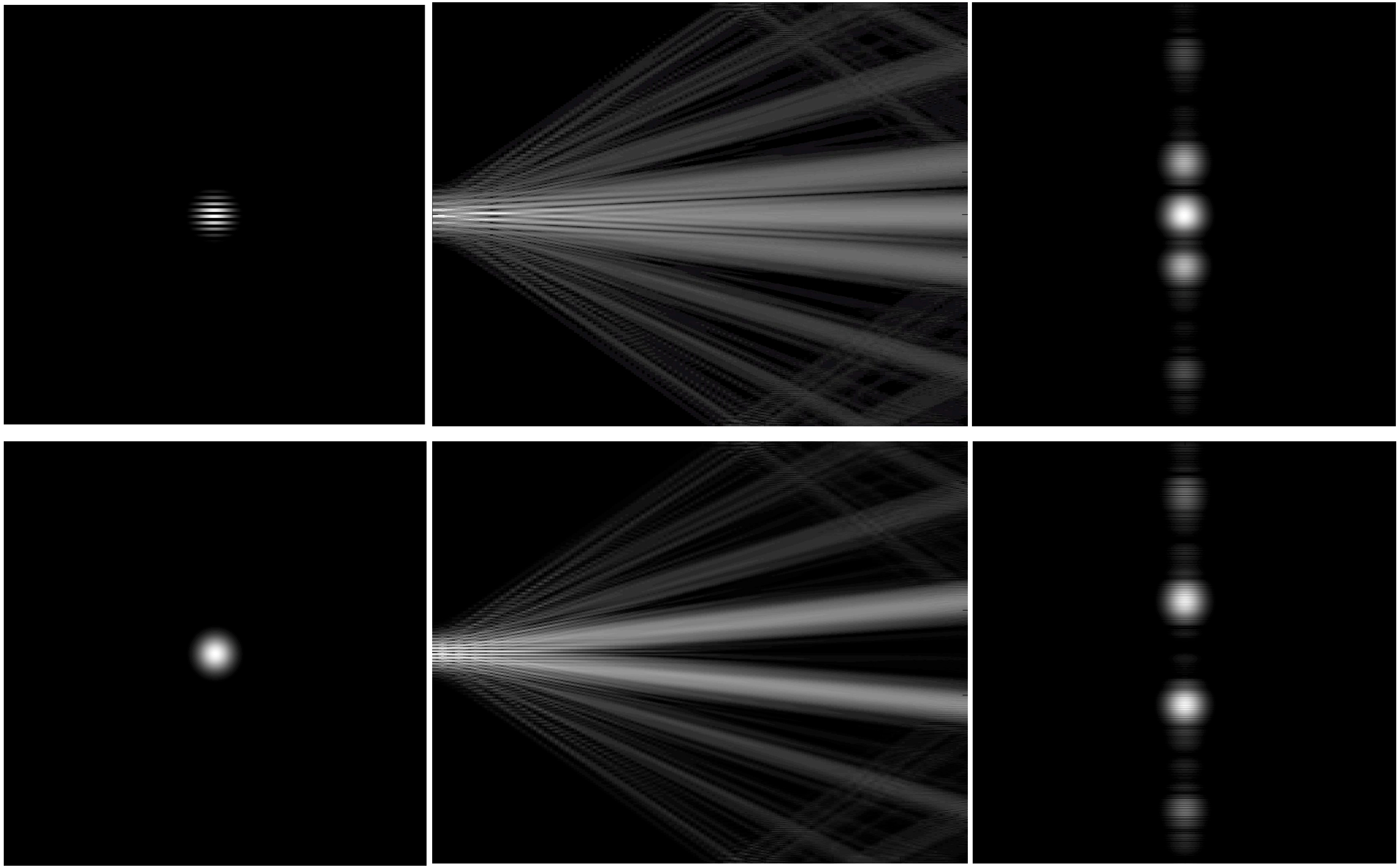
λ = 800nm

$$\phi_0 = 2\pi$$

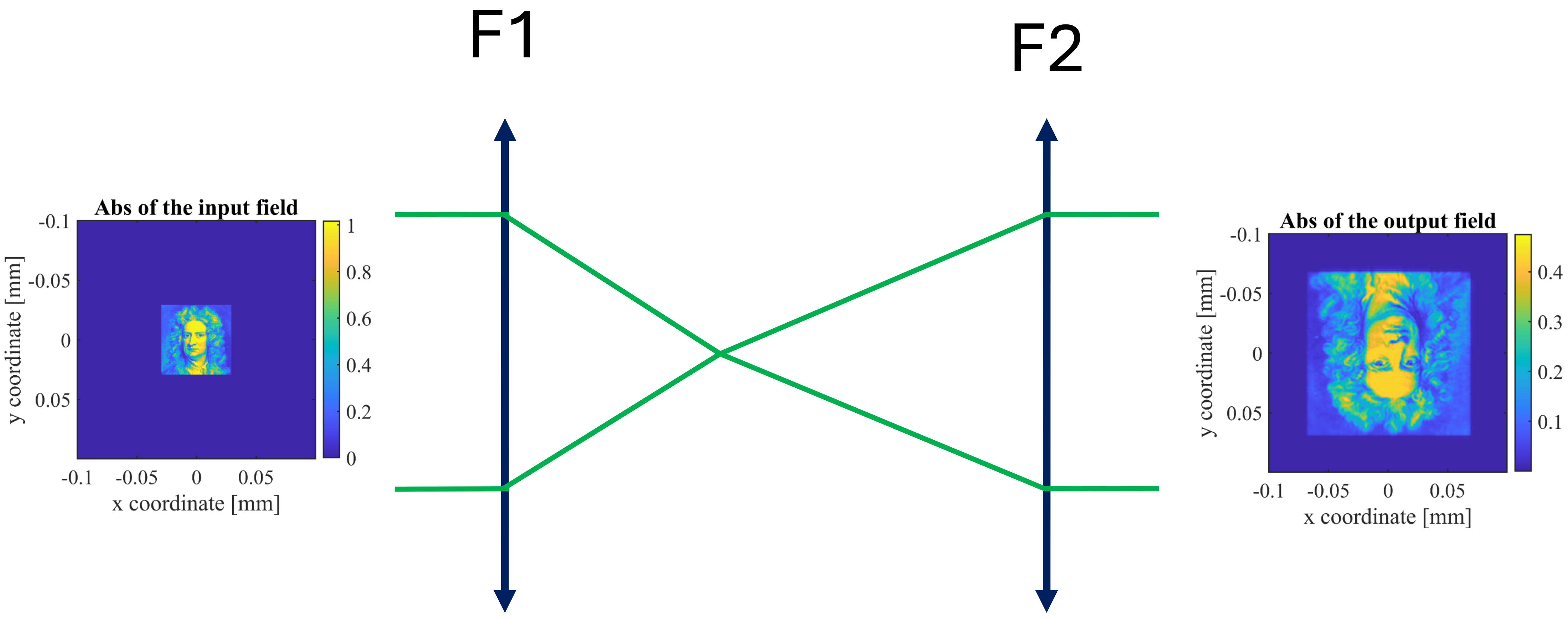


Amplitude and Phase Gratings (square profile)

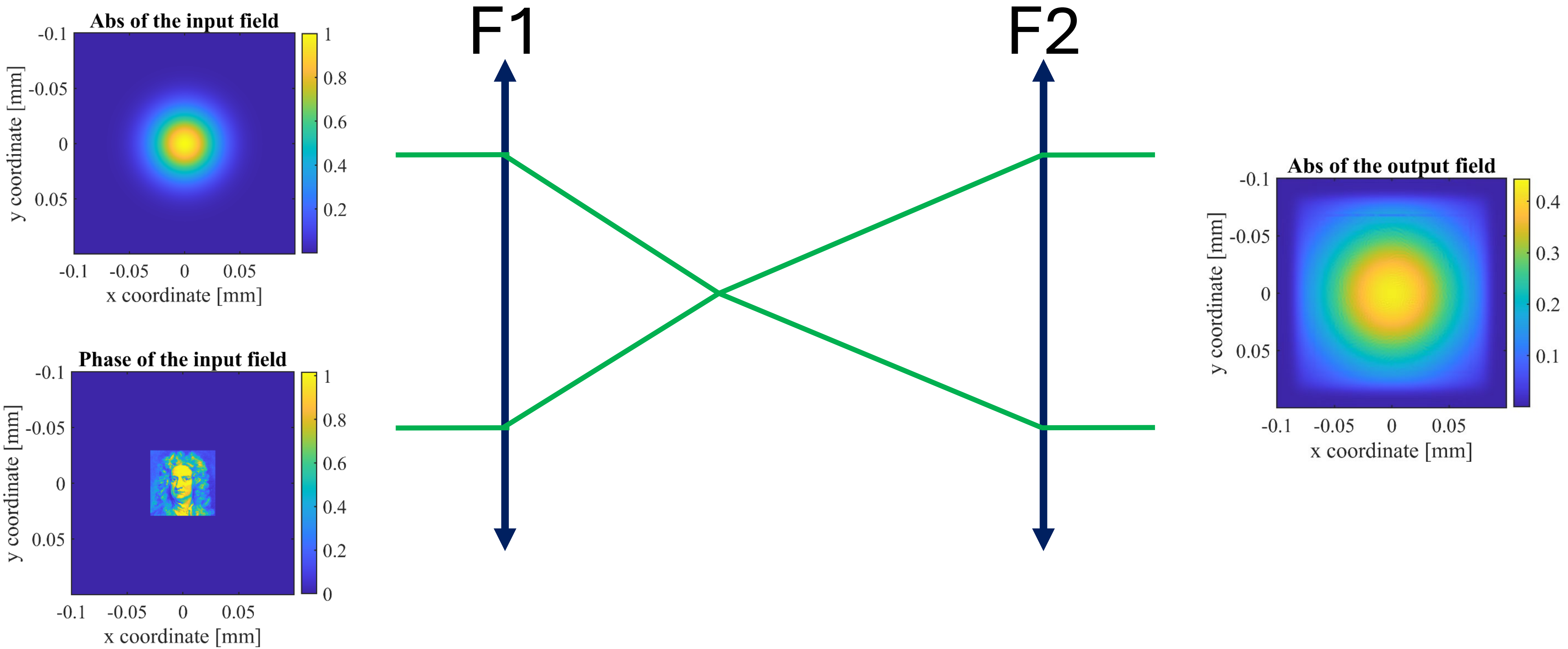
λ = 800nm



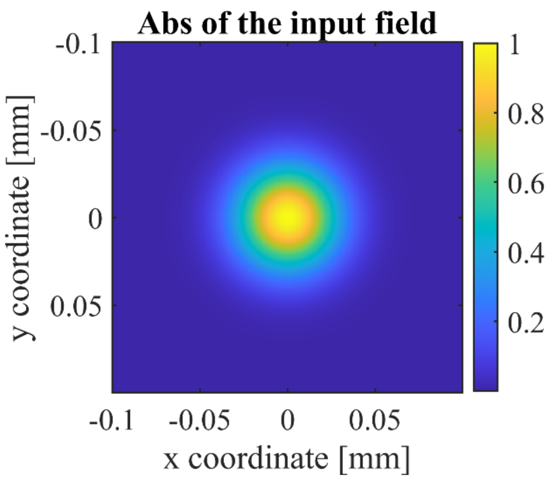
Bright Field Microscope for an amplitude object

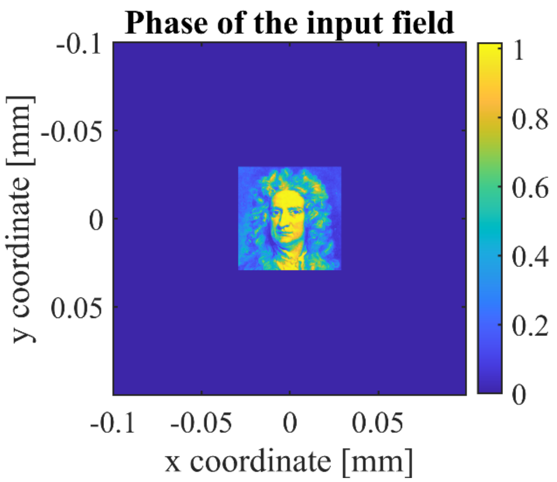


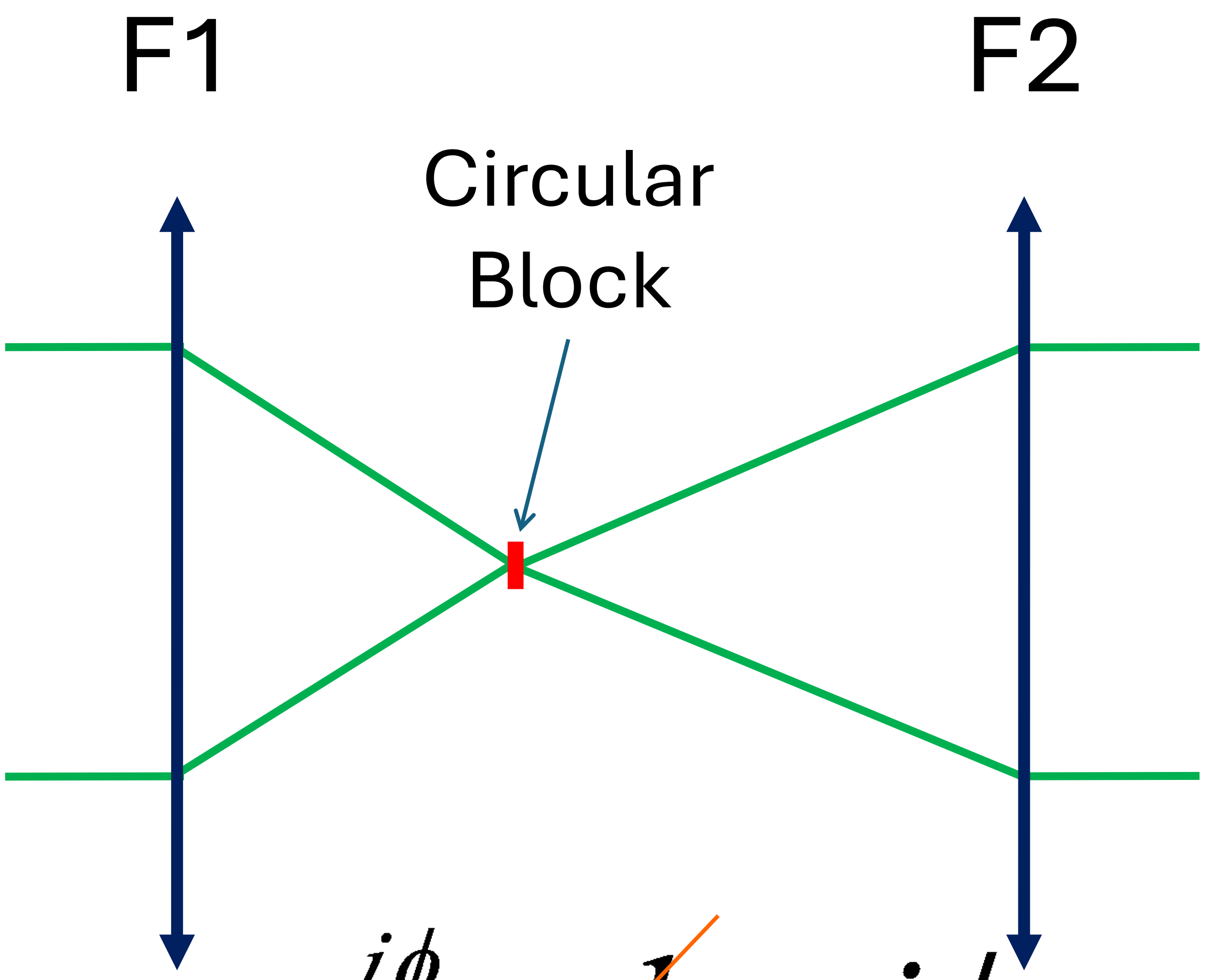
Bright Field Microscope for a phase object

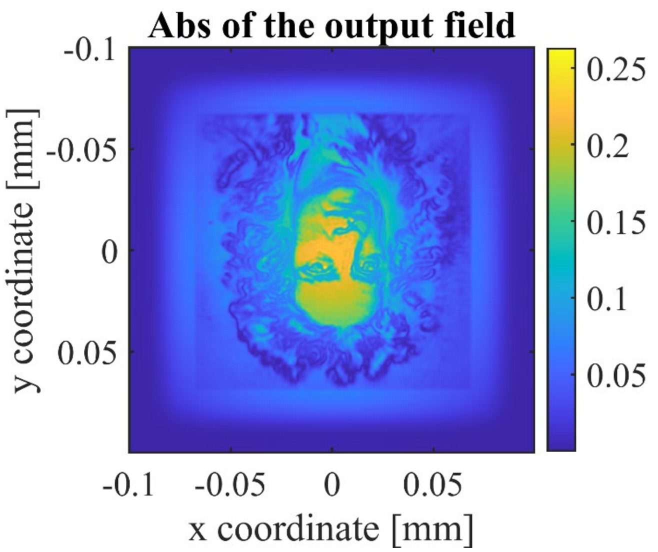


Dark Field Microscope





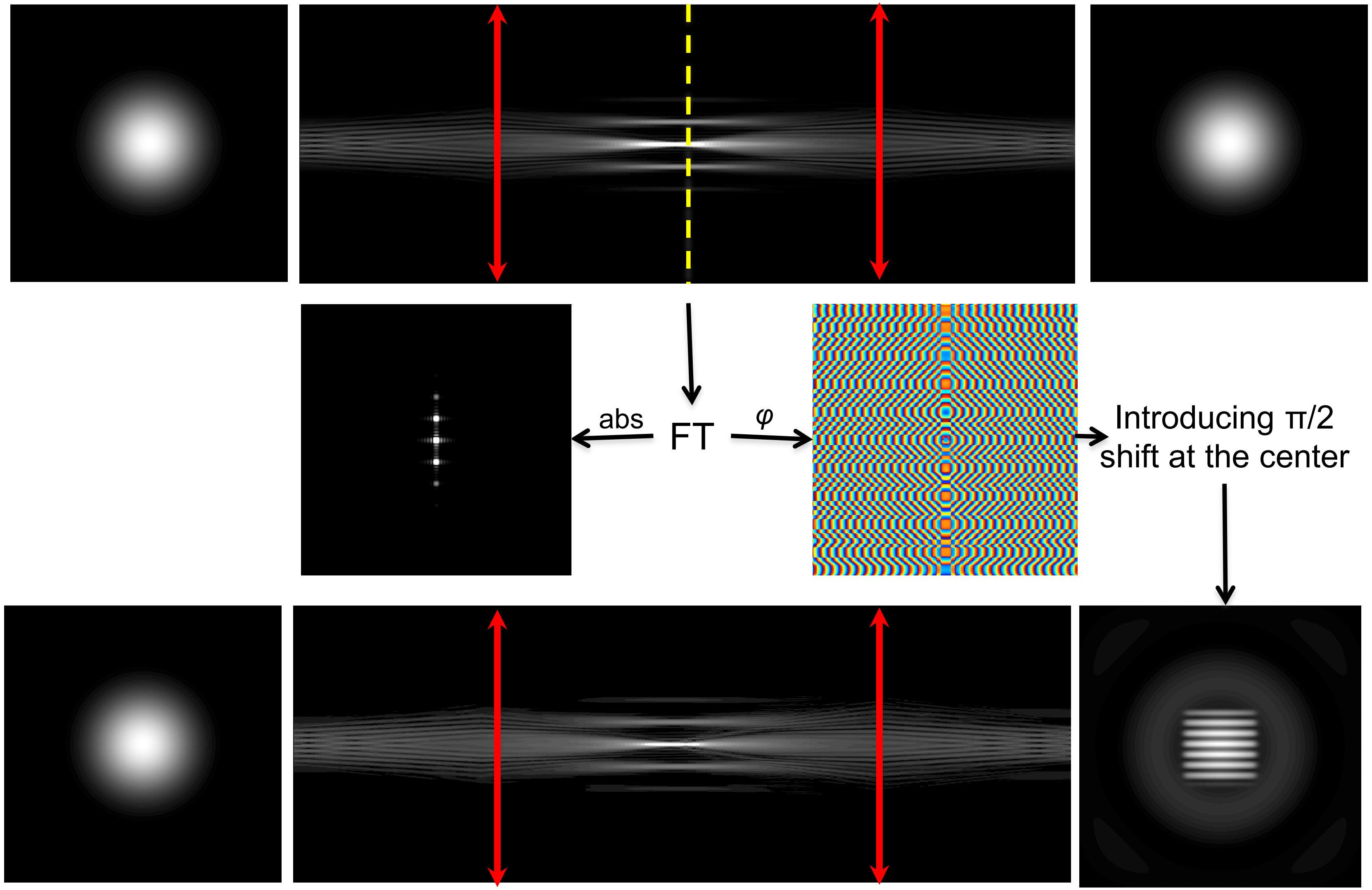




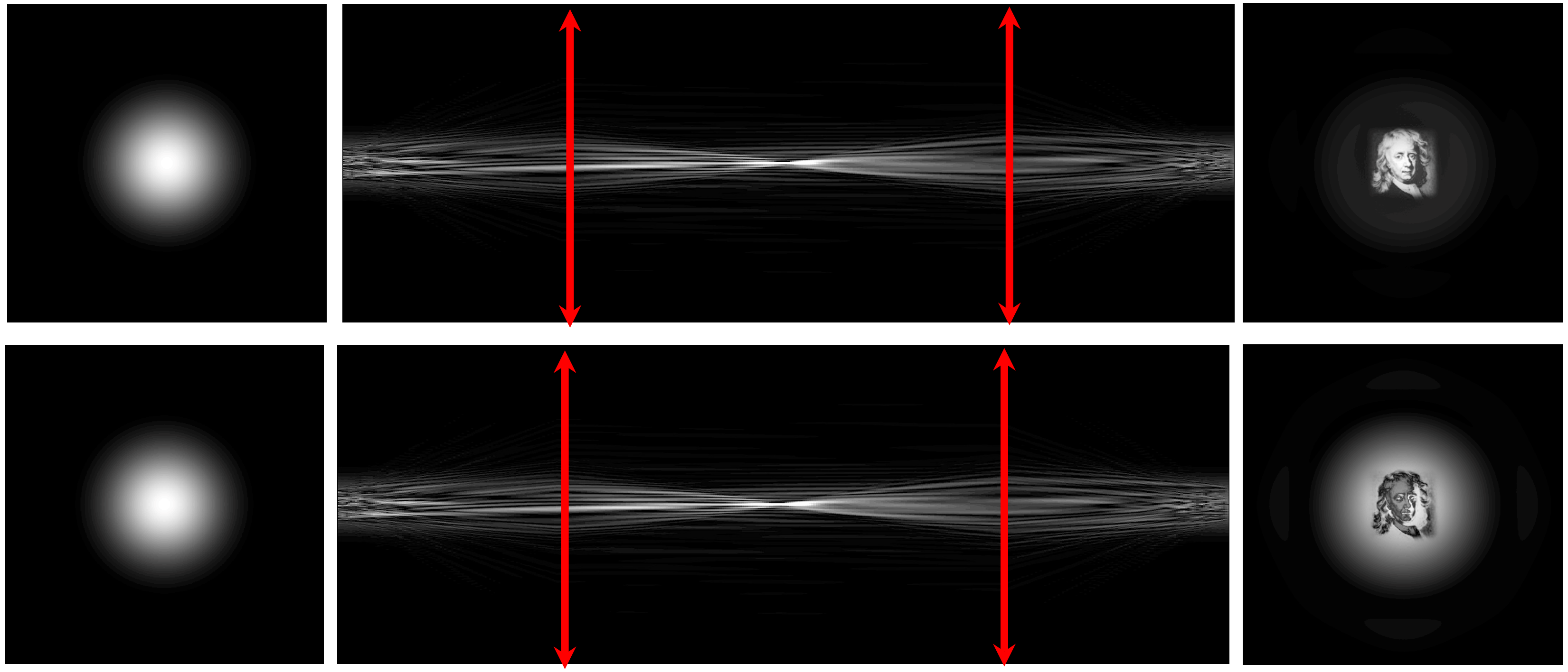
$$e^{j\phi} \approx \cancel{1} + j\phi$$

Zernike Phase Contrast microscope

Zernike Phase contrast



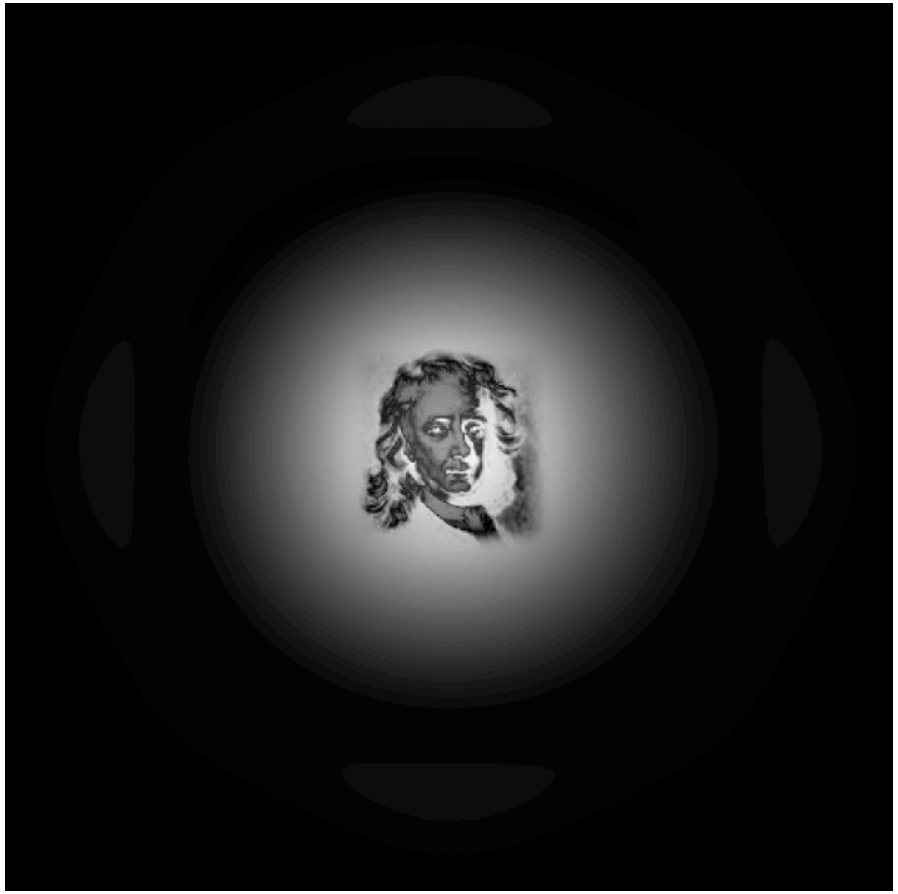
Zernike Phase contrast



Contrast inversion?

Zernike Phase contrast

π/2 shift



3π/2 shift

Exercise 2

- Zernike