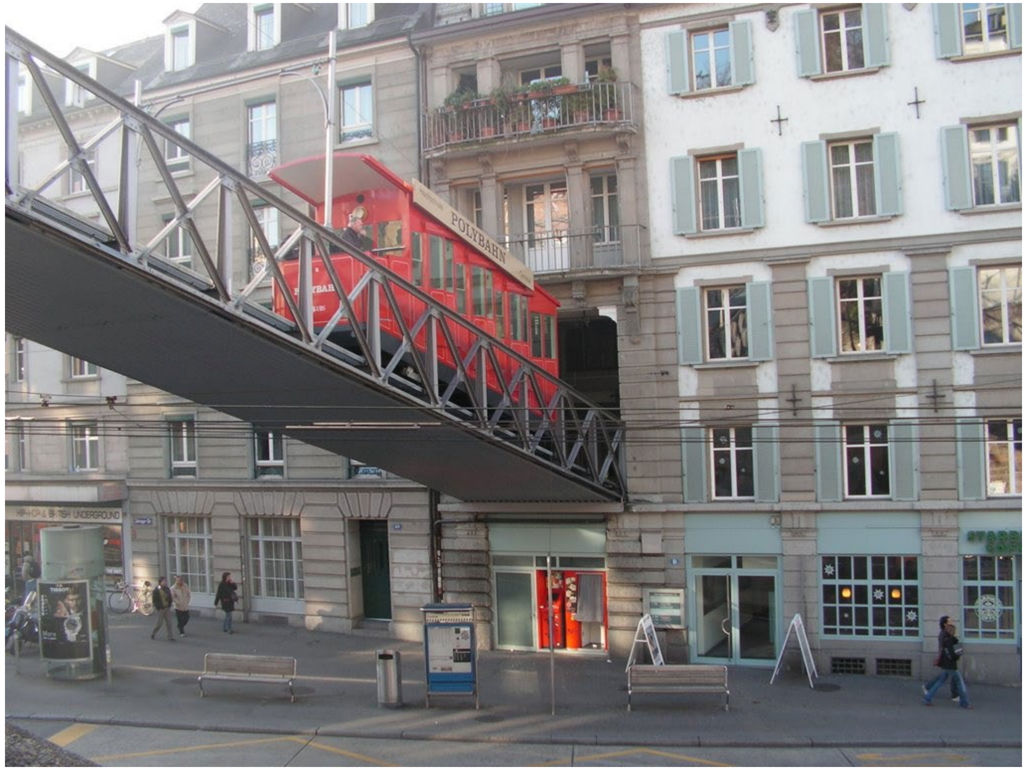
Computational Optical Imaging

Lecture 3

Outline

Beam propagation method (BPM)
Thin media
Lenses
Imaging



Polybahn at ETHZ

Reflection at air-metal interfaces

mirror

scatterer

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} + \vec{J}$$

$$\nabla \bullet \varepsilon \vec{E} = \rho$$

ε and n

$$D = \epsilon_0 E + P = \epsilon_0 E + \epsilon_1 E = \epsilon E$$

$$v = \sqrt{\frac{1}{\mu\epsilon}} \qquad n = \frac{c}{v}$$

Beam Propagation Method (BPM) in free space

$$\frac{\partial^2 E_x}{\partial x^2}+\frac{\partial^2 E_x}{\partial y^2}+\frac{\partial^2 E_x}{\partial z^2}=-\omega^2\mu\varepsilon E_x$$

Wave equation

$$E_x(x,y,z,t)=A(x,y,z)e^{j\omega t}e^{-jkz}$$

A(x,y,z) is the slow varying envelope

$$\frac{\partial^2 E_x}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial A}{\partial x}\right)e^{j\omega t}e^{-jkz}=\frac{\partial^2 A}{\partial x^2}e^{j\omega t}e^{-jkz}$$

same for y

~0

$$\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}+\frac{\partial^2 A}{\partial z^2}-2jk\frac{\partial A}{\partial z}-k^2A=-\omega^2\mu\varepsilon A$$

Recognizing that

$$-k^2A=-\omega^2\mu\varepsilon A$$

$$\frac{\partial A}{\partial z}\simeq-\frac{j}{2k}\left(\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}\right)$$

$$\frac{\partial A}{\partial z}\approx\frac{A(z+\Delta z)-A(z)}{\Delta z}\Rightarrow. A(z+\Delta z)=A(z)+\frac{\partial A}{\partial z}\Delta z$$

Paraxial Angular Spectrum

$$E(x,y,z\neq 0,t) = e^{j\omega t}\iint F(u,v)e^{-j(2\pi ux+2\pi vy+\sqrt{k^2-(2\pi u)^2-(2\pi u)^2}z)}\,du\,dv$$

Paraxial plane wave:

$$\tilde{A}(k_x,k_y,z) = e^{j\left[\frac{k_x^2+k_y^2}{2k}\right]z}\tilde{A}(k_x,k_y,z=0)$$

Convolution Theorem:

$$g(x',y') = \int\int f(x,y)h(x'-x,y-y')dxdy$$

$$G(u,v) = F(u,v)H(u,v)$$

G, F and H are Fourier transforms of g, f and h respectively

Note that Fourier Transform of the exponential is:

$$e^{-\pi(x^2+y^2)} \Leftrightarrow e^{-\pi(u^2+v^2)} \quad where \quad k_x = 2\pi u \quad k_y = 2\pi v$$

In free space paraxial BPM = Fresnel diffraction

BPM

$$\frac{\partial A}{\partial z} \simeq -\frac{j}{2k}\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}\right)$$

FT

Angular Spectrum

$$\tilde{A}(k_x,k_y,z) = e^{j\left[\frac{k_x^2+k_y^2}{2k}\right]z}\,\tilde{A}(k_x,k_y,z=0)$$

FT

$$E_x\left(x',y',z,t\right) = A(x',y',z)e^{j\omega t}e^{-jkz} = \frac{1}{j\lambda z}e^{j\omega t}e^{-jkz}\iint A(x,y,z=0)\,e^{-\frac{j\pi[(x-x')^2+(y-y')^2}{\lambda z}}dxdy$$

Fresnel Diffraction

Beam Propagation Method

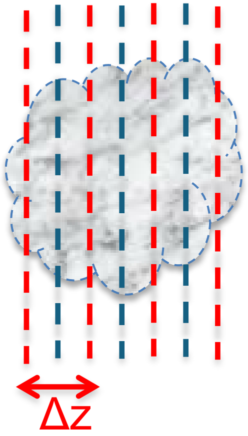
$$\frac{\partial A}{\partial z} = -\frac{j}{2k}\left(\frac{\partial A^2}{\partial x^2} + \frac{\partial A^2}{\partial y^2}\right) \boxed{- \frac{j}{2k}\omega^2 \mu \Delta\epsilon(x, y, z) A}$$

$$A(z + \Delta z) \approx A(z) - \frac{j}{2k}\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}\right)\frac{\Delta z}{2} - \frac{j}{2k}\omega^2 \mu \Delta\epsilon(x, y, z) A \frac{\Delta z}{2}$$

$$A(z + \Delta z) \approx A(z) - \frac{j}{2k}\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}\right)\frac{\Delta z}{2} - \frac{j}{2k}\omega^2 \mu \Delta\epsilon(x, y, z) A \frac{\Delta z}{2}$$

Free space

Thin phase transparency



BPM Pseudocode

Start with BPM in a medium with $\varepsilon(x,y,z)=\varepsilon_0+\Delta\varepsilon(x,y,z)$:

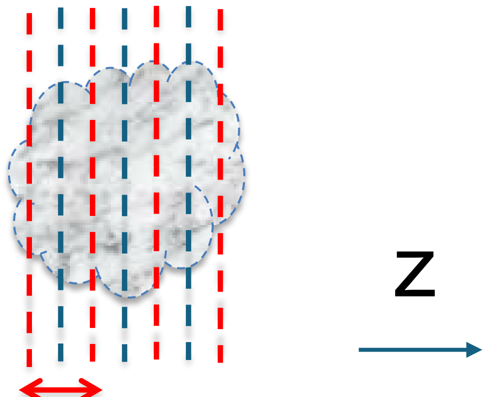
$$\frac{\partial A}{\partial z}=\frac{-j}{2k}\left(\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}\right)-\frac{j}{2k}\omega^2\mu\Delta\varepsilon(x,y,z)A$$

First step: *Take the FT in* $x-y$ *assuming* $\Delta\varepsilon\approx 0$

$$\frac{\partial \tilde{A}}{\partial z}\approx\frac{+j}{2k}(k_x^2+k_y^2)\tilde{A}(k_x,k_y,z)\Rightarrow\tilde{A}(k_x,k_y,z+\Delta z/2)=\tilde{A}(k_x,k_y,z)e^{\frac{+j}{2k}(k_x^2+k_y^2)\Delta z/2}$$

Second step: *Assume* $\Delta z/2\approx 0$ *so that diffraction is negligible*

$$\frac{\partial A}{\partial z}\approx-\frac{j}{2k}\omega^2\mu\Delta\varepsilon(x,y,z)A\Rightarrow A(x,y,z+\Delta z)=A(x,y,z+\Delta z/2)e^{-\frac{j}{2k}\omega^2\mu\Delta\varepsilon(x,y,z)\Delta z/2}$$



Beam Propagation

Run

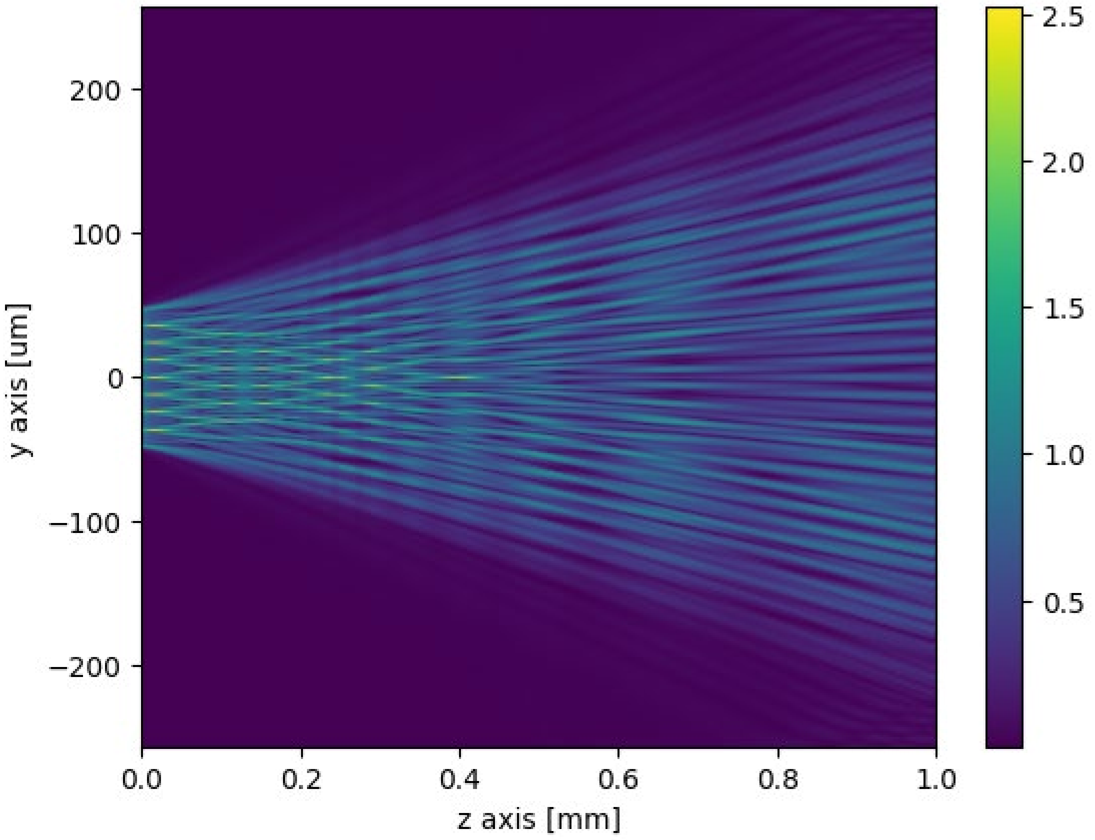
```python
import numpy as np
import matplotlib.pyplot as plt

def beam_propagation_method_grin(wavelength, dx, dz, nx, nz, n0, delta_n, beam_waist):
    k0 = 2 * np.pi / wavelength  # Free-space wavenumber
    x = np.arange(-nx//2, nx//2) * dx  # Spatial grid
    z = np.arange(0, nz) * dz  # Propagation grid

    # Define the GRIN profile (parabolic index distribution)
    n = n0 - delta_n * (x / (nx * dx / 2))**2  # Parabolic index variation

    # Initial field (Gaussian beam)
    field = np.exp(-x**2 / (2 * beam_waist**2))
    field /= np.sqrt(np.sum(np.abs(field)**2))  # Normalize power

    # Spectral components for Fourier transform
    kx = np.fft.fftfreq(nx, d=dx) * 2 * np.pi
    kx2 = kx**2

    # Propagation loop
    for zi in range(nz):
        # Compute phase shift due to index variations
        phase_shift = np.exp(1j * dz * k0 * (n**2 - n0**2) / (2 * n0))
        field *= phase_shift  # Apply phase shift in real space

        # Transform to Fourier domain
        field_fft = np.fft.fft(field)

        # Apply free-space propagation in k-space
        prop_factor = np.exp(-1j * dz * kx2 / (2 * k0 * n0))
        field_fft *= prop_factor

        # Transform back to real space
        field = np.fft.ifft(field_fft)

```

```python
    return x, np.abs(field)**2  # Return intensity profile

# Simulation parameters
wavelength = 1.55e-6  # Wavelength (m)
dx = 0.1e-6  # Spatial step (m)
dz = 1e-6  # Propagation step (m)
nx = 200  # Number of spatial points
nz = 1000  # Number of propagation steps
n0 = 1.5  # Central refractive index
delta_n = 0.05  # Index variation parameter for GRIN profile
beam_waist = 2e-6  # Initial beam waist (m)

# Run BPM simulation for GRIN lens
x, intensity = beam_propagation_method_grin(wavelength, dx, dz, nx, nz, n0, delta_n, beam_waist)

# Plot results
plt.plot(x * 1e6, intensity)
plt.xlabel("Position (µm)")
plt.ylabel("Intensity")
plt.title("Beam Propagation in GRIN Lens")
plt.show()
```

$e^{j\pi \cos(2\pi y/\Lambda)}$

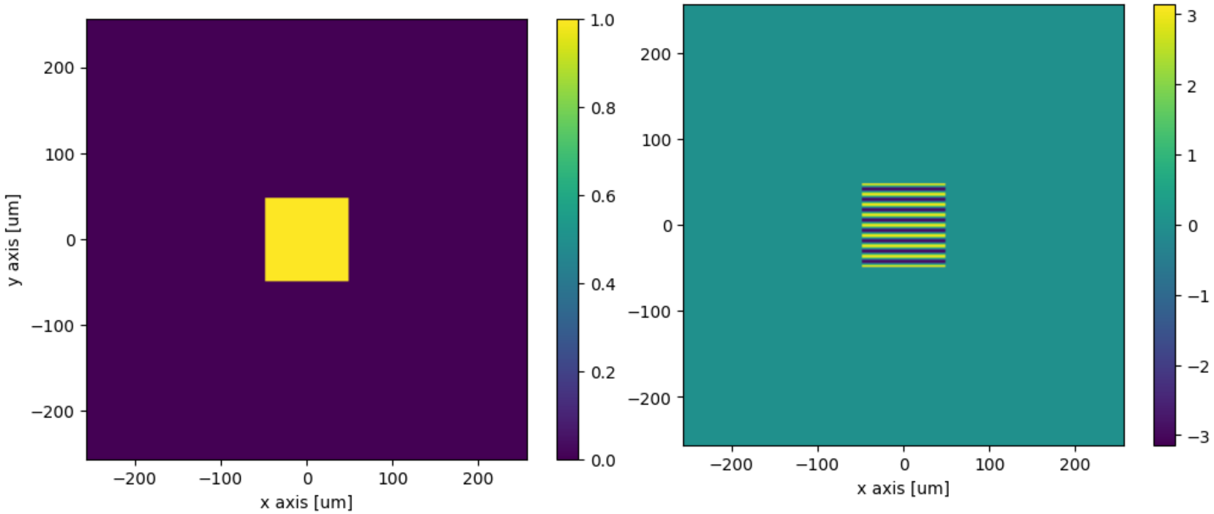
Plane wave input with λ=532 nm cropped by 96 μm by 96 μm rectangular aperture.
Λ=12 μm.

Propagation profile for 1mm



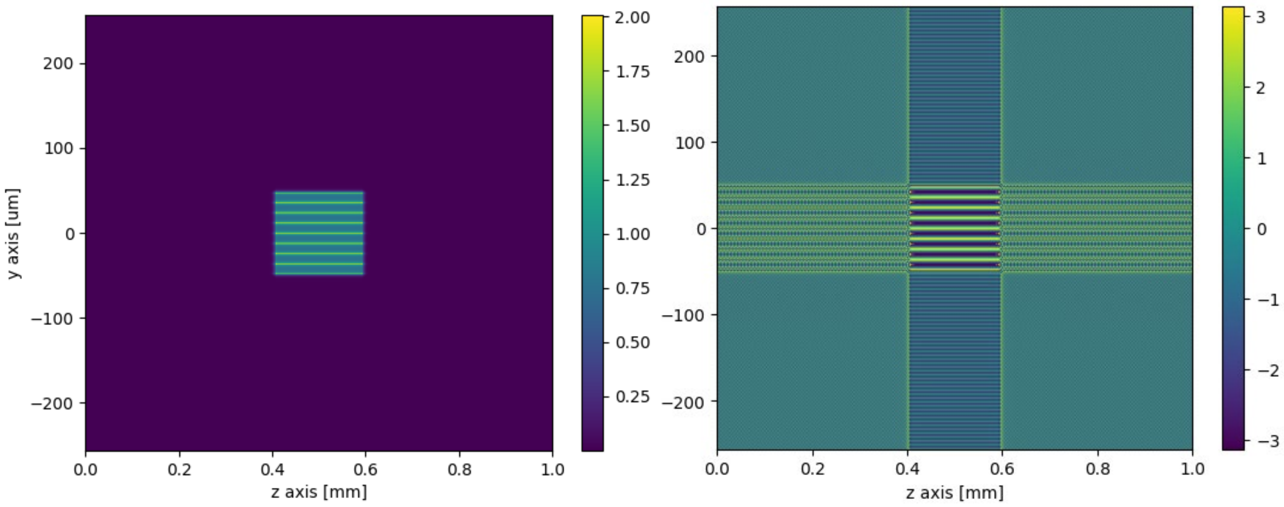
Propagation z=0

Amplitude　　Phase



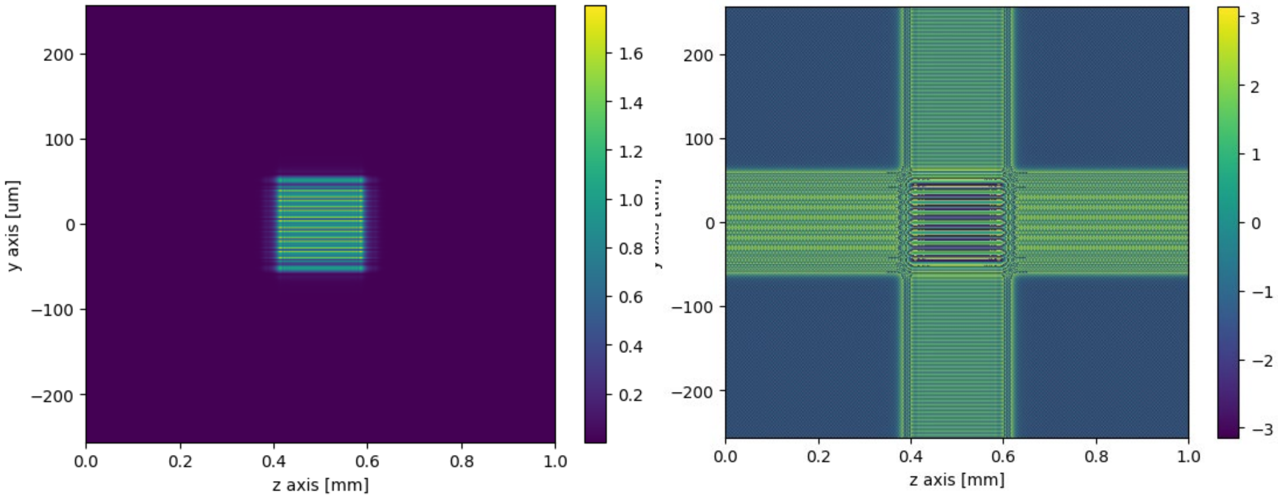
Propagation z=10 µm

Amplitude　　Phase



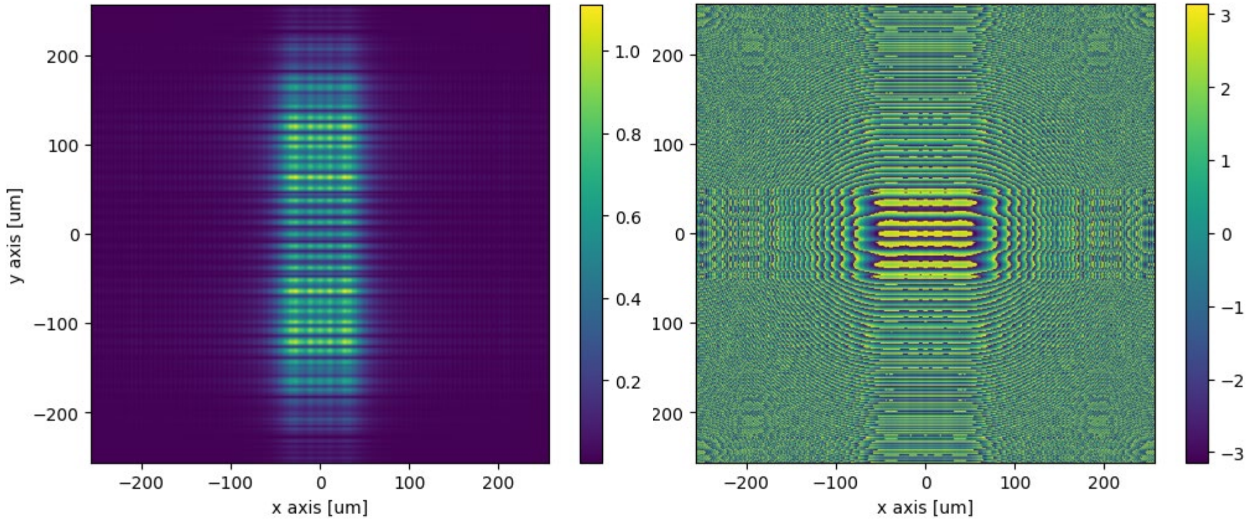
Propagation z=50 µm

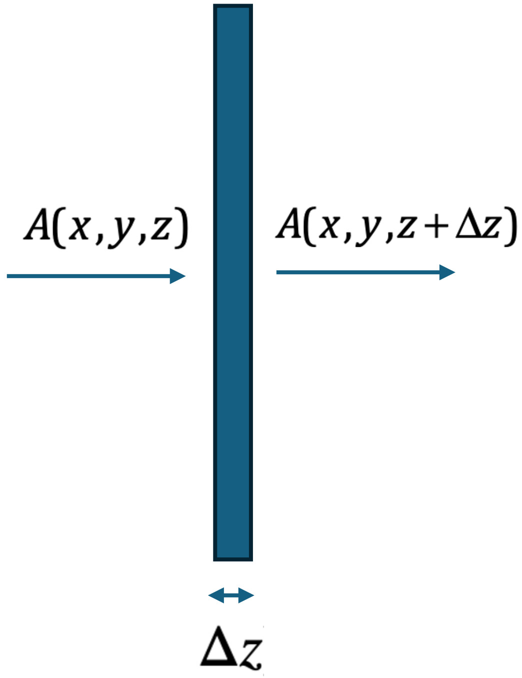
Amplitude　　Phase



Propagation z=1 mm

Amplitude　　Phase





Thin transparency

$$t(x,y)=\frac{A(x,y,z+\Delta z)}{A(x,y,z)}=e^{-\frac{j}{2k}\omega^2\mu\Delta\varepsilon(x,y,z)\Delta z}$$

Real part of $\Delta\varepsilon$ *is index; imaginary part of* $\Delta\varepsilon$ *is absorption*

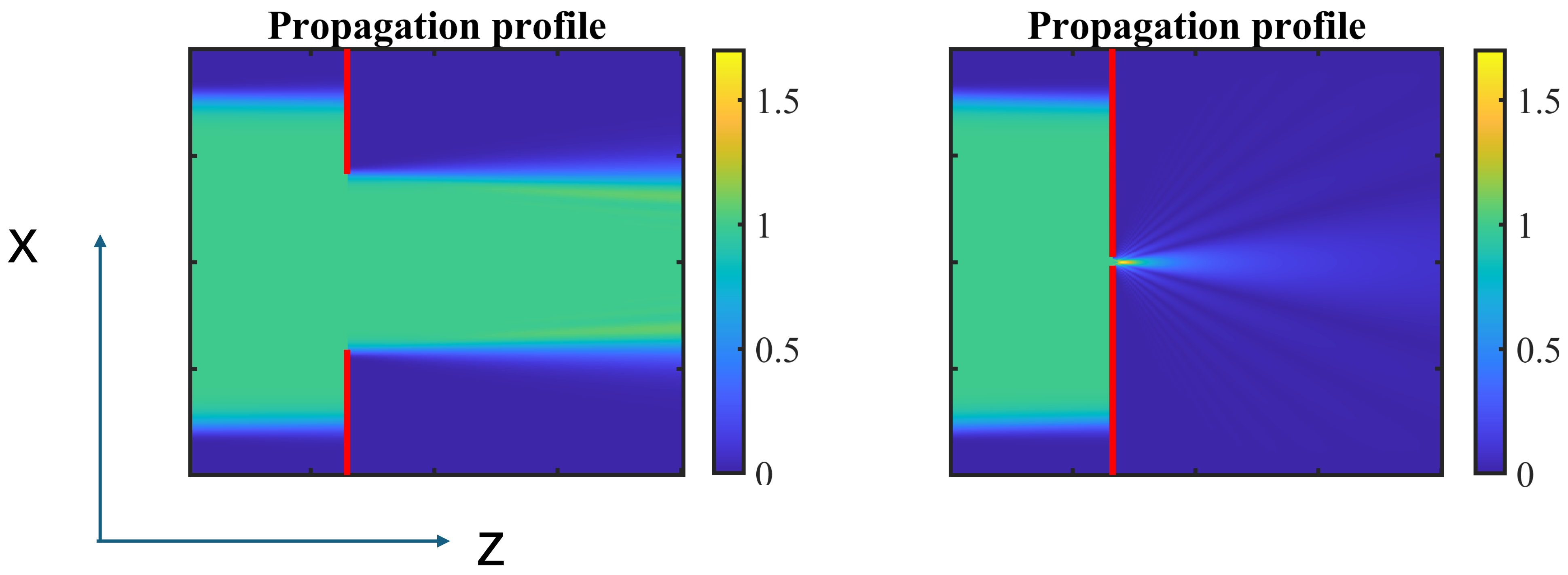
~0

$$A(z+\Delta z)\approx A(z)-\frac{j}{2k}\left(\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}\right)\frac{\Delta z}{2}-\frac{j}{2k}\omega^2\mu\Delta\epsilon(x,y,z)A\frac{\Delta z}{2}$$

Free space

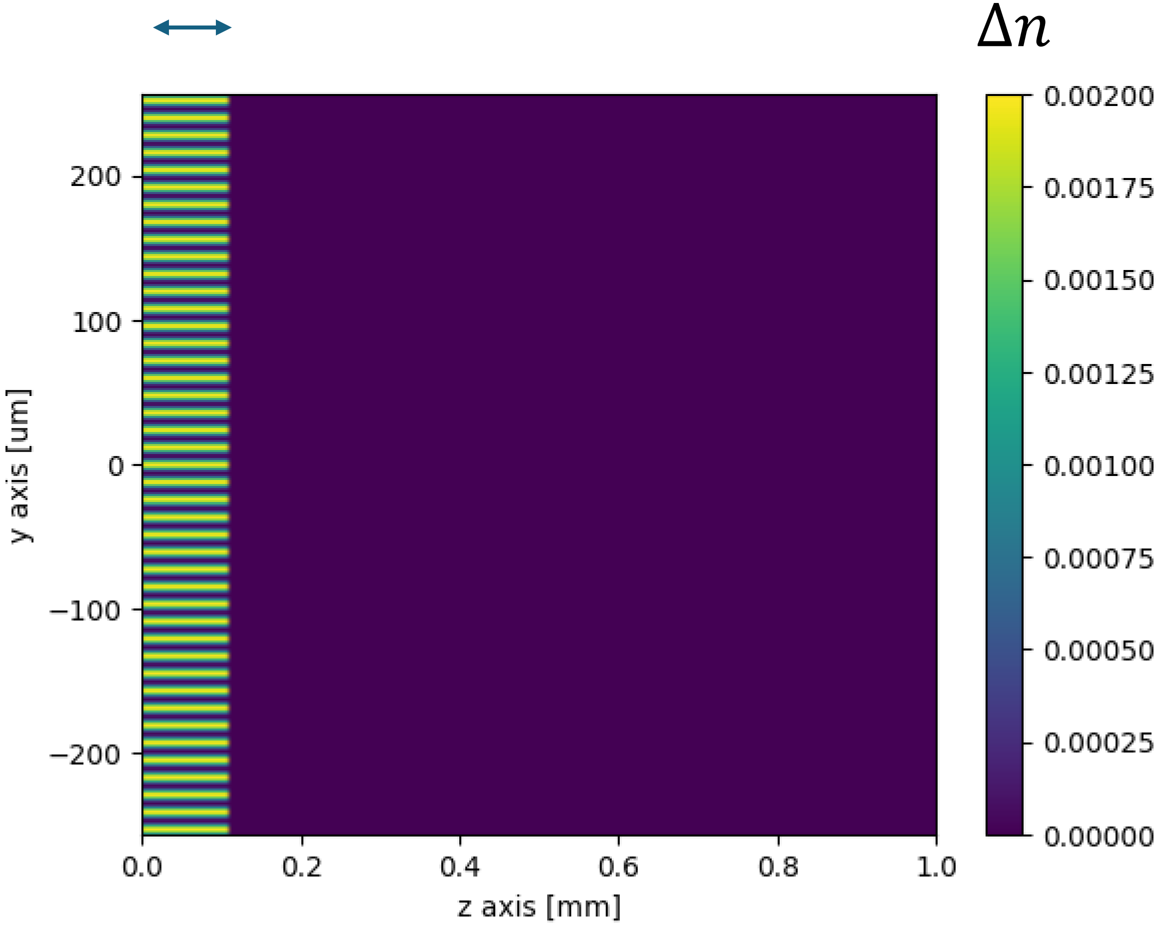
Thin phase transparency

Near field

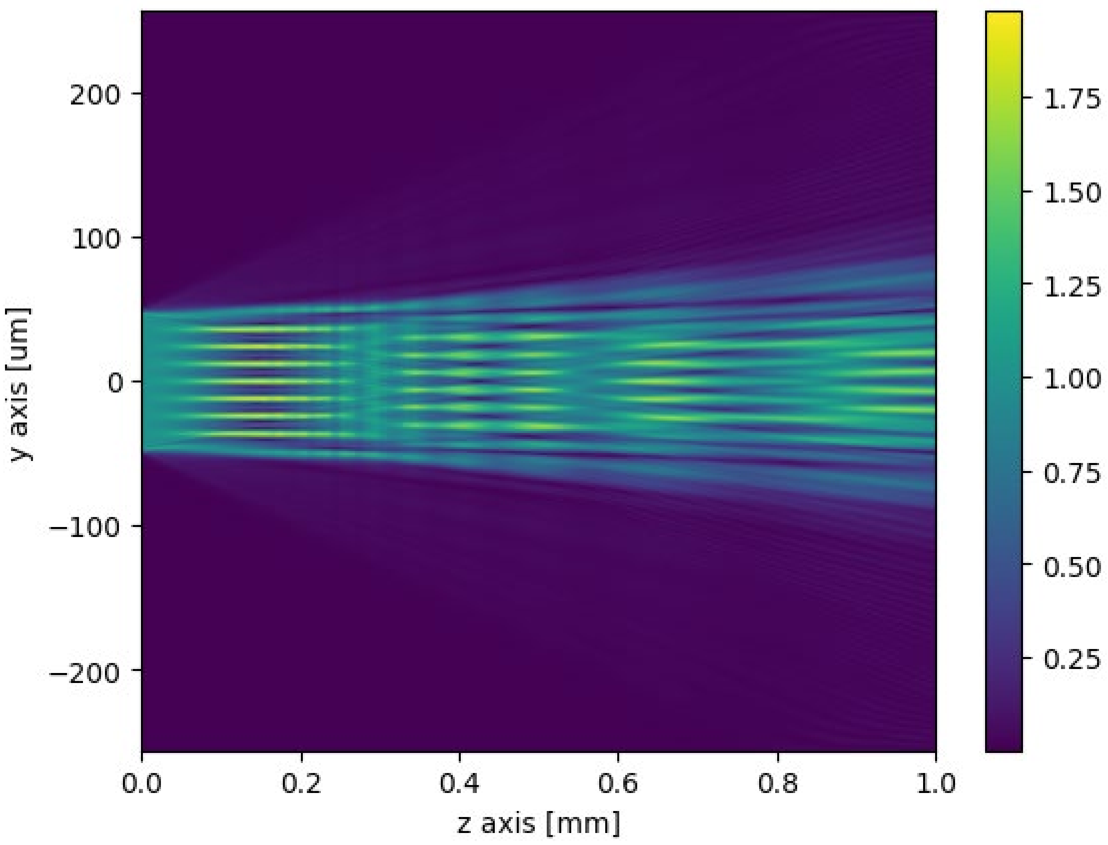

Propagation profile
1.5
1
0.5
0
Propagation profile
1.5
1
0.5
0
x
z

Plane wave input with λ=532 nm cropped by 96 μm by 96 μm rectangular aperture.
Λ=12 μm.

Thickness ≈λ/Δn

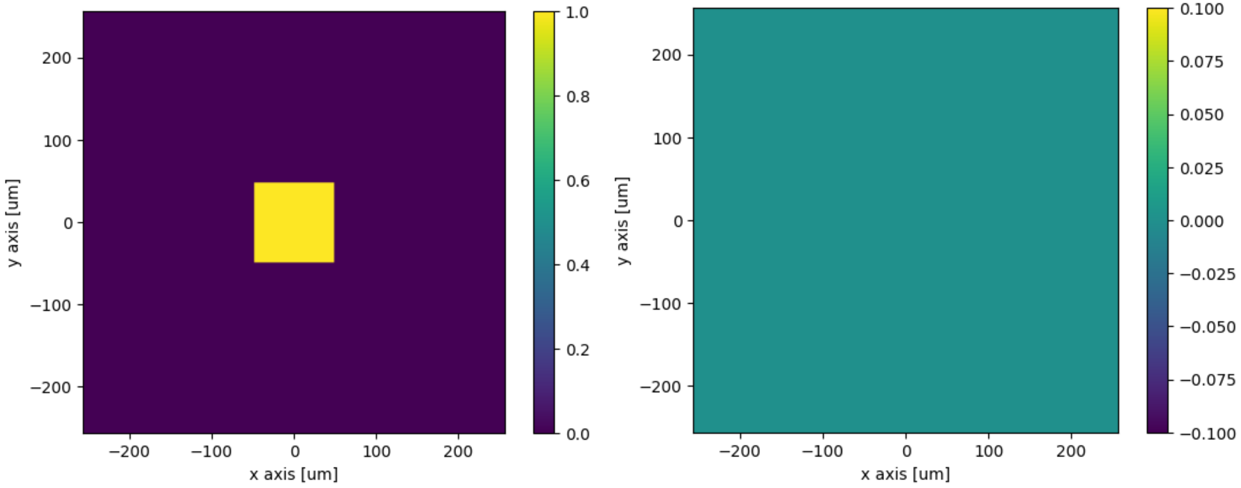


Propagation profile for 1mm



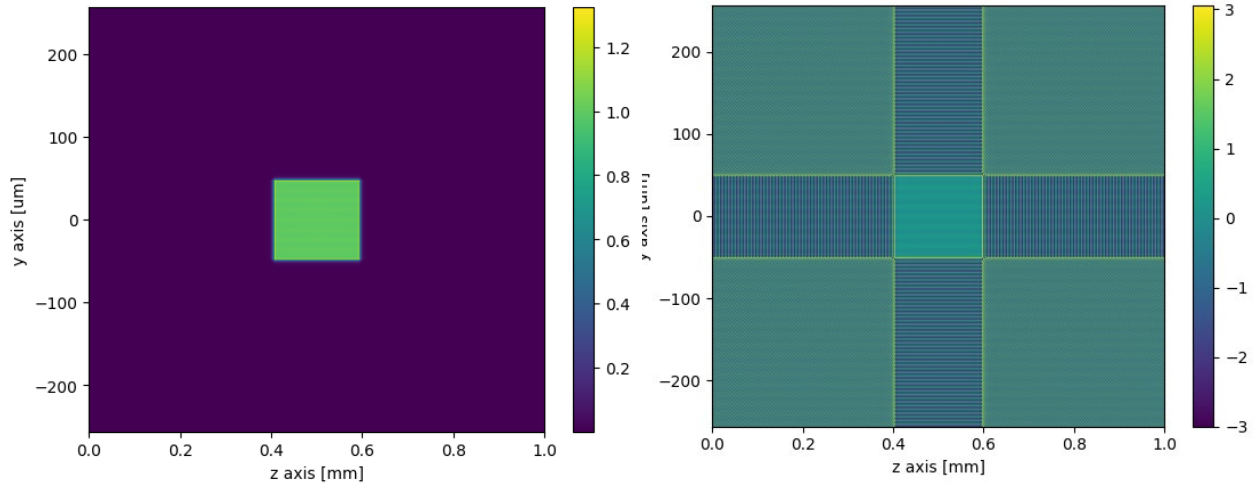
Propagation z=0

Amplitude Phase



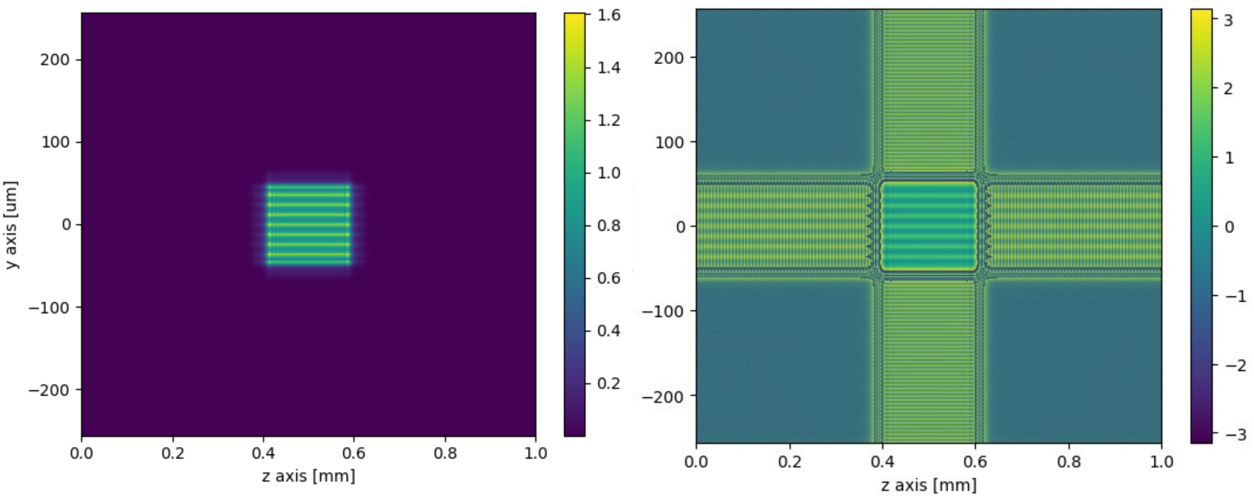
Propagation z=10 μm

Amplitude Phase



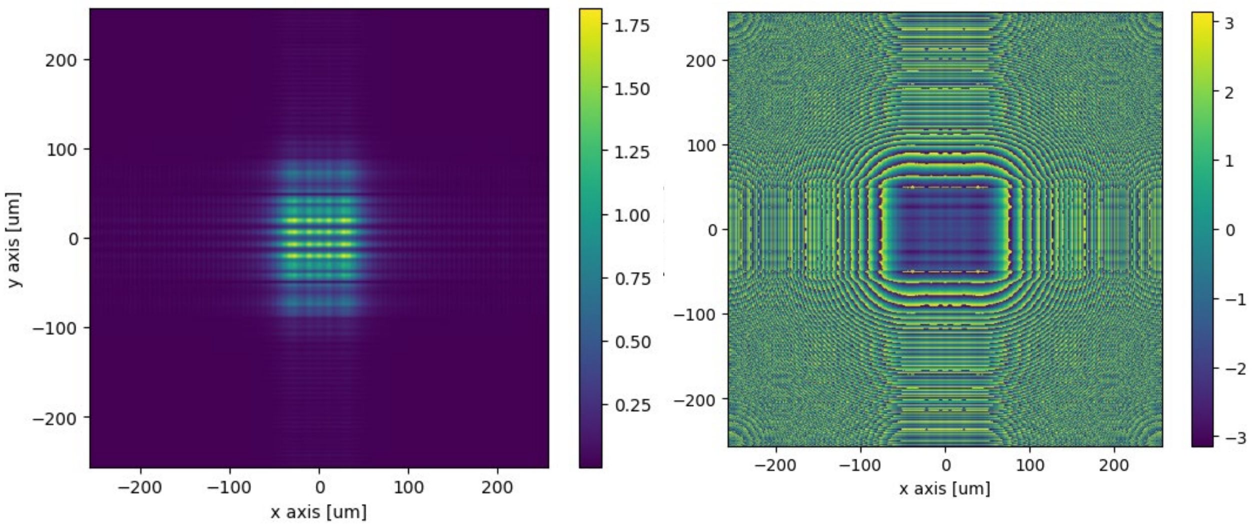
Propagation z=50 μm

Amplitude Phase

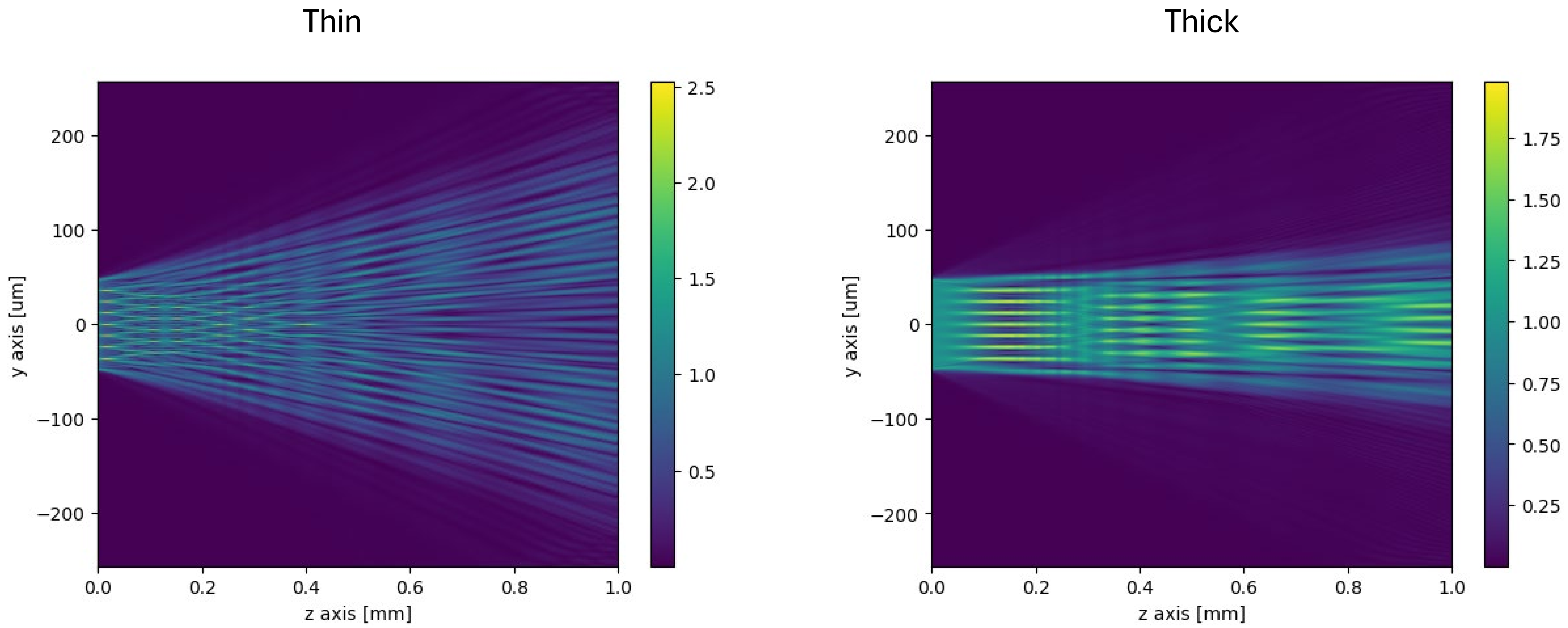


Propagation z=1 mm

Amplitude Phase



Comparison of propagation profile for 1mm

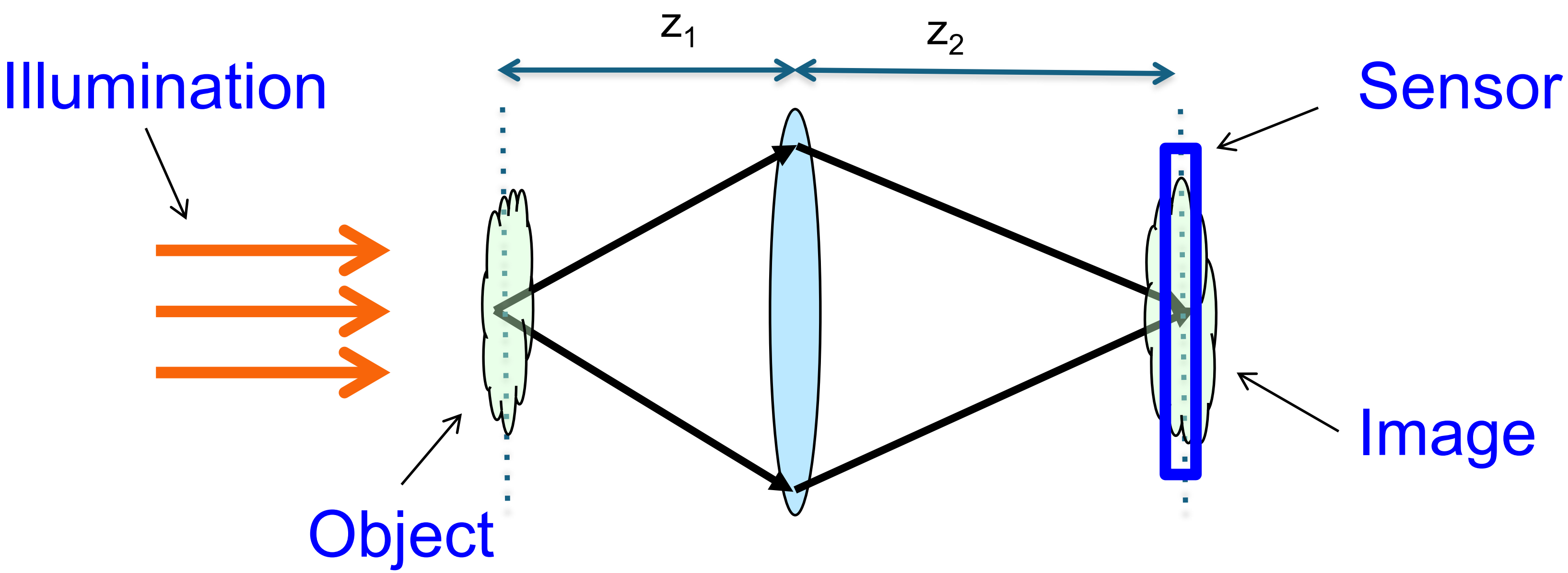


Exercise 1

BPM code

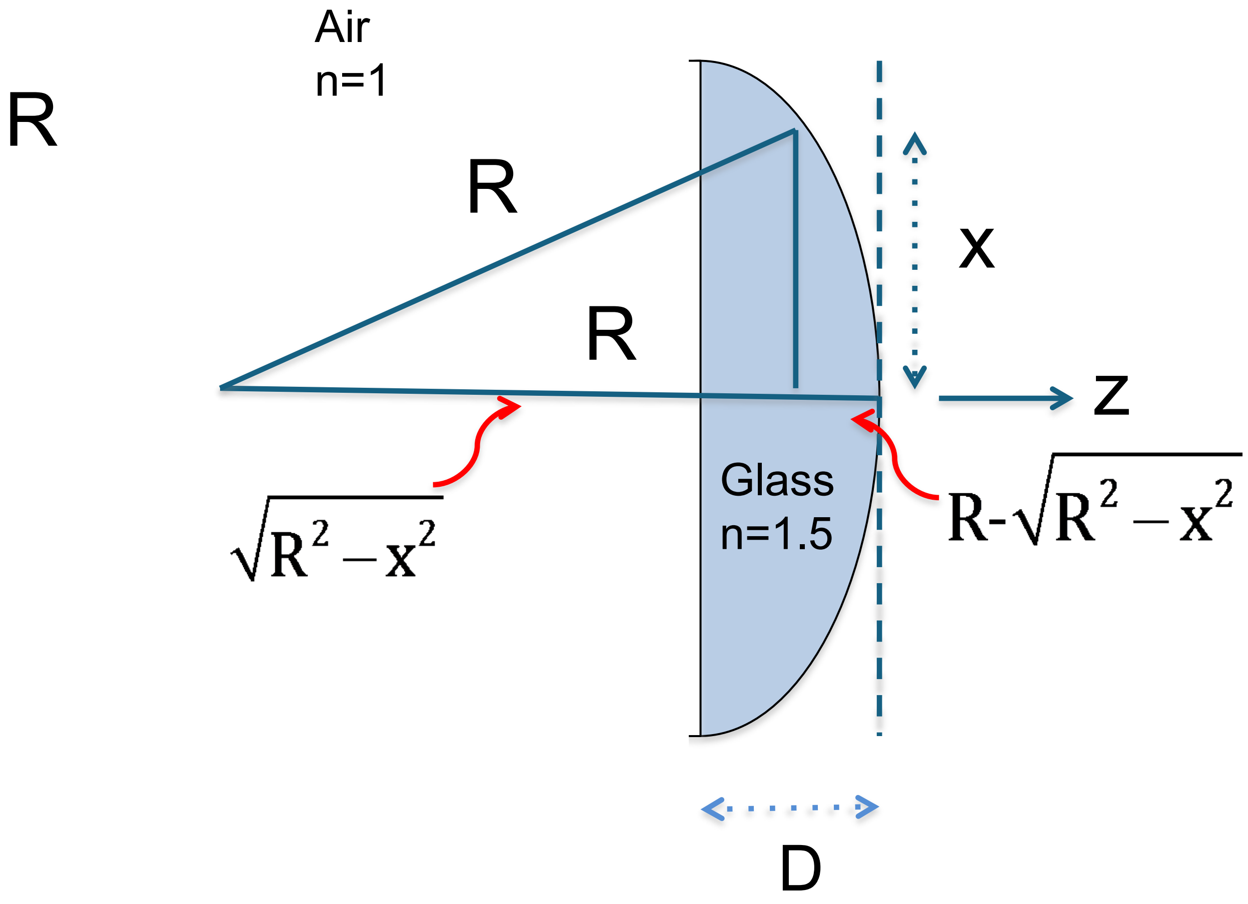
GRIN lens at variable step size

Singe Lens Imaging System



$$\frac{1}{F} = \frac{1}{z_1} + \frac{1}{z_2} \quad M = -\frac{z_2}{z_1}$$

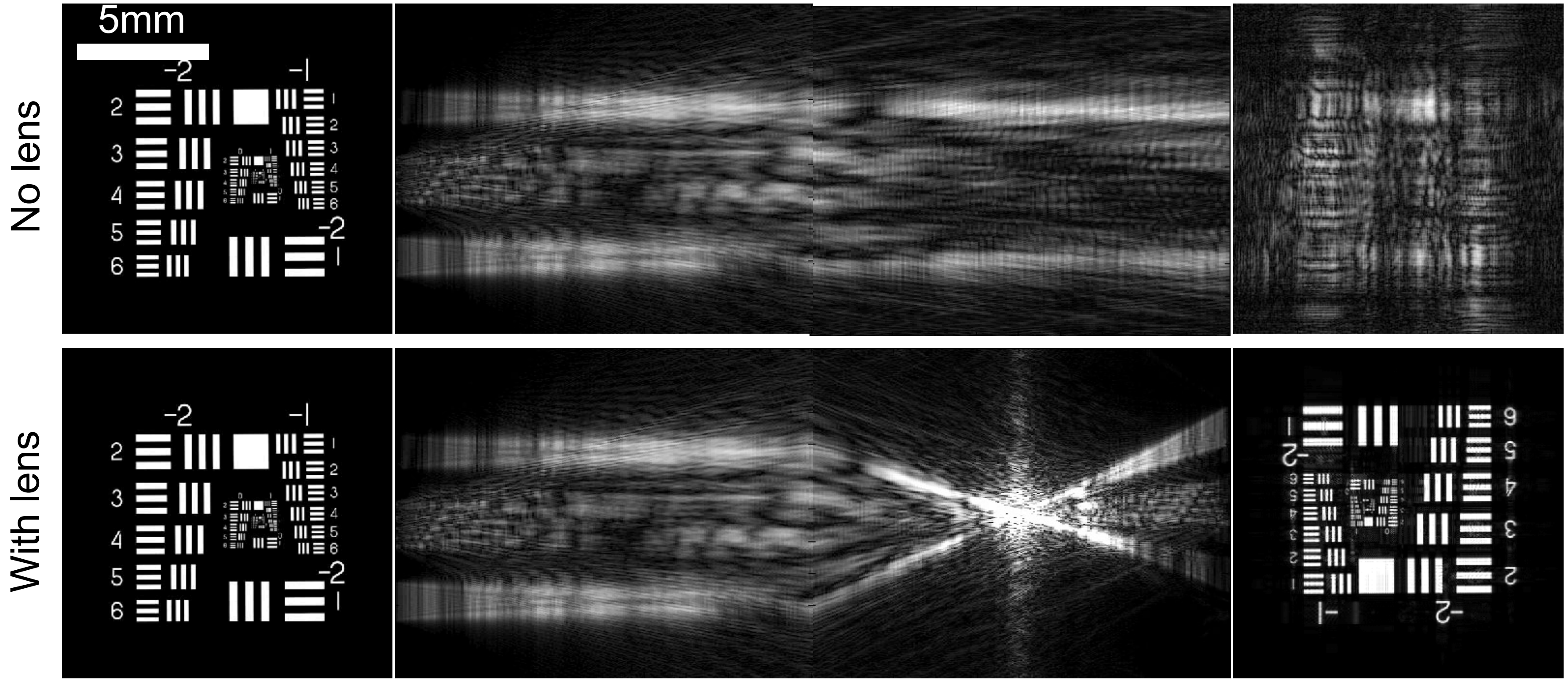
Thin Lens



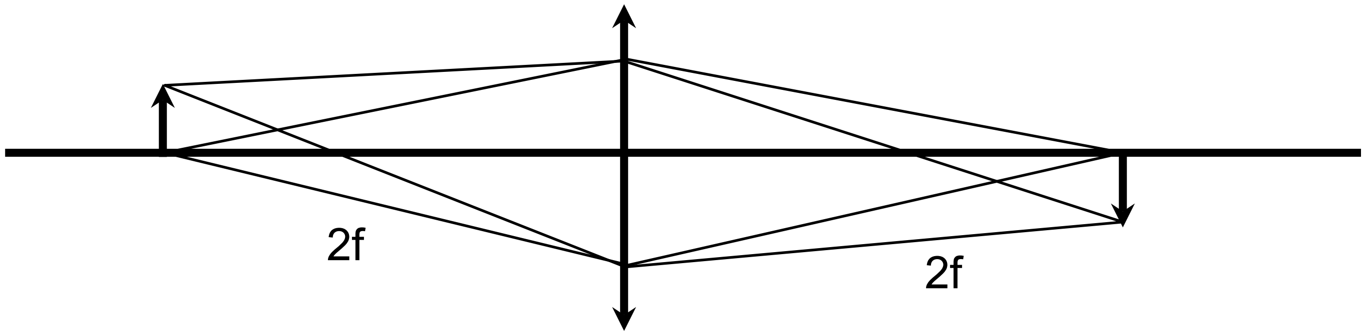
$$t_{lens} = e^{+j\frac{\pi(x^2+y^2)}{\lambda F}}$$

Single lens imaging simulation

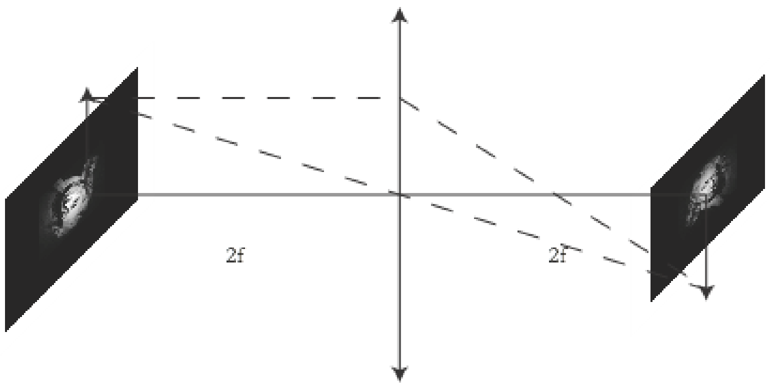
Thin lens simulated by a quadratic phase multiplication: $\exp\left[jk\frac{x^2+y^2}{2f}\right]$



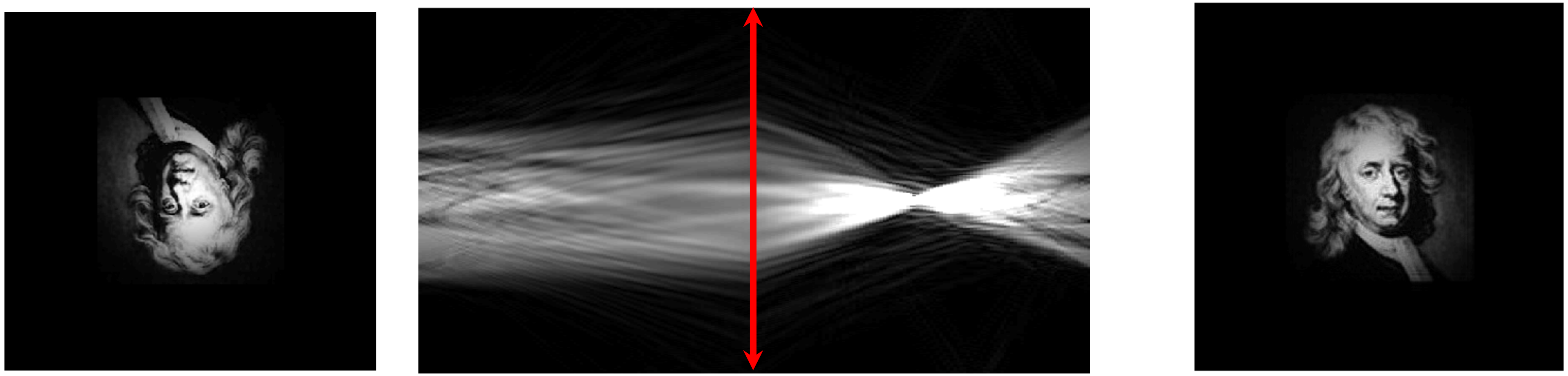
f = 500mm

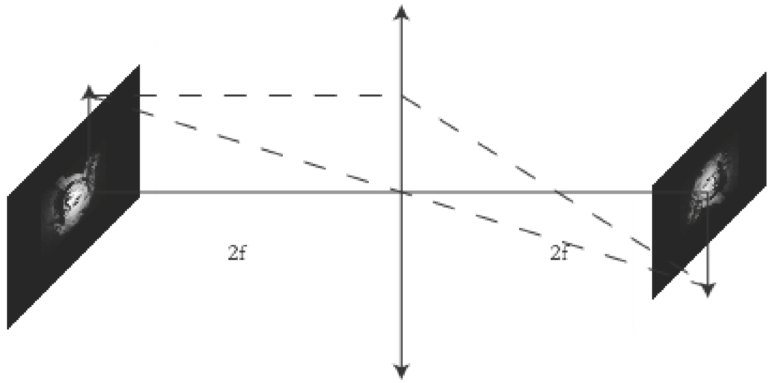


Single lens imaging simulation



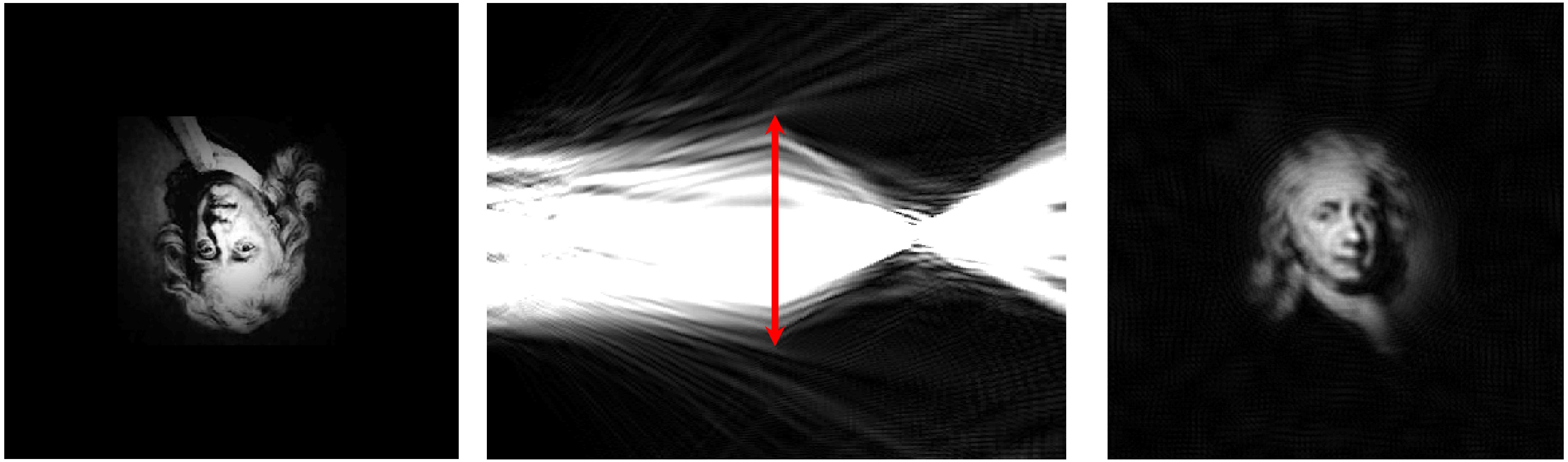
The image is reproduced (inverted) at 2f away from the lens

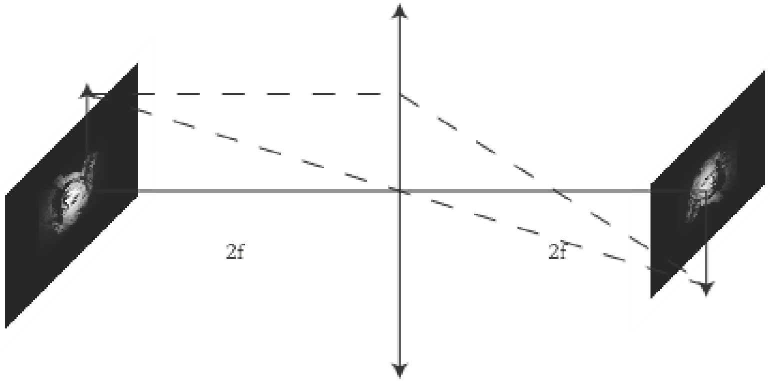




Same scheme as earlier, only the lens is now finite

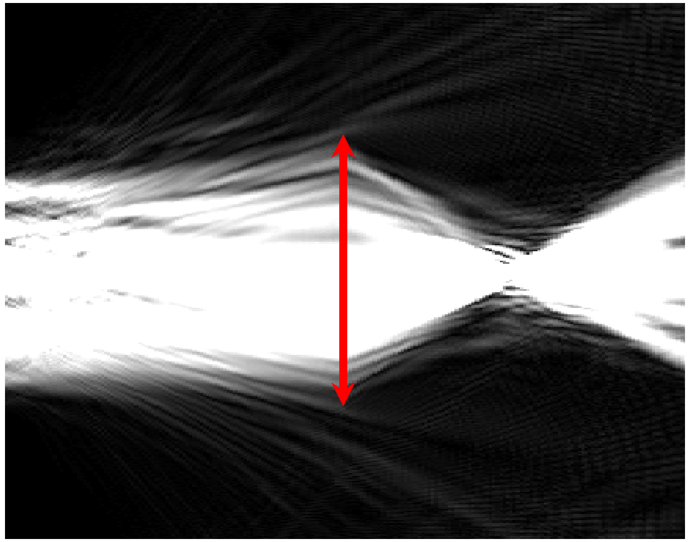
The resolution of the image is worse than before.





Same scheme as earlier, only the lens is now finite

The resolution of the image is worse than before.



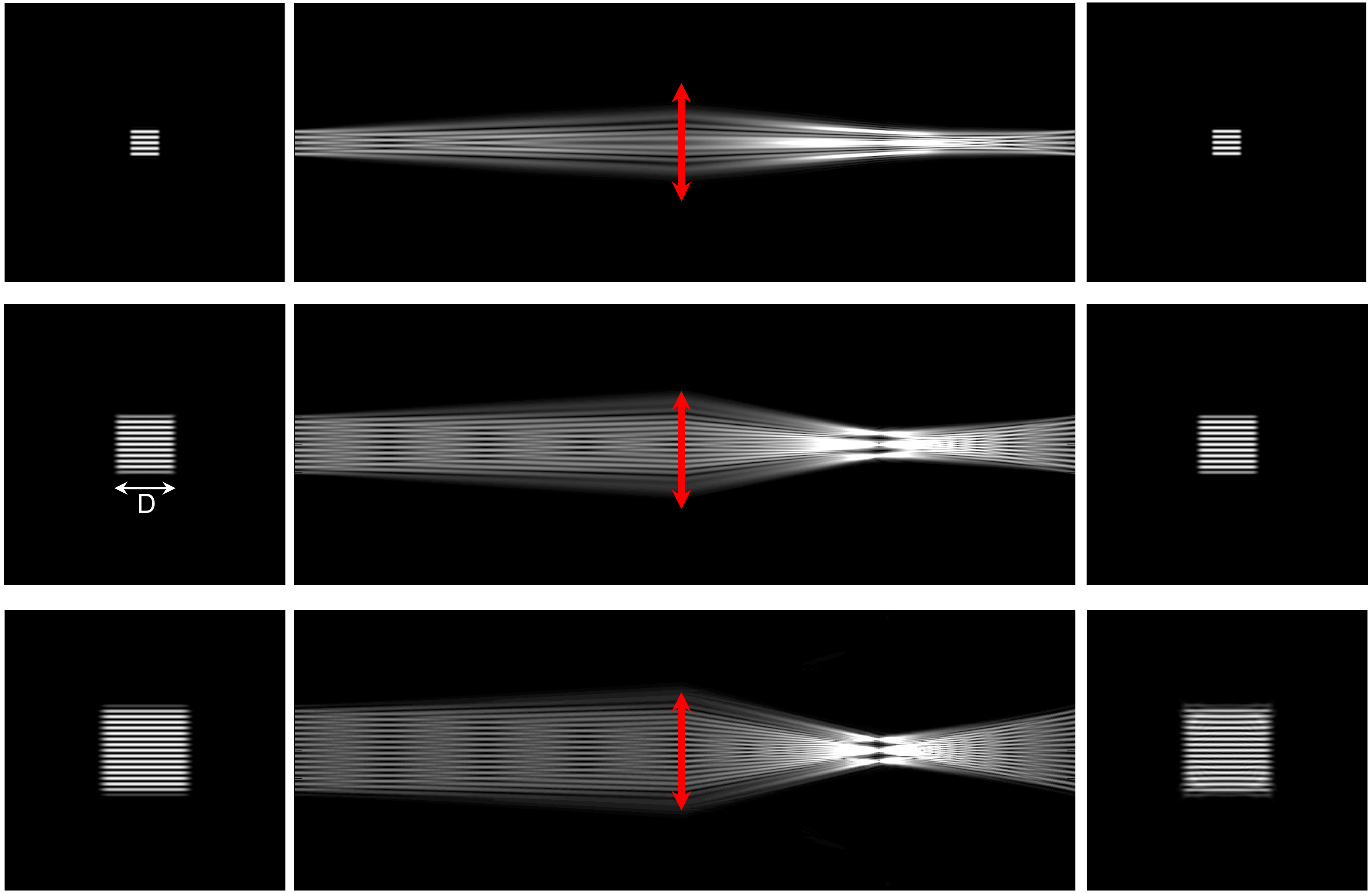
Object size vs finite aperture lens (not far field)

Lens diameter 2mm

D=500µm

D=1mm

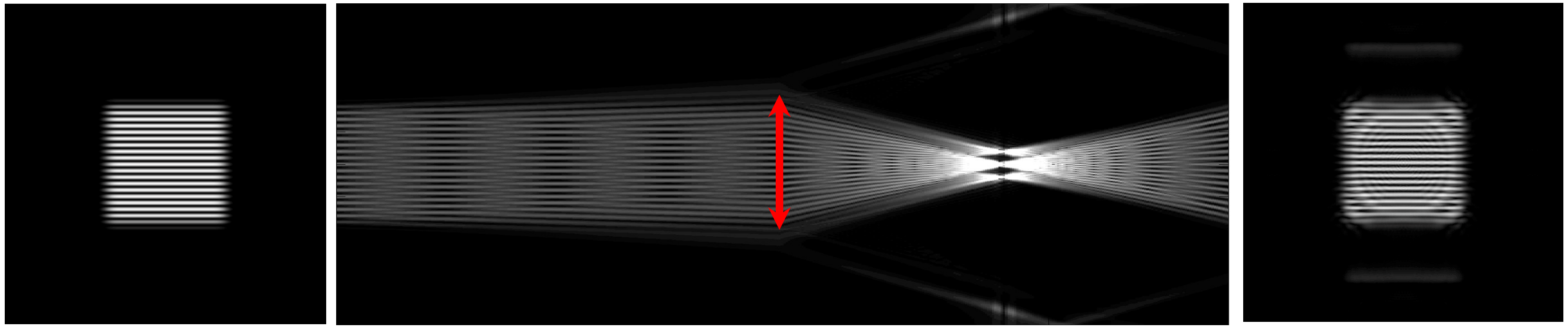
D=1.5mm



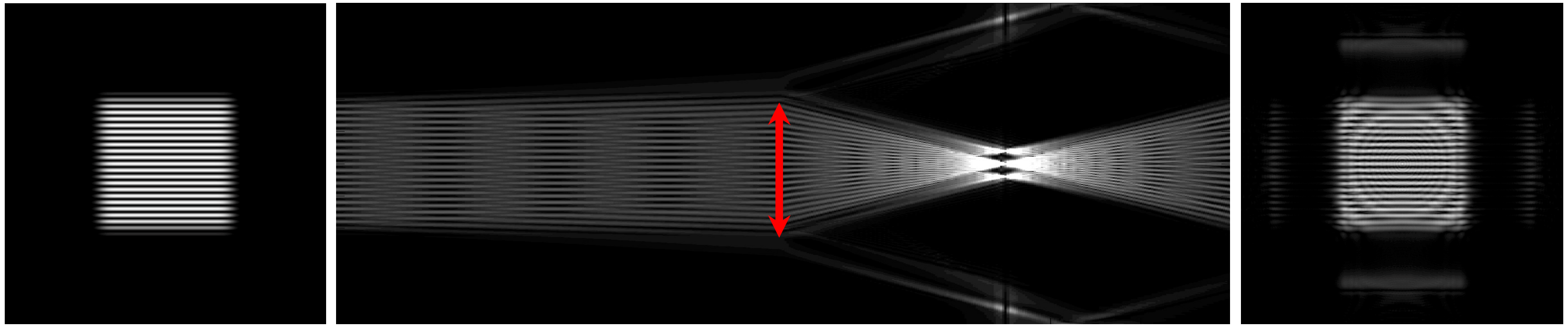
Object size vs finite aperture lens (not far field)

Lens diameter 2mm

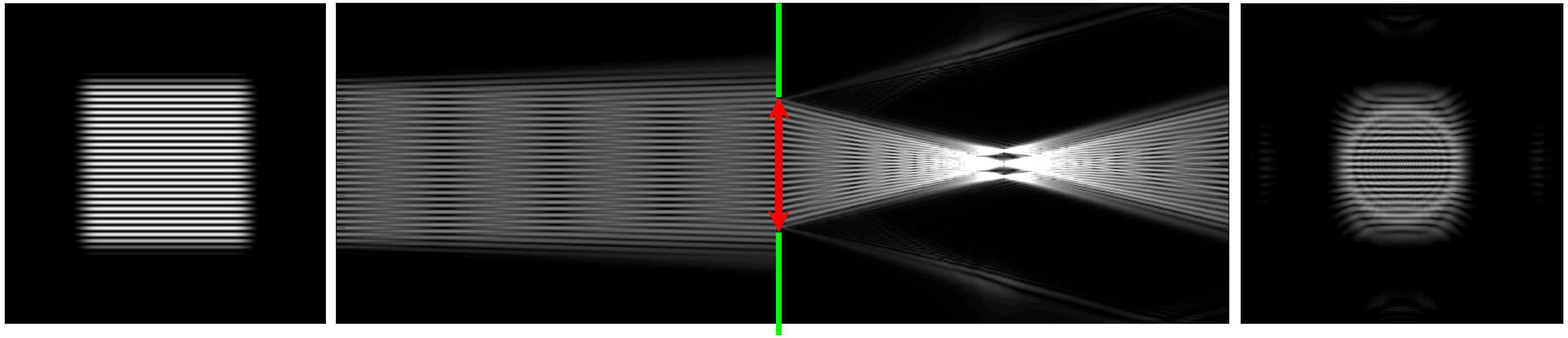
D=1.8mm



D=2mm

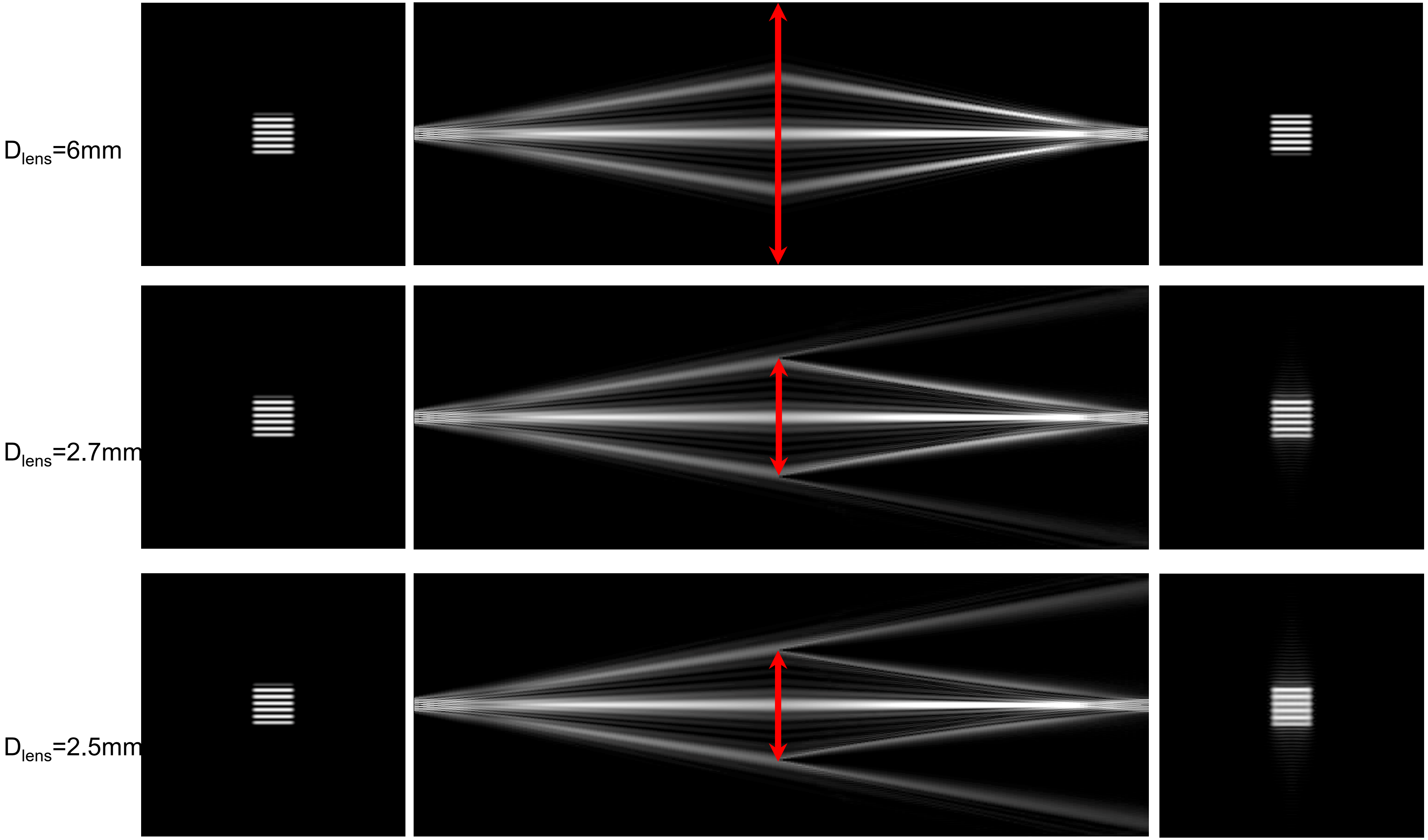


D=2.5m
m



Object resolution vs varying lens aperture

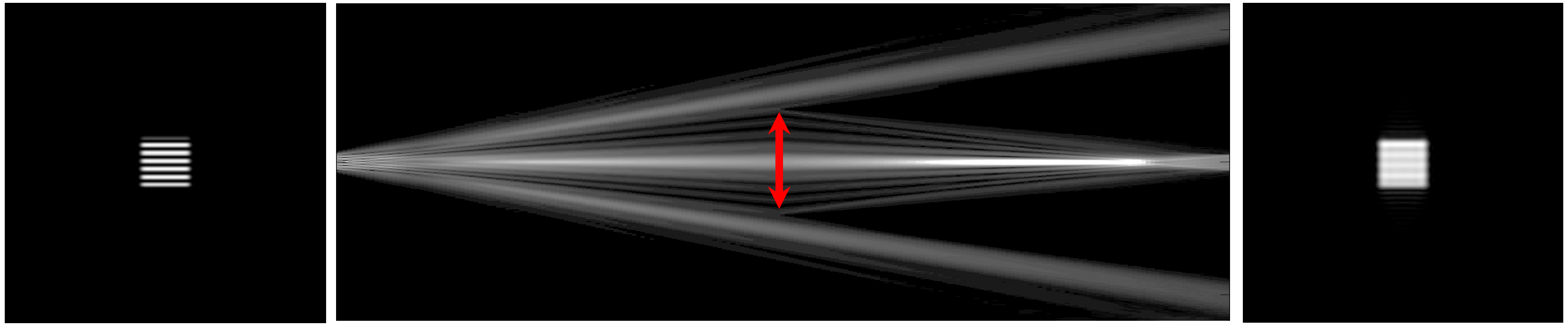
Object side 0.3mm



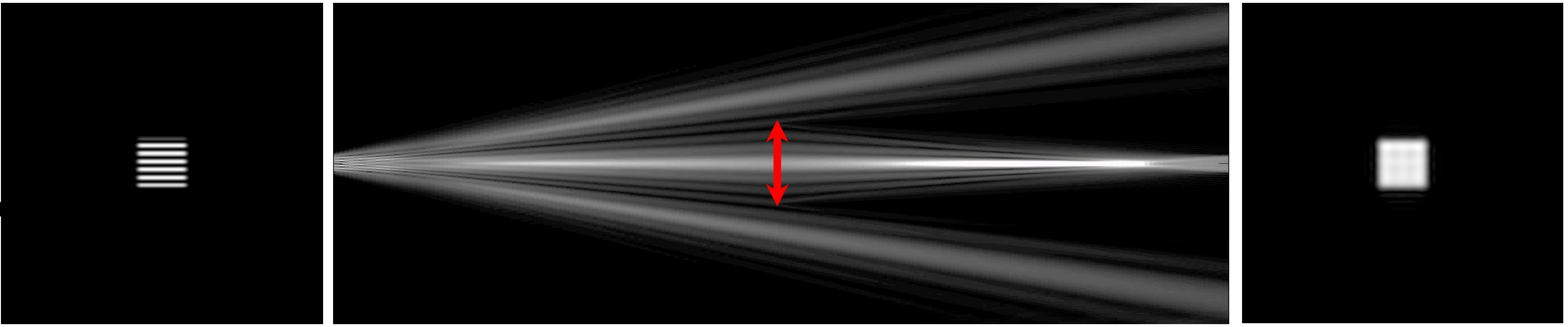
Object resolution vs varying lens aperture

Object side 0.3mm

D_{lens}=2mm



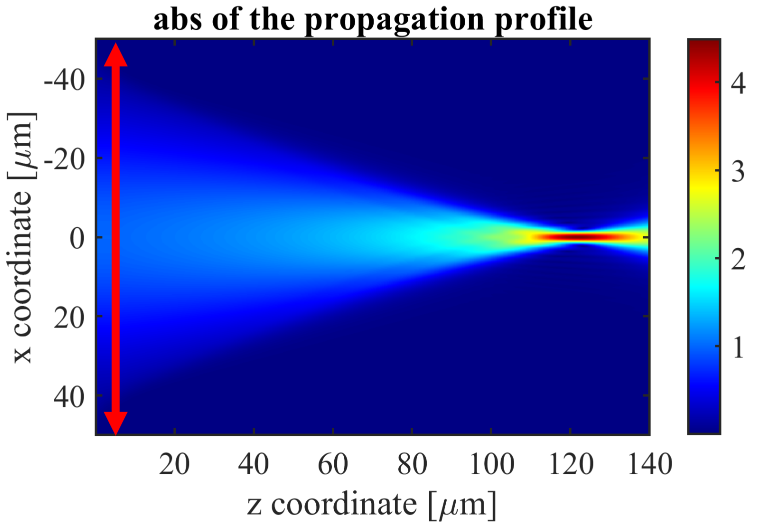
D_{lens}=1.5mm



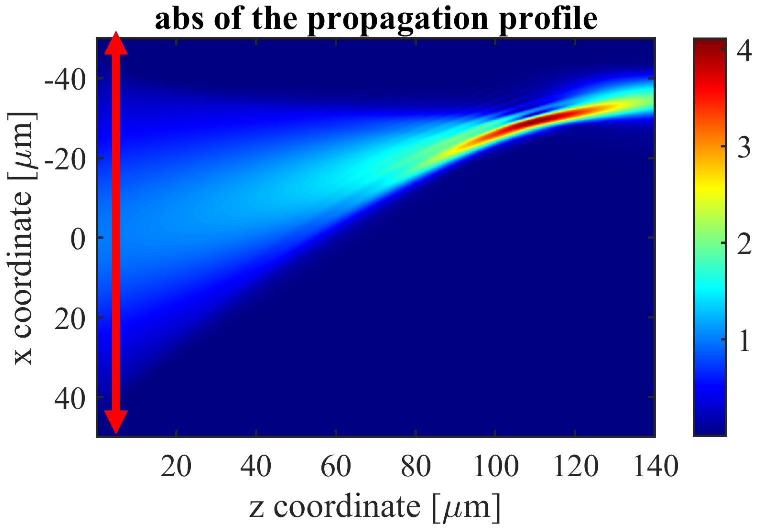
Ideal thin lens – 2D BPM

F=120μm

Θ=0°



Θ=15°



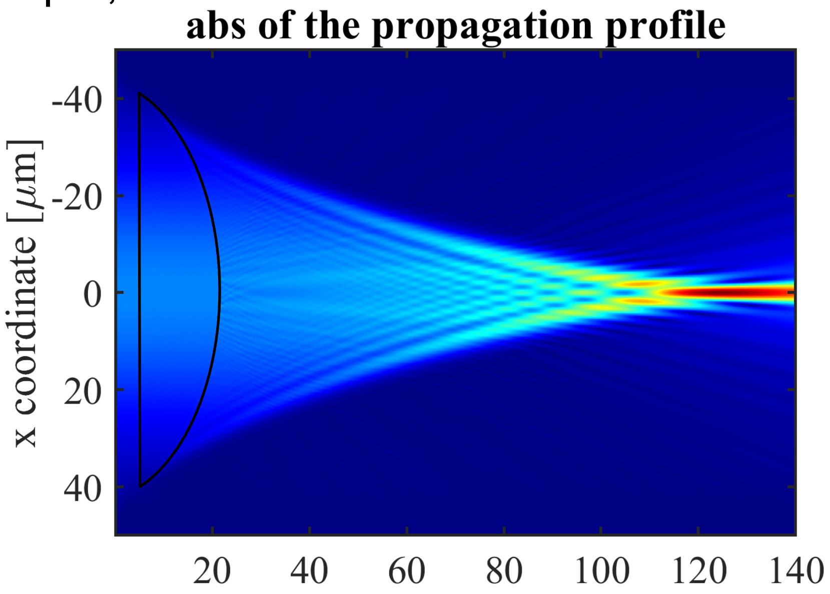
Physical implementation with quadratic surface

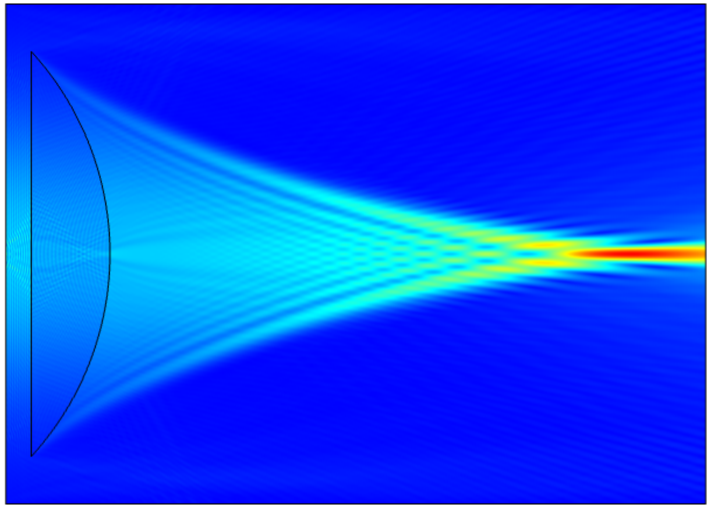
F=120μm,
t=16

2D BPM

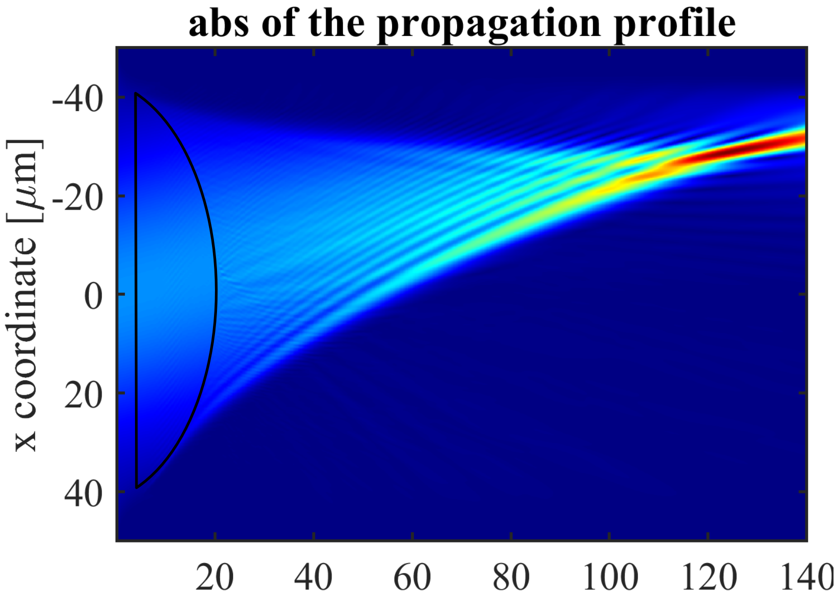
2D COMSOL

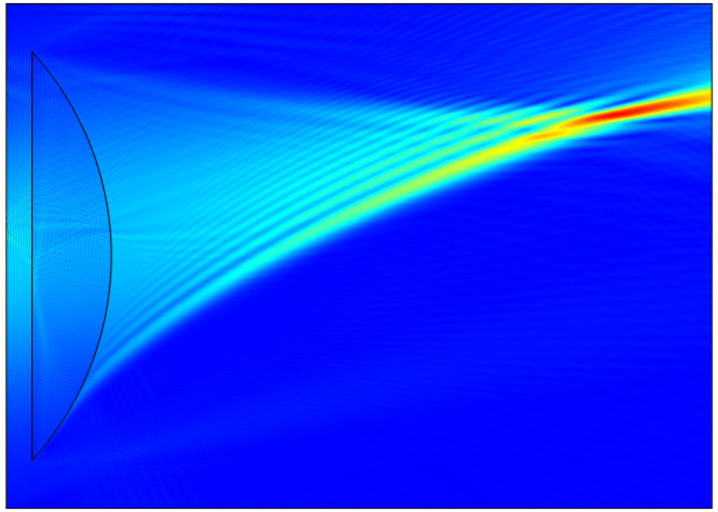
Θ=0°



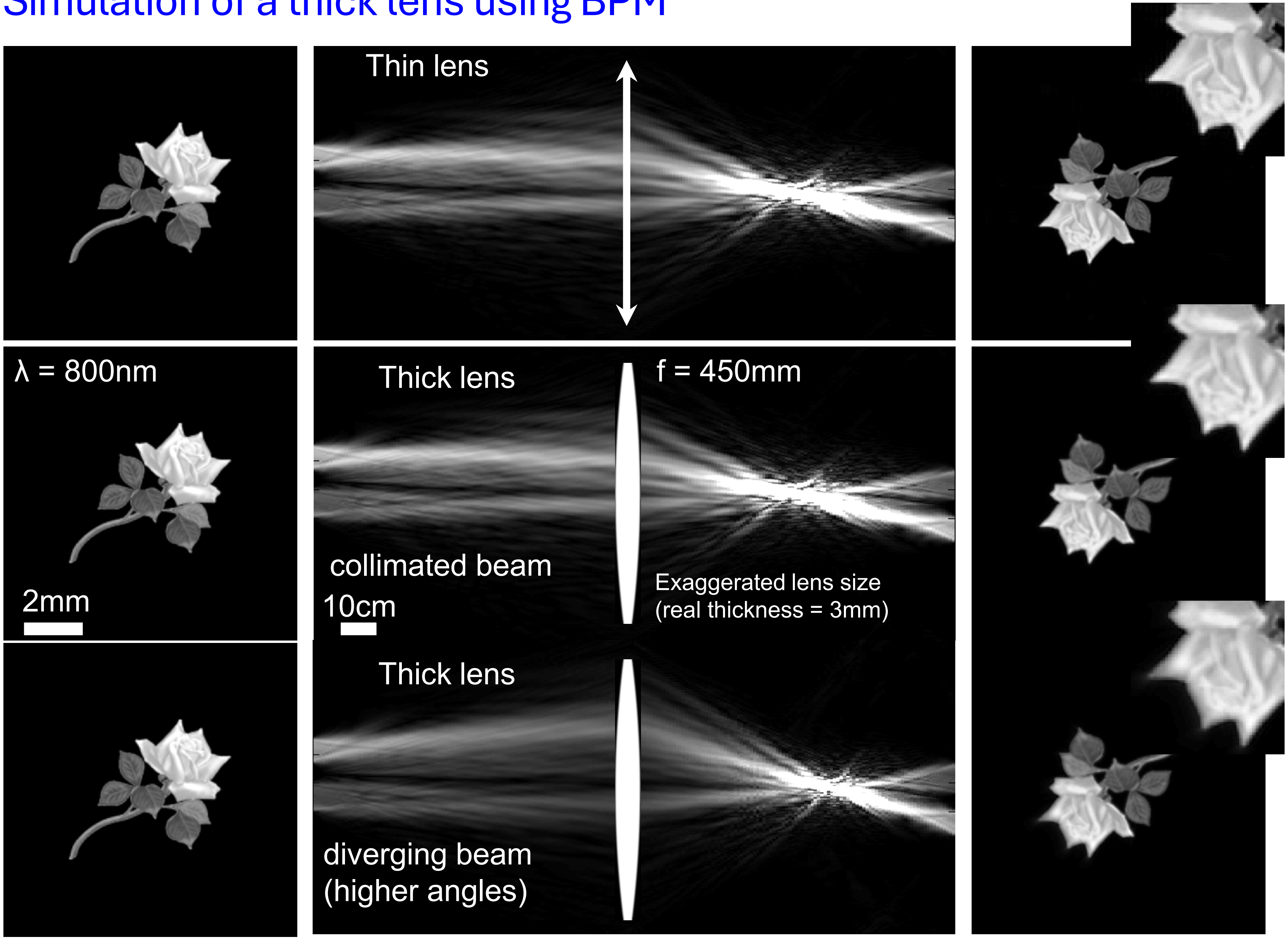


Θ=15°





Simulation of a thick lens using BPM



Exercise 2

Shape of ideal, thin lens versus thick thick lens