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Human color vision and the unsaturated blue color of the daytime sky

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The usual answer to the question "Why is the sky blue?" is based only on Rayleigh scattering from the molecules in the atmosphere and makes little mention of the contribution of color vision. We supplement this answer with a quantitative discussion of the role color vision plays in determining the appearance of the daytime sky. The anatomy of the human eye is reviewed, and its response as a function of wavelength is described via the spectral sensitivities of the cones. Color matching is examined for a mixture of monochromatic lights and for the spectrum of the daytime sky. The spectral irradiance of skylight is shown to be a metameric match to unsaturated blue light. A simple experiment is described suitable for classroom use or a student project.

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I. INTRODUCTION

The question "Why is the sky blue?" is posed and answered in physics textbooks at all levels, from the introductory, such as Serway and Jewett,[1] to the more advanced texts on electromagnetism, such as Jackson.[2] The popularity of this topic rests on the fact that students appreciate a physical explanation for such an easily observed and beautiful natural phenomenon. The typical explanation goes roughly as follows.

The electromagnetic radiation from the sun at visible wavelengths is shown in Fig. 1(a),[3–5] where the measured spectral irradiance (power per unit area per unit wavelength) is plotted as a function of the wavelength in free space, λ. The sun radiates approximately as a blackbody at a temperature of around 6000 K. Planck's radiation law is also shown in Fig. 1(a) and is given by[6]

$$I_\lambda^{\text{sun}}(\lambda)=\frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}\left(\frac{r_s}{r_{\text{se}}}\right)^2 \tag{1}$$

with the temperature $T=5762$ K. The latter is the temperature obtained by fitting the measured data with the Stefan–Boltzmann formula.[5] In Eq. (1), h is Planck's constant, k is Boltzmann's constant, c is the speed of light in free space, $r_s\approx 6.96\times 10^5$ km is the radius of the sun, and $r_{\text{se}}\approx 1.50\times 10^8$ km is the distance from the sun to the Earth. The results for the blackbody are seen to be a reasonable representation of the sun's radiation.

When the sun's radiation enters the Earth's atmosphere, it is scattered by mainly oxygen and nitrogen molecules. This scattering is known as Rayleigh scattering, and the scattered energy is proportional to $1/\lambda^4$. On a very clear day, the skylight seen at zenith when the sun is well above the horizon is this scattered radiation. Thus, the spectral irradiance for skylight, I_λ^{sky}, shown in Fig. 1(b), is approximately proportional to the product of the spectral irradiance from the sun, the blackbody curve in Fig. 1(a), and $1/\lambda^4$. We will refer to this result as the "Rayleigh sky."

For comparison, the spectral irradiance of zenith skylight measured at sea level on a clear day also is shown in Fig. 1(b).[7] Note that the measured results and those for the Rayleigh sky exhibit the same trend: an increase in irradiance with decreasing wavelength. However, at shorter wavelengths, the measurements fall below the theory. The consequences of this difference will be discussed in more detail in Sec. III.

As a guide, we have indicated at the bottom of Fig. 1(b) the color that the average observer associates with monochromatic light at a given wavelength, for example, blue for wavelengths around 460 nm.[8] Clearly, the scattered radiation is greatest at the shorter wavelengths, the blue end of the spectrum, so this observation often is offered as the explanation for the blue sky. However, from these considerations only, we could equally well say that the sky is violet. The important point is that it is not always obvious what color an observer will assign to a given spectral irradiance.

In many textbooks, the explanation for the blue sky ends at this point; however, there is another factor that should enter the argument, that is, the response of the human visual system. This response is sometimes mentioned, but it is seldom supported by a detailed, quantitative explanation.[9]

The purpose of this article is to provide a modest amount of quantitative information on the human visual system which can be used to supplement the typical textbook discussion of the cause of the blue daytime sky. Because most students, and possibly some instructors, are unfamiliar with the basics of color vision and color matching, we include a brief tutorial on color vision optics (Sec. II). This material is then used in Sec. III to examine the visual response to daytime skylight. The response is shown not to be equivalent to that for any spectral light (monochromatic light), such as blue or violet. Rather, it is equivalent to a mixture of a spectral blue light and white light, which is known as unsaturated blue light. In Sec. IV, simple experiments are described that provide practical demonstrations of the this discussion and analysis. The experiments are suitable for classroom use or a student project. Two problems that reinforce key ideas in the article are in the Appendix.

In this article we consider only the blue color of the zenith sky on a clear day at a time when the sun is well above the horizon. The color of skylight can then be attributed to Rayleigh scattering. Under other conditions additional mechanisms contribute to the color of skylight. For example, at sunset and twilight, absorption by ozone in the atmosphere is a major cause of the blue color of the zenith sky.[10]

Color vision is a complex subject that has been studied for

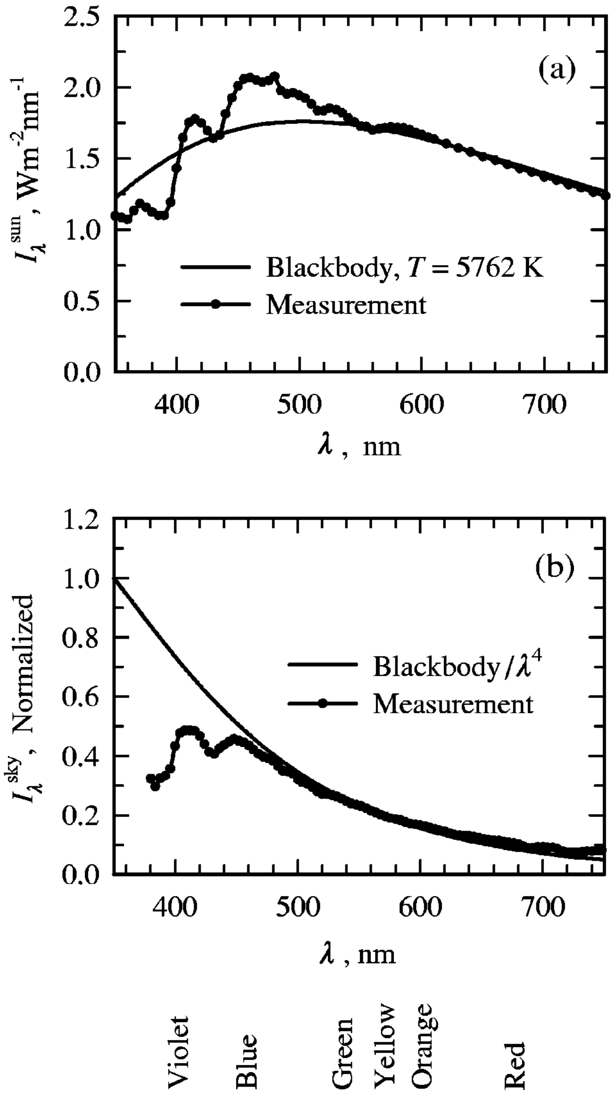

Fig. 1. (a) Spectral irradiance of sunlight outside the Earth's atmosphere: Blackbody at $T = 5762$ K and measurement of Ref. 3. (b) Spectral irradiance of skylight: Rayleigh sky [blackbody from (a) multiplied by $1/\lambda^4$] and measurement of Ref. 7.

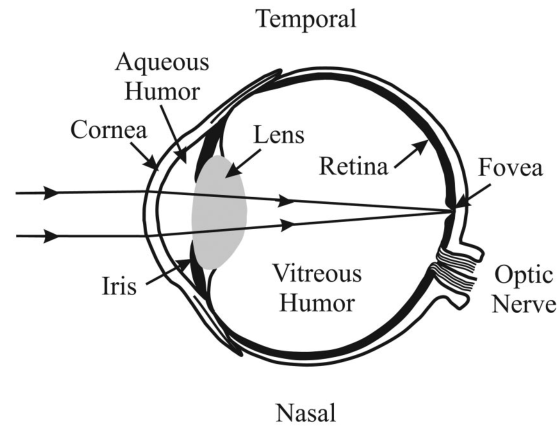

Fig. 2. Schematic diagram of the human eye.

many years and is still an active area of research. Only the most rudimentary concepts are discussed in this article. For more detailed information, the reader is referred to the excellent books that provide a comprehensive overview of this subject.[11–16]

II. HUMAN EYE AND COLOR MATCHING

It is important to stress that color is not a property of light itself, but is a sensation produced by the human visual system. This fact has been recognized for some time and is often put in historical context by quoting from Newton's famous treatise:[17] "... For the Rays to speak properly are not coloured. In them there is nothing else than a certain Power and Disposition to stir up a Sensation of this or that Color." So, when we look for the answer to the question, "Why is the sky blue?," we must begin by examining the process of color vision for the average human observer.

Figure 2 is a schematic diagram that shows the major elements of the human eye.[11,12] Light enters the eye and is brought to a focus on the retina. Most of the refraction (bending) of the light rays occurs at the front surface of the cornea, where the index of refraction changes from $n \approx 1.00$ in air to $n \approx 1.37$. The shape of the lens is changed by the action of the surrounding muscles (accommodation) to bring the light rays from an object at a particular distance into focus on the retina. The lens is made thicker for objects that are closer than it is for objects that are far away.

There are two types of photoreceptors that make up the human retina: rods and cones.[11–14] Each photoreceptor absorbs light and produces a neural (electrical) signal. The signals from all of the photoreceptors are processed within a network of biological cells both within the eye and the brain to produce a visual sensation. The rods are sensitive to low levels of light and do not take part in color vision; the cones are sensitive to higher levels of light (ordinary daylight) and are responsible for color vision. In a typical retina there are roughly 100 million rods and 5 million cones. There are three types of cones; each contains a pigment that absorbs light within a specific range of wavelengths. For reasons that we will see shortly, they are referred to as long-wavelength (L), medium-wavelength (M), and short-wavelength (S) cones. Over 90% of the cones are of the L and M types with the ratio $L{:}M$ within the range 1–4. The fovea is a small depressed region of the retina used for high-resolution color vision.[18] It is covered by a layer called the macula lutea (yellow spot) that acts as a filter that absorbs short wavelengths. The central area of the fovea contains only cones, and these cones are smaller in diameter; hence, their density is higher than in the rest of the retina. When we view an object, such as a word on this page, we adjust the positions of our eyes and head so that the light from the object is brought to a focus on the fovea.

For our purposes, we can think of the eye as a system in which the input is the light entering the cornea, and the outputs are the responses of the cones. We can then construct system functions for the eye that relate the response of the three types of cones to the spectral irradiance of the incident light for wavelengths in the visible range. The system functions account for the physical processes that occur within the eye: absorption by the pigments in the lens and the macula, the conversion of light to an electrical signal in the cones, etc. There are a number of ways to determine these system functions.[11–13,19] Some approaches are based entirely on physical measurements in which the system functions are obtained by combining the measured absorptions in the lens and macula with the measured responses of single excised

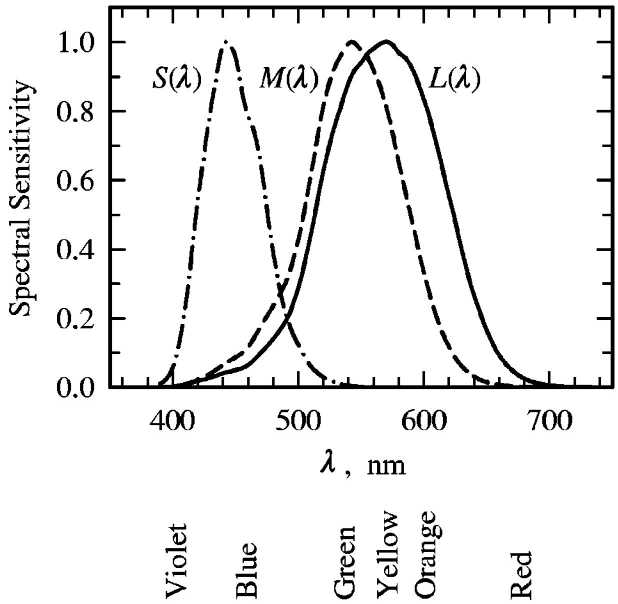

Fig. 3. Spectral sensitivities of human cones for a centrally viewed target two degrees in diameter. The results are for an average, normal observer. Each curve is normalized so that the maximum value of the sensitivity is one. Data from Ref. 27.

cones. The response of a cone can be a spectrophotometric measurement of the absorption by the photopigment[20] or the spectral sensitivity determined from electrode measurements of the photocurrent.[21–23] Other approaches make use of psychophysical techniques—techniques in which the relation between a measured physical stimulus (the light entering the cornea) and the sensation it produces for normal and color-defective observers is established.[19,24] When appropriately adjusted, the system functions obtained by the various methods are in reasonable agreement.[12,24]

Plots of the system functions, the cone spectral sensitivities $L(\lambda)$, $M(\lambda)$, and $S(\lambda)$, for the three types of cones of an average, normal observer are shown in Fig. 3. These results were obtained by Stockman and Sharpe[24–27] using psychophysical techniques in which observers viewed a centrally located target. Each curve in Fig. 3 is normalized to make the maximum value of the sensitivity equal to one. This normalization does not affect the conditions for a color match, as will be shown.

Each curve in Fig. 3 can be thought of as the relative response of the cone (output) as the wavelength is varied for monochromatic light of fixed spectral irradiance (input). The peak sensitivity is at $\lambda \approx 570$ nm for the long wavelength cones (L), $\lambda \approx 543$ nm for the medium wavelength cones (M), and $\lambda \approx 442$ nm for the short wavelength cones (S). Note that the response of a single type of cone contains no information about the specific wavelength of excitation (the ‘‘principle of spectral univariance’’).[11] For example, consider the response of the L cone in Fig. 3. Monochromatic light of wavelength 570 nm with one unit of amplitude produces the same response as monochromatic light of wavelength 513 or 623 nm with two units of amplitude, so it is impossible to associate a particular wavelength with a given level of response. The processing of the signals from all three types of cones at a higher level in the eye/brain is required to associate a particular response (color) with monochromatic light.

The discovery of three types of cones and the measurement of their spectral sensitivities are modern developments. However, *trichromatic theories* of color vision, in which there are three receptors in the eye, for example, one each for red, green, and blue light, have been discussed for many years. They date back at least to the seminal contribution of Thomas Young (1802),[28,29] who was the first to combine the physical concept of a continuously variable wavelength with the physiological concept of three receptors.[30] Now we recognize that the cone sensitivities are the basis for these earlier theories.

We next consider a general light source with the spectral irradiance $I_\lambda(\lambda)$ incident on the eye. At the level of the cones, the sensation of color is determined by the three cone responses R_L, R_M, and R_S

$$R_L = K_L \int_{\lambda_{\min}}^{\lambda_{\max}} L(\lambda) I_\lambda(\lambda)\, d\lambda, \tag{2a}$$

$$R_M = K_M \int_{\lambda_{\min}}^{\lambda_{\max}} M(\lambda) I_\lambda(\lambda)\, d\lambda, \tag{2b}$$

$$R_S = K_S \int_{\lambda_{\min}}^{\lambda_{\max}} S(\lambda) I_\lambda(\lambda)\, d\lambda, \tag{2c}$$

where K_L, K_M, and K_S are constants, and $\lambda_{\min} \approx 350$ nm and $\lambda_{\max} \approx 750$ nm. Because these responses involve integrals of the spectral irradiance, a given set of three responses can arise from different spectral irradiances. In other words, an observer will say two stimuli, a and b, with different spectral irradiances, $I_\lambda^a(\lambda)$ and $I_\lambda^b(\lambda)$, have the same color, that is, their colors match, provided that the stimuli produce the same cone responses

$$R_L^a = R_L^b, \quad R_M^a = R_M^b, \quad R_S^a = R_S^b. \tag{3}$$

This phenomenon is known as *metamerism*, and two stimuli that have different spectral irradiances but produce the same visual response are known as *metamers*.[11,12]

Now we can see why the normalization of the spectral sensitivities in Fig. 3 does not affect the conditions for a color match. Consider one of the equalities in Eq. (3). If we change the normalization of the spectral sensitivities, both sides of the equality would be multiplied by the same constant; the equality, which determines the color match, is unchanged.

Notice that the cone responses in Eq. (2) are a linear function of the spectral irradiance, so a match in Eq. (3) is maintained when the amplitudes of both stimuli are changed in the same proportion.[11] When the irradiance is very high, the responses of the cones saturate, and the linearity expressed by Eq. (2) no longer holds.

The phenomenon of metamerism is nicely illustrated by a well-known color matching experiment involving monochromatic light, actually quasimonochromatic light. Figure 4 shows the arrangement of the apparatus for the experiment. Three lamps project beams of light onto a matte white screen. Each lamp contains a filter of narrow bandwidth $\Delta\lambda^i$ centered about the wavelength λ^i. For simplicity, we will assume that the envelope of the filter, $F^i(\lambda)$, is a rectangular pulse

$$F^i(\lambda) = U\left[\lambda - \left(\lambda^i - \frac{1}{2}\Delta\lambda^i\right)\right] - U\left[\lambda - \left(\lambda^i + \frac{1}{2}\Delta\lambda^i\right)\right], \tag{4}$$

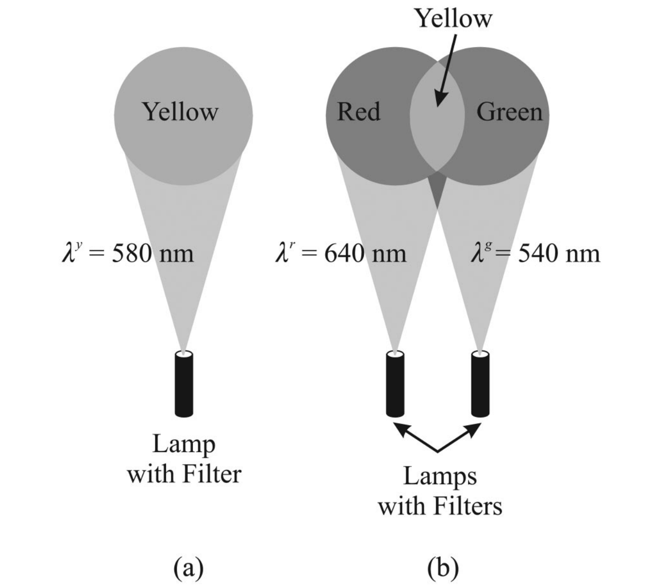

Fig. 4. Schematic drawing for the color matching experiment. (a) Monochromatic yellow light. (b) Monochromatic red and green lights overlap to form yellow.

where U is the Heaviside step function. For the L cones, the response to the filtered light is

$$R_L = K_L \int_{\lambda_{\min}}^{\lambda_{\max}} L(\lambda) F^i(\lambda) I_\lambda(\lambda)\, d\lambda \approx K_L L(\lambda^i) I_\lambda(\lambda^i) \Delta\lambda^i \equiv K_L L(\lambda^i) I_\lambda^i \Delta\lambda^i, \tag{5}$$

with similar results for the M and S cones. In the examples that follow we will take $\Delta\lambda^i = 10$ nm for all filters.

The wavelength for the lamp in Fig. 4(a) is $\lambda^y = 580$ nm, and an observer viewing the screen sees a yellow spot. The wavelengths for the two lamps in Fig. 4(b) are $\lambda^r = 640$ nm and $\lambda^g = 540$ nm, and the amplitudes for the lamps are adjusted as described in the following. Where the two beams from these lamps do not overlap, the observer viewing the screen sees two spots: a red one on the left and a green one on the right. However, where the two beams do overlap, the observer sees yellow with a hue identical to that for the spot in Fig. 4(a) (a metameric match).[31]

For this match, the amplitudes of the lamps must be properly adjusted to obtain equality of the cone responses as given by Eq. (3). We see from Fig. 3 that the wavelengths for all three lamps are long enough so that the responses of the S cones are essentially zero, and hence only the responses of the L and M cones must be matched

$$R_L^y \approx K_L L(\lambda^y) I_\lambda^y \Delta\lambda^y = R_L^r + R_L^g \approx K_L L(\lambda^r) I_\lambda^r \Delta\lambda^r + K_L L(\lambda^g) I_\lambda^g \Delta\lambda^g, \tag{6a}$$

$$R_M^y \approx K_M M(\lambda^y) I_\lambda^y \Delta\lambda^y = R_M^r + R_M^g \approx K_M M(\lambda^r) I_\lambda^r \Delta\lambda^r + K_M M(\lambda^g) I_\lambda^g \Delta\lambda^g. \tag{6b}$$

We will assume that the bandwidths for all of the filters are the same ($\Delta\lambda^y = \Delta\lambda^r = \Delta\lambda^g$), and solve Eq. (6) for the ratios of the spectral irradiances of the lights

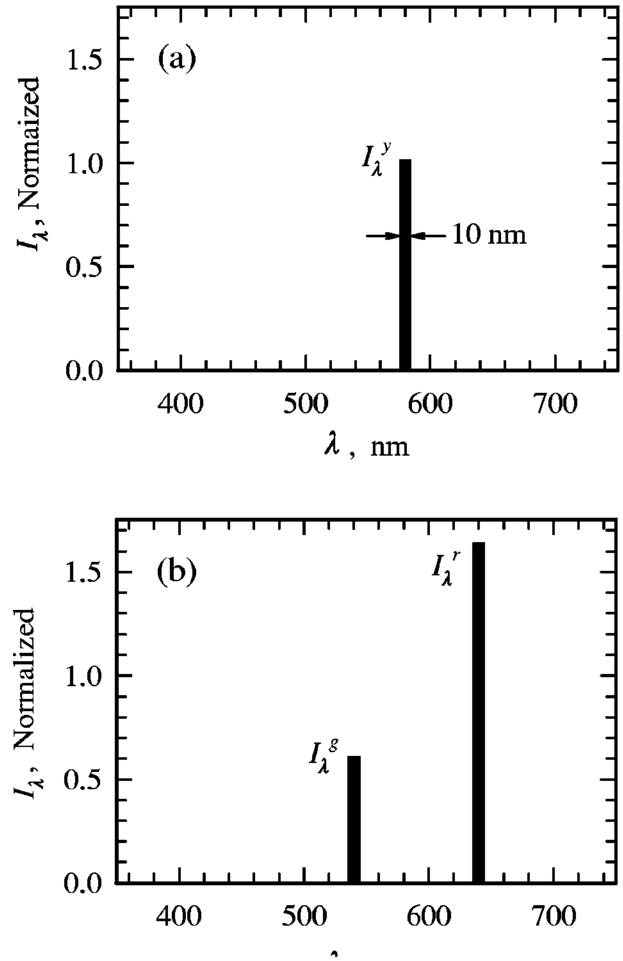

Fig. 5. Spectral irradiances for two lights that produce the same response for the average observer, a metameric match. (a) Monochromatic yellow light, $\lambda^y = 580$ nm. (b) The combination of monochromatic red and green lights, $\lambda^r = 640$ nm and $\lambda^g = 540$ nm.

$$\frac{I_\lambda^r}{I_\lambda^y} = \frac{L(\lambda^g)M(\lambda^y) - L(\lambda^y)M(\lambda^g)}{L(\lambda^g)M(\lambda^r) - L(\lambda^r)M(\lambda^g)}, \tag{7a}$$

$$\frac{I_\lambda^g}{I_\lambda^y} = \frac{L(\lambda^y)M(\lambda^r) - L(\lambda^r)M(\lambda^y)}{L(\lambda^g)M(\lambda^r) - L(\lambda^r)M(\lambda^g)}. \tag{7b}$$

When we substitute numerical values for the cone responses taken from Fig. 3 into Eq. (7), we find that $I_\lambda^r / I_\lambda^y \approx 1.64$, $I_\lambda^g / I_\lambda^y \approx 0.605$, or $I_\lambda^r / I_\lambda^g \approx 2.71$. The spectral irradiances for the two stimuli (metamers) are shown in Fig. 5. We say that the combination of monochromatic red and green lights shown in Fig. 5(b) is yellow because it produces the same response for the average observer as the monochromatic yellow light shown in Fig. 5(a).

The experiment that we described is sometimes called a ‘‘Rayleigh match’’ after Lord Rayleigh (John William Strutt, 1842–1919). In 1881 Lord Rayleigh described an experiment in which observers matched a mixture of red and green lights to a yellow light.[32,33] He found that the ratio of amplitudes of the red and green lights required for the match, $I_\lambda^r / I_\lambda^g$ in our notation, was nearly the same for most observers. However, he noticed for observers who suffered from a color vision deficiency (color blindness), including his Cambridge University colleague J. J. Thomson, that the ratio could be quite different. Rayleigh's procedure was later incorporated into a device for detecting anomalies of color vision known as a Nagel anomaloscope.[34]

III. COLOR OF THE DAYTIME SKY

The example in Sec. II showed that the average observer assigns a color (yellow) to the spectral irradiance for a stimulus (mixture of monochromatic red and green lights) when the cone responses for the stimulus match those for a spectral light (monochromatic yellow). As you might expect, the same procedure can be used with the spectral irradiance in Fig. 1(b) to show that the average observer will assign the color blue to daytime skylight. However, before we can carry out this procedure, we need to introduce a few additional concepts.

For the average observer, a stimulus with a uniform spectral irradiance, that is, $I_\lambda^w = \text{constant}$ for visible wavelengths, is seen as white light. A mixture of white light with a monochromatic light is called an unsaturated color. The degree of saturation is determined by the relative amount of the monochromatic light, with complete saturation occurring when there is no white light. An example is white light mixed with a monochromatic red light to produce pink light.

Now we will try to obtain a metameric match for skylight with the spectral irradiance I_λ^{sky} to a mixture of white light and a monochromatic light of unknown wavelength λ^u. The spectral irradiance for the mixture is

$$I_\lambda^{\text{mix}}(\lambda) = I_\lambda^w + I_\lambda^u \{U[\lambda - (\lambda^u - \Delta\lambda^u/2)] - U[\lambda - (\lambda^u + \Delta\lambda^u/2)]\}. \tag{8}$$

The objective is to determine λ^u and the ratio I_λ^w/I_λ^u for the match. Because the spectral irradiance of skylight, Fig. 1(b), is given only to within a constant multiplier, Eq. (3) implies only that the ratios of the individual cone responses for the two stimuli must be equal

$$\frac{R_L^{\text{mix}}}{R_L^{\text{sky}}} = \frac{R_M^{\text{mix}}}{R_M^{\text{sky}}} = \frac{R_S^{\text{mix}}}{R_S^{\text{sky}}}. \tag{9}$$

We substitute Eq. (8) into Eq. (2) and insert the results into the equality on the left-hand side of Eq. (9) and obtain

$$\left.\frac{I_\lambda^w}{I_\lambda^u}\right|_{L,M} = \frac{\Delta\lambda^u[M(\lambda^u)(R_L^{\text{sky}}/K_L) - L(\lambda^u)(R_M^{\text{sky}}/K_M)]}{(R_M^{\text{sky}}/K_M)\int_{\lambda_{\min}}^{\lambda_{\max}} L(\lambda)d\lambda - (R_L^{\text{sky}}/K_L)\int_{\lambda_{\min}}^{\lambda_{\max}} M(\lambda)d\lambda} \tag{10}$$

and similarly for the equality on the right-hand side of Eq. (9)

$$\left.\frac{I_\lambda^w}{I_\lambda^u}\right|_{S,M} = \frac{\Delta\lambda^u[M(\lambda^u)(R_S^{\text{sky}}/K_S) - S(\lambda^u)(R_M^{\text{sky}}/K_M)]}{(R_M^{\text{sky}}/K_M)\int_{\lambda_{\min}}^{\lambda_{\max}} S(\lambda)d\lambda - (R_S^{\text{sky}}/K_S)\int_{\lambda_{\min}}^{\lambda_{\max}} M(\lambda)d\lambda}. \tag{11}$$

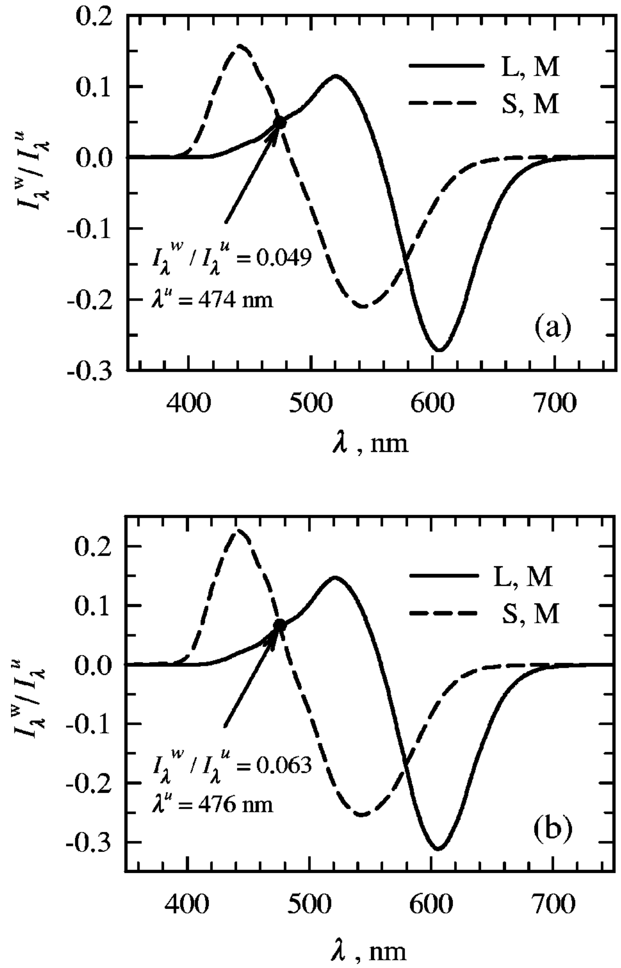

Fig. 6. Graphs for determining the mixture of white light and monochromatic light of wavelength λ^u that is a metameric match to skylight. (a) Rayleigh sky, and (b) measured skylight.

For a match, Eq. (10) must equal Eq. (11), so a solution can be found by plotting both Eqs. (10) and (11) as a function of λ^u and finding the values of λ^u and I_λ^w/I_λ^u at which the two curves intersect. In Fig. 6(a) this has been done using the spectral irradiance for the Rayleigh sky given in Fig. 1(b). Note that there is only one point of intersection for which I_λ^w/I_λ^u is positive, and this intersection occurs at $\lambda^u \approx 474$ nm and $I_\lambda^w/I_\lambda^u \approx 0.049$. Clearly, λ^u is in the range of wavelengths that the average observer would call blue light (see the scale at the bottom of Fig. 3). The spectral irradiance for this mixture of a monochromatic blue light ($\lambda^u = \lambda^b$) and white light is shown in Fig. 7(a). Note that the vertical scale is logarithmic.

This calculation was also performed using the measured spectral irradiance for skylight given in Fig. 1(b); the results are shown in Figs. 6(b) and 7(b). The values, $\lambda^u \approx 476$ nm and $I_\lambda^w/I_\lambda^u \approx 0.063$, are close to those for the Rayleigh sky. So the aforementioned difference between the spectra for the Rayleigh sky and the measurements, shown in Fig. 1(b), has little effect on the perception of the color of skylight. The sensitivity of the S cones is small at the wavelengths for which the difference is the greatest.

Now we have the answer to the initial question. The average observer sees daytime skylight as blue because the response of the eye to the spectrum of skylight is the same as

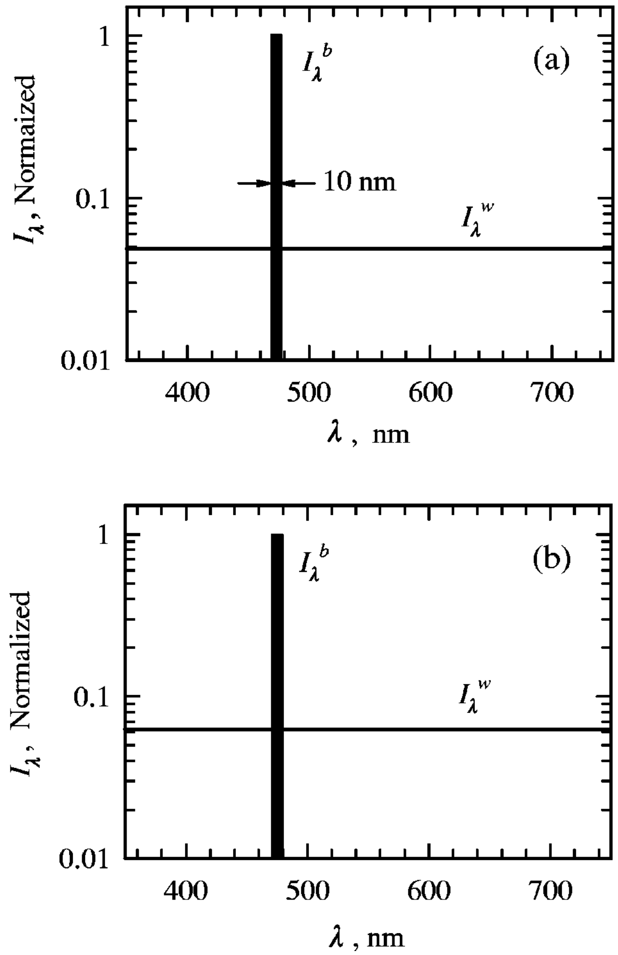

Fig. 7. Spectral irradiance for a mixture of white light and monochromatic blue light that is a metameric match to skylight. (a) Rayleigh sky ($\lambda^b = 474$ nm), and (b) measured skylight ($\lambda^b = 476$ nm).

that to a mixture of monochromatic blue light (spectral blue light) and white light, that is, it is the same as the response to unsaturated blue light.

IV. DEMONSTRATION

The discussion is well illustrated by a relatively simple and inexpensive demonstration that is essentially an implementation of the experiment shown schematically in Fig. 4. Lamps suitable for the demonstration are available from Arbor Scientific (Model P2-9700). Each lamp contains a bulb (type MR-16), a holder with a paper filter, and a lens.[35] The paper filter is replaced by a narrow bandwidth interference filter to produce quasimonochromatic light. Filters with the following specifications are available from Spectra-Physics and Edmund Optics: diameter$\approx$2.5 cm, center wavelength $\lambda^i \pm 2$ nm, and full width at half maximum bandwidth $\Delta\lambda^i_{\mathrm{FWHM}} = 10$ nm± 2 nm. A simple adapter is required to mount the interference filter in the holder. The voltage supplied to each lamp is controlled by a variable transformer (Staco Energy Products, Model 3PN1010B). In a darkened room, light from the lamps is projected onto a matte white screen (Da-Lite Screen) at a distance of about 1.5–2 m.

To match a mixture of monochromatic red light plus monochromatic green light to monochromatic yellow light as described in Sec. II, the center wavelengths for the three filters are $\lambda^r = 640$ nm, $\lambda^g = 540$ nm, and $\lambda^y = 580$ nm. The spectral irradiances for the individual lights are measured at the screen with a suitable meter and adjusted to the values predicted for the average observer. In our demonstration, we use an International Light radiometer (model IL1400BL with SEL033/F/HMR/W detector). This meter measures the irradiance I^i, has a fairly uniform response over the range of visible wavelengths ($\lambda'_{\min} \approx 400$ nm$\leqslant \lambda \leqslant \lambda'_{\max} \approx 700$ nm), and has negligible response outside this range. Hence, the irradiance for a lamp/filter measured with this meter is

$$I^i \approx I^i_\lambda \Delta\lambda^i. \tag{12}$$

If we assume that all of the filters have approximately the same bandwidth, the ratio of the measured irradiances for two lamps (i and j) is approximately equal to the ratio of their spectral irradiances

$$\frac{I^i}{I^j} \approx \frac{I^i_\lambda \Delta\lambda^i}{I^j_\lambda \Delta\lambda^j} \approx \frac{I^i_\lambda}{I^j_\lambda}. \tag{13}$$

When we adjust the irradiances of the lamps to the values predicted for the average observer ($I^r_\lambda / I^y_\lambda \approx 1.64$, $I^g_\lambda / I^y_\lambda \approx 0.605$), the color match is fairly good. However, an individual with normal color vision can generally obtain a slightly better match, at least for him/her, by making small adjustments to the irradiances of the lamps.

The lamps also can be used to illustrate the equivalence of a mixture of monochromatic blue light and white light to skylight, as described in Sec. III. For monochromatic blue light, a filter with a center wavelength of $\lambda^b = 470$ nm or $\lambda^b = 480$ nm is used. Filters are only available at 10 nm increments in the center wavelength, so we cannot use a filter with precisely the same wavelength as predicted by our earlier calculations for the average observer ($\lambda^b = 474$ nm).

There is no bulb that can be used in one of the lamps to produce strictly white light. However, there are bulbs that are designed to have a fairly uniform spectral irradiance at visible wavelengths (EIKO, model SOLUX Q50MR16/CG/47/17). These bulbs use a small halogen lamp to illuminate a coated reflector. The halogen lamp produces more light at the longer wavelengths (red) than at the shorter wavelengths (blue). The reflector is designed to reflect light at the shorter wavelengths more than light at the longer wavelengths.[36] Thus, the combination of halogen lamp plus coated reflector produces light with a fairly uniform spectrum. One of the lamps equipped with this bulb and a neutral density filter (optical density $\approx$1.5–2.0) is used as a substitute for a source of white light. The neutral density filter is used to adjust the irradiance of the lamp rather than changing the voltage supplied to the lamp, because the latter might change the spectrum of the light.

If we assume that the light from the lamp equipped with the special bulb is roughly equivalent to white light, we can estimate the ratio of the measured irradiances required for the two lamps from our previous calculations

$$\frac{I^w}{I^b} \approx \frac{(\lambda'_{\max} - \lambda'_{\min})}{\Delta\lambda^b} \frac{I^w_\lambda}{I^b_\lambda}. \tag{14}$$

For the Rayleigh sky and the average observer ($I^w_\lambda / I^b_\lambda = 0.049$, $\Delta\lambda^b = 10$ nm), Eq. (14) gives $I^w / I^b \approx 1.47$. When we adjust the lamps to give this ratio, their overlapping beams produce light that resembles skylight in color, or, more precisely, what we recall for the color of skylight, because a direct comparison to skylight is not being made. However, an individual with normal color vision can generally obtain a better match to his/her perception of skylight by adjusting the irradiance of the blue light.[37] The two blue filters, $\lambda^b = 470$ nm and $\lambda^b = 480$ nm, produce different hues, but both resemble skylight. The light for the $\lambda^b = 470$ nm filter is a little more violet than the light for the $\lambda^b = 480$ nm filter.

V. SUMMARY

The question, ''Why is the sky blue?,'' has been considered with particular emphasis on the role of the human visual system. After introducing the concept of color matching, the spectral irradiance of daytime skylight was shown to be a metameric match to a mixture of a monochromatic blue light plus white light (unsaturated blue light). Thus, for the average human observer the visual sensation produced by the two is the same. A relatively simple and inexpensive demonstration of color matching that can be used to support classroom discussion of this question was described.

Those familiar with color science may ask why we didn't use the Commission Internationale de l'Eclairage (CIE) system of colorimetry to describe color in this article. We believe that the effort required of the student to become familiar with the CIE system is unnecessary, and that the use of the cone responses, given in Fig. 3, provides a more direct path to the results we wished to present.

We have only considered the blue color of daytime skylight. There are several other interesting phenomena associated with skylight, such as the color at sunrise and sunset and the state of polarization. The causes of these phenomena are described in the literature on atmospheric optics.[38,39]

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APPENDIX A: PROBLEMS

The tables of cone spectral sensitivities (cone fundamentals) available on the Web may be of help in solving these problems.[27]

Problem 1: About 8% of the population has anomalous color vision; the incidence in men is much higher than in women. One group, known as red-green dichromats, lacks either the response of the L cones (protanopia) or the response of the M cones (deuteranopia).[24,40] (a) Obtain relations like those given in Sec. II describing the match of a mixture of monochromatic red light and monochromatic green light to monochromatic yellow light for observers with protanopia and deuteranopia. For these observers, show that there is not a unique mixture for the match (not one set of values for I_λ^r/I_λ^y and I_λ^g/I_λ^y), and that they can obtain a match using only red light or only green light. (b) Make a graph of I_λ^r/I_λ^y vs I_λ^g/I_λ^y required for a match for each type of observer; also include the point for an observer with normal color vision. Do you think that measured data like those in the graph can be used to distinguish observers that have protanopia and deuteranopia from normal observers and from each other?

Problem 2: A planet identical to the Earth, including its atmosphere, orbits a red giant star. At visible wavelengths, the star radiates as a blackbody at the temperature $T = 2500$ K. An observer on the surface of the planet looks at the sky in a direction away from the direct radiation from the star. (a) Make graphs, like those in Fig. 1, for the normalized spectral irradiance of the direct radiation from the star outside the atmosphere of the planet and for the normalized spectral irradiance of the skylight seen by the observer. What is your guess for the color of the skylight? (b) By following the same procedure as in Sec. III, determine the parameters for a mixture of monochromatic light and white light (λ^u and the ratio I_λ^w/I_λ^u), which is a metameric match to the skylight seen by the observer. What color is the skylight?

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[36]When the reflector of one of these bulbs is viewed with the power off, it has a blue hue.
[37]It is important to realize that we are only adjusting the irradiance of the monochromatic blue light relative to that of the white light to match the ratio of irradiances for skylight. We cannot adjust the overall irradiance of the mixture to equal that of skylight, because the maximum irradiance for the white light is fixed by the output of the bulb.
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