Exercise: Principles and Numerical Analysis of Melt Electrowriting (MEW)

Part 1: Conceptual Questions

1. Define the following key phenomena in MEW and discuss whether they are desirable or problematic. If they are issues, suggest possible solutions:
    - Arcing
    - Whipping
    - Fiber Pulsing
    - Critical Translational Speed (CTS)
2. How does collector speed influence fiber diameter in MEW? What challenges arise when increasing collector speed beyond a certain threshold?

Part 2: Numerical Problem

A Melt Electrowriting system is used to print Polycaprolactone (PCL) fibers with the following parameters:

- Flow rate (Q) = 10 μL/h
- Nozzle diameter (D_o) = 200 μm
- Voltage applied (V) = 8 kV
- Collector speed (v_c) = 500 mm/min

Use mass conservation to solve for the value of fiber diameter D_f.

Useful Information to solve the question:

The mass flow rate ($\dot{m}$) at the nozzle and the fiber can be expressed as:

$$\dot{m} = \rho \times Q$$

where:

- ρ = polymer density (assumed constant)
- Q = volumetric flow rate (in μL/h or mm^3/s)

Since the volume must remain conserved, we can also write:

$$Q = A_0 v_0 = A_f v_C$$

where:

- $A_0 = \pi(D_0/2)^2$ is the cross-sectional area of the nozzle
- v_0 is the velocity of the polymer at the nozzle
- $A_f = \pi(D_f/2)^2$ is the cross-sectional area of the final fiber
- v_c is the collector speed (or fiber deposition speed)

Questions:

1. Calculate the theoretical fiber diameter D_f.
2. If the applied voltage is increased to 12 kV, explain qualitatively how this would influence the fiber diameter and jet stability.

Solution:

Part 1: Conceptual Questions

1. Definitions and Solutions for Common MEW Phenomena:
   - **Arcing:** Arcing occurs when an electrical discharge jumps between the nozzle and the collector due to excessive voltage.
     - Issue? Yes, it can damage equipment and disrupt fiber deposition.
     - Solution: Reduce the applied voltage, increase the nozzle-collector distance, or improve insulation.
   - **Whipping:** A rapid, chaotic oscillation of the polymer jet due to electrostatic instabilities.
     - Issue? Yes, as it leads to inconsistent fiber placement and non-uniform structures.
     - Solution: Optimize the electric field, increase polymer viscosity, or adjust the extrusion rate.
   - **Fiber Pulsing:** Periodic fluctuations in fiber diameter due to unstable flow rates or inconsistent extrusion.
     - Issue? Yes, it reduces print precision and mechanical uniformity of structures.
     - Solution: Stabilize the extrusion system by controlling pressure, heating, and polymer properties.
   - **Critical Translational Speed (CTS):** The minimum collector speed required to prevent fiber sagging or looping.
     - Issue? If the collector speed is too low, fibers will not stretch properly, leading to uneven deposition.
     - Solution: Increase the collector speed to match fiber extrusion, ensuring smooth fiber deposition.
2. Effect of Collector Speed on Fiber Diameter:
   - As collector speed (v_c) increases, the fiber is stretched more, reducing its diameter.
   - If the speed is too high, fibers may break or exhibit defects.
   - Optimal speed balances fiber uniformity and mechanical properties.

Part 2: Numerical Problem

Given Data:

- Flow rate: Q = 10 µL/h
- Nozzle diameter: D_0 = 200 µm = 0.2 mm
- Collector speed: v_c=500 mm/min
- Cross-sectional area of the nozzle:

$$A_0 = \frac{\pi D_0^2}{4}$$

Step 1: Compute Theoretical Fiber Diameter

Using the formula:

$$D_f = D_0 \times \sqrt{\frac{Q}{v_C \times A_0}}$$

Substitute values and obtain the answer: 20.6 µm

Step 2: Effect of Increasing Voltage to 12 kV

- Higher voltage increases electrostatic forces, which enhances fiber stretching, leading to a smaller fiber diameter.
- If voltage is too high, instabilities like whipping may occur, leading to non-uniform deposition.
- Solution: Optimize the voltage to balance fiber stretching and stability.

Final Answers:

1. Theoretical fiber diameter D_f = 20.6 µm
2. Increasing voltage to 12 kV → Decreases fiber diameter but may cause jet instabilities like whipping.

Derivation of the formula:

Step-by-Step Derivation:

The principle of mass conservation states that the mass flow rate of the polymer must remain constant between the nozzle and the final deposited fiber.

Step 1: Define the Mass Flow Rate

The mass flow rate ($\dot{m}$) at the nozzle and the fiber can be expressed as:

$$\dot{m} = \rho \times Q$$

where:

- ρ = polymer density (assumed constant)
- Q = volumetric flow rate (in µL/h or mm^3/s)

Since the volume must remain conserved, we can also write:

$$Q = A_0 v_0 = A_f v_C$$

where:

- $A_0 = \pi(D_0/2)^2$ is the cross-sectional area of the nozzle
- v_0 is the velocity of the polymer at the nozzle
- $A_f = \pi(D_f/2)^2$ is the cross-sectional area of the final fiber
- v_c is the collector speed (or fiber deposition speed)

Since mass is conserved,

$$A_0 v_0 = A_f v_C$$

Step 2: Expressing in Terms of Fiber Diameter

Rewriting the areas:

$$\pi \left(\frac{D_0}{2}\right)^2 v_0 = \pi \left(\frac{D_f}{2}\right)^2 v_C$$

Cancelling same terms on each side:

$$D_0^2 v_0 = D_f^2 v_C$$

Since $v_0 = Q/A_0$, we substitute:

$$D_0^2 \left(\frac{Q}{A_0}\right) = D_f^2 v_C$$

Rearrange for D_f:

$$D_f = D_o \times \sqrt{\frac{Q}{v_C \times A_0}}$$

where $A_0 = \pi(D_0/2)^2$, which is the cross-sectional area at the nozzle.