



# **MICRO 372 - Advanced Mechanisms for Extreme Environments**

**Chapter 8**

**(Ultra-) High precision**

**Florent Cosandier**

# (Ultra-) High precision

- Calibration
- Flexure's tolerance effect
  - Manufacturing
  - Assembly
- Kinematic couplings

# Calibration of robot or mechanisms

- **Kinematics:**

Calibration of the robot's kinematics involves adjusting parameters related to its **joint angles**, **link lengths**, and **coordinate systems**. This ensures that the robot moves accurately according to its intended trajectory.

- **Accuracy and repeatability:**

Robots often undergo calibration procedures to **improve their accuracy and repeatability**.

- **Sensor calibration:**

If the robot is equipped with sensors such as **cameras**, **force/torque sensors**, or **proximity sensors**, these sensors may need calibration to ensure accurate data acquisition and integration into the robot's control system.

- **Environmental factors:**

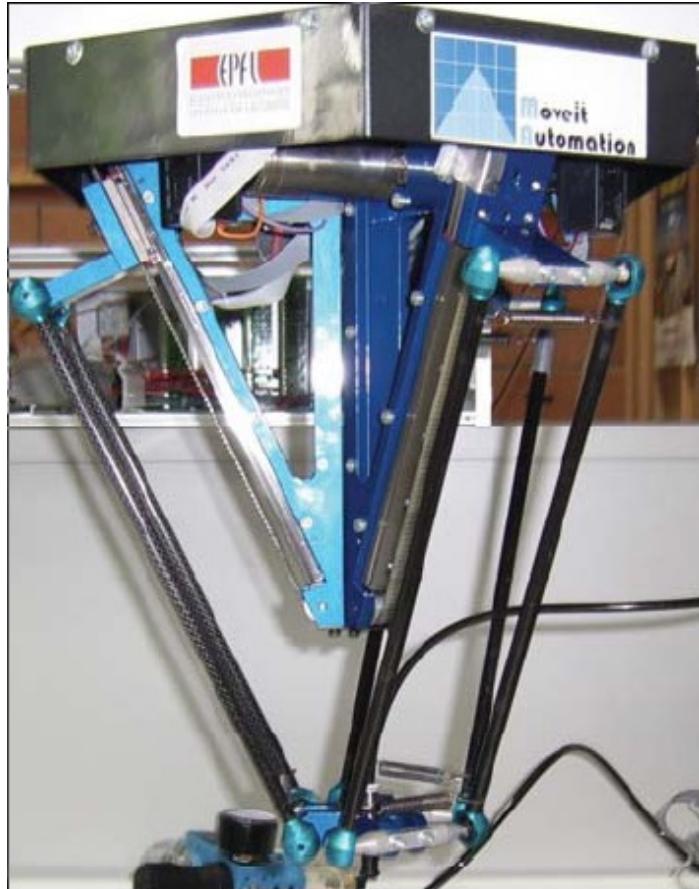
Calibration may also account for environmental factors such as **temperature variations**, **humidity levels**, and **gravitational effects**

- **Load and payload calibration**

calibration may involve adjusting parameters related to **varying loads** or payloads.

# Calibration example: Kheops robot

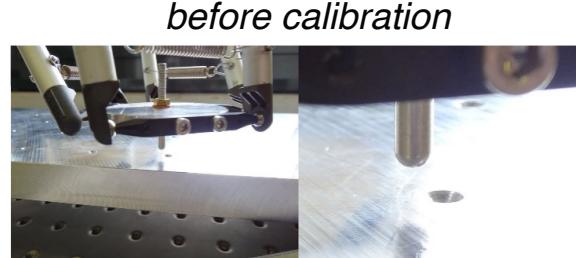
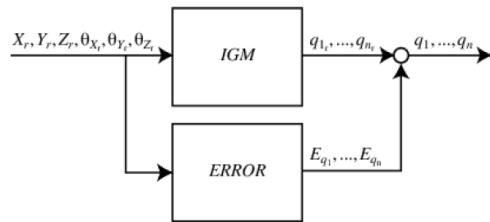
- **Calibrate the robot** makes possible to bring the **actual output** (position of the tool) **as close as possible to the desired theoretical position**.
- The purpose to calibrate a robot is to **correct its positioning errors** due to differences between **theoretical values and actual values**.
- These errors can be due to an **inaccuracy of the geometric model**, like imprecision of the lengths of the robot arms.
- **Other types of error** that can be calibrated: **sensor drift, nonlinearity, hysteresis, noise, interference**.
- Or to an influence of the **robot's environment**: **temperature, humidity, pressure**.



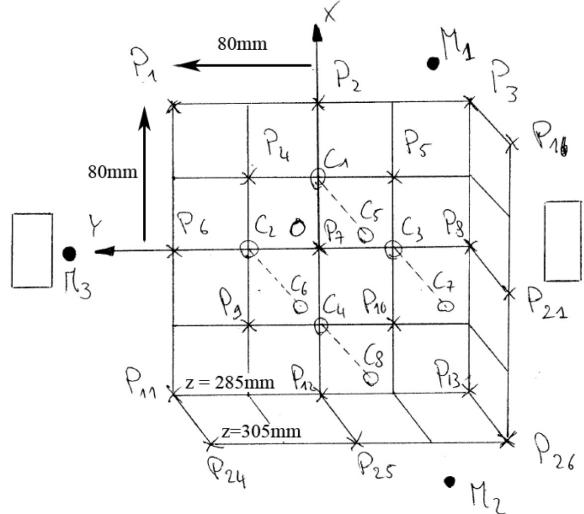
# Calibration example: Kheops robot

## Calibration procedure:

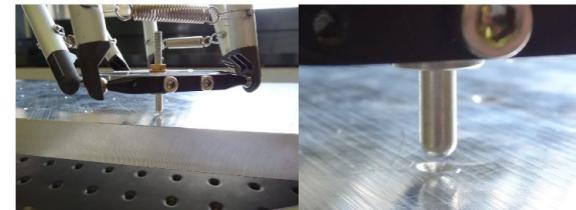
- Calculate the IGM
- Manual positioning the robot on precise points (motors off)
- Measurement of the motor coordinates at these points
- Determination of a polynomial fit of measured errors
- Addition to the IGM
- Validation of the correction by comparing the motor theoretical values and measured values on control points (motors still off)



Calibration points ( $P_1-P_{26}$ ) and control points ( $C_1-C_8$ )



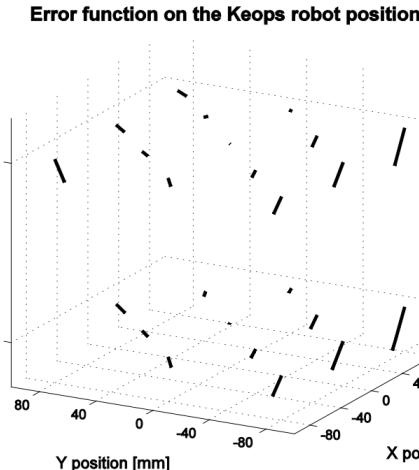
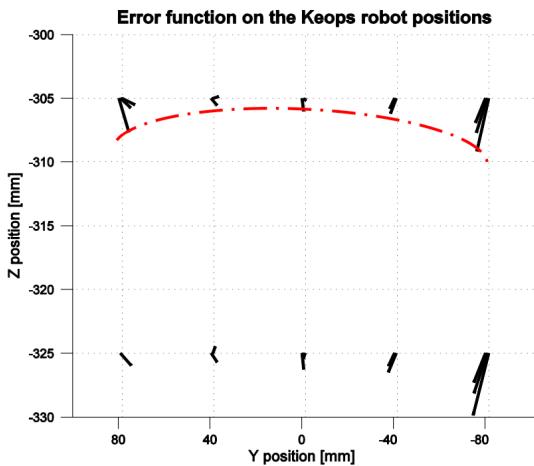
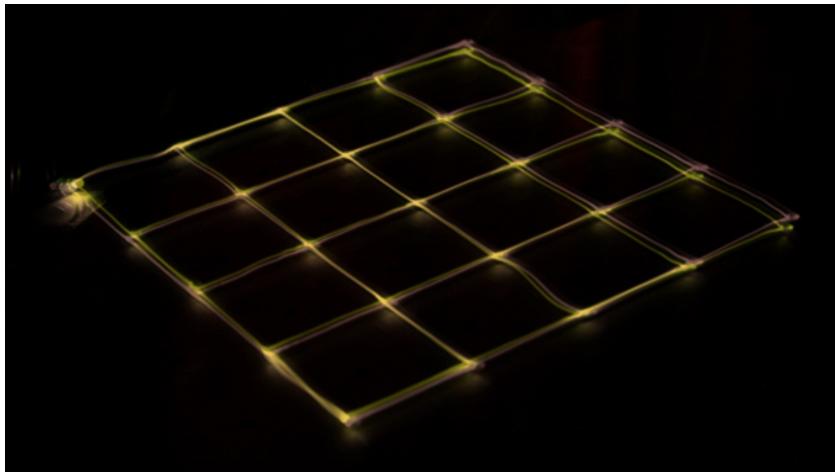
*after calibration*



# Calibration example: Kheops robot

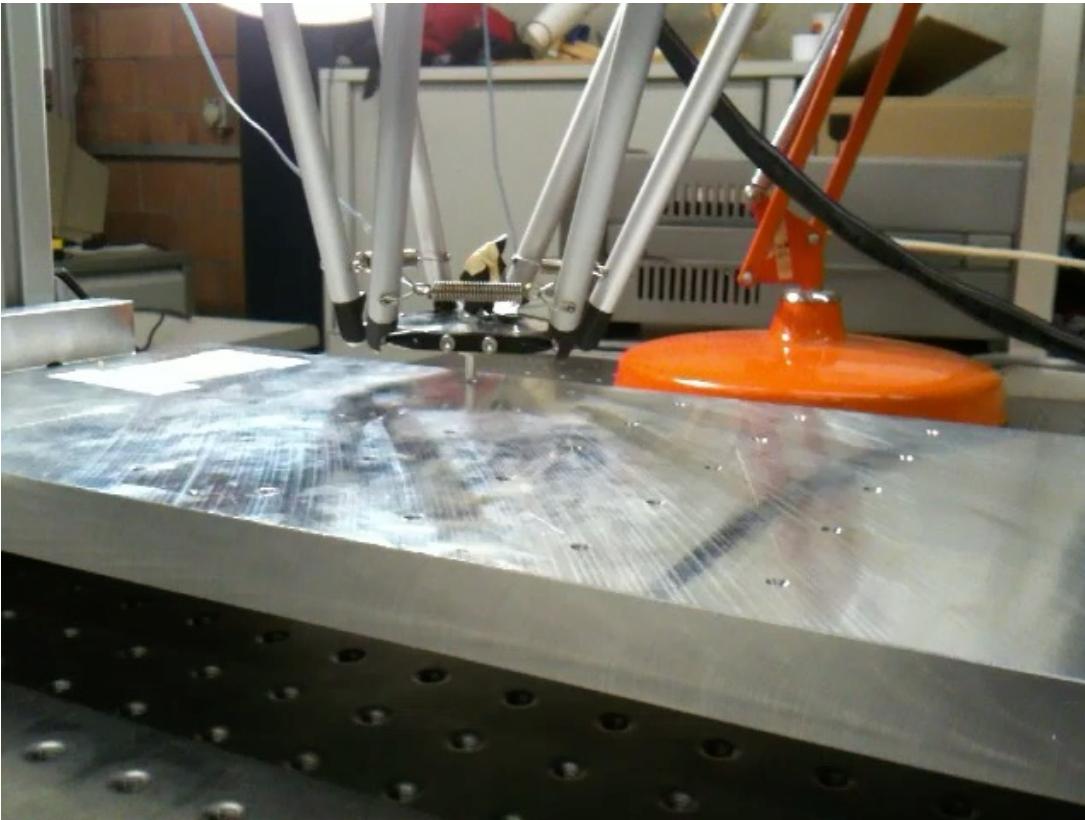
## Results:

- Prior to calibration, **the workspace of the robot is distorted**, mainly due to bad length identification of the robot arms.
- **Twisting, tilting and scaling effects** are removed from the workspace of the robot
- Resulting **absolute accuracy** over the workspace  $< 20 \text{ }\mu\text{m}$



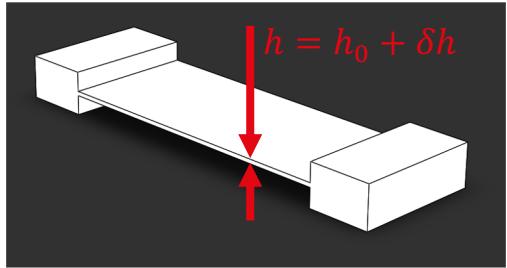
# Calibration example: Kheops robot

*Kheops robot's tool stays at 0.2 mm of the surface*



# Flexure's manufacturing tolerance calculation

$\delta h$  depends mainly on the manufacturing technology



## STIFFNESS

$$K = K_0 \cdot \left( \frac{h_0 + \delta h}{h_0} \right)^3 = K_0 \cdot \left( 1 + 3 \frac{\delta h}{h_0} + 3 \frac{\delta h^2}{h_0^2} + \frac{\delta h^3}{h_0^3} \right) \cong K_0 \cdot \left( 1 + 3 \frac{\delta h}{h_0} \right)$$

$\frac{\delta h}{h_0} \ll 1$

$$\Delta K \cong 3 \frac{\delta h}{h_0}$$

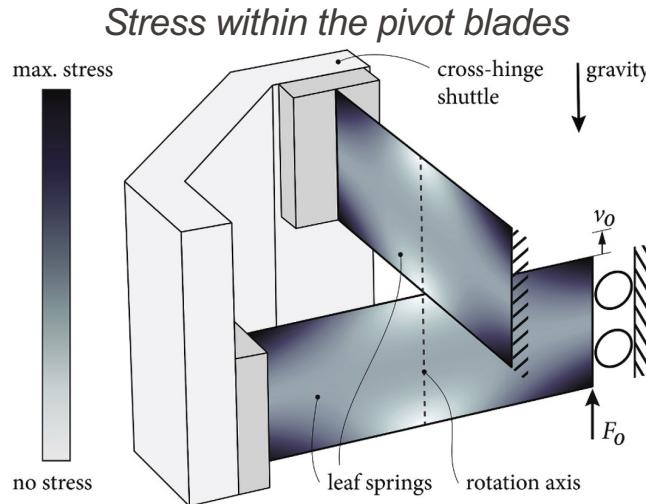
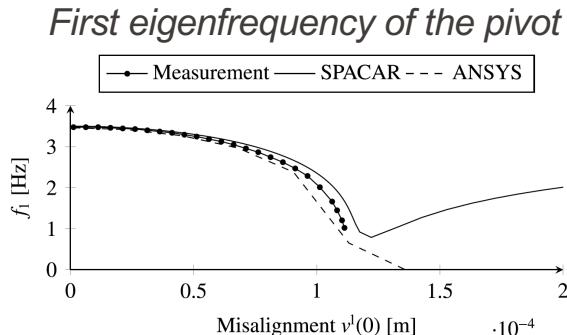
## STRESS

$$\sigma \sim h$$

$$\Delta \sigma \cong \frac{\delta h}{h_0}$$

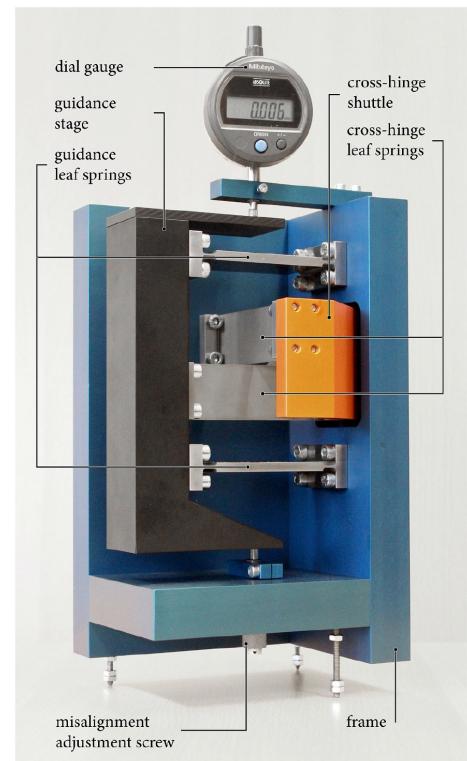
# Flexure's assembly tolerance calculation

- The effect of assembly tolerances is assessed
- Analytical and FEM values are compared with measurements



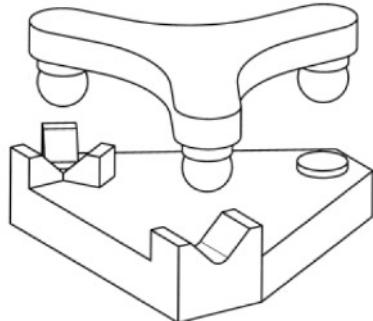
Overconstrained cross-hinge design consisting of two leaf springs. Gray scales in the leaf springs represent the von Mises stress distribution caused by misalignment  $v_0$  due to a force  $F_0$ .

## Experimental setup

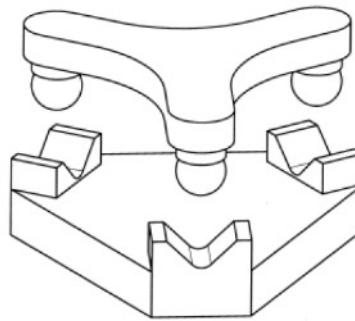


# Kinematic coupling: Kelvin and Maxwell Clamps

- Kinematic couplings are **exact constraint design couplings** because they use six known contact points to locate one component with respect to another.
- They have long been known to provide an **economical and dependable method** for attaining high repeatability in fixtures.
- **Kinematic couplings are deterministic** because they only make contact at a number of points equal to the number of degrees of freedom that are to be restrained.



**Kelvin clamp**



**Maxwell clamp**

# Kinematic coupling: Hertzian contact

- Heinrich Hertz is a mathematician famous for his work in the frequency domain
- He also created the first analytical solution for determining the stress between two bodies in point contact
- Exact constraint design often creates contact at single points

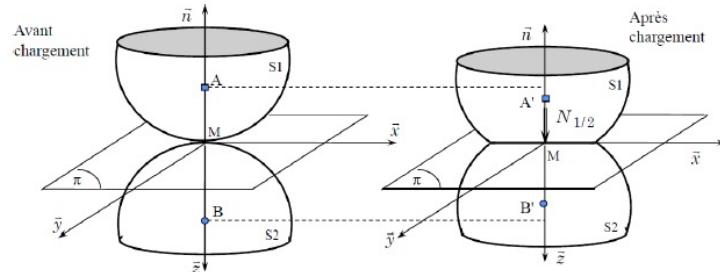


FIGURE 3 – Modélisation des déformations

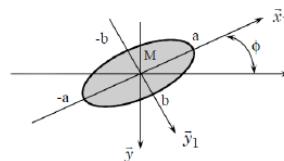


FIGURE 4 – Zone de contact

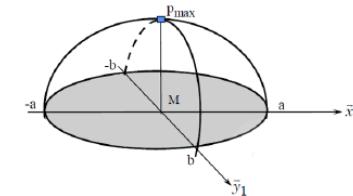
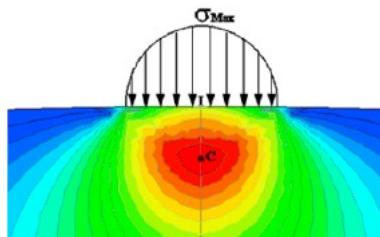


FIGURE 5 – Répartition de pression (ellipsoïde)

On détermine alors :

$$p_{\text{moy}} = \frac{N_{1 \rightarrow 2}}{\pi ab} \quad \text{et} \quad p_{\text{max}} = \frac{3}{2} p_{\text{moy}} \quad \Rightarrow \quad p_{\text{max}} = \frac{3 N_{1 \rightarrow 2}}{2 \pi ab}$$

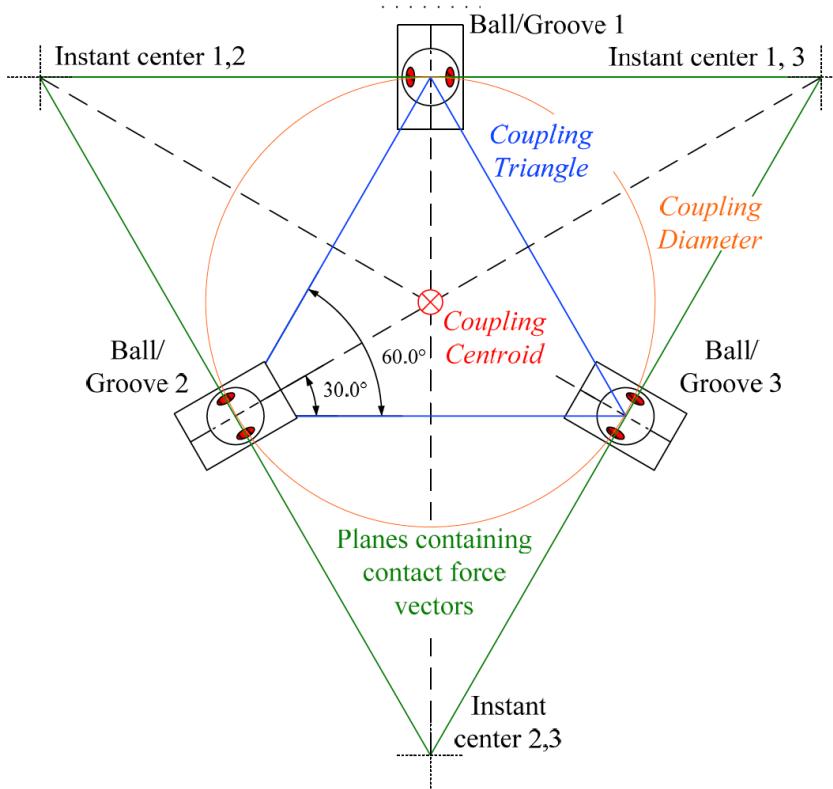


# Kinematic coupling: Hertzian contact

		Contact ponctuel		Contact linéique	
Types de contacts		Solides quelconques	Sphère/sphère	Sphère/plan	Cylindre/cylindre
Surface de contact	Forme	Ellipse ( $a, b$ )	Cercle ( $a$ )	Cercle ( $a$ )	Rectangle ( $\infty, b$ )
	Dimensions	$a = m \sqrt[3]{\frac{3\pi}{2} \frac{k_1 + k_2}{C_1 + C_1' + C_2 + C_2'} N}$ $b = \frac{n}{m} a$	$a = \sqrt[3]{\frac{3\pi}{4} (k_1 + k_2) \frac{R_1 R_2}{R_1 + R_2} N}$	$a = \sqrt[3]{\frac{3\pi}{4} (k_1 + k_2) R N}$	Rectangular ( $\infty, b$ ) $b = 2 \sqrt{(k_1 + k_2) \frac{R_1 R_2}{R_1 + R_2} q}$
Rapprochement $\delta$		$\delta = r \frac{3\pi}{4a} (k_1 + k_2) N$	$\delta = \sqrt[3]{\frac{9\pi^2}{16} (k_1 + k_2)^2 \frac{R_1 + R_2}{R_1 R_2} N^2}$	$\delta = \sqrt[3]{\frac{9\pi^2}{16} \frac{(k_1 + k_2)^2}{R} N^2}$	$\delta = 3,84 \cdot 10^{-5} \frac{N^{0,9}}{L^{0,8}}$ (acier)
Pression max $p_{\max}$		$p_{\max} = \frac{3}{2} \frac{N}{\pi a b}$	$p_{\max} = \frac{3}{2} \frac{N}{\pi a^2}$	$p_{\max} = \frac{3}{2} \frac{N}{\pi a^2}$ $p_{\max} = 0,59 \sqrt{\frac{E_1 E_2}{E_1 + E_2} \frac{R_1 + R_2}{R_1 R_2} q}$ Si $\nu = 0,3$ : $p_{\max} = \frac{2N}{\pi L b} = \frac{2q}{\pi b}$ $p_{\max} = \frac{1}{\pi} \sqrt{\frac{1}{k_1 + k_2} \frac{R_1 + R_2}{R_1 R_2} q}$	$p_{\max} = 0,59 \sqrt{\frac{E_1 E_2}{E_1 + E_2} \frac{1}{R} q}$ Si $\nu = 0,3$ : $p_{\max} = \frac{2N}{\pi L b} = \frac{2q}{\pi b}$ $p_{\max} = \frac{1}{\pi} \sqrt{\frac{1}{k_1 + k_2} \frac{1}{R} q}$
Profondeur pour $\tau_{\max}$ (pour $\nu = 0,3$ )	Fonction de l'excentration de l'ellipse	$h = 0,5a$	$h = 0,5a$	$h = \frac{\pi}{4} b$	$h \approx 0,78b$

\* avec :  $k_i = \frac{1 - \nu_i^2}{\pi E_i}$

# Kinematic coupling parameters

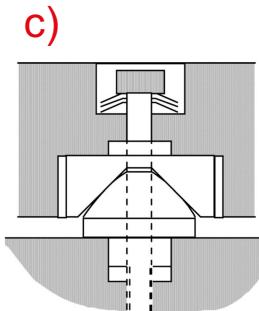
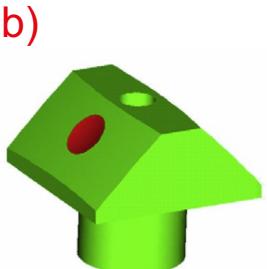


**Figure 5** Geometric parameters for a three groove kinematic coupling.

# Kinematic coupling: Canoe ball

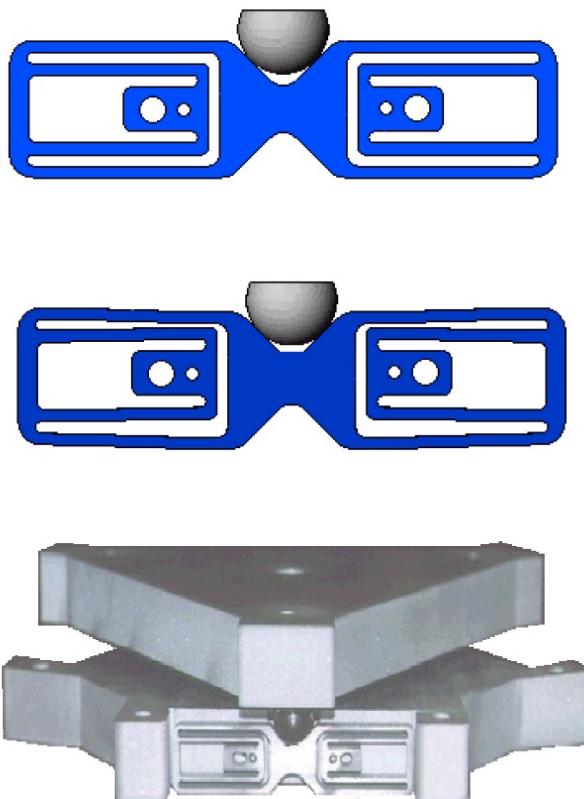
- Acts like a **ball 1 meter in diameter**
- Has **100 times the stiffness** and **load capacity** of a normal 1" ball
- The **large shallow Hertzian zone** is very repeatable

- a) Modular “Canoe ball” and vee
- b) Solid model showing Hertz contact zone used in the kinematic coupling.
- c) Preloading a kinematic coupling through the coupling elements to avoid deforming the rest of the structure.



# Flexural Kinematic coupling

- Combines the **repeatable location** of traditional kinematic couplings with the **stiffness characteristics** of many other coupling methods
- After the kinematic members (3 "balls" and 3 "grooves") are mated, **force is applied to bring the two components together.**
- Parts of the kinematic coupling (flexures attached to one of the major components) displace and allow the components to mate, thereby creating a **classical surface-to-surface, high load capacity joint.**
- When properly designed, this can be accomplished **without compromising the kinematic nature of the coupling.**

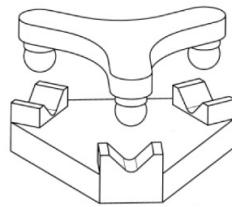
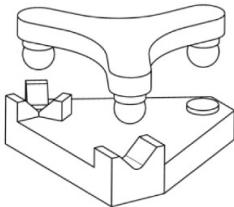


# Kinematic coupling: repeatability calculation

**Nonrepeatability** ( $\rho$ ) due to friction decreases linearly with decreasing coefficient of friction between the balls and grooves, following the general relation between the friction coefficient ( $\mu$ ), ball radius ( $R$ ), the applied load ( $P$ ) and the elastic modulus ( $E$ ):

$$\rho = \mu \left( \frac{2}{3R} \right)^{\frac{1}{3}} \left( \frac{P}{E} \right)^{\frac{2}{3}}$$

Kelvin clamp

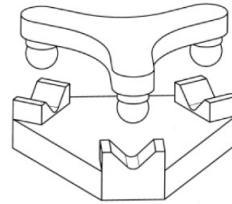
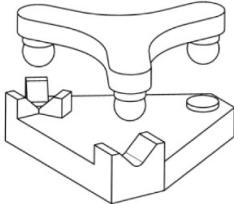


Maxwell clamp

# Kinematic coupling: conclusions

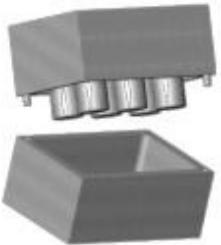
- Offers **very high positioning accuracy**
- **Repeatability** of the positioning is in the **sub-micron range**, generally in the order of the **surface roughness**
- **Stability zone** is limited
- Requires a **repeatable preload**
- Stiffness is limited by **hertzian contacts** and by the **rigid bodies' rigidity**
- **More stiffness when the contact radius is increased**

Kelvin clamp



Maxwell clamp

# Common coupling methods



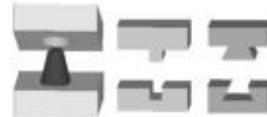
**Pinned Joints**

No Unique Position



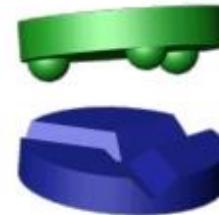
**Flexural Kin. Couplings**

Kinematic Constraint



**Elastic Averaging**

Non-Deterministic



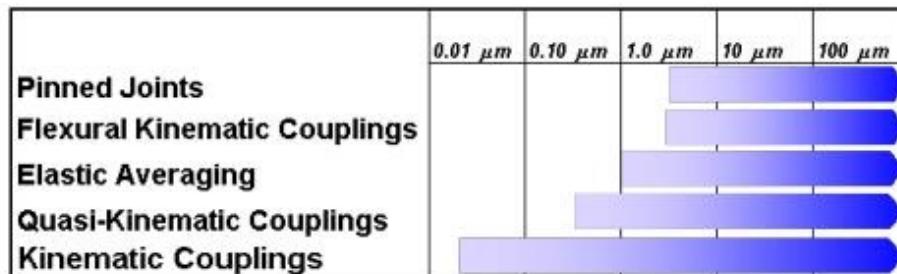
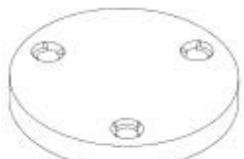
**Kinematic Couplings**

Kinematic Constraint

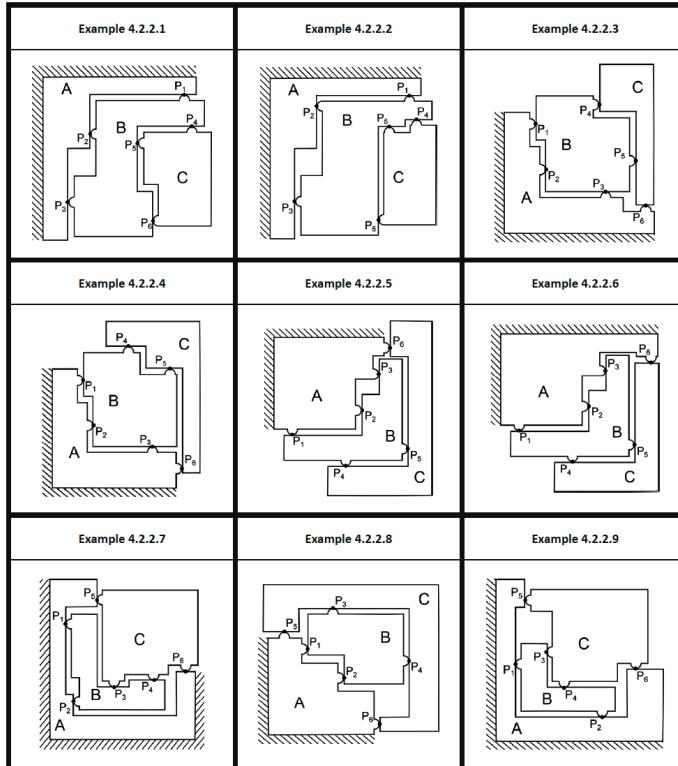


**Quasi-Kinematic Couplings**

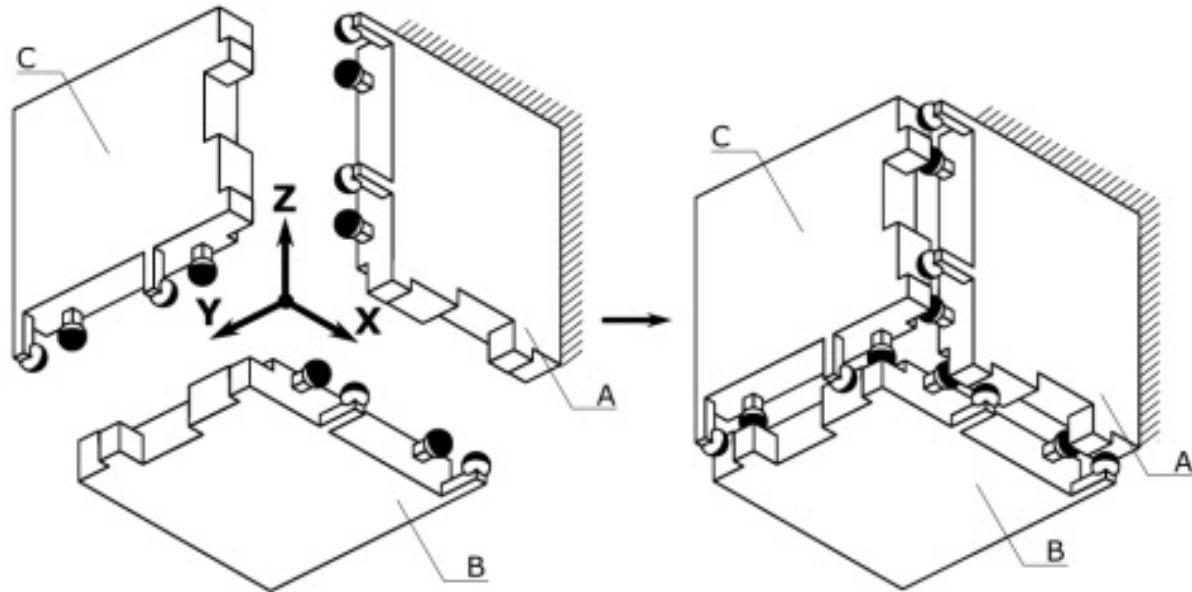
Near Kinematic Constraint



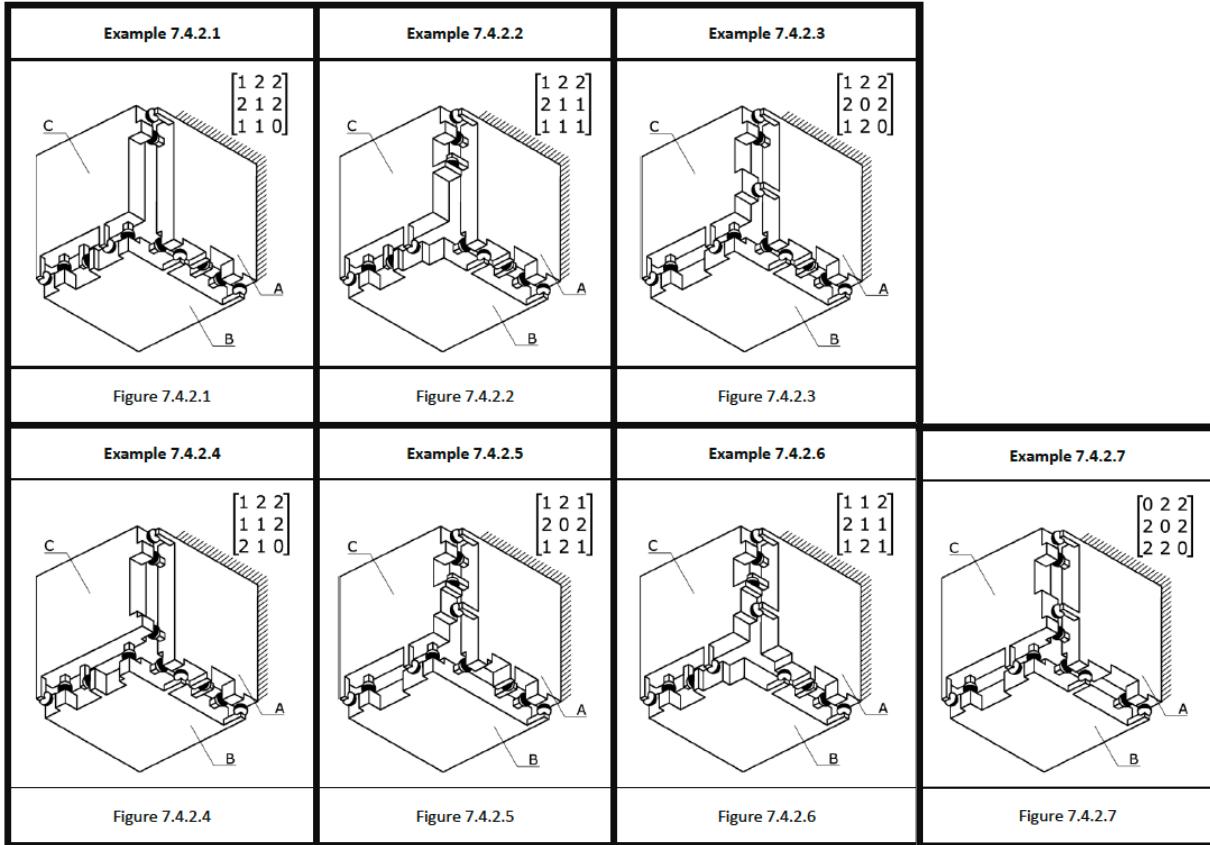
# Kinematic coupling (3 bodies - 2D)



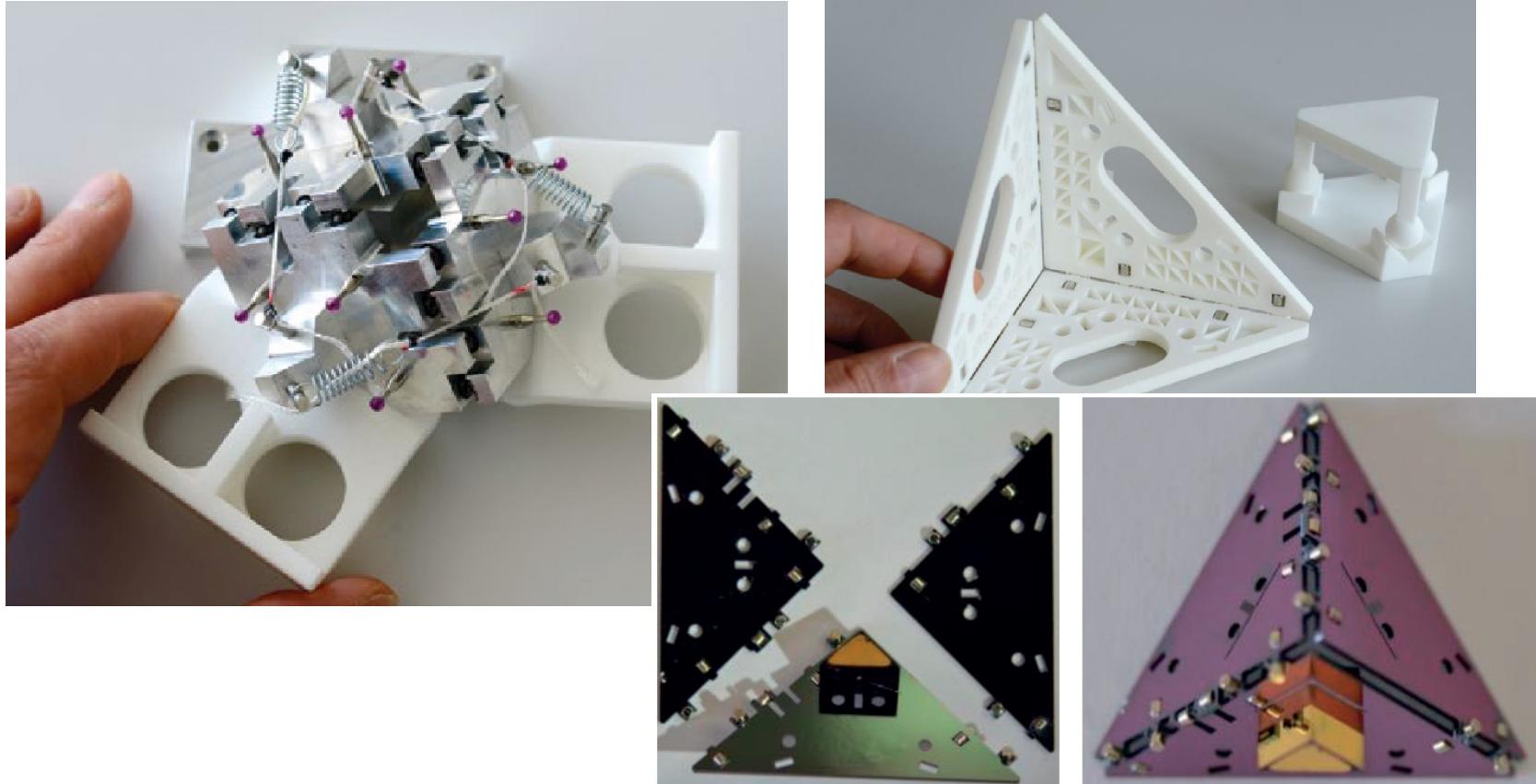
# Kinematic coupling (3 bodies – 3D)



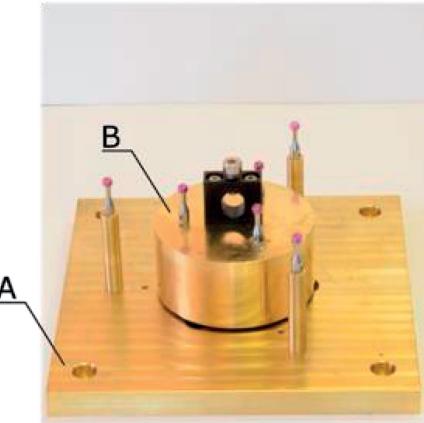
# Kinematic coupling (3 bodies - 3D)



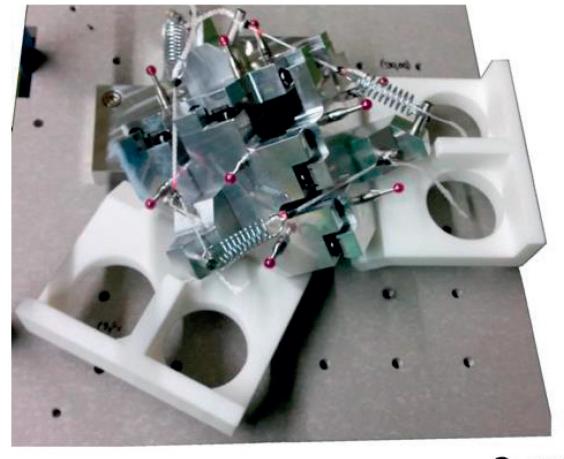
# Kinematic coupling (3 bodies – 3D)



# Kinematic coupling (3 bodies - 3D)



Average				
Three-Vee		Example 7.3.2.7		
Body	B	C		
Degree of Freedom (DOF)	D	0.178 $\mu\text{m}$	0.902 $\mu\text{m}$	0.712 $\mu\text{m}$
	$\alpha$	5 $\mu\text{rad}$	42 $\mu\text{rad}$	31 $\mu\text{rad}$
Number of measurements		10	3	



## Exercise on MOODLE:

- EXO10\_Balancing\_II.pdf

## Homework:

- No homework