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MICRO 372 - Advanced Mechanisms for Extreme Environments

Chapter 6

Dynamic aspects of mechanisms

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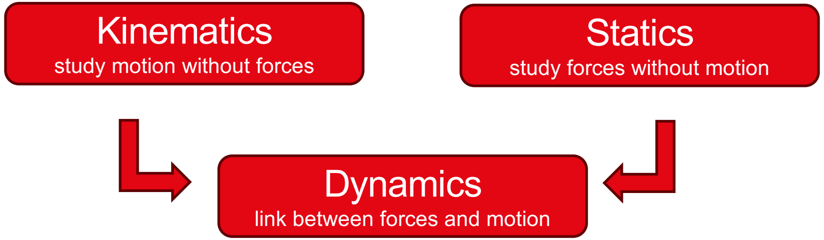
https://www.festo.com/be/fr/e/journal/la-precision-d-une-montre-suisse-id_23722/

Dynamics – Introduction

- From ancient Greek **δυναμικός**: powerfull, efficient
- **Discipline of classical mechanics** which studies bodies in motion under the influence of mechanical actions applied to them.
- It combines **statics** which studies the balance of bodies at rest, and **kinematics** which studies movement.



- **Newton's second law**: $\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}$

  The change of motion of an object is proportional to the force impressed; and is made in the direction of the straight line in which the force is impressed

- **Euler's second law:** $\mathbf{M} = \mathbf{r}_{\text{cm}} \times \mathbf{a}_{\text{cm}} m + J\alpha$

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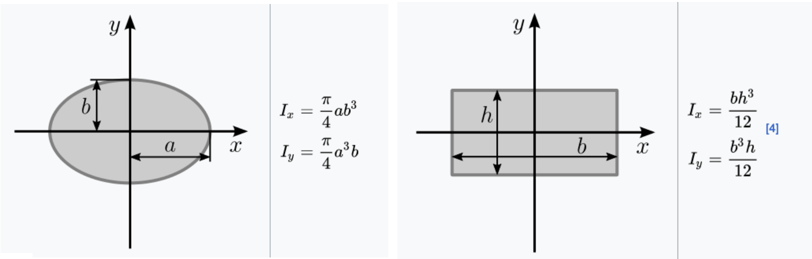
Clarification on inertia(s): second moments of area (I)

- Also known as **area moment of inertia**
- It is a **geometrical property** of an area which reflects how its points are distributed with respect to an arbitrary axis
- The unit of dimension of the second moment of area is **length to fourth power** (L^4 : e.g. mm^4)
- It should not be confused with the **mass moment of inertia**.

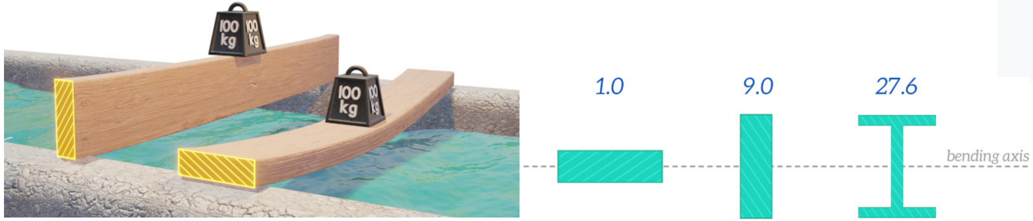
$$I_x = \iint_A y^2 \, dx \, dy$$

and

$$I_y = \iint_A x^2 \, dx \, dy.$$



https://en.wikipedia.org/wiki/List_of_second_moments_of_area



https://efficientengineer.com/area-moment-of-inertia/

Used for example in:

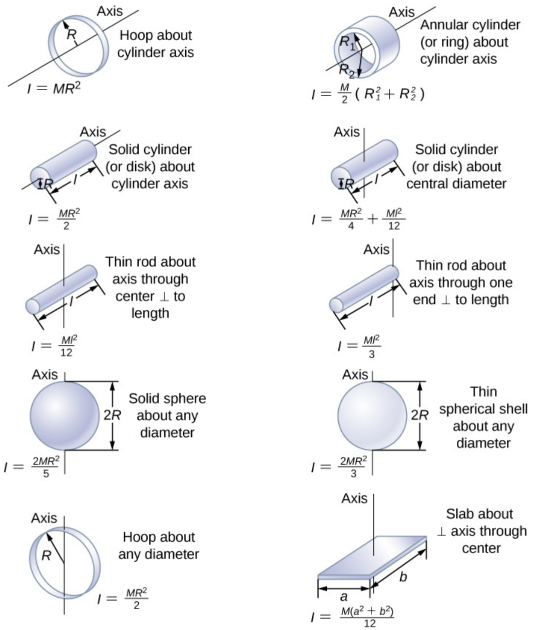
$$M = -EI\frac{d^2w}{dx^2}$$

Clarification on inertia(s): moment of inertia (J)

- Otherwise known as the **mass moment of inertia**, **angular/rotational mass**, **second moment of mass**, or most accurately, **rotational inertia**
- Quantitative measure of the **rotational inertia** of a body.
- i.e., the **opposition that the body exhibits to having its speed of rotation about an axis altered by the application of a torque**.
- The moment of inertia (J), is always specified with respect to a **specific axis** and **unit is [kg*m^2].**
- And is defined as the **sum of the products** obtained by **multiplying the mass of each particle of matter** in a given body **by the square of its distance from the axis**.

www.britannica.com

$$J = \int r^2 dm$$



pressbooks.online.ucf.edu

Used for example in Euler second law of motion:

$$\sum M = J\ddot{\theta}$$

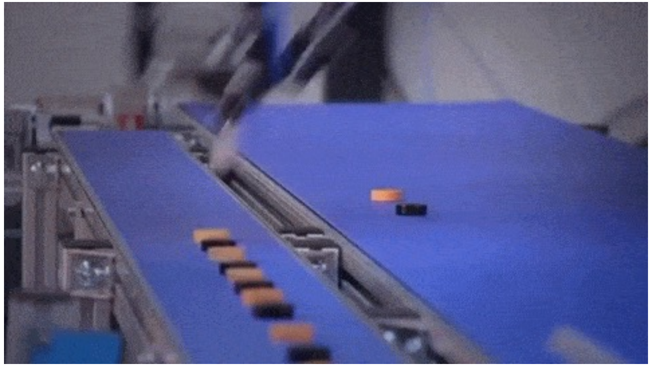
High dynamics mechanics

A mechanism with **high dynamics** is defined by its ability to maintain high performance levels in terms of:

- **Acceleration**
- **Speed**
- **Responsiveness**
- Accurate control under varying and demanding conditions

Thus, is is generally **characterized by**:

- **Low mobile masses and inertia**
- **High stiffnesses**
- **High actuation torque**
- **Efficient control**



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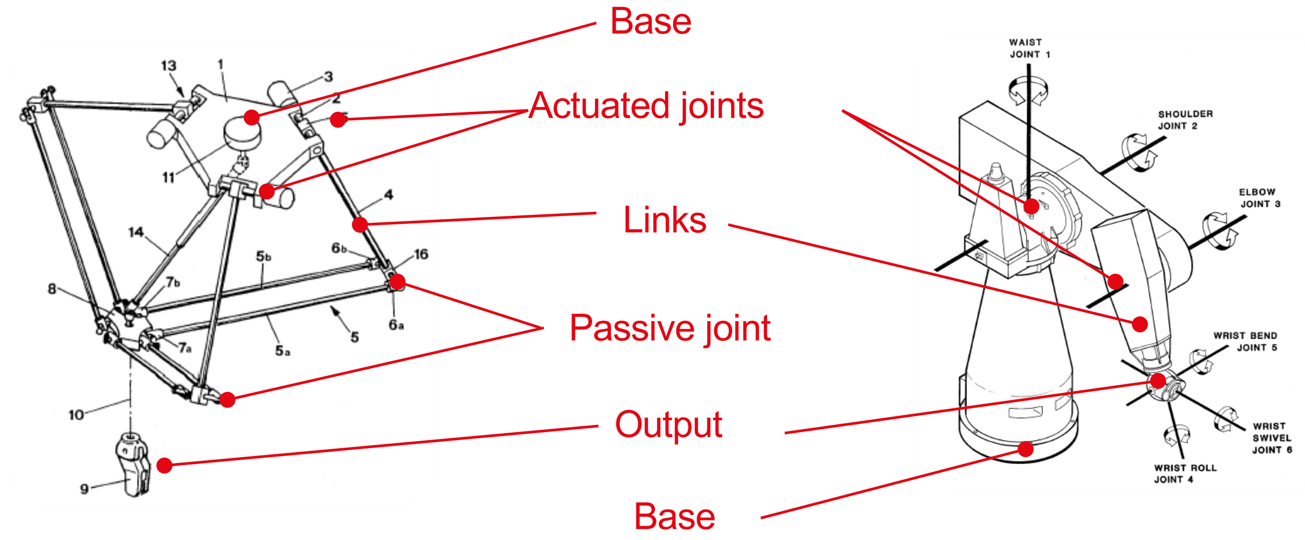
Parallel vs serial architecture

- **Parallel kinematics** have a mobile output linked to the base by **several kinematic chains** while **serial kinematics** link the base to the output by only **one kinematic chain**.

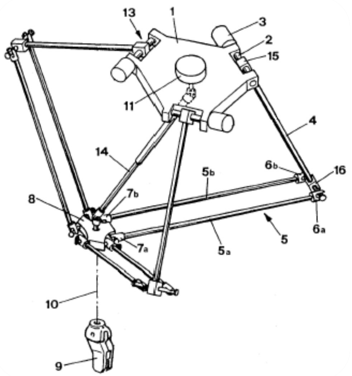


[R. Clavel, 1991, "Conception d'un robot parallèle rapide à 4 degrés de liberté", Ph.D. Thesis, EPFL, Lausanne, Switzerland]

http://industrialrobot.info/robot-history

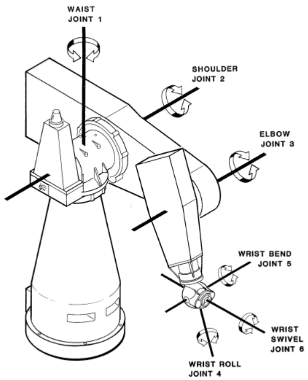
Parallel vs serial architecture

- No error accumulation at the output
- No mobile actuator
- Moving mass is small
- Dynamic is higher
- Workspace is reduced
- Complex geometric model



[R. Clavel, 1991, "Conception d'un robot parallèle rapide à 4 degrés de liberté", Ph.D. Thesis, EPFL, Lausanne, Switzerland]

- Error accumulation at output
- Actuators are moving
- Moving mass is big
- Dynamic is lower
- Higher workspace
- Less complex geometric model



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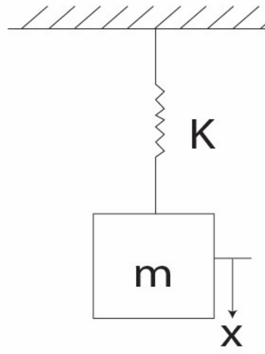
High dynamics and limitations for mechanisms

QUIZZ

In a mechanism, **several aspects can limit high dynamics**, such as:

Vibrational mechanics theory

Undamped mechanical oscillator

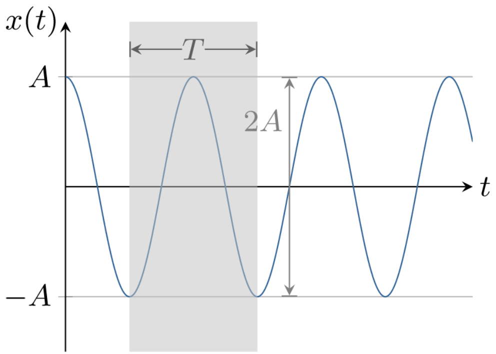


- Differential equation:

$$\ddot{x} + \omega_0^2 x = 0 \quad \text{with} \quad \omega_0 = \sqrt{\frac{k}{m}} \quad [\mathrm{rad.s^{-1}}]$$

- Solution:

$$x(t) = A\cos(\omega_0 t + \varphi)$$



- Eigenfrequency:

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- Note that the **natural frequency depends** on the characteristics of the elastic pendulum (k and m) but **not on the amplitude of the oscillations**: we speak of **isochronism** of the oscillations

https://femto-physique.fr/mecanique/oscillateurs-mecaniques.php

Vibrational mechanics theory

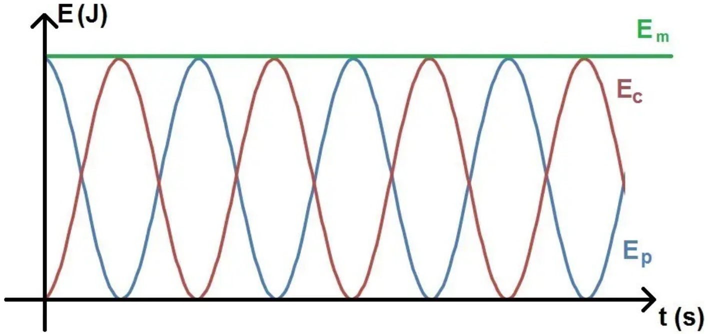
Undamped mechanical oscillator: total mechanical energy

- Is composed of **potential energy**:

$$\mathcal{E}_\mathrm{p} = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega_0 t + \varphi)$$

- And **kinetic energy**:

$$\mathcal{E}_\mathrm{c} = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}kA^2\sin^2(\omega_0 t + \varphi)$$



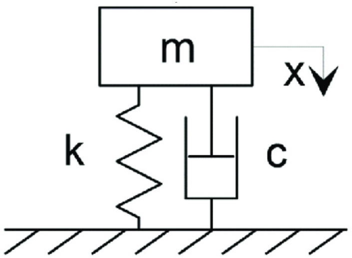
- The **total mechanical energy** of a harmonic oscillator is constant and proportional to the square of the amplitude:

$$\mathcal{E}_\mathrm{m} = \mathcal{E}_\mathrm{c} + \mathcal{E}_\mathrm{p} = \tfrac{1}{2}kA^2$$

N.B.: $\cos^2(X) = \frac{1}{2}(1 + \cos 2x)$

Vibrational mechanics theory

Damped mechanical oscillator



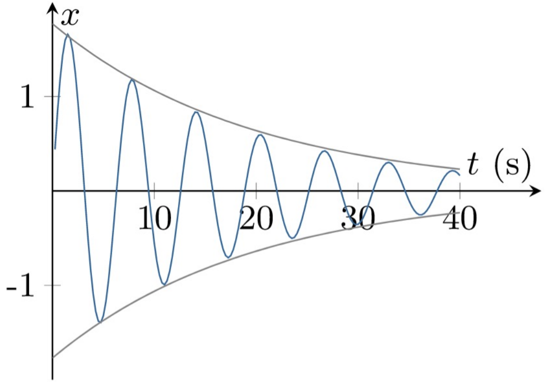
- Differential equation:

$$m\ddot{x} + \alpha\dot{x} + kx = 0$$

or $$\ddot{x} + 2\lambda\,\dot{x} + {\omega_0}^2 x = 0$$ with $\omega_0 = \sqrt{k/m}\ [\mathrm{rad.s^{-1}}]$ and $2\lambda = \alpha/m\ [\mathrm{s^{-1}}]$

- Solution:

$$x(t) = A\mathrm{e}^{-\lambda\,t}\,\cos\left(\omega t + \varphi\right)$$



- Pseudo-period:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{{\omega_0}^2 - \lambda^2}}$$

- Note that the **natural frequency depends** on the characteristics of the elastic oscillator (k, m and α) but **not on the amplitude of the oscillations**: we speak of **isochronism** of the oscillations

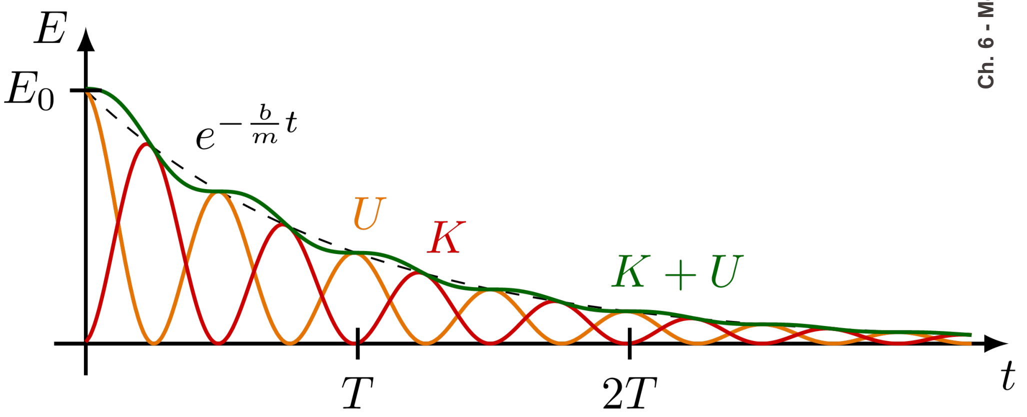
https://femto-physique.fr/mecanique/oscillateurs-mecaniques.php

Vibrational mechanics theory

Damped mechanical oscillator: total mechanical energy

- Is composed of **potential energy**:

$$\mathcal{E}_{\mathrm{p}} = \frac{1}{2}kx^2$$



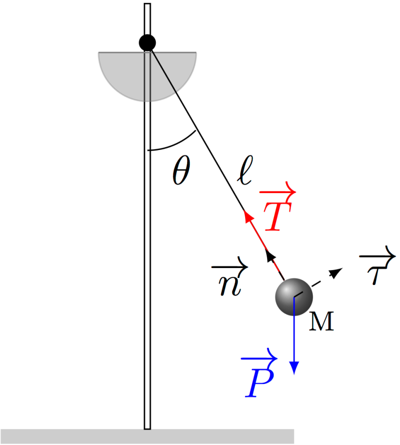
- And **kinetic energy**:

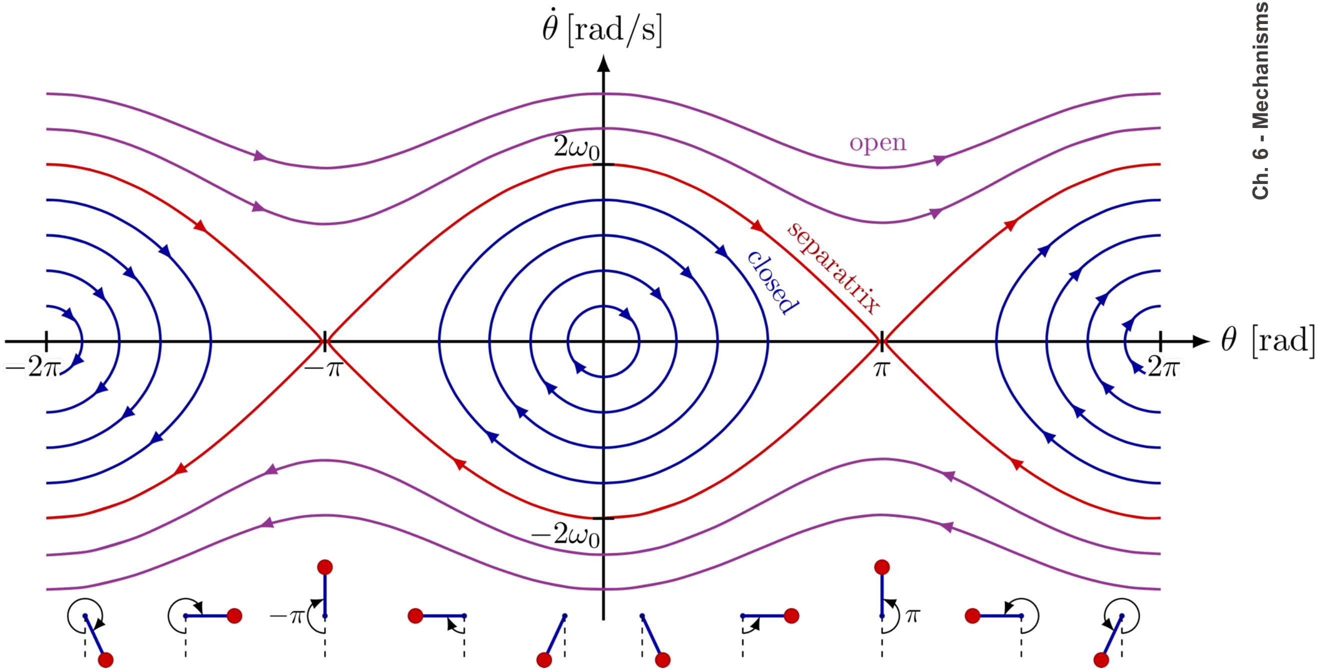
$$\mathcal{E}_{\mathrm{c}} = \frac{1}{2}m\dot{x}^2$$

- The **total mechanical energy** is:

$$\mathcal{E}_{\mathrm{m}} = \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2$$

Phase space (or phase portrait) of an undamped pendulum

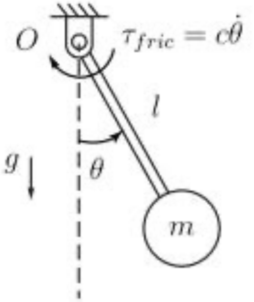




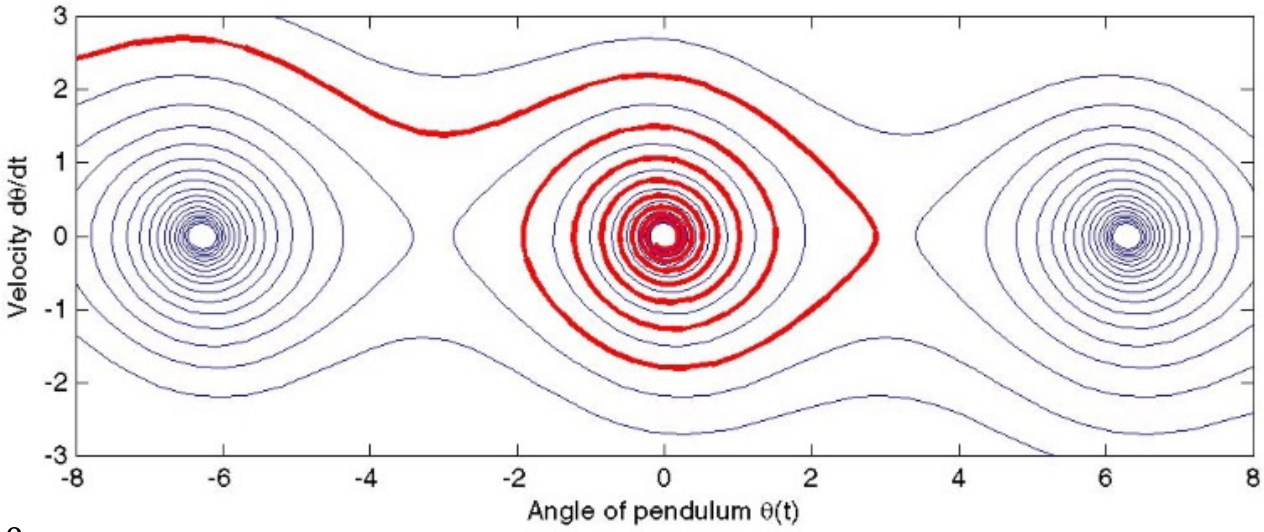
$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$$

https://tikz.net/dynamics_phaseportrait/

Phase space (or phase portrait) of a damped pendulum



$$\frac{d^2\theta}{dt^2} + c\frac{d\theta}{dt} + \frac{g}{l}sin\theta = 0$$



https://www.dam.brown.edu/people/mumford/beyond/coursenotes/2006PartIIb.pdf

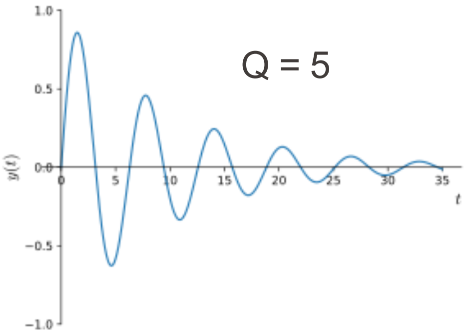
Quality factor

- Physically speaking, *Q* is approximately **2π times the ratio** of the **total energy stored** and the **energy lost in a single cycle:**

$$Q \stackrel{\text{def}}{=} 2\pi \times \frac{\text{energy stored}}{\text{energy dissipated per cycle}} = 2\pi f_r \times \frac{\text{energy stored}}{\text{power loss}}$$

- For large values of *Q*, the ***Q* factor is approximately the number of oscillations** required for a freely oscillating system's energy to **fall off to $e^{-2\pi}$**, or about **1⁄535** or **0.2%,** of its original energy.

- This means the **amplitude falls** off to approximately $e^{-\pi}$ **or 4.32%** of its original amplitude.



wikipedia.org

Damping ratio and related quantities

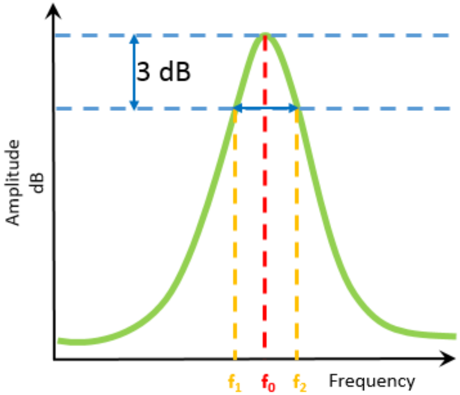
- The **damping factor** or damping ratio is the ratio of the actual damping coefficient (C) to the critical damping coefficient (C_{CR}) is known as:

$$\zeta = \frac{c}{c_{cr}} = \frac{c}{2m\omega}$$

- The **loss factor** is the inverse of the **quality factor**:

$$\eta = \frac{1}{Q} = 2\zeta = \frac{\Delta\omega_{3dB}}{\omega_0}$$

η: loss factor
Q: damping factor or quality factor
ζ: damping ratio



$$Q = \frac{f_0}{f_2 - f_1}$$

Equations of motion: Newton VS Lagrange

$$\sum F = ma$$

$$\mathcal{L} = E_c - E_p = T - V$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \dot{q}_k} = \frac{\partial \mathcal{L}}{\partial q_k}$$

Newtonian mechanics

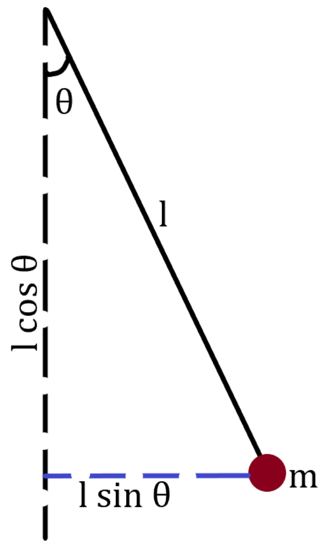
- **Motion** is described by **forces**
- Based on **Newton's laws of motion**
- Involves constraint forces
- Does not have a systematic method for deriving conservation laws
- Uses **vectors**
- Can handle **non-conservative forces** quite well
- Mainly applicable to **classical physics** and in describing everyday phenomena

Lagrangian mechanics

- **Motion** is described by **energies**
- Based on the **principle of least action**
- Generalized coordinates are used instead of constraint forces
- Conservation laws can be derived easily (Noether's theorem)
- Does not use vectors (energy being a **scalar**)
- Not ideal for non-conservative forces (such as friction)
- Used widely in **all areas of physics**

https://profoundphysics.com/lagrangian-vs-newtonian-mechanics-the-key-differences/

Lagrange example : pendulum



$$L = K - V$$

Kinetic energy | Potential energy

$$L = \frac{1}{2}ml^2\dot{\theta}^2 - mgl(1-\cos\theta)$$

$$L = \frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta - mgl$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$\frac{\partial L}{\partial \dot{\theta}} = ml^2\dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = ml^2\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = -mgl\sin\theta$$

$$ml^2\ddot{\theta} = -mgl\sin\theta$$

$$\rightarrow \quad \ddot{\theta} = -\frac{g}{l}\sin\theta$$

for small angles:

$$\lim_{\theta\to 0} \sin\theta = \theta$$

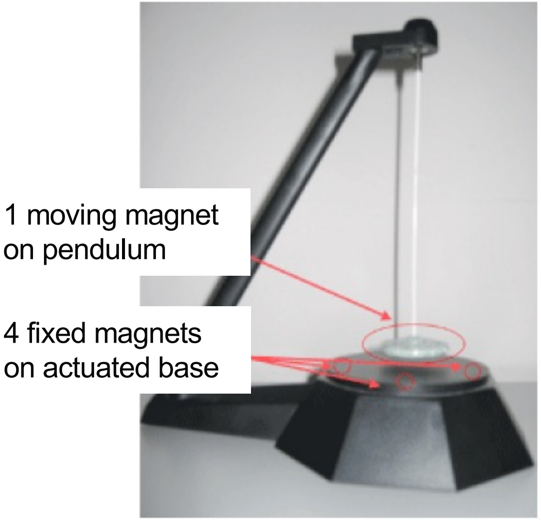
$$\rightarrow \quad \ddot{\theta} = -\frac{g}{l}\theta$$

mohammad-yasir.medium.com

Lagrange example : 2 DOF magnetic pendulum

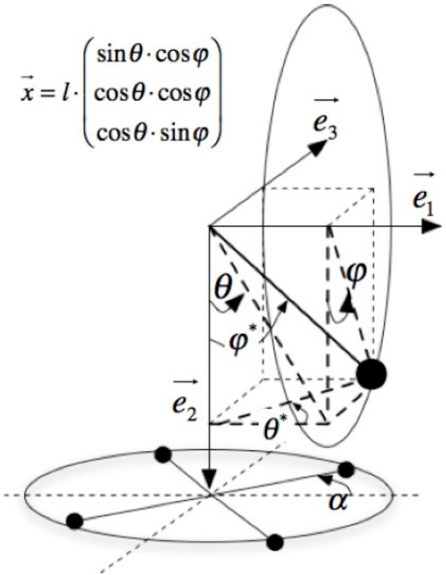
Lagrange example : 2 DOF magnetic pendulum

setup

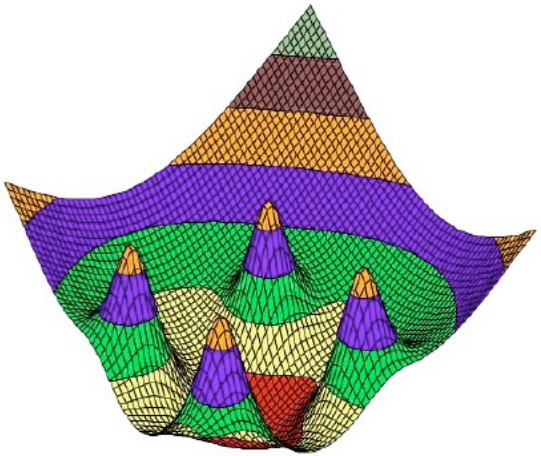


coordinates

$$\vec{x} = l \cdot \begin{pmatrix} \sin\theta \cdot \cos\varphi \\ \cos\theta \cdot \cos\varphi \\ \cos\theta \cdot \sin\varphi \end{pmatrix}$$



potential field



https://www.epfl.ch/labs/la/wp-content/uploads/2018/08/Florent.Cosandier.pdf

Lagrange example : 2 DOF magnetic pendulum

Lagrangian

$$L = T - V$$

kinetic energy

$$T = \frac{1}{2} m \sum_{i=1}^{3} \dot{x}_i^2 + \frac{1}{2} I_{mot} \dot{\alpha}^2$$

$$\dot{\vec{x}} = \begin{pmatrix} \frac{\partial x_1}{\partial \theta} & \frac{\partial x_1}{\partial \varphi} \\ \frac{\partial x_2}{\partial \theta} & \frac{\partial x_2}{\partial \varphi} \\ \frac{\partial x_3}{\partial \theta} & \frac{\partial x_3}{\partial \varphi} \end{pmatrix} \begin{pmatrix} \dot{\theta} \\ \dot{\varphi} \end{pmatrix}$$

potential energy

$$V = E_{hauteur} + E_{magntique}$$

$$E_{hauteur} = mgl(1 - cos(\theta)cos(\varphi))$$

potential energy of mobile mass height

$$E_{magntique} = K \sum_{i=1:4} \|\vec{r}_i\|^{-1}$$

magnetic potential energy

$$\vec{r}_i = \vec{x}^T - x_{aimants}(i,:)$$

$$x_{aimants} = \begin{pmatrix} Rcos(\alpha) & h & Rsin(\alpha) \\ -Rsin(\alpha) & h & Rcos(\alpha) \\ -Rcos(\alpha) & h & -Rsin(\alpha) \\ Rsin(\alpha) & h & -Rcos(\alpha) \end{pmatrix}$$

https://www.epfl.ch/labs/la/wp-content/uploads/2018/08/Florent.Cosandier.pdf

Lagrange example : 2 DOF magnetic pendulum

Euler-Lagrange equation

$$\frac{d}{dt}\frac{\partial L(\vec{q},\dot{\vec{q}})}{d\dot{\vec{q}}} - \frac{\partial L(\vec{q},\dot{\vec{q}})}{d\vec{q}} = Q$$

external work

$$(Q = 0)$$

$$\vec{q} = \begin{pmatrix} \theta \\ \varphi \\ \alpha \end{pmatrix}$$

generalized coordinates

intermediate function

$$F = \frac{\partial L(\vec{q},\dot{\vec{q}})}{\partial \dot{\vec{q}}}$$

partial derivative

$$\frac{dF(\vec{q},\dot{\vec{q}})}{dt} = \frac{\partial F}{\partial \vec{q}}\frac{\partial \vec{q}}{\partial t} + \frac{\partial F}{\partial \dot{\vec{q}}}\frac{\partial \dot{\vec{q}}}{\partial t} = \frac{\partial F}{\partial \vec{q}}\dot{\vec{q}} + \frac{\partial F}{\partial \dot{\vec{q}}}\ddot{\vec{q}}$$

$$\frac{\partial^2 L}{\partial \dot{\vec{q}}^2}\ddot{\vec{q}} + \frac{\partial^2 L}{\partial \vec{q}\partial \dot{\vec{q}}}\dot{\vec{q}} - \frac{\partial L}{\partial \vec{q}} = Q$$

handy notation

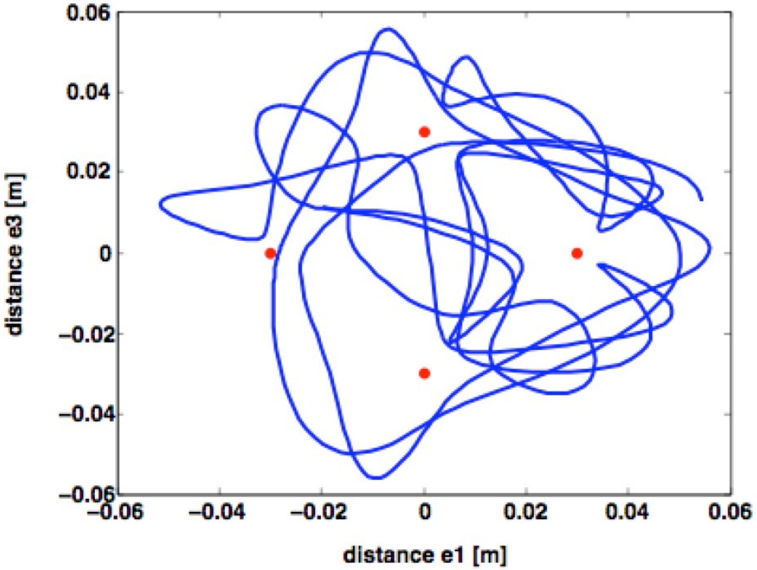
$$A = \frac{\partial^2 L}{\partial \dot{\vec{q}}^2} \qquad \text{et} \qquad B = Q - \frac{\partial^2 L}{\partial \vec{q}\partial \dot{\vec{q}}}\dot{\vec{q}} + \frac{\partial L}{\partial \vec{q}}$$

equation of motion

$$A\ddot{\vec{q}} = B \qquad \Leftrightarrow \qquad \ddot{\vec{q}} = A^{-1}B$$

Lagrange example : 2 DOF magnetic pendulum

trajectory of the chaotic pendulum



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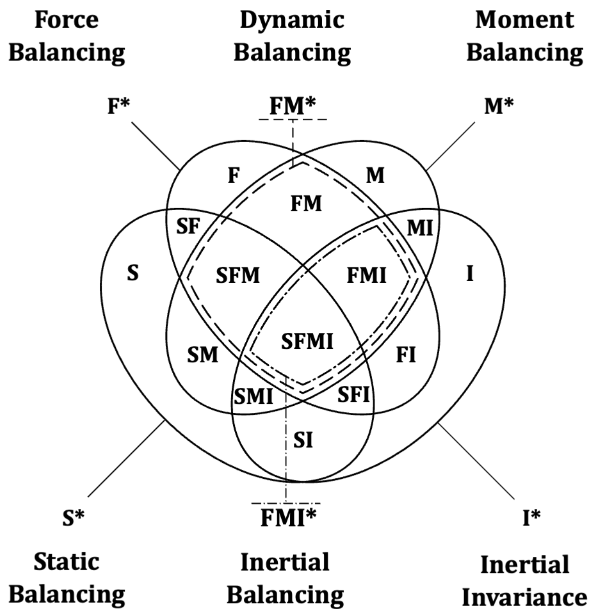
Mechanisms balancing

- Balancing Taxonomy
  - Balancing types
  - Proposed balancing taxonomy
- Study case of the Wattwins oscillator
- Conclusion



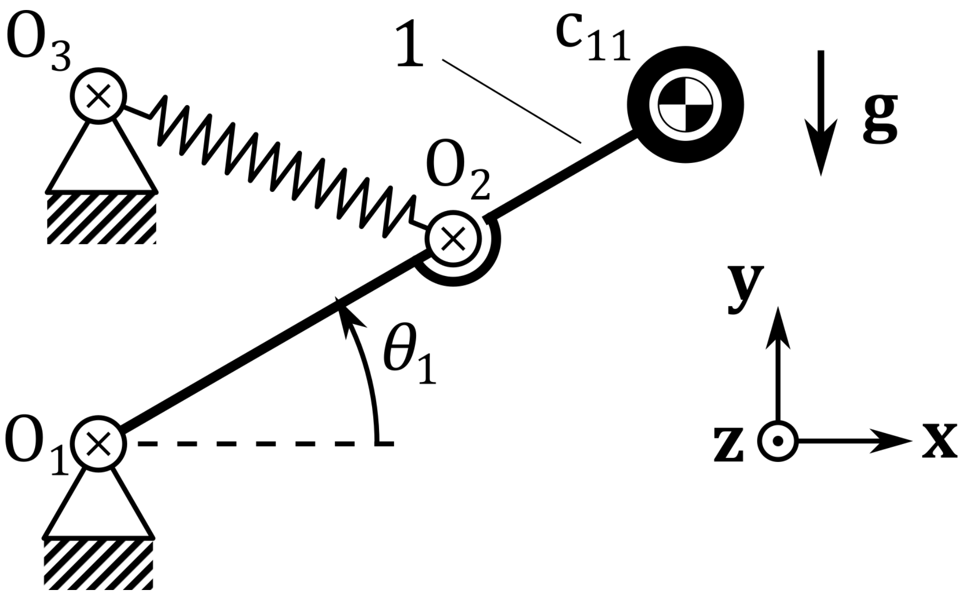
Nomenclature

- $\mathcal{R}$ — Inertial frame of reference.
- $\mathcal{F}$ — Frame of reference attached to the chassis of the mechanism.
- V — Total potential energy of the mechanism.
- q_i — Generalized coordinate of the i-th degree-of-freedom of the mechanism.
- $\dot{q}_i$ — Generalized velocity of the i-th degree-of-freedom of the mechanism.
- m_i — mass of link i.
- m_{tot} — Total mass of the mechanism.
- $\boldsymbol{c}_{A,i}$ — Center of mass (CoM) position vector of link i relative to point A.
- $\dot{\boldsymbol{c}}_{i/\mathcal{F}}$ — CoM linear velocity vector of link i relative to $\mathcal{F}$.
- $\dot{\mathbf{c}}_{\text{tot}/\mathcal{F}}$ — Total CoM linear velocity vector of the mechanism relative to $\mathcal{F}$.
- $\boldsymbol{p}_{\text{tot}/\mathcal{F}}$ — Total linear momentum vector of the mechanism relative to $\mathcal{F}$.
- $\boldsymbol{\sigma}_{A,i/\mathcal{F}}$ — Angular momentum vector of link i given at point A relative to $\mathcal{F}$.
- $\boldsymbol{\sigma}_{A,\text{tot}/\mathcal{F}}$ — Total angular momentum vector of the mechanism given at point A relative to $\mathcal{F}$.
- $\boldsymbol{\Omega}_{i/\mathcal{F}}$ — Angular velocity vector of link i relative to $\mathcal{F}$.
- $\bar{\bar{\boldsymbol{J}}}_i$ — Inertia tensor of link i given at its CoM.
- $\bar{\bar{\boldsymbol{J}}}_{A,\text{tot}/\mathcal{F}}$ — Total inertia tensor of the mechanism given at A relative to $\mathcal{F}$.

What is balancing ?

Source: Youtube - James Fuhrman

Type S: Static Balancing



Ⓥ Constant potential energy

Ⓣ Zero-force mechanism

Eq Neutral equilibrium

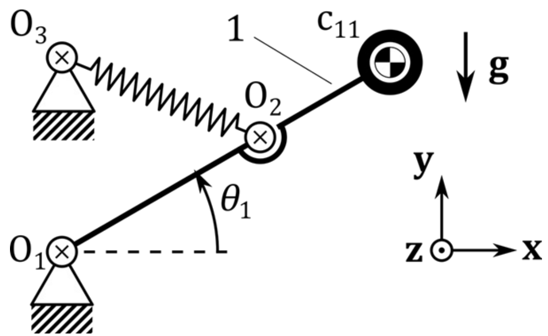
Static balancing condition:
$m_{11} l_{c11} g = l_{12} l_{13} k$

Type S*: Static Balancing

Definition **Static balancing** is achieved when a mechanism has a total **potential energy** which is constant over its workspace. The chassis of the mechanism ($\mathcal{F}$) is considered fixed relative to the inertial frame of reference $\mathcal{R}$.

$$V = \text{const.}$$

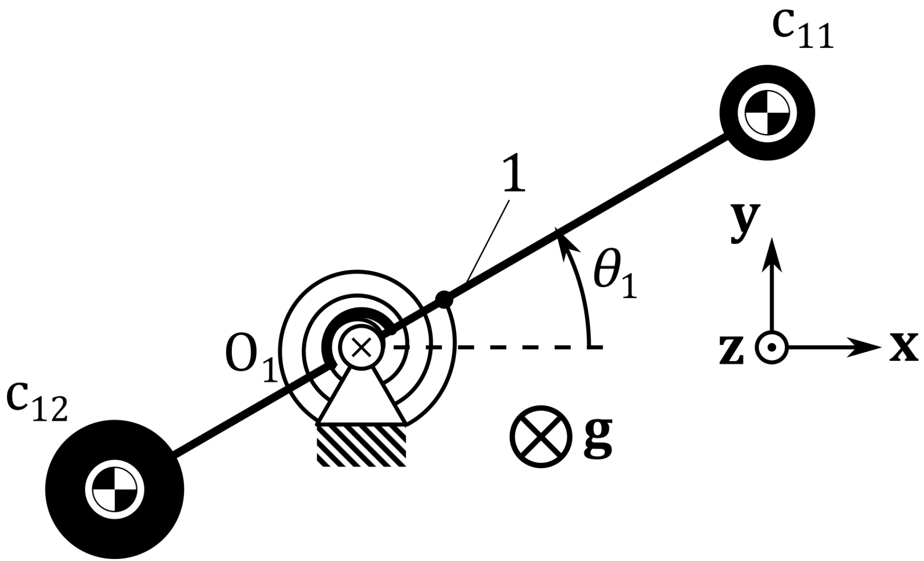
Where V is the total potential energy of the mechanism.



Type S: Static Balancing

Source: www.dailymotion.com/video/x4ezcx8

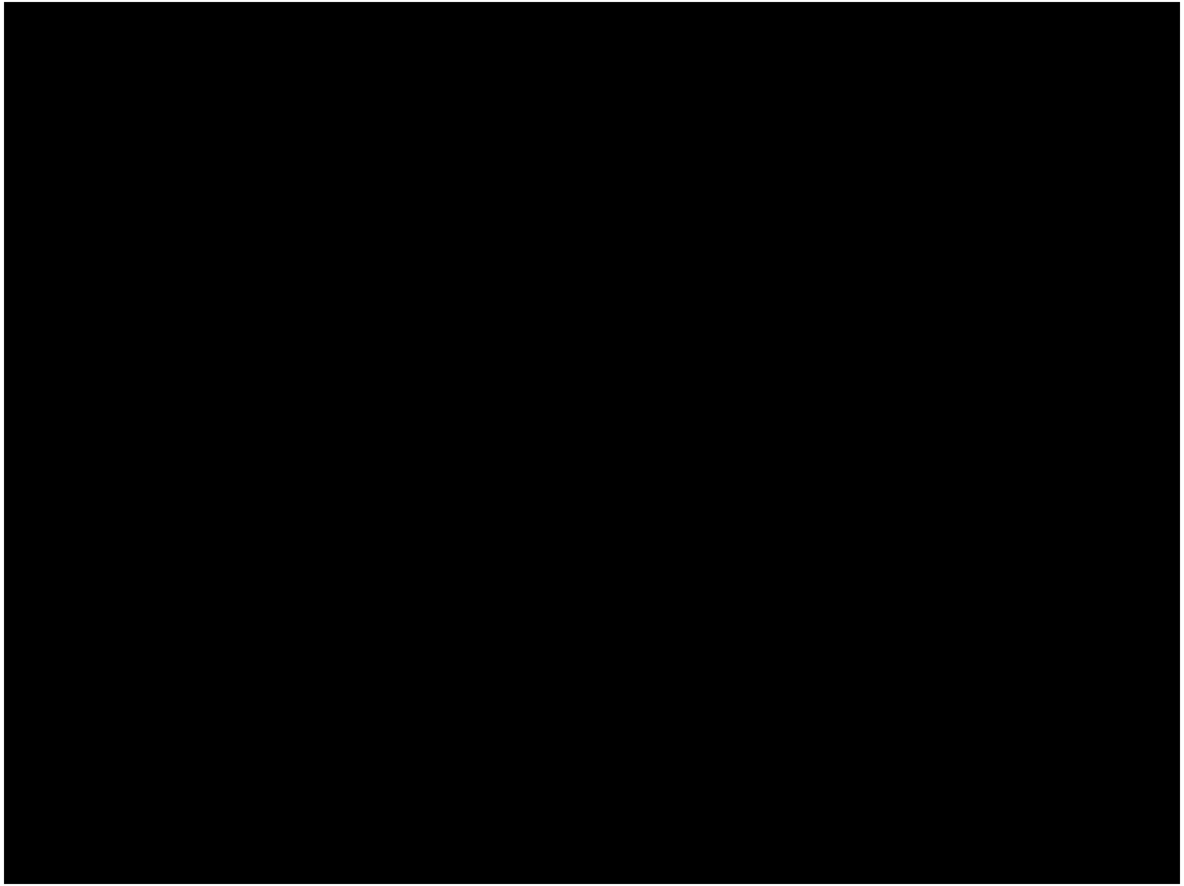
Type F*: Shaking Force Balancing



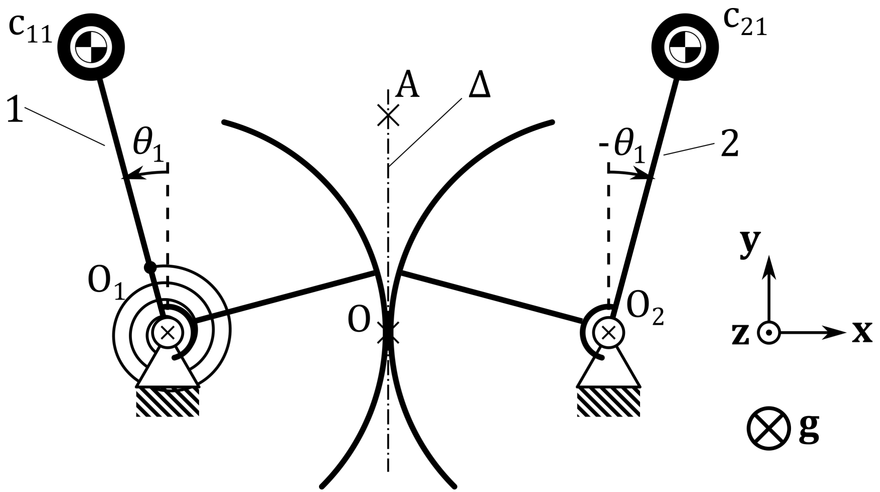
- (p) Constant Linear Momentum
- (G) Fixed CoM relative to the chassis
- (φ_g) Insensitivity to gravity orientation
- (F) No exported forces to the chassis
- (V_g) Constant gravitational potential
- (γ) Insensitivity to linear accelerations

Force balancing condition:
$m_{11} l_{c11} = m_{12} l_{c12}$

Type F*: Shaking Force Balancing



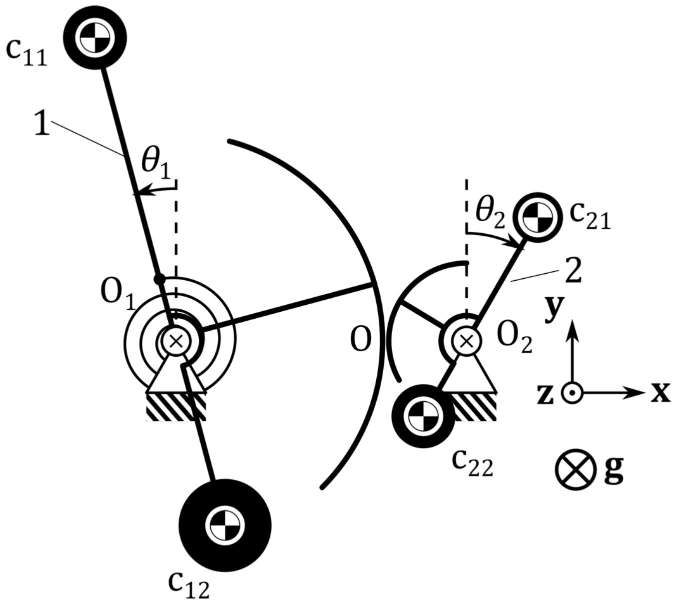
Type M^*: Shaking Moment Balancing



- σ(A) Constant angular momentum at A
- M(A) No exported moments to the chassis at A
- α(A) Insensitivity to angular acceleration at A

Moment balancing condition around $A \in \Delta$:
$m_{11} = m_{12}$ and $l_{c11} = l_{c12}$

Type FM*: Dynamic Balancing



Dynamic balancing conditions:

$$m_{11}l_{c11} = m_{12}l_{c12}$$
$$m_{21}l_{c21} = m_{22}l_{c22}$$
$$m_{11}l_{c11}^2 + m_{12}l_{c12}^2 = \frac{r_1}{r_2}(m_{11}l_{c21}^2 + m_{12}l_{c22}^2)$$

Force Balancing

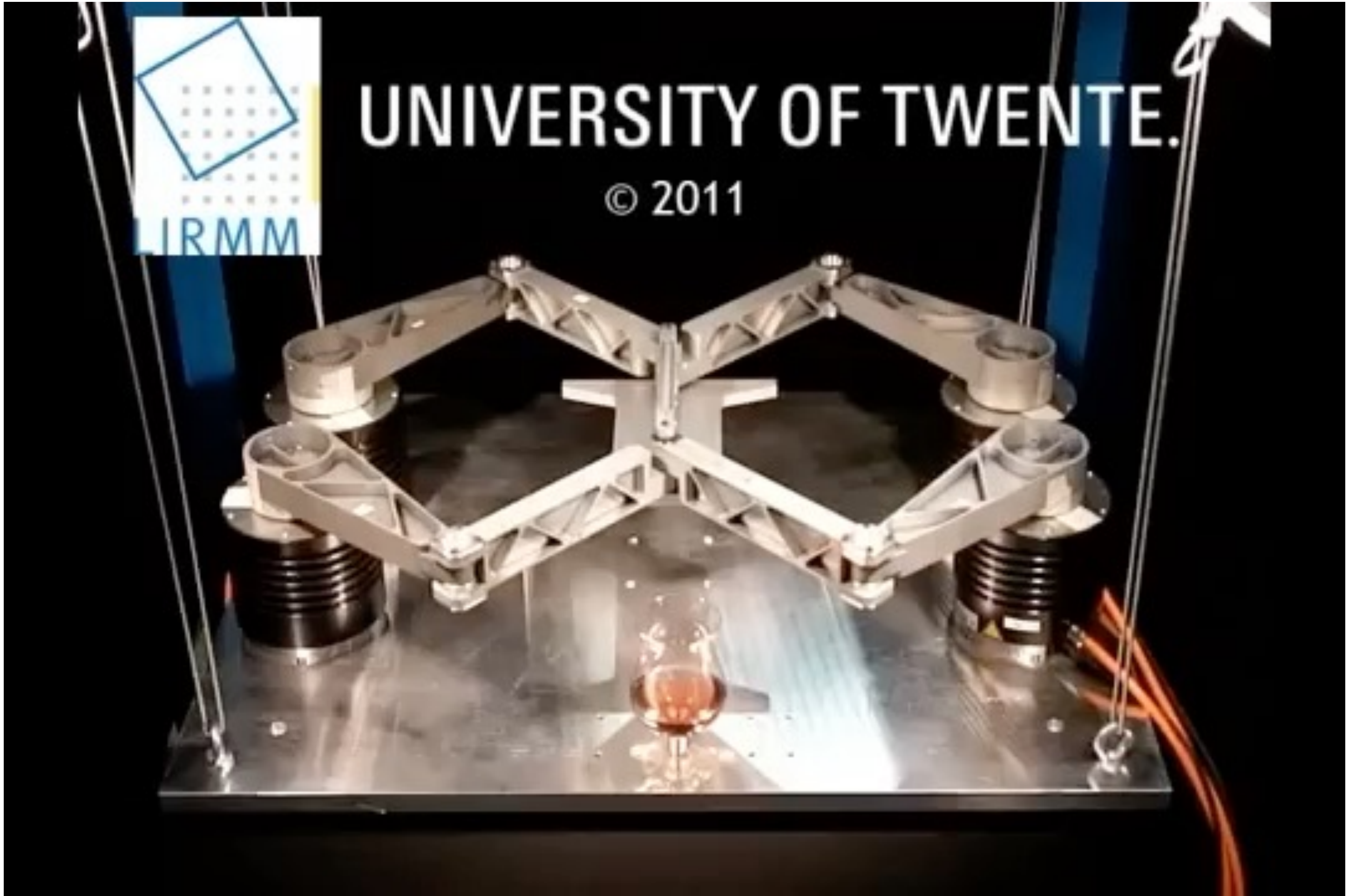
- (p) Constant Linear Momentum
- (G) Fixed CoM relative to the chassis
- (φ_g) Insensitivity to gravity orientation
- (F) No exported forces to the chassis
- (V_g) Constant gravitational potential
- (γ) Insensitivity to linear accelerations

+

Moment Balancing

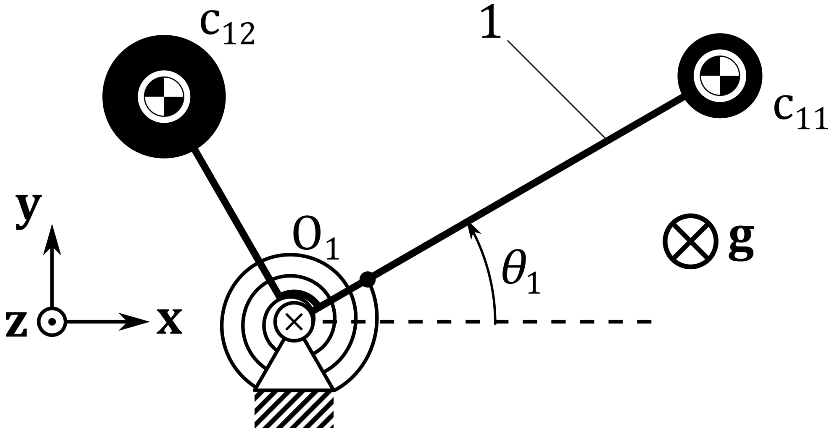
- (σ) Constant angular momentum
- (M) No exported moments to the chassis
- (α) Insensitivity to angular acceleration

Type FM*: Dynamic Balancing



video link

Type I*: Inertial Invariance



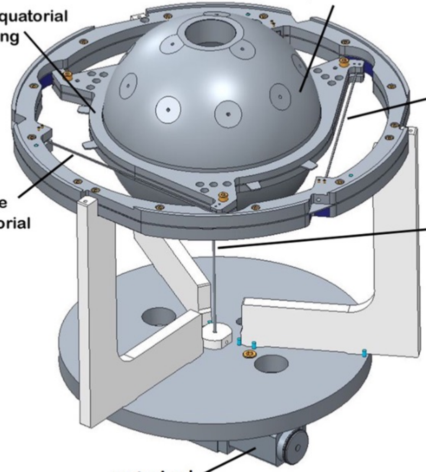
- Angular momentum pose independent
- Configuration-invariant total inertia tensor at A
- Insensitivity to angular velocities at A

Inertial Invariance condition around O_1:

$$m_{11} l_{c11}^2 = m_{12} l_{c12}^2$$

Type I*: Inertial Invariance

Foucault pendulum properties of spherical oscillators



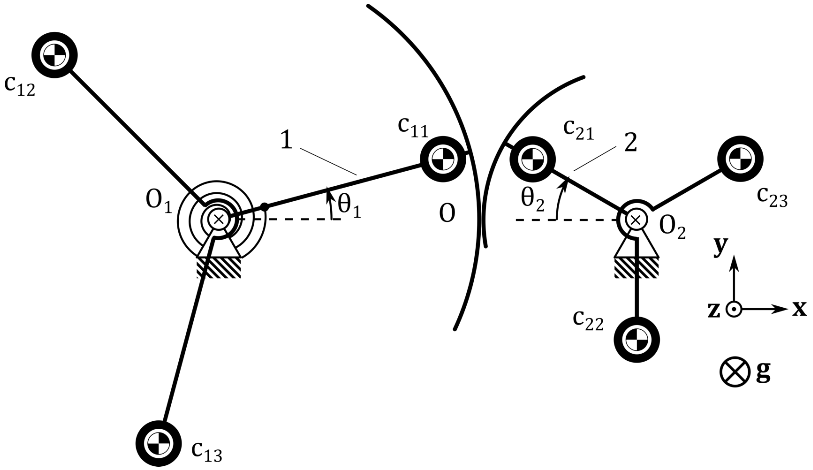
2-DoF flexure-based oscillator whose moving mass is a **sphere wobbling** (tip & tilt) around its center of mass.

The spherical oscillator **is** then **force balanced** but **is not moment balanced**.

However, even though it has a **configuration-invariant inertia tensor** (spherical inertia tensor), its angular momentum is NOT pose independent. It is therefore **sensitive to angular velocities** and can be used as a Foucault pendulum.

P. Flückiger, I. Vardi, and S. Henein, "Foucault pendulum properties of spherical oscillators," *Review of Scientific Instruments*, vol. 91, no. 9, p. 095115, Sep. 2020, doi: 10.1063/5.0010759.

Type FMI*: Inertial Balancing



Inertial balancing conditions:

$$\left.\begin{aligned} m_{11} &= m_{12} = m_{13} \\ m_{21} &= m_{22} = m_{23} \\ l_{c11} &= l_{c12} = l_{c13} \\ l_{c21} &= l_{c22} = l_{c23} \end{aligned}\right\} \text{Force balancing \& Inertial Invariance}$$

$$\left. m_{11} l_{c11}^2 = \frac{r_1}{r_2} m_{12} l_{c12}^2 \right\} \text{Moment balancing}$$

Dynamic Balancing

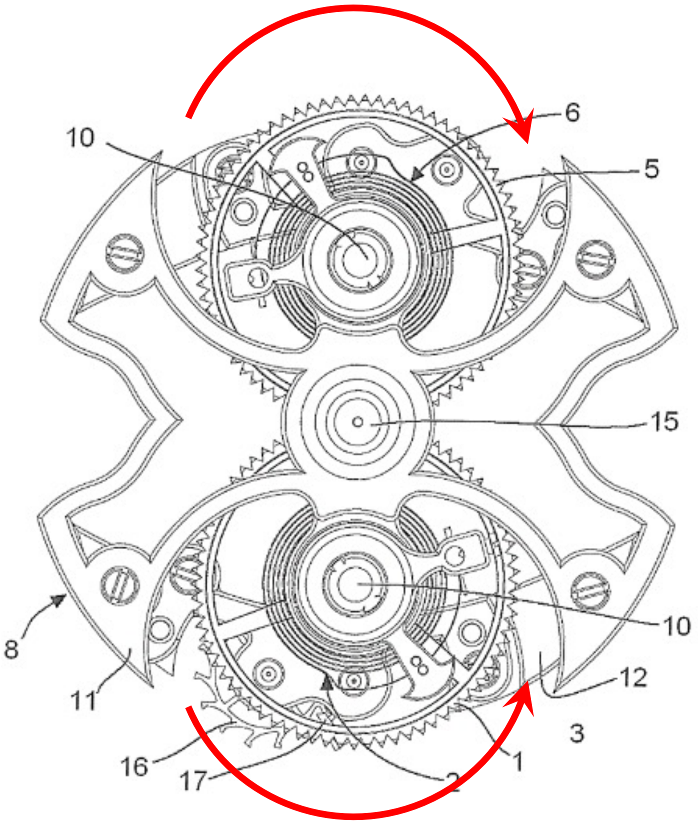
- (p) Constant Linear Momentum
- (G) Fixed CoM relative to the chassis
- (φ_g) Insensitivity to gravity orientation
- (F) No exported forces to the chassis
- (V_g) Constant gravitational potential
- (γ) Insensitivity to linear accelerations
- (σ) Constant angular momentum
- (M) No exported moments to the chassis
- (α) Insensitivity to angular acceleration

+

Inertial Invariance

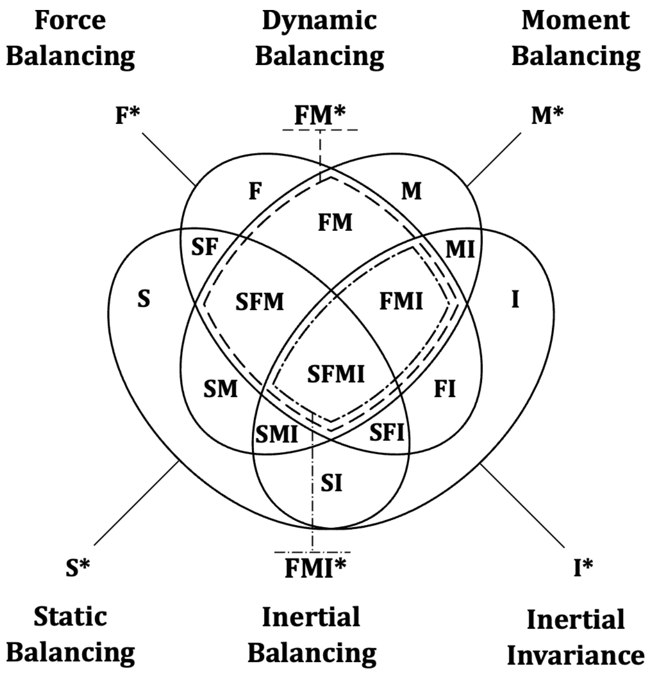
- ($\sigma_\perp$) Angular momentum pose independent
- (J) Configuration-invariant total inertia tensor
- (Ω) Insensitivity to angular velocities at A

Type FMI*: Inertial Balancing



R. Mika, "Mechanical oscillator for use in mechanical resonator of horological movement of wristwatch, has balance wheels associated with respective hairsprings, where balance wheels are toothed and arranged in engagement with each other," CH700747(B1), Jul. 31, 2014.

Proposed Balancing Taxonomy: physical properties

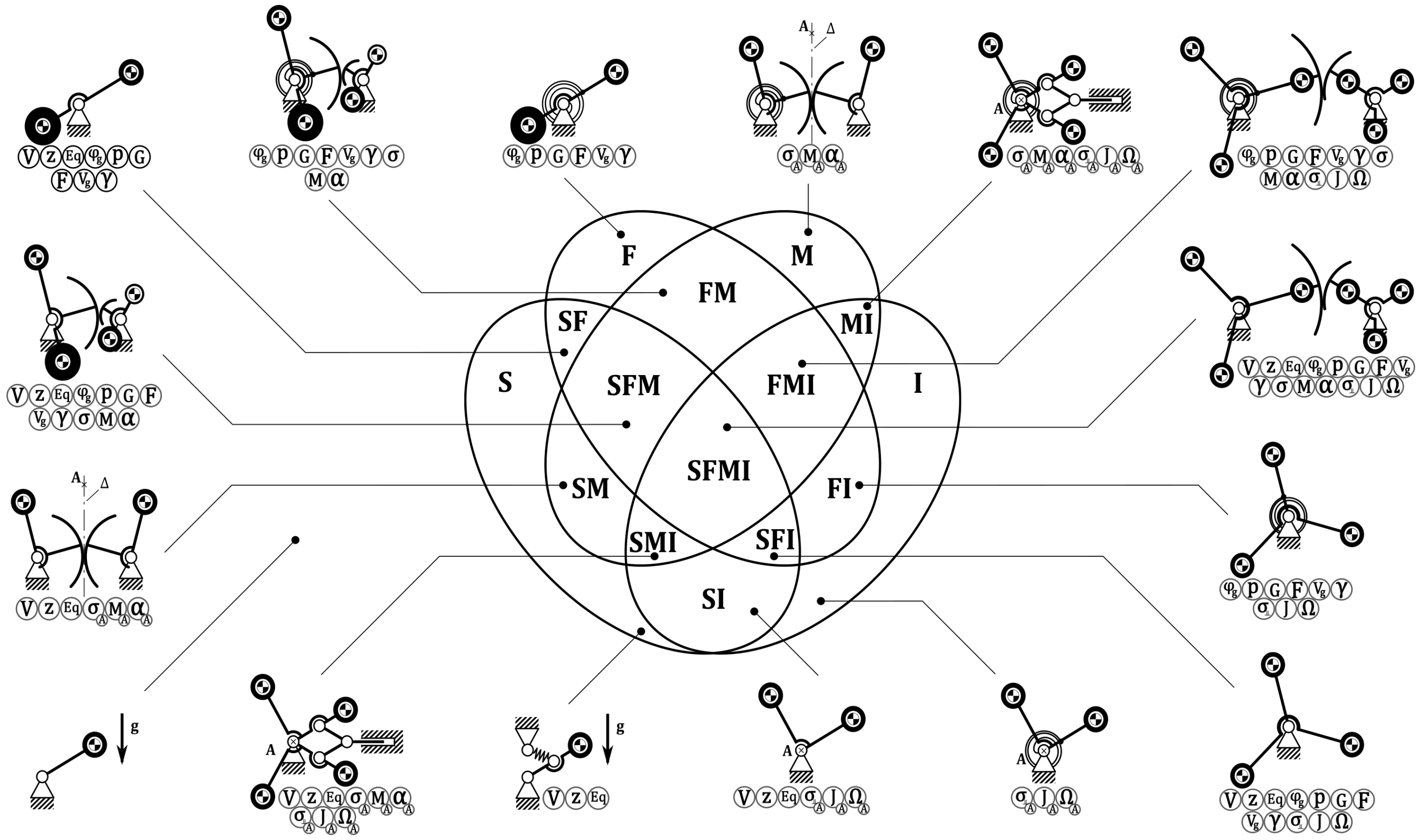


BALANCING TYPES

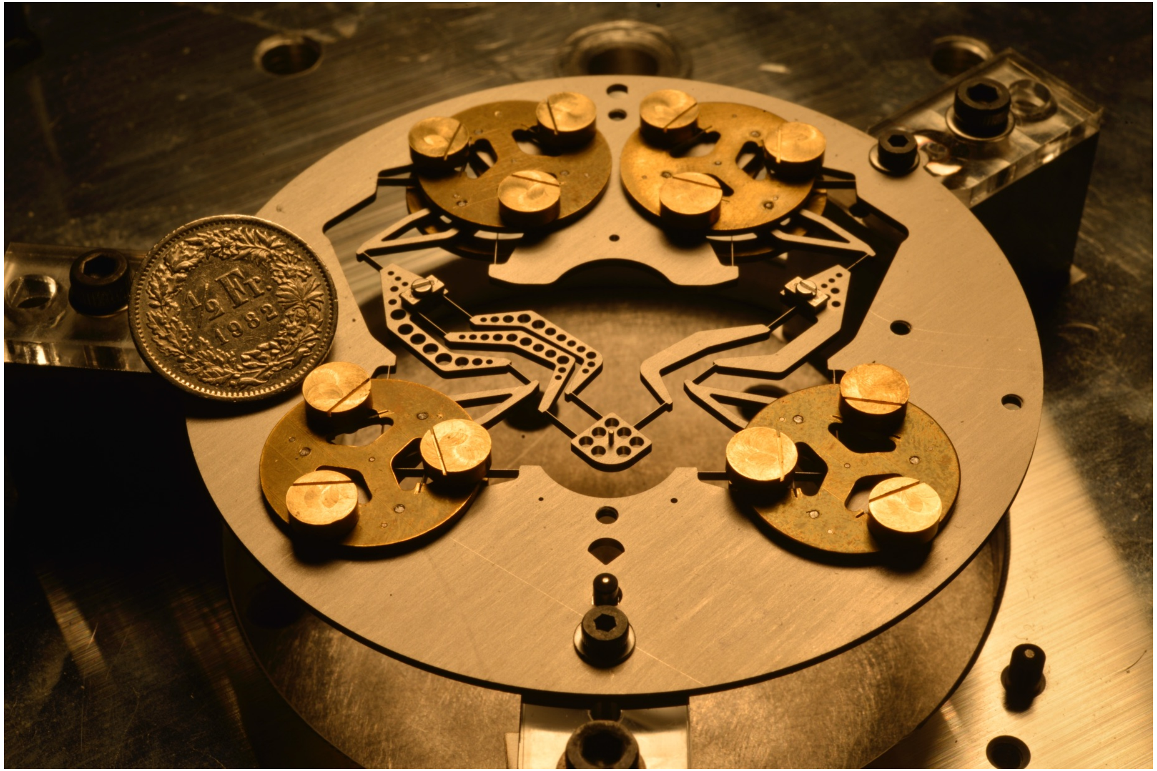
BALANCING PROPERTIES		I	S	SI	F	FI	SF	SFI	M	MI	SM	SMI	FM	FMI	SFM	SFMI
σ, M, α	M*								σ_A, M_A, α_A	σ_A, M_A, α_A	σ_A, M_A, α_A	σ_A, M_A, α_A	σ, M, α	σ, M, α	σ, M, α	σ, M, α
φ_g, p, G, F, V_g, γ	F*				φ_g, p, G, F, V_g, γ	φ_g, p, G, F, V_g, γ	φ_g, p, G, F, V_g, γ	φ_g, p, G, F, V_g, γ					φ_g, p, G, F, V_g, γ	φ_g, p, G, F, V_g, γ	φ_g, p, G, F, V_g, γ	φ_g, p, G, F, V_g, γ
V, z, Eq	S*		V, z, Eq	V, z, Eq			V, z, Eq	V, z, Eq			V, z, Eq	V, z, Eq			V, z, Eq	V, z, Eq
$\sigma_\perp$, J, Ω	I*	$\sigma_{\perp A}$, J_A, Ω_A		$\sigma_{\perp A}$, J_A, Ω_A		$\sigma_\perp$, J, Ω		$\sigma_\perp$, J, Ω		$\sigma_{\perp A}$, J_A, Ω_A		$\sigma_{\perp A}$, J_A, Ω_A		$\sigma_\perp$, J, Ω		$\sigma_\perp$, J, Ω

Increased number of balancing properties

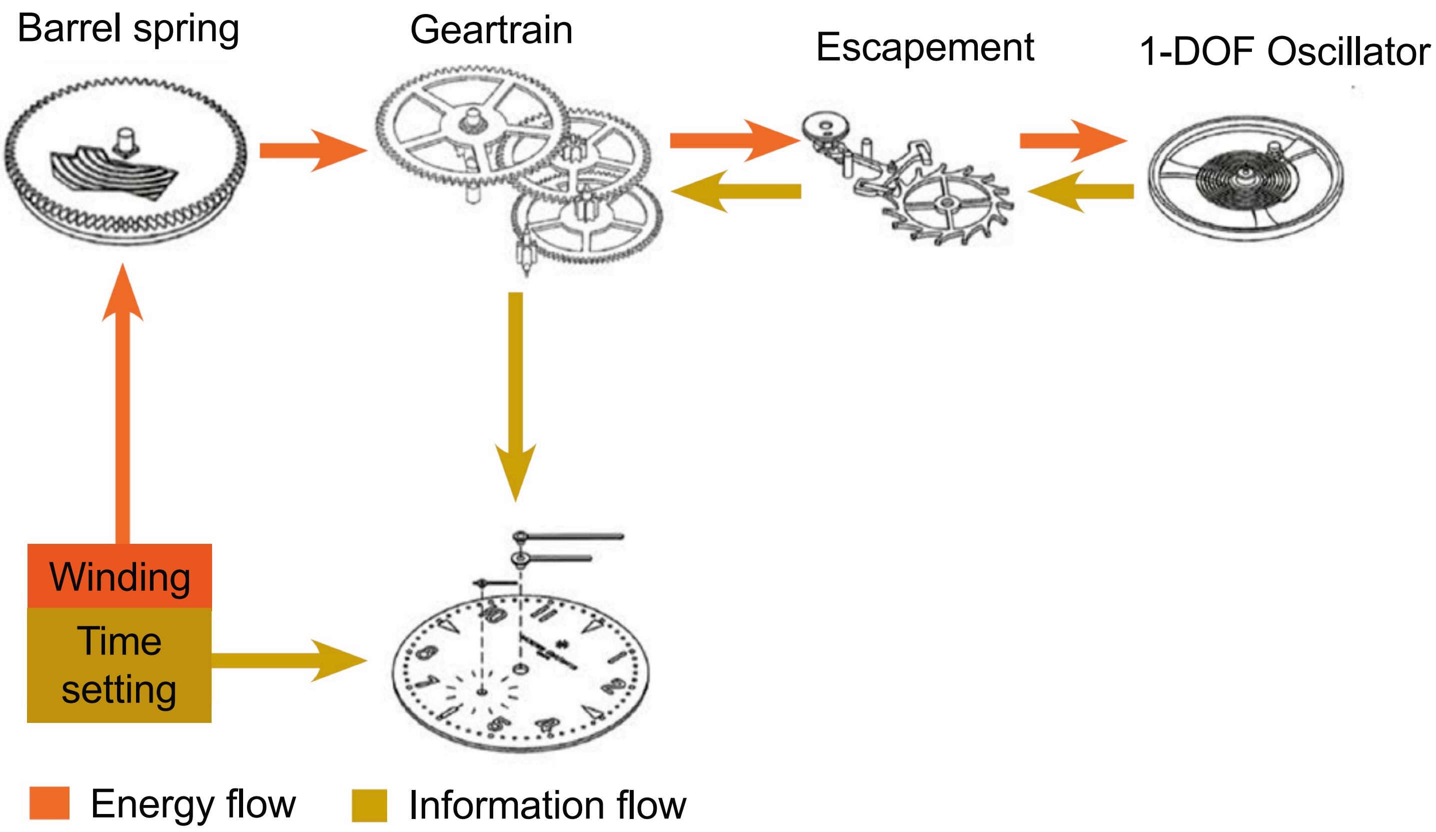
Proposed Balancing Taxonomy



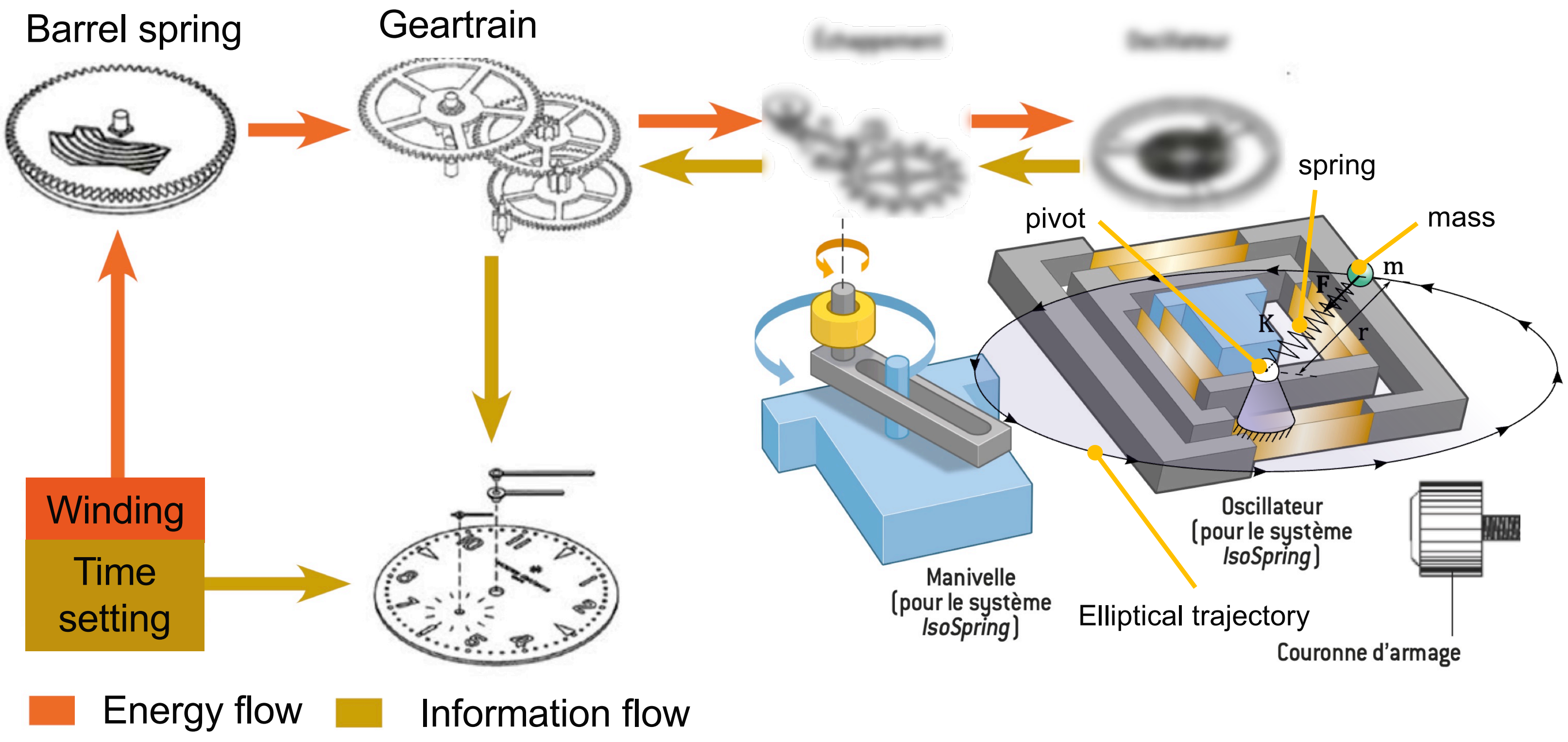
Study case of the Wattwins oscillator



Timepiece using a 1-DOF oscillator



Timepiece using a 1-DOF oscillator

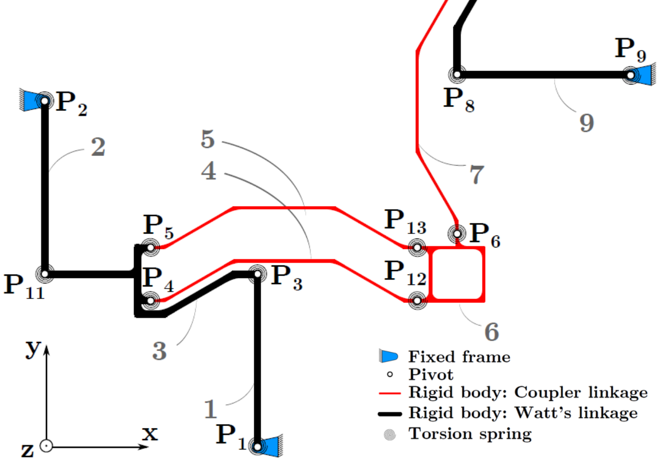


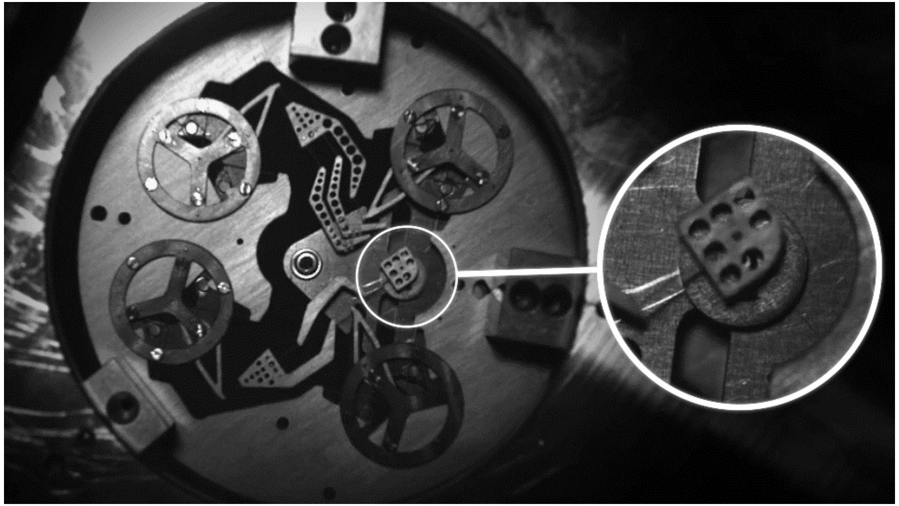
Wattwins: 2 DOF oscillator

Wattwins titanium prototype (external diam. 75 mm)

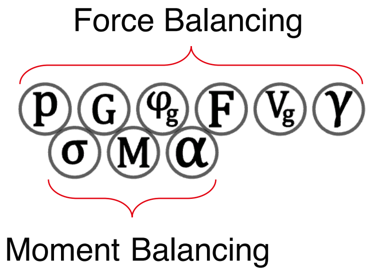
- Flexure-based 2 DOF oscillator.
- Dynamically balanced oscillator in its nominal configuration.
- Tuning mechanism that allow for a better separation of COM and inertia adjustment of the oscillator.

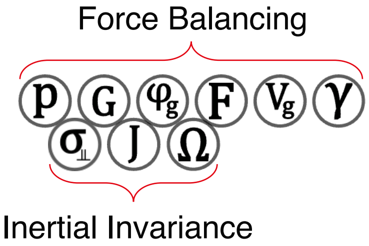
Wattwins description

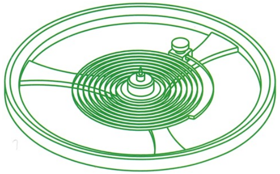


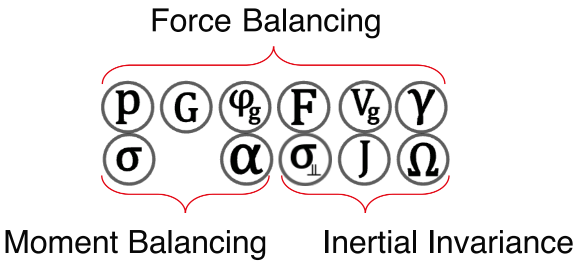


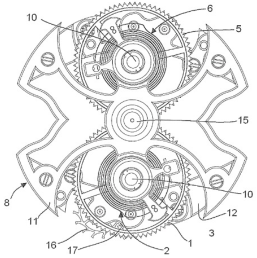
Conclusion

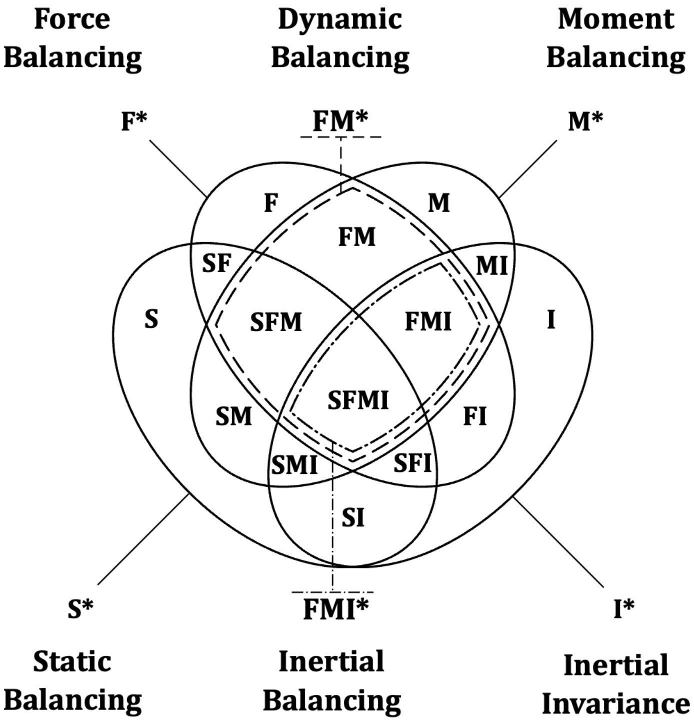

Force Balancing
p G φg F Vg γ
σ M α
Moment Balancing


Force Balancing
p G φg F Vg γ
σ⊥ J Ω
Inertial Invariance

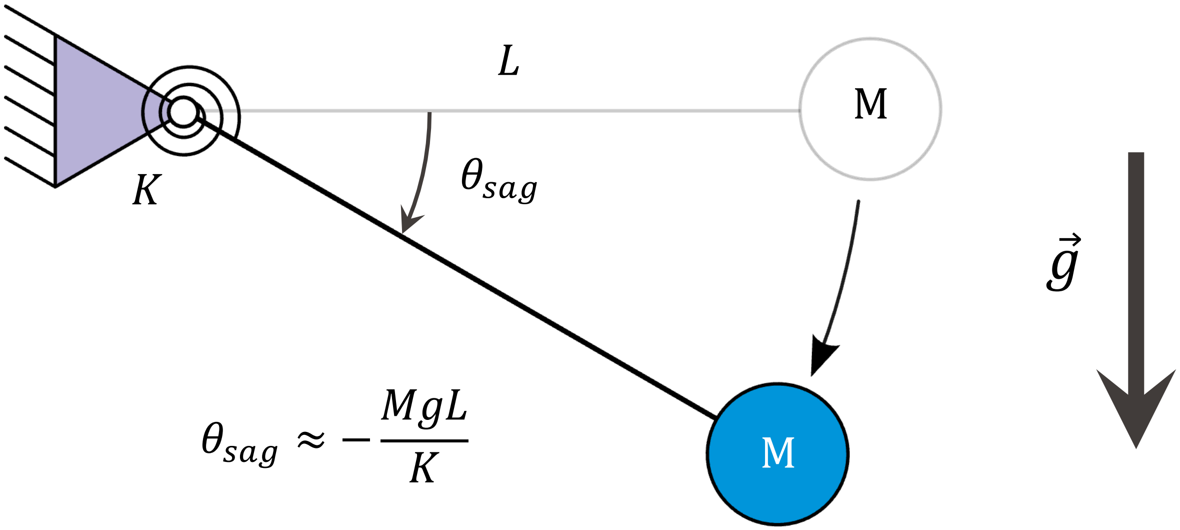



Force Balancing
p G φg F Vg γ
σ α σ⊥ J Ω
Moment Balancing
Inertial Invariance




Force Balancing
Dynamic Balancing
Moment Balancing
F*
FM*
M*
F
M
FM
SF
MI
S
SFM
FMI
I
SFMI
SM
FI
SMI
SFI
SI
S*
FMI*
I*
Static Balancing
Inertial Balancing
Inertial Invariance

1st order gravity effect

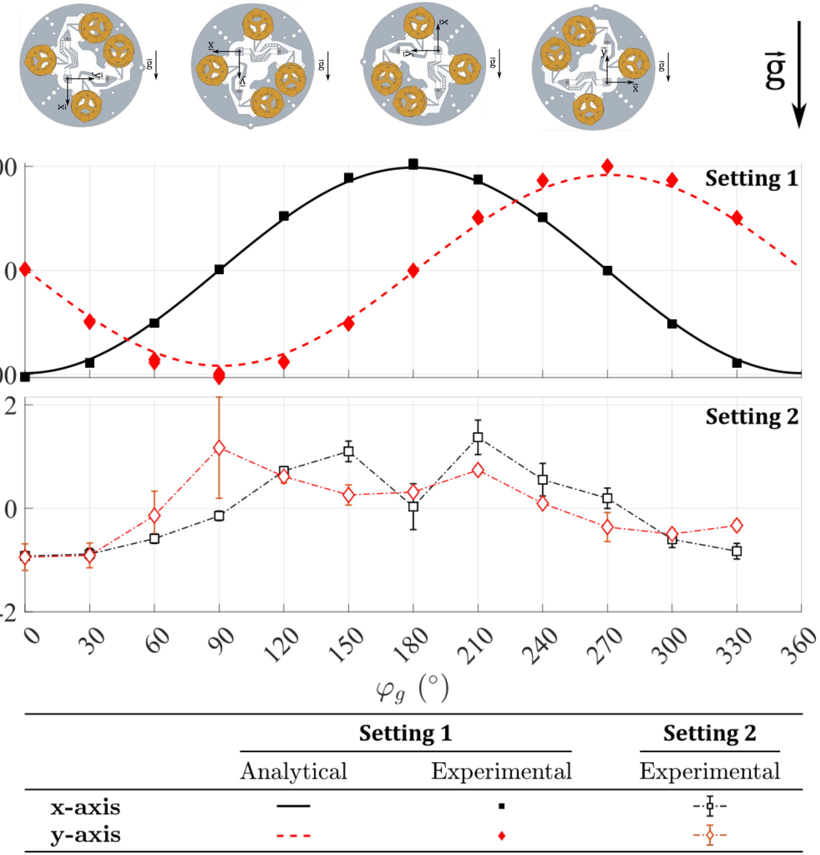


$$\theta_{sag} \approx -\frac{MgL}{K}$$

Sag cancelation: 1st order gravity effect

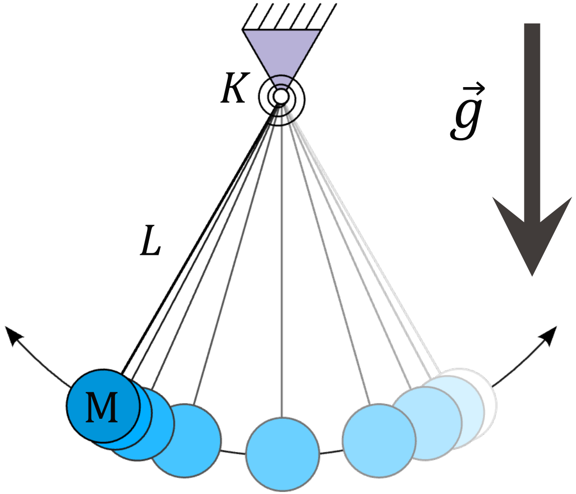
Before tuning
(Setting 1)

After tuning
(Setting 2)

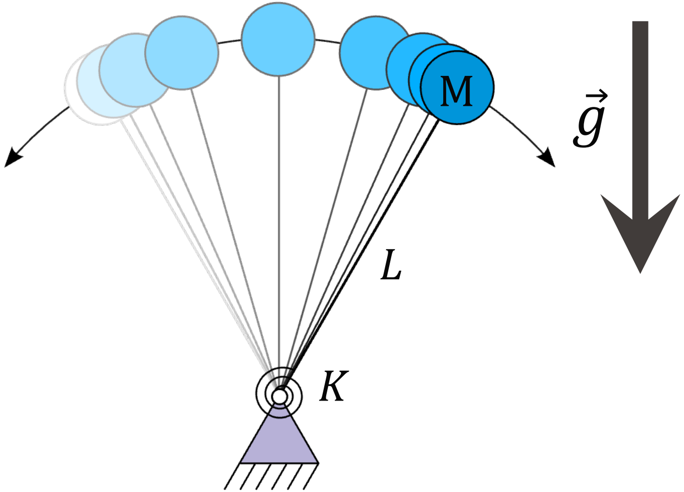


2nd order gravity effect

STIFFNESS DISCRIPANCY



$$f \approx \frac{1}{2\pi}\sqrt{\frac{K + MLg}{ML^2}}$$

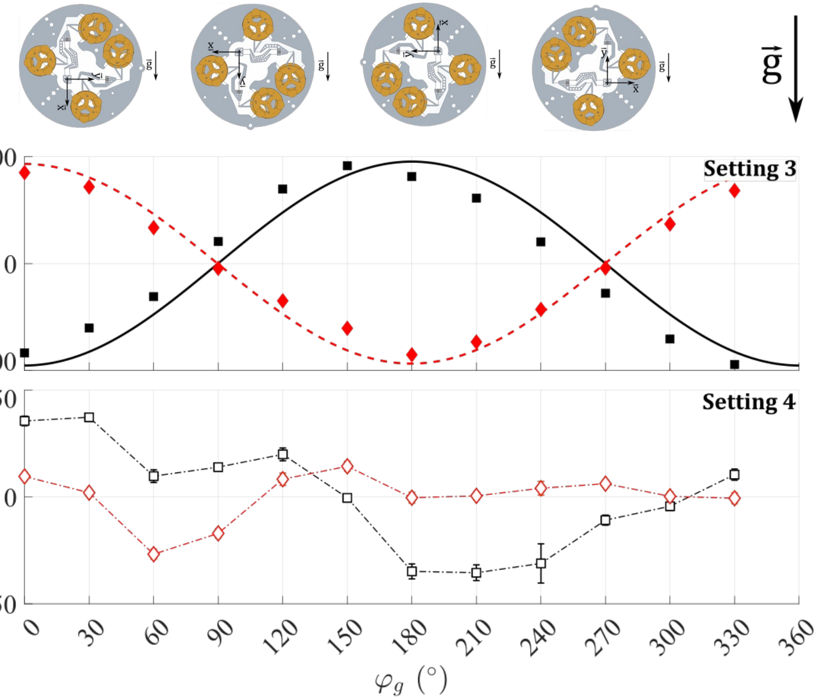


$$f \approx \frac{1}{2\pi}\sqrt{\frac{K - MLg}{ML^2}}$$

Stiffness variation cancelation: 2nd order gravity effect

Before tuning
(Setting 3)

After tuning
(Setting 4)



	Setting 3		Setting 4
	Analytical	Experimental	Experimental
x-axis	— f_0 =15.81 Hz	▪ f_0 =16.01 Hz	-⊡- f_0 =15.98 Hz
y-axis	- - - f_0 =16.57 Hz	♦ f_0 =16.63 Hz	-◇- f_0 =17.12 Hz

High dynamics and limitations for mechanisms

In a mechanism, **several aspects can limit high dynamics**, such as:

QUIZZ

- **Inertia and mass**
  Heavier components require more time to accelerate or decelerate
- **Structural rigidity and deformations**
  Structural flexing or bending can introduce errors in positioning and tracking
- **Vibrations and resonance**
  Unwanted vibrations can disrupt normal operation
- **Balancing**
  Force balanced mechanisms have generally more mass, but higher dynamics as center of mass of mobile bodies do not move
- **Clearances and mechanical play**
  Gaps between moving parts can lead to undesired motion or loss of precision
- **Drive systems and power transmission**
  Limitations in speed, torque, or response time of drive systems can restrict the mechanism's dynamic capabilities.
- **Friction and energy losses**
  Complicates control and reduces the system's dynamic performance.
- **Environmental conditions**
  Temperature, humidity, and external vibrations can influence dynamic performance by affecting component behavior or introducing external disturbances

Week 9 exercises and homework

Exercise on MOODLE:

- EXO9_Balancing_I.pdf

Homework:

- Read “Le facteur de qualité en horlogerie mécanique.pdf”