



MICRO 372 - Advanced Mechanisms for Extreme Environments

Chapter 4a

Advanced mechanisms design

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Basics of beam theory

2 common models:

- Euler-Bernoulli
 - Bending only
- Timoshenko-Ehrenfest
 - Bending and shear

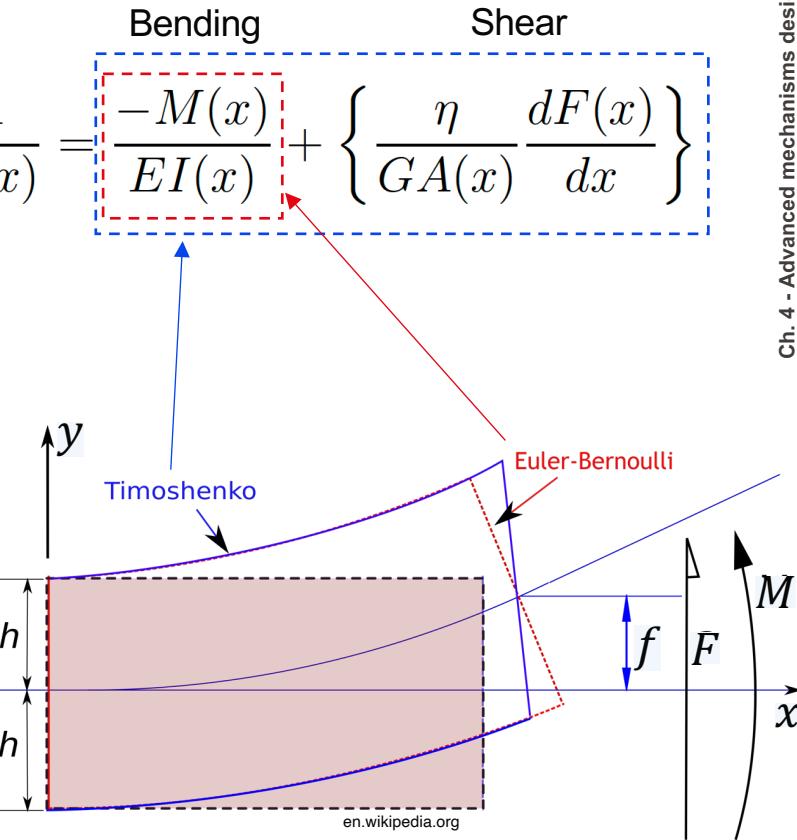
Assumptions:

- Continuity, homogeneity and isotropy of solid material
- Deflections proportional to applied stress
- Low magnitude wrt the beam dimensions

Parameters:

- y is the beam's deflection
- ρ its radius of curvature
- M the bending moment
- E the Young's modulus
- I the moment of inertia of the beam cross section
- F the shear force
- η a shape factor (6/5 for rectangular sections)
- G the shear modulus and
- A the beam cross-sectional area

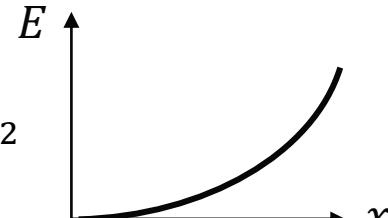
$$y''(x) \simeq \frac{1}{\rho(x)} =$$



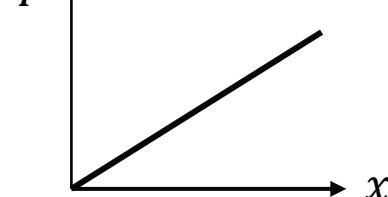
Elastic energy, force and stiffness

- Elastic energy is a potential energy
- Elasticity is reversible by nature
- Stable system, where equilibrium is at neutral position (if $K > 0$)
- The stiffness is the slope of the force profile, and the force is the slope of the energy profile
- The force is the integration of the stiffness curve and the energy if the integration of the force curve
- Stiffness is not well defined at neutral position (0 divided by 0)

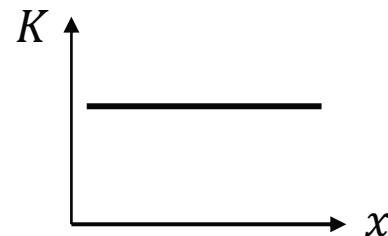
$$E_{el} = \int F(x)dx = \frac{1}{2}Kx^2$$



$$F = \frac{dE_{pot}}{dx} = Kx$$

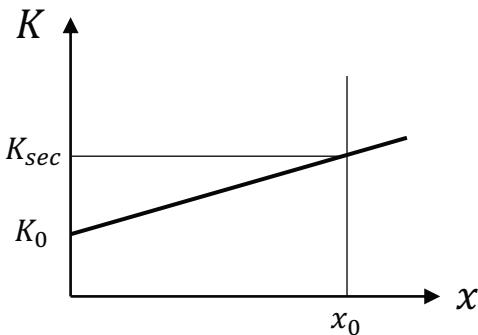
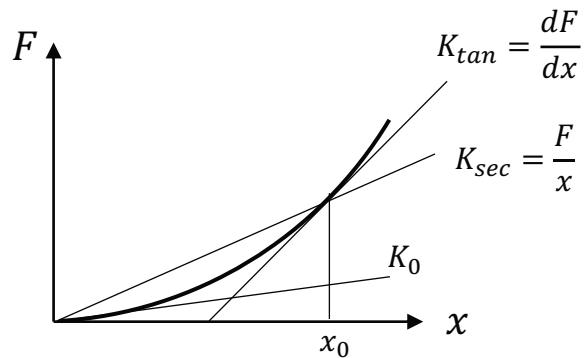


$$K = \frac{dF}{dx} = \frac{d^2E_{pot}}{dx^2} = cste$$



Nonlinearity and stiffness definition

- Non-linear force or torque profile is common in compliant mechanisms
- Generally, the nonlinearity is positive (stiffer at large deflection)
- When a nonlinearity exists, the secant stiffness differs from the tangent stiffness
- In practice, the secant stiffness is generally used
- For some applications, the nonlinear effect is not a problem (guiding with actuation and control)
- For watch oscillators, the chronometric performance is given by a very linear torque of spiral spring ($1 \text{ sec/day} \approx 10 \text{ ppm}$)



Stiffnesses combination

Stiffnesses in series

Inverse of equivalent stiffness of springs is given by addition of inverse of individual stiffness

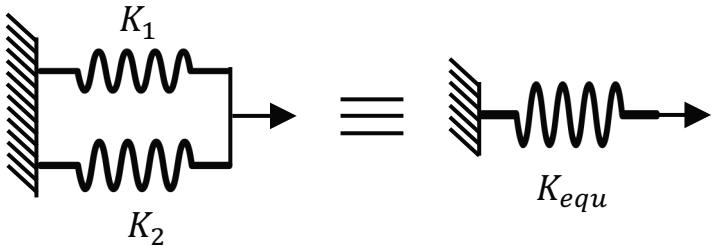
$$\frac{1}{K_{equ}} = \frac{1}{K_1} + \frac{1}{K_2}$$



Stiffnesses in parallel

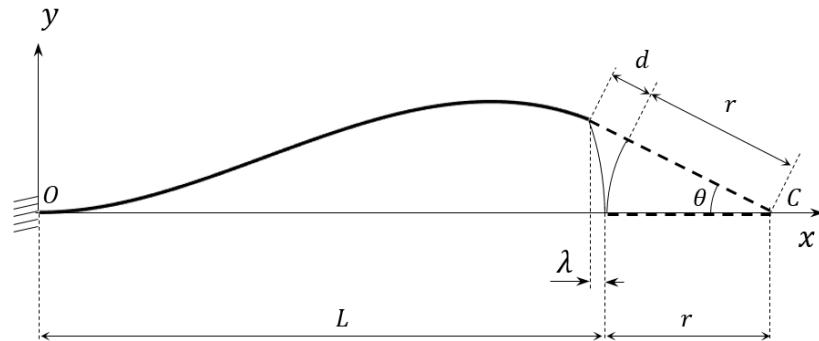
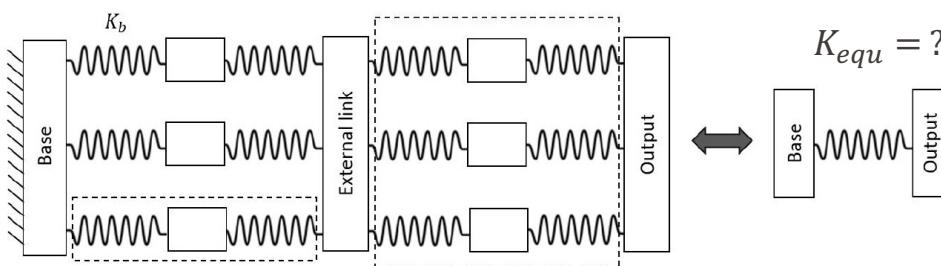
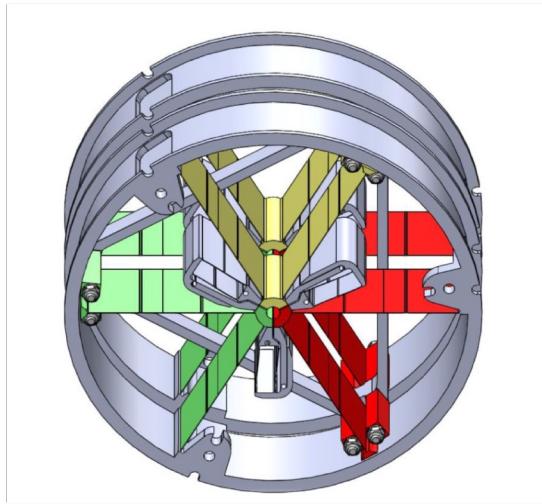
Equivalent stiffness of springs is given by addition of individual stiffnesses

$$K_{equ} = K_1 + K_2$$



Stiffnesses combination: example

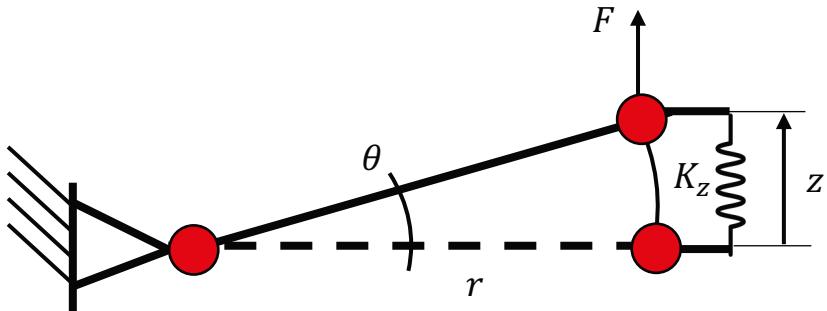
- What is the rotation stiffness of the Large Angle Flexure Pivot?



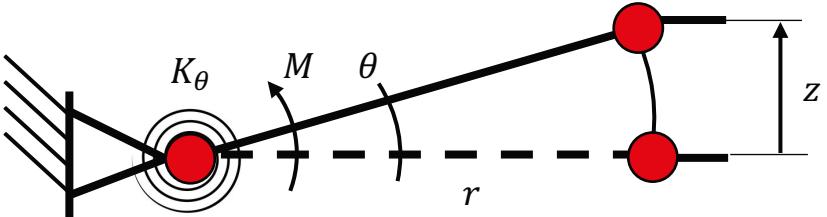
$$\rho = \frac{r}{L}$$

$$K_b = \frac{K_{RCC}}{2} = \frac{4EI}{L} (1 + 3\rho + 3\rho^2)$$

Stiffnesses in translation to rotation conversion



≡



$$K_z = \frac{F}{z}$$

$$K_\theta = \frac{M}{\theta}$$

$$M = F \cdot r$$

$$\theta = \text{atan}\left(\frac{z}{r}\right) \approx \frac{z}{r}$$

$$K_\theta = r^2 \cdot \frac{F}{z} = r^2 \cdot K_z$$

Force method for stiffness calculation

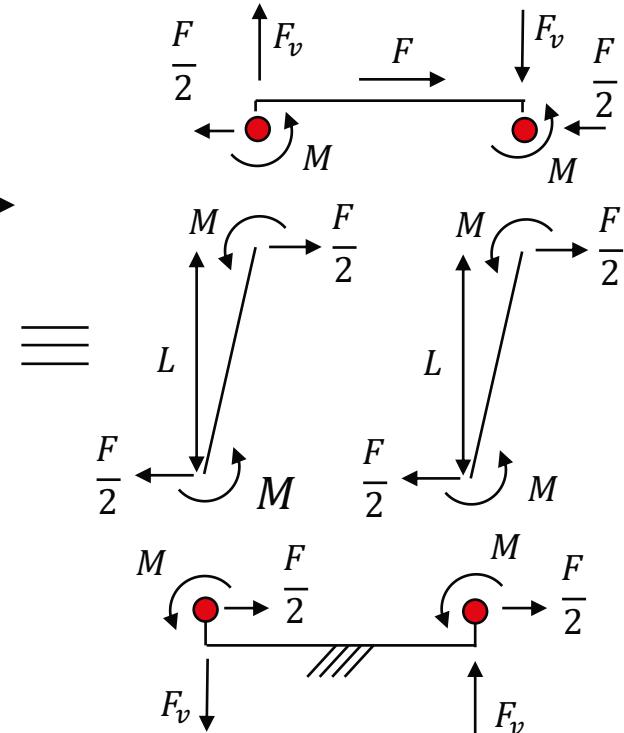
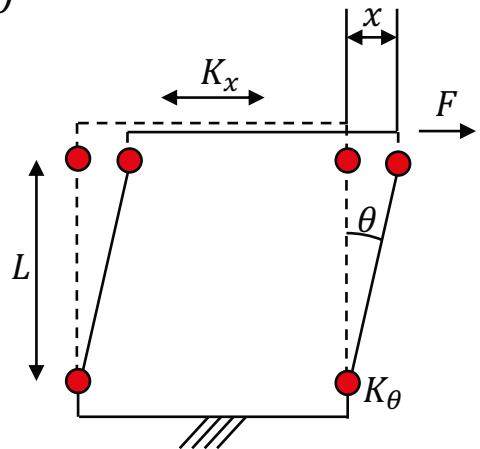
- Stage stiffness: $K = F/x$
- Pivot stiffness: $K_\theta = M/\theta$
- Equilibrium condition give the elastic torque:

$$\sum M = 0 \rightarrow M = \frac{FL}{4}$$

- As we have $\theta = \frac{x}{L}$
- Then

$$K = \frac{F}{x} = \frac{4M}{L^2\theta} \rightarrow K = \frac{4K_\theta}{L^2}$$

with $K_\theta = \frac{Ebh^3}{12l}$



Energy method for stiffness calculation

The energy of the whole stage:

$$E_{stage} = \frac{1}{2} K x^2$$

..equals the sum of energies at pivots level:

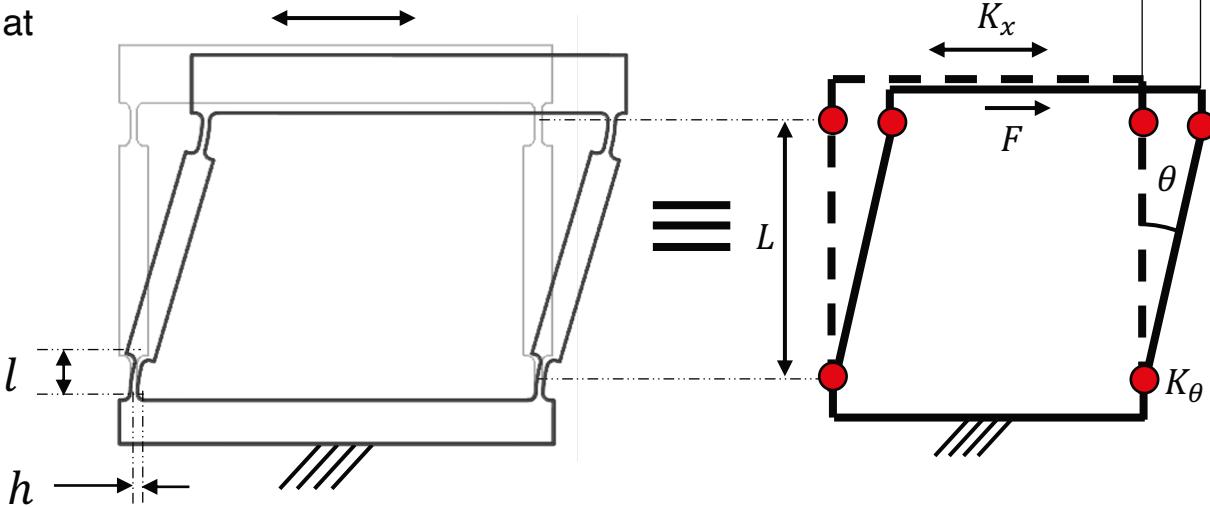
$$E_{pivots} = 4 \cdot \frac{1}{2} K_\theta \theta^2$$

As we have:

$$\theta = \frac{x}{L}$$

It comes:

$$E_{stage} = E_{pivots}$$



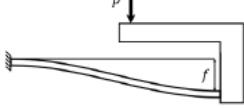
\leftrightarrow

$$K = \frac{4K_\theta}{L^2}$$

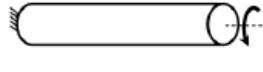
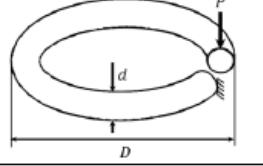
with

$$K_\theta = \frac{Ebh^3}{12l}$$

Flexure calculation form (1)

LOAD CASE	STIFFNESS	STROKE	FORCES & TORQUES	ENERGY
	$K = \frac{E \cdot b \cdot h}{l}$	$f = \frac{\sigma \cdot l}{E}$	$P = \sigma \cdot b \cdot h$	$U = \frac{\sigma^2 \cdot b \cdot h \cdot l}{2 \cdot E}$
	$K = \frac{\pi \cdot E \cdot d^2}{4 \cdot l}$	$f = \frac{\sigma \cdot l}{E}$	$P = \frac{\pi \cdot \sigma \cdot d^2}{4}$	$U = \frac{\pi \cdot \sigma^2 \cdot d^2 \cdot l}{8 \cdot E}$
	$K_{\alpha M} = \frac{E \cdot I}{l} = \frac{E \cdot b \cdot h^3}{12 \cdot l}$	$\alpha = \frac{2 \cdot \sigma \cdot l}{E \cdot h}$	$M = \frac{\sigma \cdot b \cdot h^2}{6}$	$U = \frac{\sigma^2 \cdot b \cdot h \cdot l}{6 \cdot E}$
	$K_{f M} = \frac{2 \cdot E \cdot I}{l^2} = \frac{E \cdot b \cdot h^3}{6 \cdot l^2}$	$f = \frac{\sigma \cdot l^2}{E \cdot h}$		
	$K_{f P} = \frac{3 \cdot E \cdot I}{l^3} = \frac{E \cdot b \cdot h^3}{4 \cdot l^3}$	$f = \frac{2 \cdot \sigma \cdot l^2}{3 \cdot E \cdot h}$	$P = \frac{\sigma \cdot b \cdot h^2}{6 \cdot l}$	$U = \frac{\sigma^2 \cdot b \cdot h \cdot l}{18 \cdot E}$
	$K_{\alpha P} = \frac{2 \cdot E \cdot I}{l^2} = \frac{E \cdot b \cdot h^3}{6 \cdot l^2}$	$\alpha = \frac{\sigma \cdot l}{E \cdot h}$		
	$K_{f P} = \frac{12 \cdot E \cdot I}{l^3} = \frac{E \cdot b \cdot h^3}{l^3}$	$f = \frac{\sigma \cdot l^2}{3 \cdot E \cdot h}$	$P = \frac{\sigma \cdot b \cdot h^2}{3 \cdot l}$	$U = \frac{\sigma^2 \cdot b \cdot h \cdot l}{18 \cdot E}$

Flexure calculation form (2)

LOAD CASE	STIFFNESS	STROKE	FORCES & TORQUES	ENERGY
	$K_{\alpha M} = \frac{4 \cdot E \cdot I}{l} = \frac{E \cdot b \cdot h^3}{3 \cdot l}$	$\alpha = \frac{\sigma \cdot l}{2 \cdot E \cdot h}$	$M = \frac{\sigma \cdot b \cdot h^2}{6}$	$U = \frac{\sigma^2 \cdot b \cdot h \cdot l}{24 \cdot E}$
	$K_{\alpha M} = \frac{4 \cdot E \cdot I}{l} \cdot \left(1 + \frac{3 \cdot r}{l} + \frac{3 \cdot r^2}{l^2}\right)$	$\alpha = \frac{\sigma \cdot l^2}{E \cdot (2 \cdot h \cdot l + 3 \cdot h \cdot r)}$		
	$K_{TORS} = \frac{b \cdot h^3 \cdot G}{3 \cdot l}$	$\alpha = \frac{\tau \cdot l}{h \cdot G}$	$(\tau = \frac{\sigma}{\sqrt{3}})$	
	$K_{TORS} = \frac{G \cdot I_p}{l} = \frac{G \cdot \pi \cdot d^4}{32 \cdot l}$	$\alpha = \frac{2 \cdot \tau \cdot l}{G \cdot d}$	$M = \frac{2 \cdot \tau \cdot l \cdot p}{d}$ $= \frac{\pi \cdot \tau \cdot d^3}{16}$	$U = \frac{\pi \cdot \tau^2 \cdot d^2 \cdot l}{16 \cdot G}$
	$K = \frac{G \cdot d^4}{8 \cdot n \cdot D^3}$	$f = \frac{\pi \cdot \tau \cdot n \cdot D^2}{G \cdot d}$	$P = \frac{\pi \cdot \tau \cdot d^3}{8 \cdot D}$	$U = \frac{4 \cdot n \cdot P^2 \cdot D^3}{G \cdot d^4}$ $= \frac{\pi^2 \cdot \tau^2 \cdot d^2 \cdot n \cdot D}{16 \cdot G}$

Linear stage parasitic motion

- Vertical parasitic motion is a quadratic function of the horizontal displacement for linear stages composed of blades, and a cosine function for linear stages composed of hinges
- Linear stage with flexure blades:

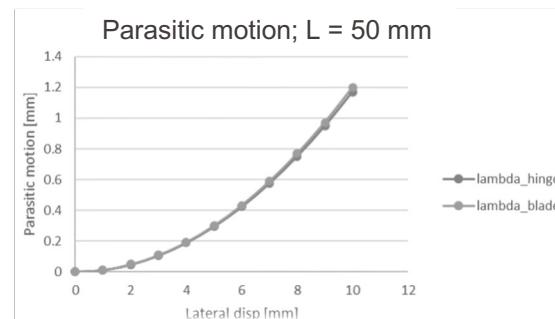
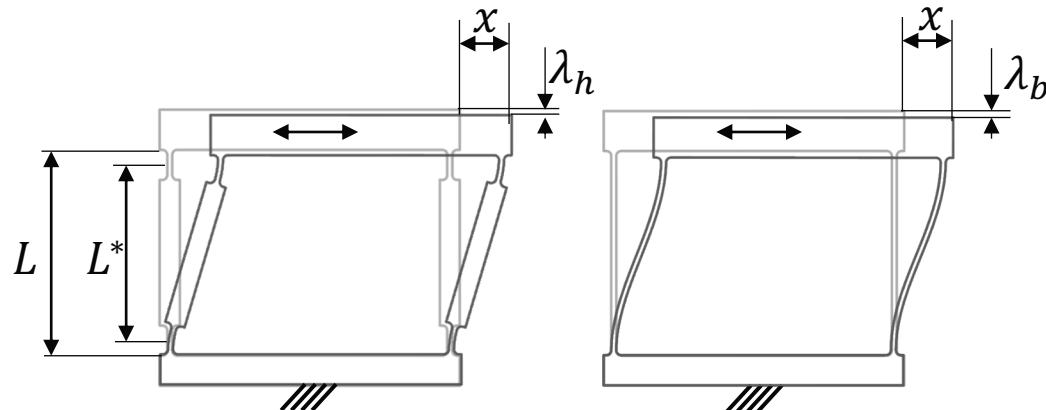
$$\lambda_b = \frac{3x^2}{5L}$$

- Linear stage with hinges:

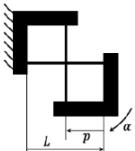
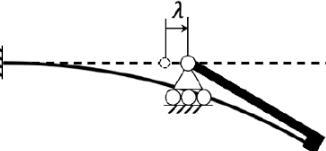
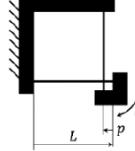
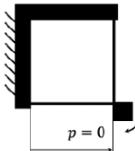
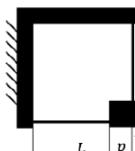
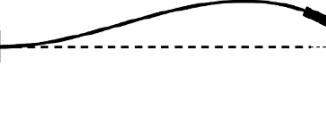
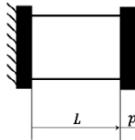
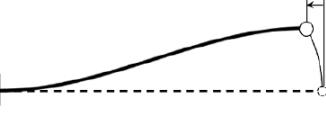
$$\lambda_h = L^*(1 - \cos \frac{x}{L^*})$$

$$L^* \cong 0.85 \cdot L$$

(for a stiffener of 70% of length)

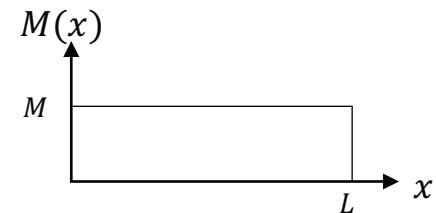
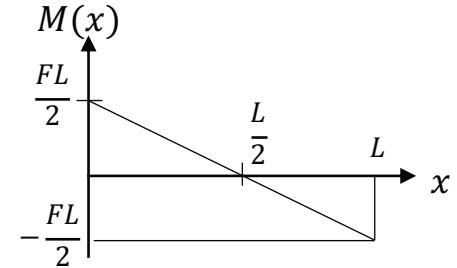
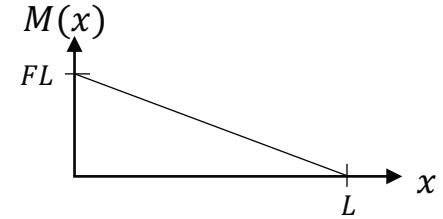
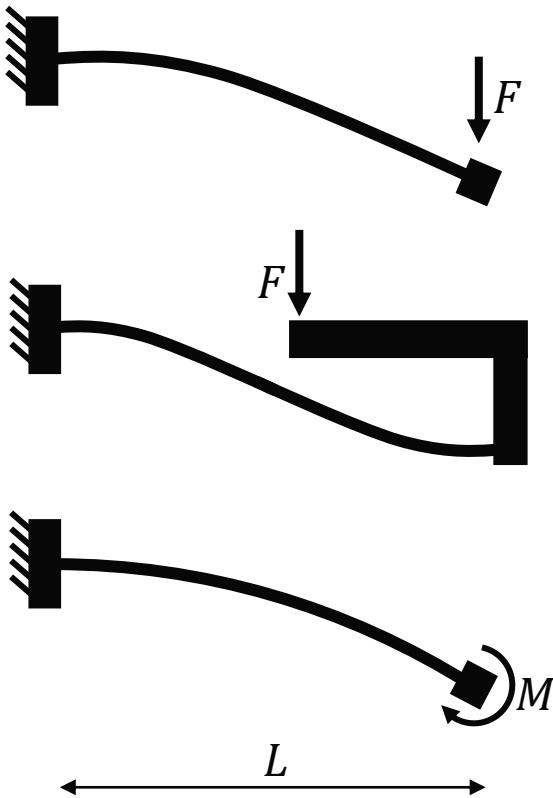


Blade shortening – Generic case

TYPE DE PIVOT	CAS DE CHARGE	RAPPORT DE CROISEMENT DES LAMES	DÉFORMÉE DE LA LAME	RACCOURCISSEMENT APPARENT DE LA LAME
PIVOT À LAMES CROISÉES SÉPARÉES		$\rho = \frac{p}{L} = -\frac{1}{2}$		$\lambda = -\frac{La^2}{12}$
PIVOT WITTRICK		$\rho = \frac{p}{L} = \frac{\sqrt{5}-3}{6} \approx -\frac{1}{8}$		$\lambda = 0$
PIVOT À LAMES EN COIN		$\rho = \frac{p}{L} = 0$		$\lambda = \frac{La^2}{15}$
PIVOT RCC (CAS GÉNÉRIQUE)		$\rho = \frac{p}{L} > 0$		$\lambda = \frac{La^2}{15} (1 + 9\rho + 9\rho^2)$
TABLE À LAMES PARALLÈLES		$\rho = \frac{p}{L} = \infty$		$\lambda = \frac{3f^2}{5L}$ ($\rho \gg 1$ et $f = p \cdot \alpha$)

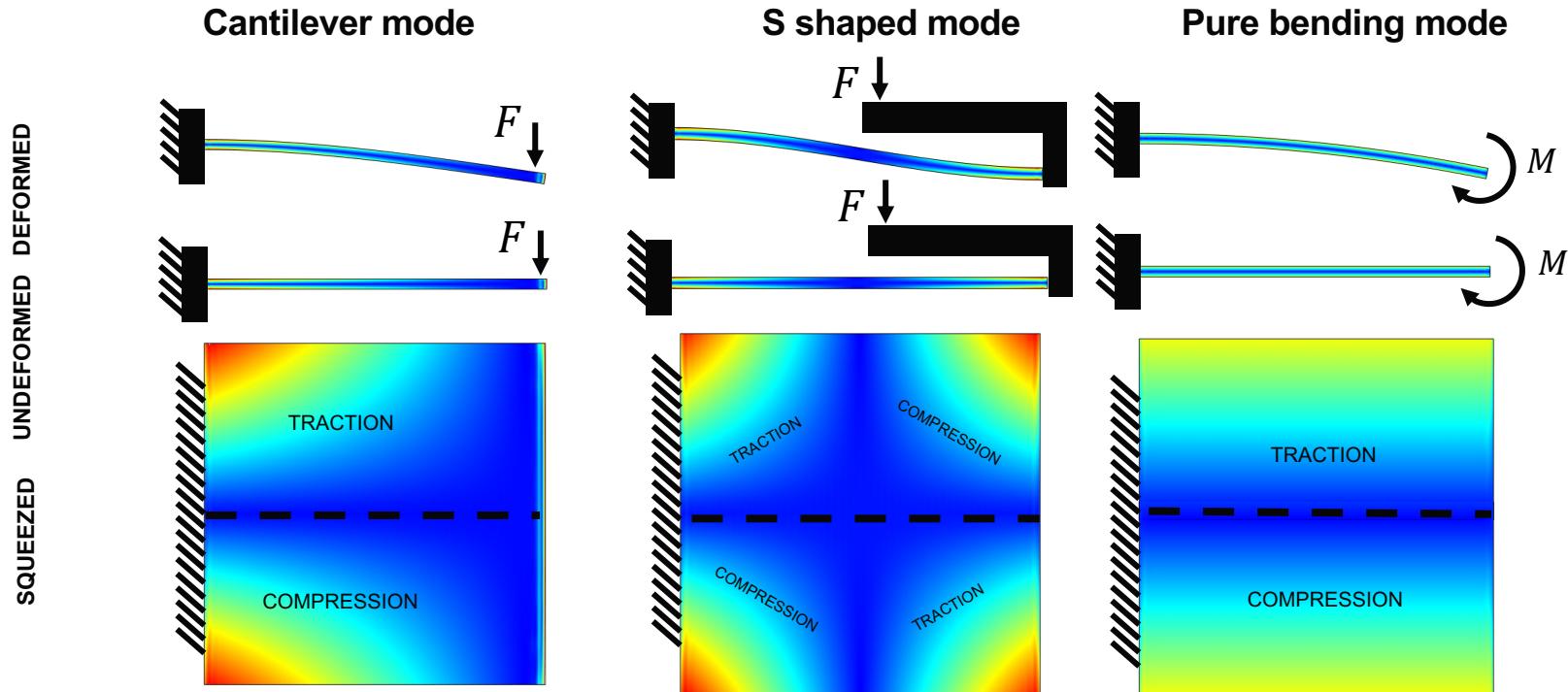
Blade deformation modes

- Cantilever mode
- S shaped mode
- Pure bending mode



Stress distribution in a bended blade

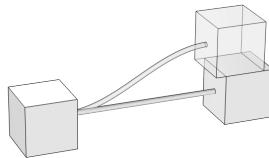
- Stress is proportional to bending torque and curvature
- Stress is proportional to distance from neutral axis



Flexure building blocs (bricks, joints, mechanisms)

- Flexure **bricks**:

Elementary flexible elements



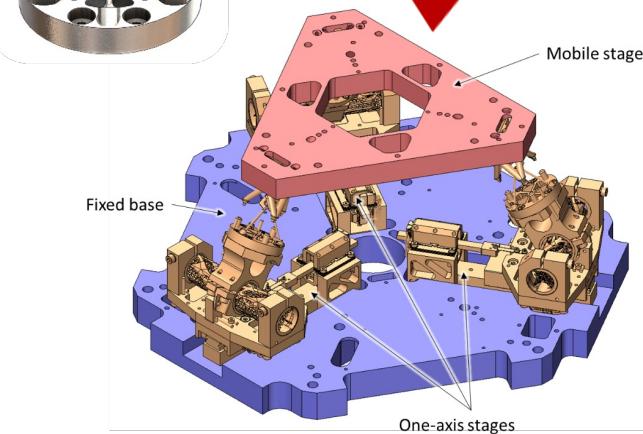
- Flexure **joints**:

Combination of flexure bricks



- Flexure **mechanisms**:

Assembly of compliant joints



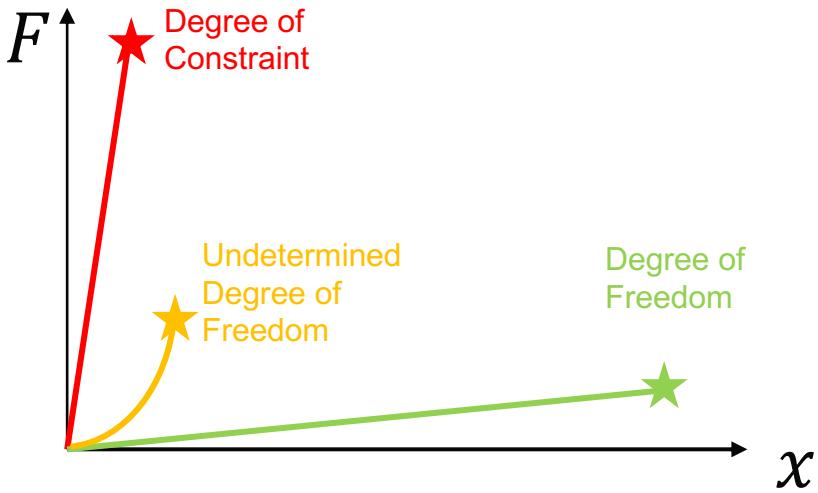
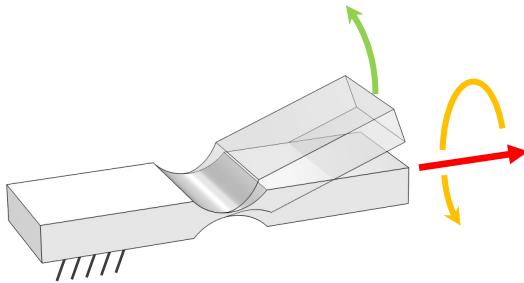
Flexure building blocs (bricks, joints, mechanisms)

		1 DoF	2 DoF	3 DoF	4 DoF	5 DoF	6 DoF
Translation	1	Linear stage Pair of membranes Sarrus Orthogryres Pseudorthogryres	2	Planar joint * XY stage bellow Serial XY stage Linear Isospring	3 Delta Serial XYZ stage	3	
Rotation	0		0		1	2	Bellow
Translation	0	Hinge * Torsion bar Pivot	1	Double hinge * Blade and rod	2 Spatial parallelo. * XYRz Stage	2 Double uni-joint *	
Rotation	1		1		2	3 Rod L-shaped blade	
Translation			0	Universal joint *** Circular hinge *** Spherical Isospring	1 Blade Membrane Tripod Tip-tilt piston	1	
Rotation			2		3		
Translation				0	Gimbal		
Rotation				3			

Flexures undetermined DOFs: *(**) DOF(s) not stiff

- Degree of Constraint **DoC**: high stiffness, low stroke
- Degree of Freedom **DOF**: low stiffness, high stroke
- Undetermined Degree of Freedom **uDoF**: low stiffness, low stroke

HINGE *



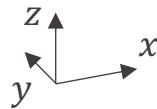
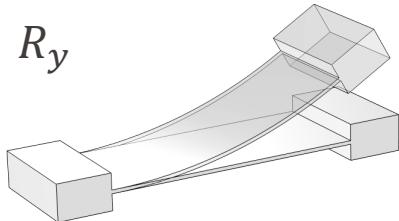
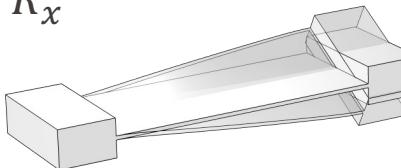
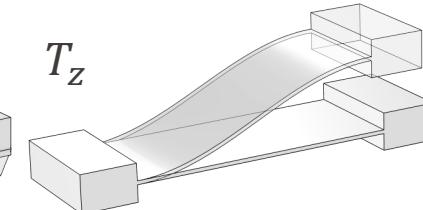
Flexure building blocs (bricks, joints, mechanisms)

		1 DoF	2 DoF	3 DoF	4 DoF	5 DoF	6 DoF	
Translation	1	Linear stage Pair of membranes Sarrus Orthogryres Pseudorthogryres	2	Planar joint * XY stage bellow Serial XY stage Linear Isospring	3	Delta Serial XYZ stage	3	Bellow
Rotation	0		0		1		2	
Translation	0		1		2		3	
Rotation	1	Hinge * Torsion bar Pivot	1	Double hinge * Blade and rod	1	Spatial parallelo. * XYRz Stage	2	
Translation			0	Universal joint *** Circular hinge ***	1	Blade Membrane	2	
Rotation			2	Spherical Isospring	2	Tripod Tip-tilt piston	3	
Translation				0		Gimbal		
Rotation				3				

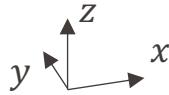
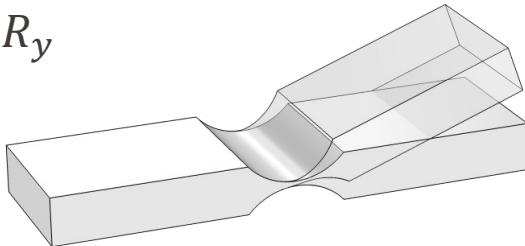
*(**) DOF(s) not stiff

Flexure building blocs (bricks, joints, mechanisms)

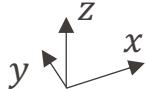
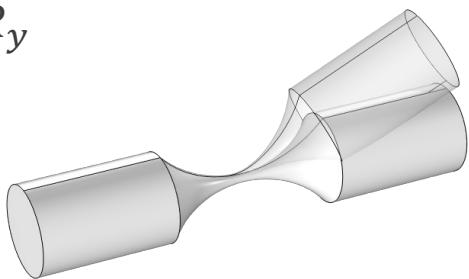
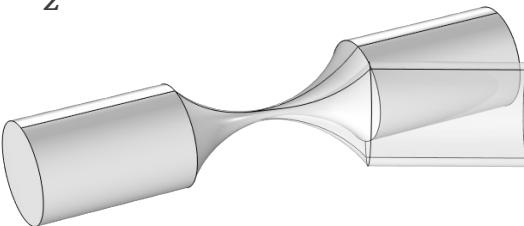
BLADE

 R_y  R_x  T_z 

HINGE *

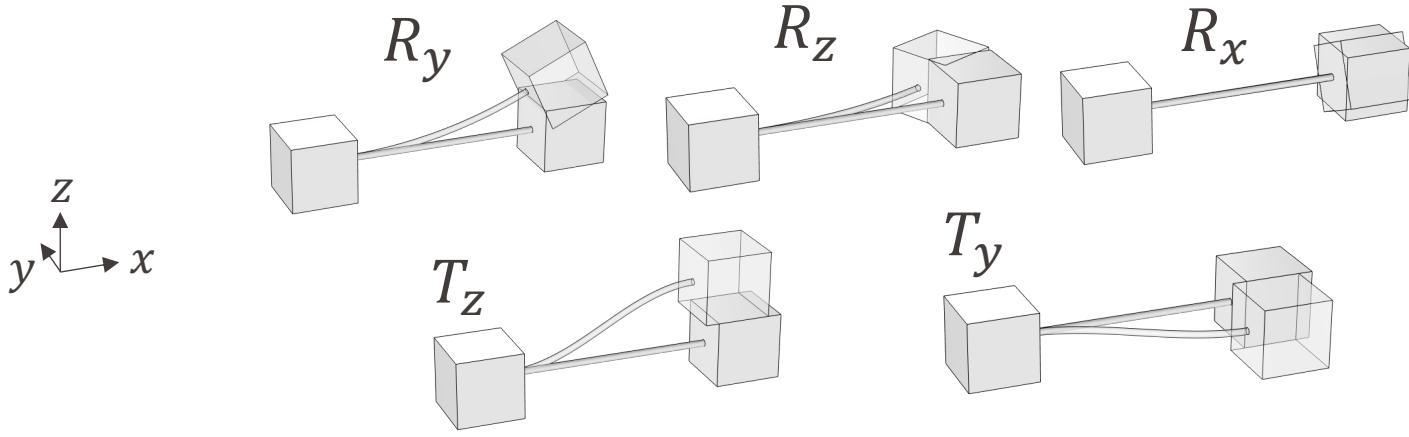
 R_y * R_x not stiff

CIRCULAR HINGE ***

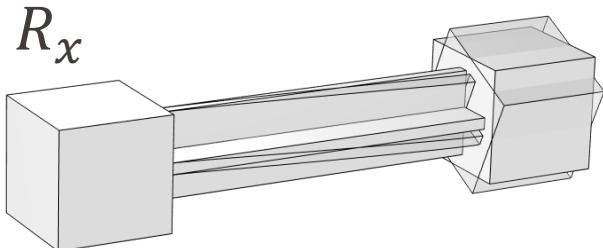
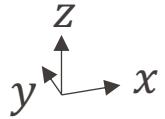
 R_y  R_z *** $y z R_x$ not stiff

Flexure building blocs (bricks, joints, mechanisms)

ROD

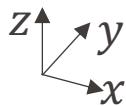
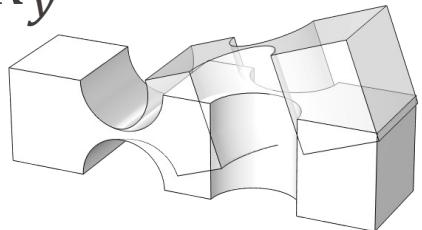
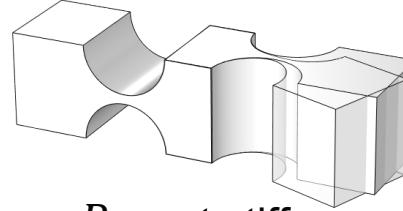


TOSRION BAR



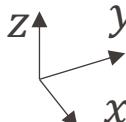
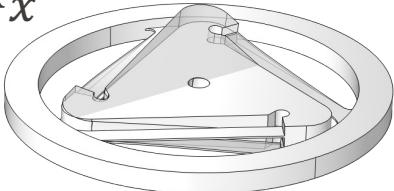
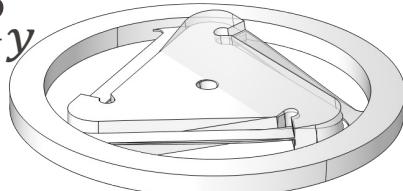
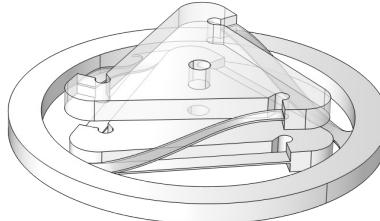
Flexure building blocs (bricks, joints, mechanisms)

UNIVERSAL JOINT ***

 R_y  R_z 

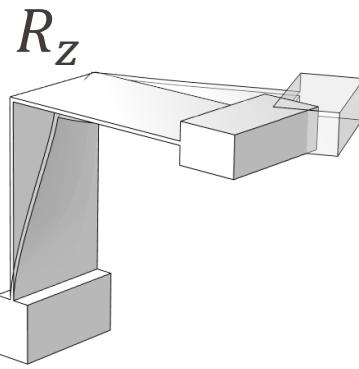
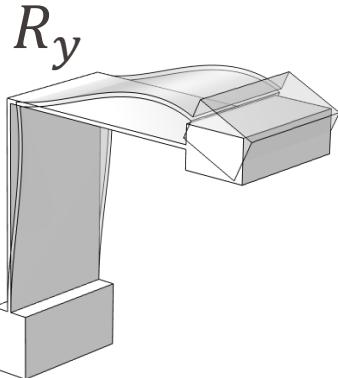
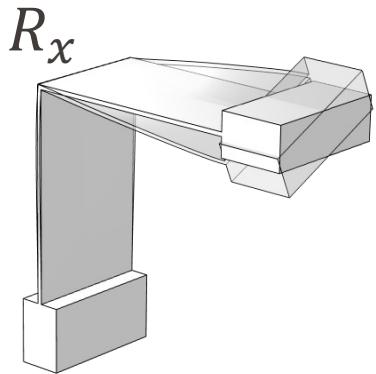
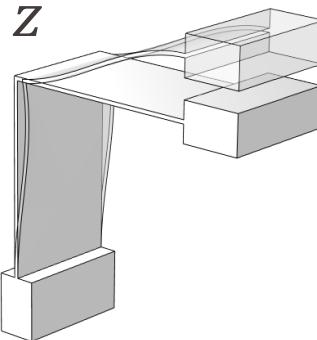
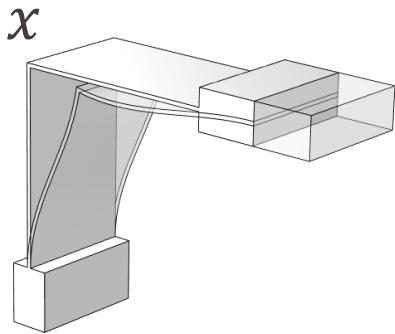
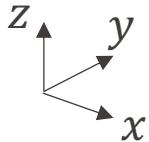
*** $y z R_x$ not stiff

MEMBRANE

 R_x  R_y  Z 

Flexure building blocs (bricks, joints, mechanisms)

L-SHAPED BLADE (or CORNER BLADE)

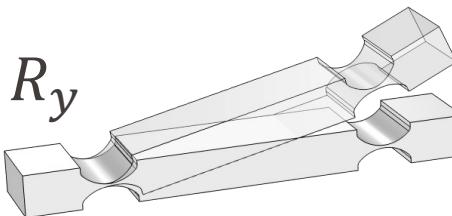
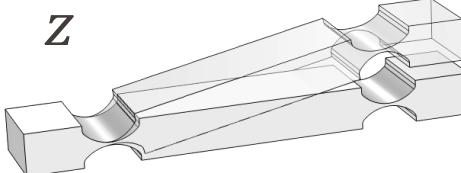
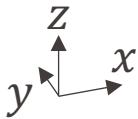


		1 DoF		2 DoF		3 DoF		4 DoF		5 DoF		6 DoF		
Translation	1	Linear stage Pair of membranes		2	Planar joint * XY stage bellow Serial XY stage		3	Delta Serial XYZ stage		3	Bellow		3	
Rotation	0	Sarrus Orthogryres Pseudiorthogryres		0	Serial Isospring		0			1			2	
Translation	0	Hinge * Torsion bar Pivot		1	Double hinge * Blade and rod		2	Spatial parallelo. * XYRz Stage		2	Double uni-joint *		2	
Rotation	1			1			1			2	Rod L-shaped blade		3	
Translation				0	Universal joint *** Circular hinge ***		1	Blade Membrane Tripod		1				
Rotation				2	Spherical Isospring		2	Tip-tilt piston		3				
Translation				0			0							
Rotation				3			3	Gimbal						

***(**) DOF(s) not stiff**

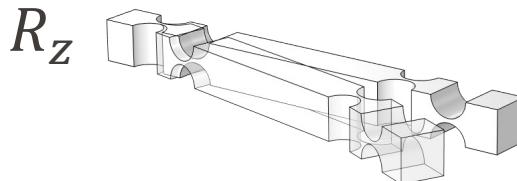
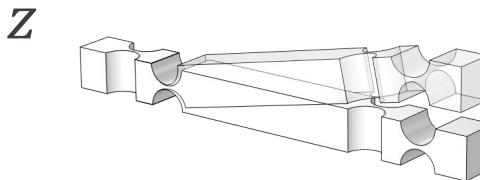
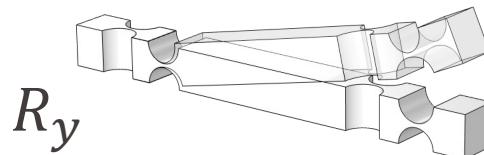
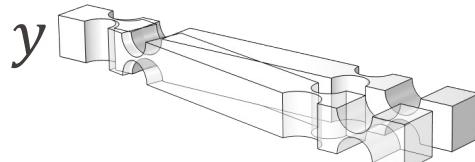
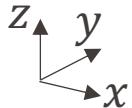
Flexure building blocs (bricks, joints, mechanisms)

DOUBLE HINGE *



* R_x not stiff

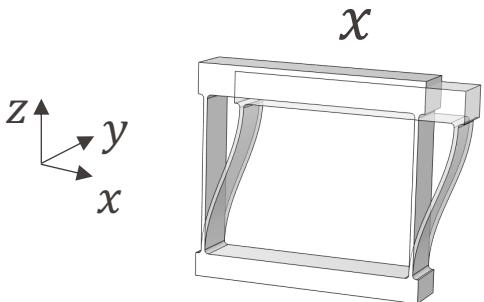
DOUBLE UNIVERSAL JOINT *



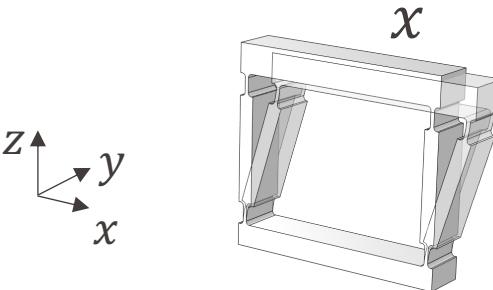
* R_x not stiff

Flexure building blocs (bricks, joints, mechanisms)

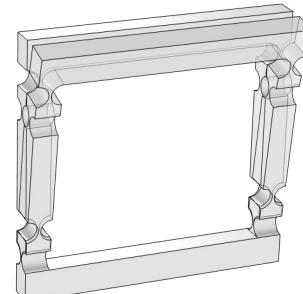
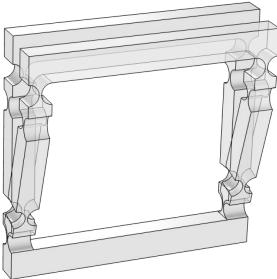
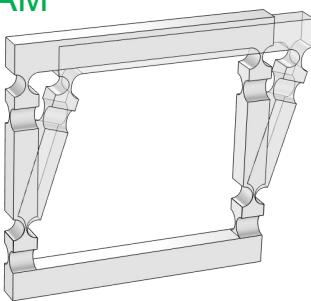
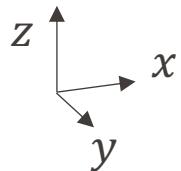
BLADE PARALLELOGRAM



HINGE PARALLELOGRAM



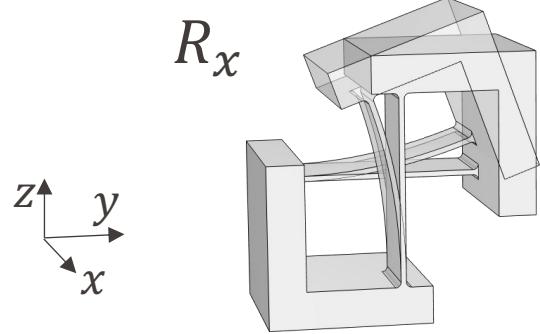
SPATIAL PARALLELOGRAM *



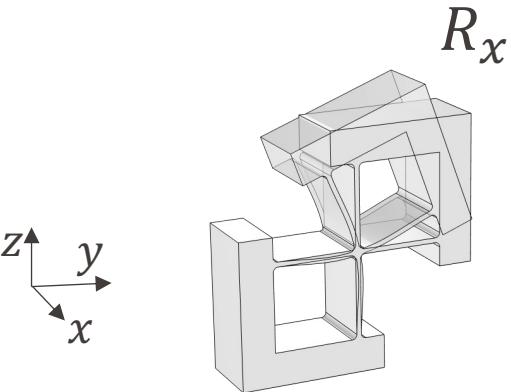
* R_z not stiff

Flexure building blocs (bricks, joints, mechanisms)

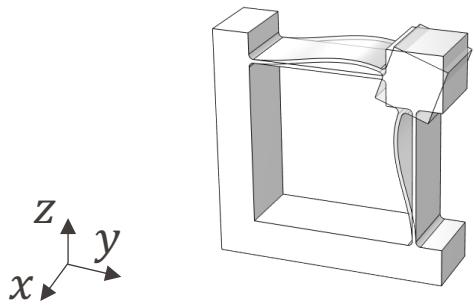
SEPARATED BLADES PIVOT



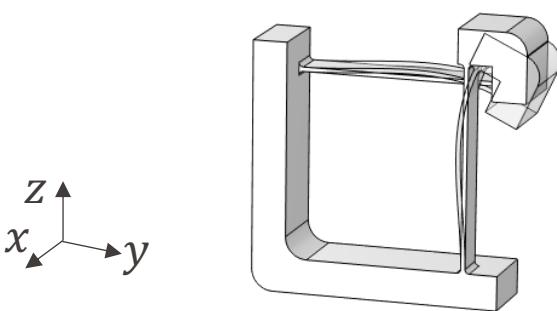
CARTWHEEL PIVOT



RCC PIVOT

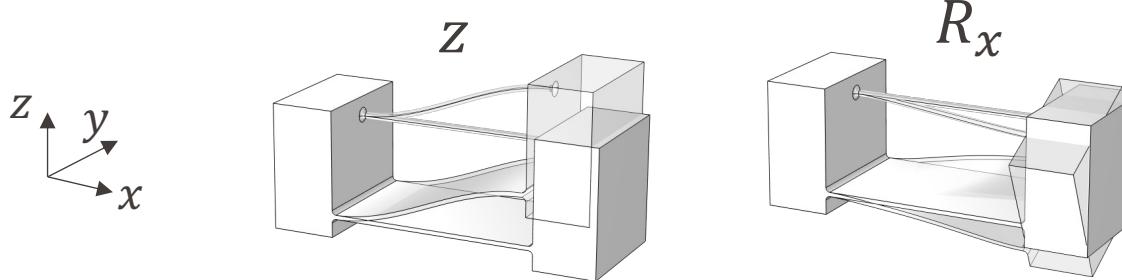


WITTRICK PIVOT

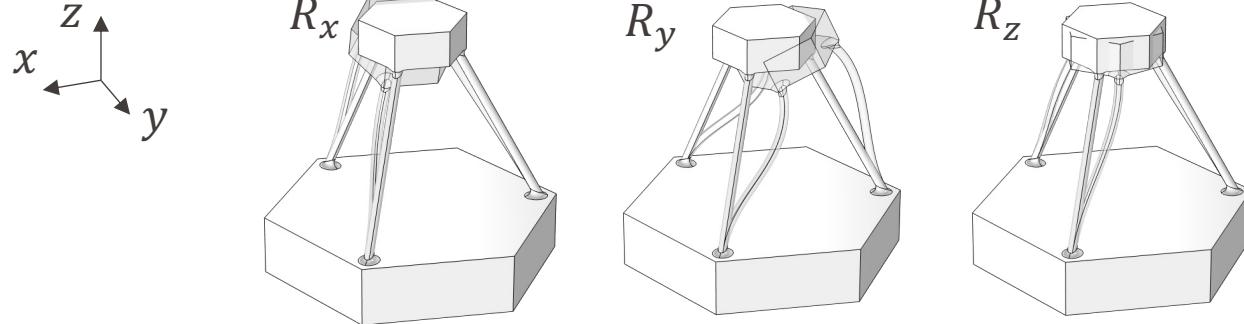


Flexure building blocs (bricks, joints, mechanisms)

BLADE AND ROD

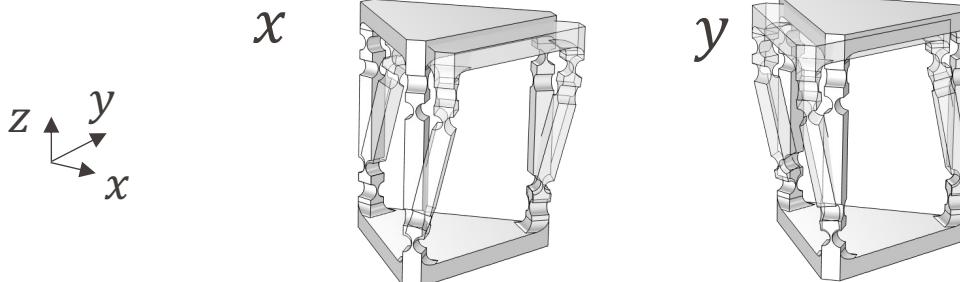
 R_x

GIMBAL



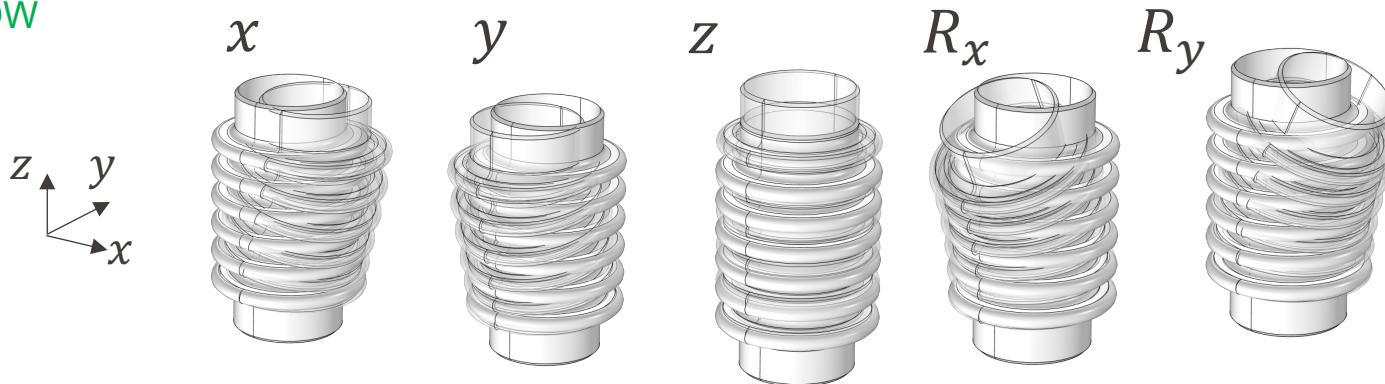
Flexure building blocs (bricks, joints, mechanisms)

PLANAR JOINT *



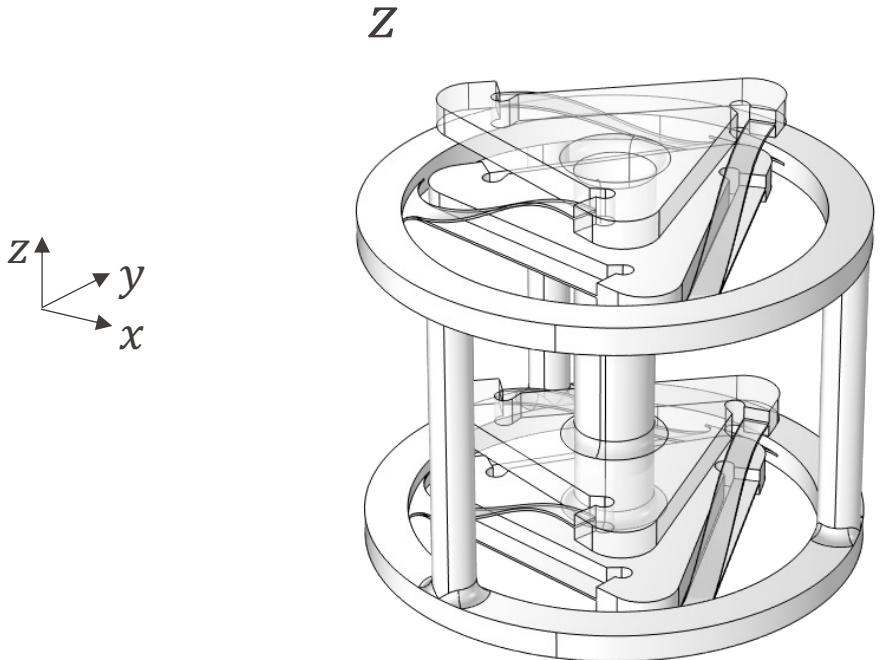
* R_z not stiff

BELLOW



Flexure building blocs (bricks, joints, mechanisms)

MEMBRANES PAIR



Flexure building blocs (bricks, joints, mechanisms)

		1 DoF	2 DoF	3 DoF	4 DoF	5 DoF	6 DoF
Translation	1	Linear stage Pair of membranes Sarrus Orthogryres Pseudorthogryres	2	Planar joint * XY stage bellow Serial XY stage Linear Isospring	3 Delta Serial XYZ stage	3	
Rotation	0		0		1	2	3 Bellow 3 Hexapod
Translation	0	Hinge * Torsion bar Pivot	1	Double hinge * Blade and rod	2 Spatial parallelo. * XZRz Stage	2 Double uni-joint *	2 Rod L-shaped blade
Rotation	1		1		1	2	
Translation			0	Universal joint *** Circular hinge *** Spherical Isospring	1 Blade Membrane Tripod Tip-tilt piston	1	
Rotation			2		2	3	
Translation				0	Gimbal		
Rotation				3			

*(**) DOF(s) not stiff

PIVOTS (1 DOF)

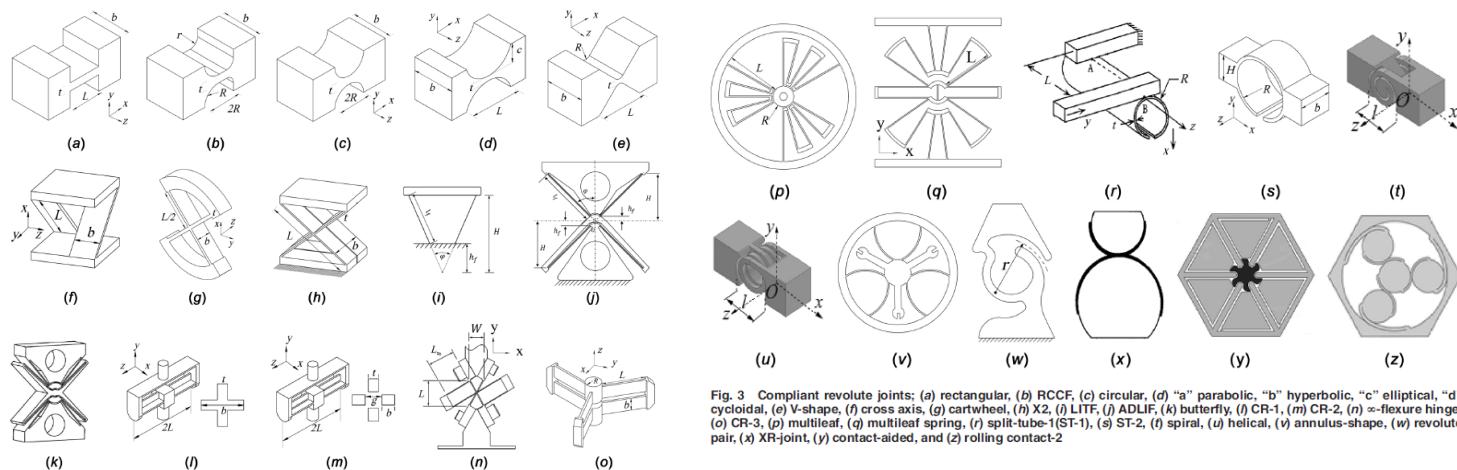


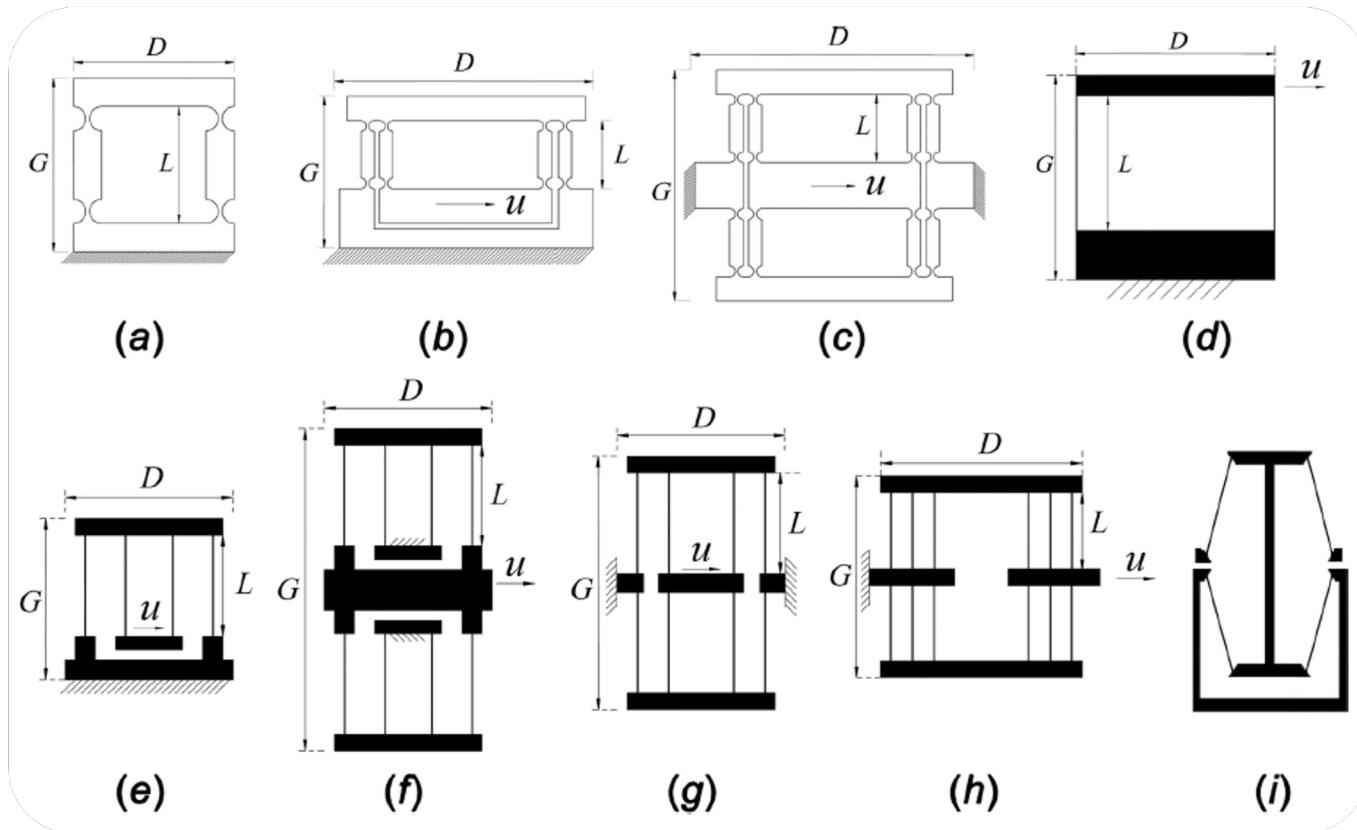
Fig. 3 Compliant revolute joints: (a) rectangular, (b) RCCF, (c) circular, (d) "a" parabolic, "b" hyperbolic, "c" elliptical, "d" cycloidal, (e) V-shape, (f) cross axis, (g) cartwheel, (h) X2, (i) LITF, (j) ADLIF, (k) butterfly, (l) CR-1, (m) CR-2, (n) \approx -flexure hinge, (o) CR-3, (p) multileaf, (q) multileaf spring, (r) split-tube-1(ST-1), (s) ST-2, (t) spiral, (u) helical, (v) annulus-shape, (w) revolute pair, (x) XR-joint, (y) contact-aided, and (z) rolling contact-2

D. Farhadi, J. Herder, *A Review on Compliant Joints and Rigid-Body Constant Velocity Universal Joints Toward the Design of Compliant Homokinetic Couplings*, In Journal of Mechanical Design, March 2015

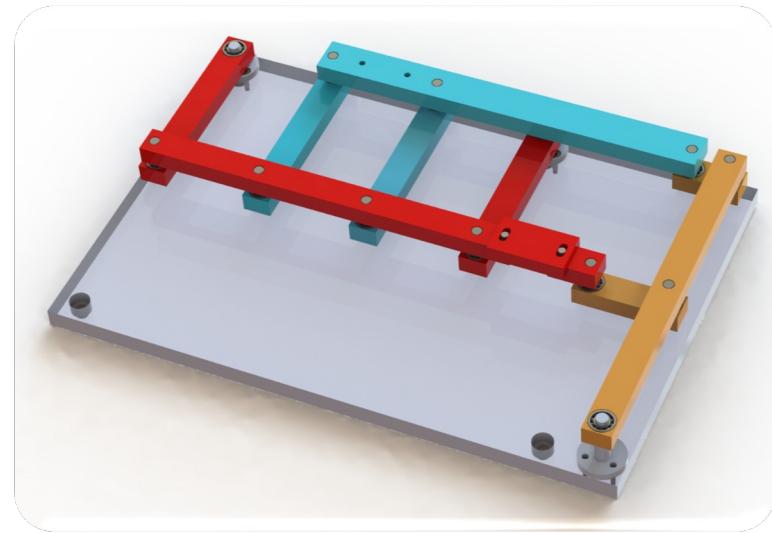
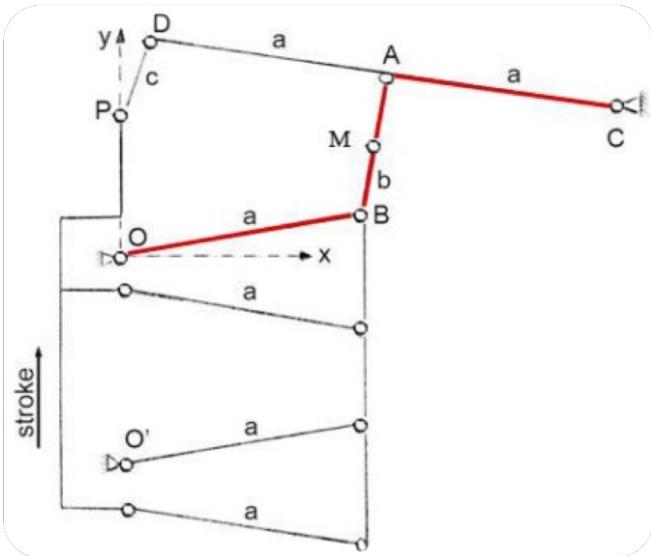


Flexure building blocs (bricks, joints, mechanisms)

LINEAR STAGES (1DOF)

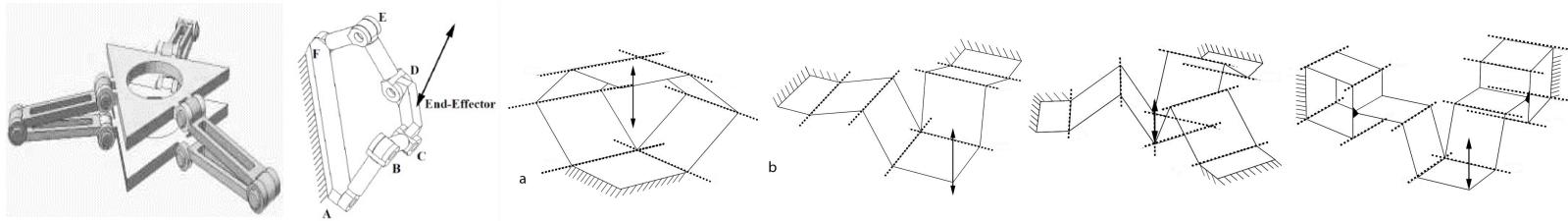


13 HINGE STAGE (1DOF)

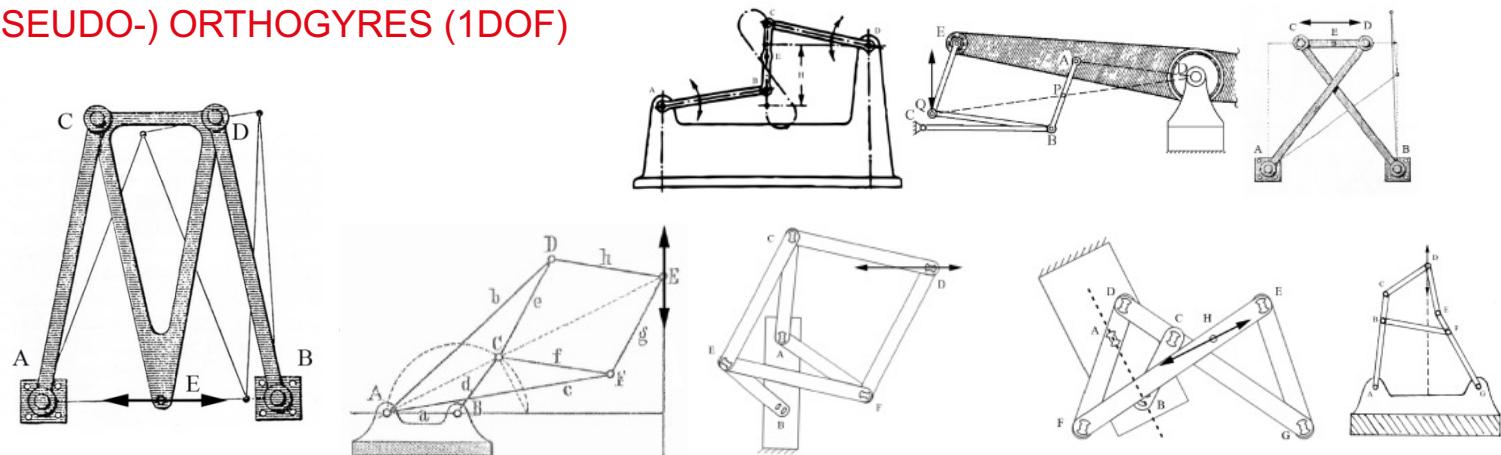


Flexure building blocs (bricks, joints, mechanisms)

SARRUS (1 DOF)



(PSEUDO-) ORTHOGYRES (1DOF)

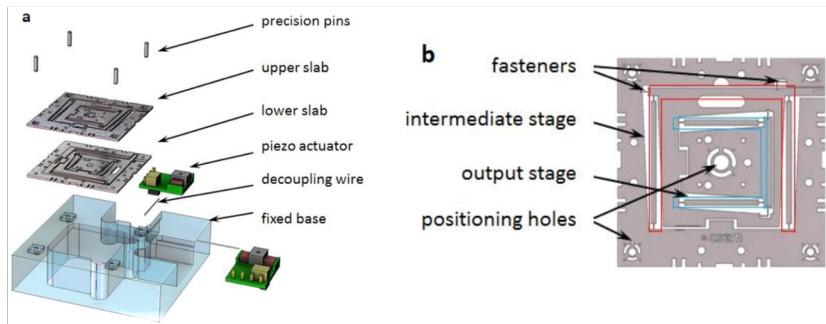
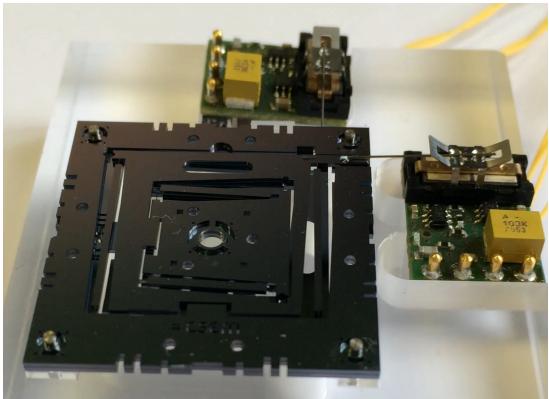


SERIAL XY STAGE (2 DOFs)

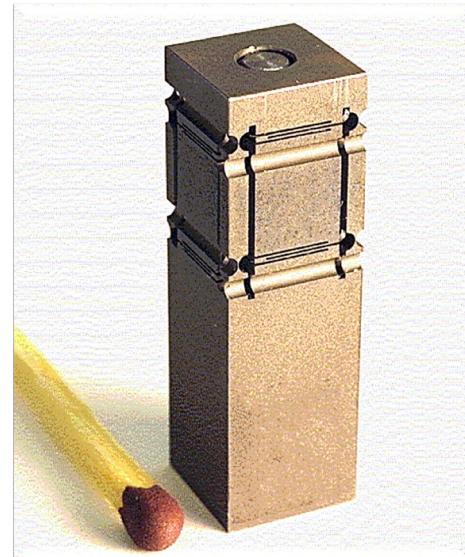


Flexure building blocs (bricks, joints, mechanisms)

SERIAL XY STAGE (2 DOFs)



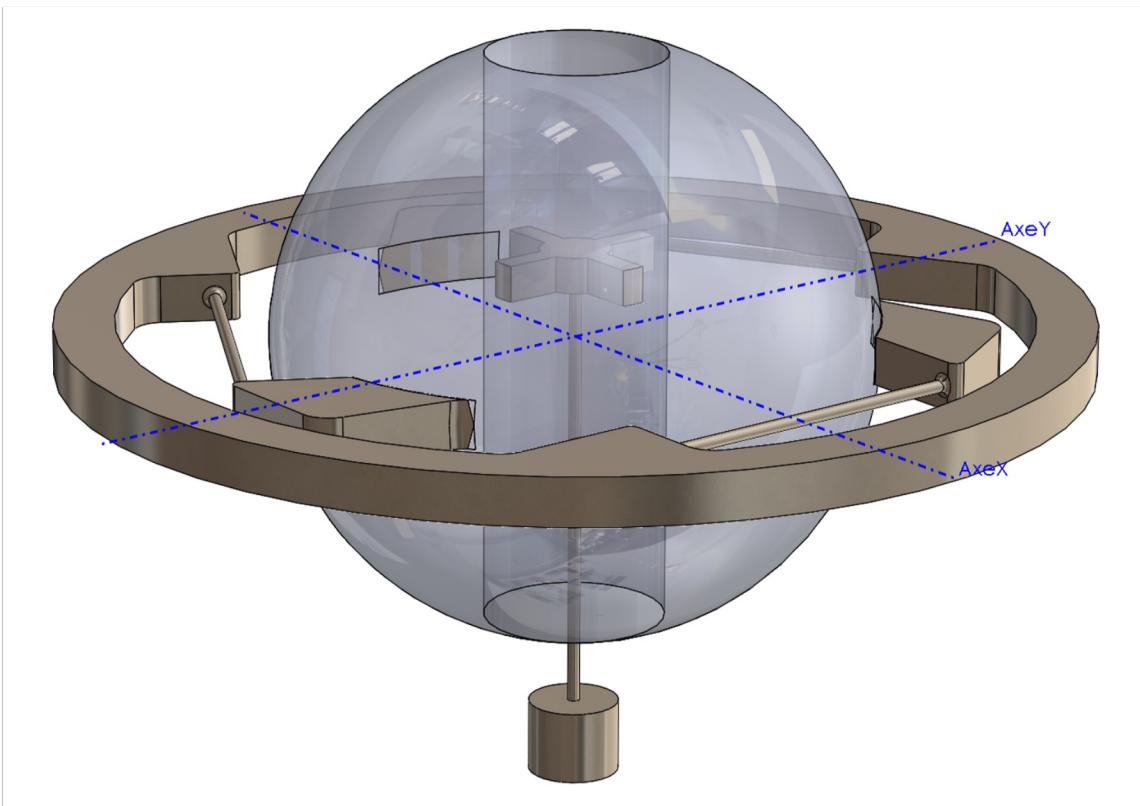
XY STAGE BELOW (2 DOFs)



[*Compact Light Weight Mechanisms for High Precision Manipulators*, Jean-Marc Breguet, Yves Bellouard, Simon Henein, Cédric Aymon, Stefano Bottinelli, Reymond Clavel]

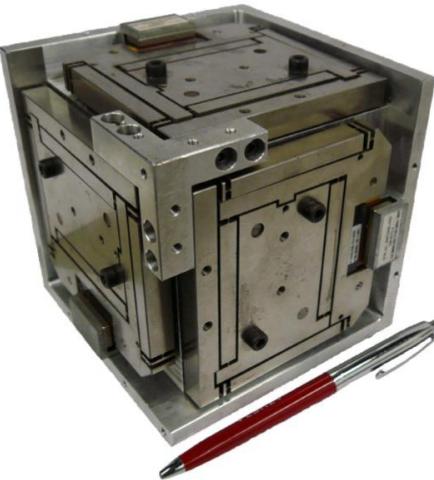
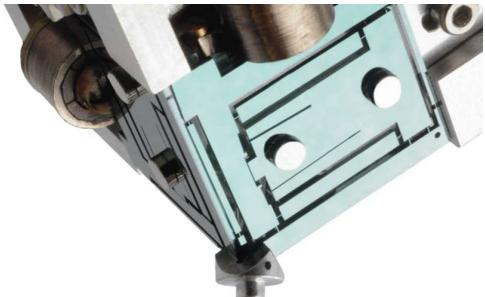
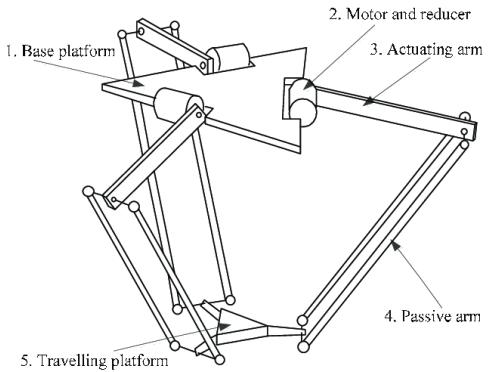
Flexure building blocs (bricks, joints, mechanisms)

SHERICAL ISOSPRING (2 DOFs)

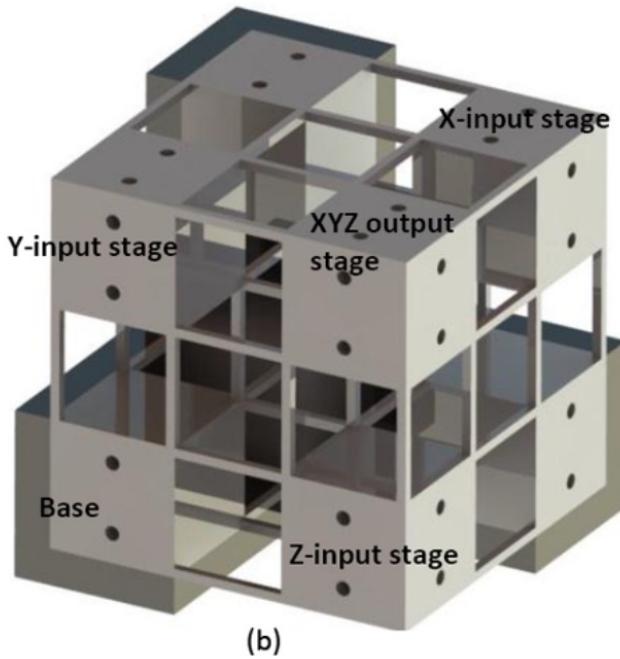


Flexure building blocs (bricks, joints, mechanisms)

DELTA (3 DOFs)

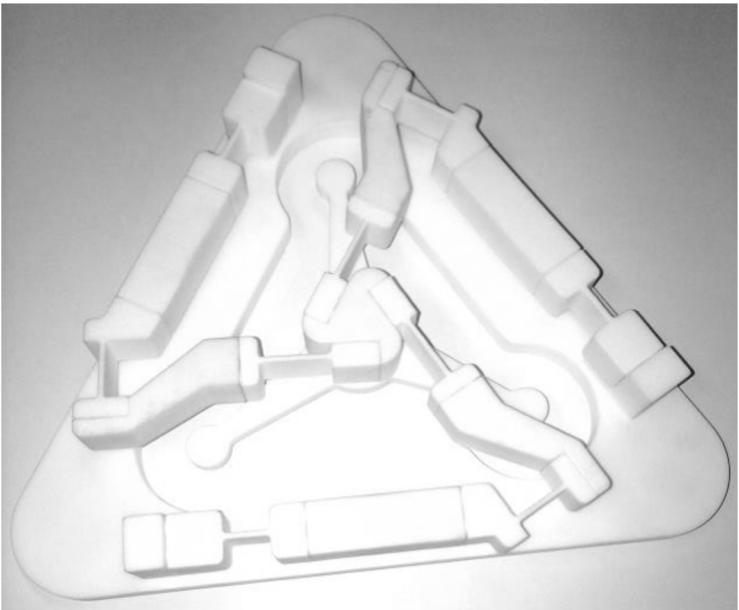
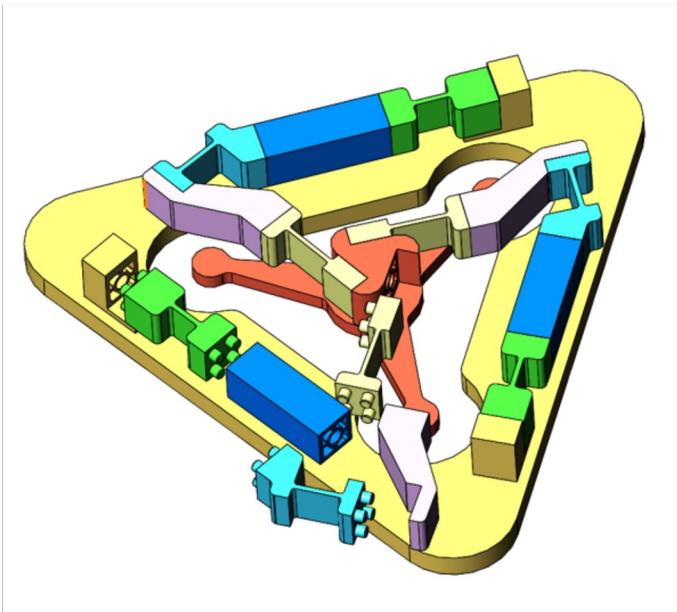


PARALLEL XYZ STAGE



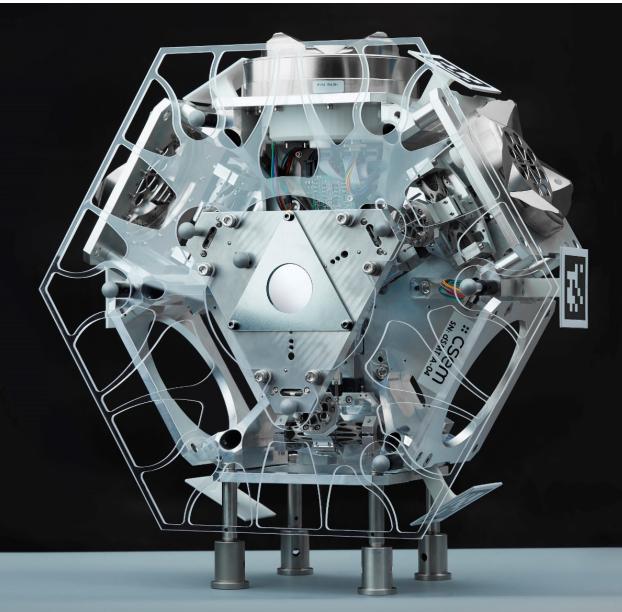
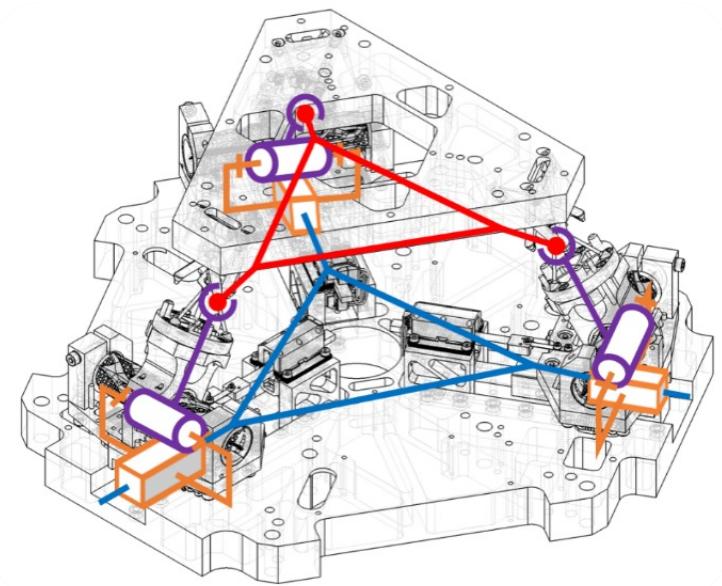
Flexure building blocs (bricks, joints, mechanisms)

XYRz STAGE (3 DOFs)



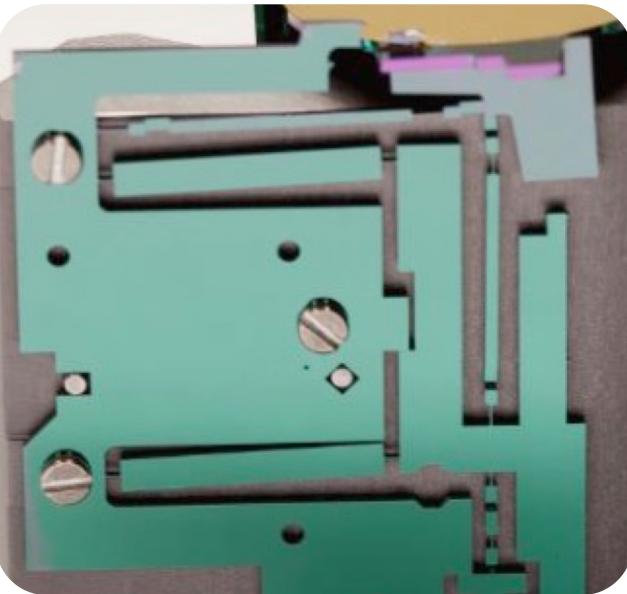
Flexure building blocs (bricks, joints, mechanisms)

TRIPOD (3 DOFs)

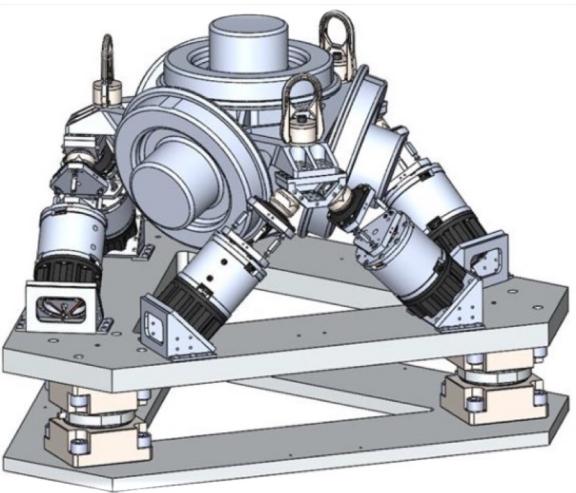
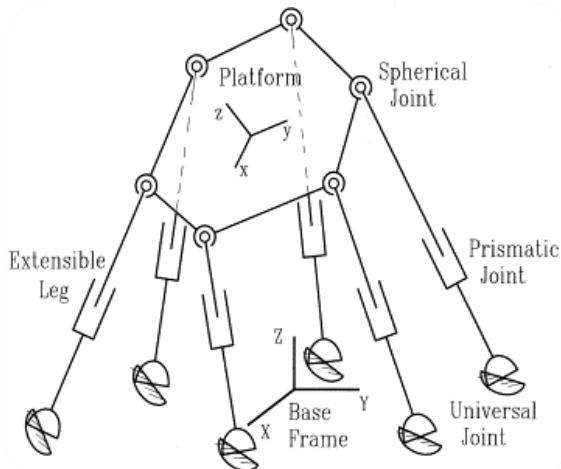


Flexure building blocs (bricks, joints, mechanisms)

TIP-TILT PISTON (3 DOFs)

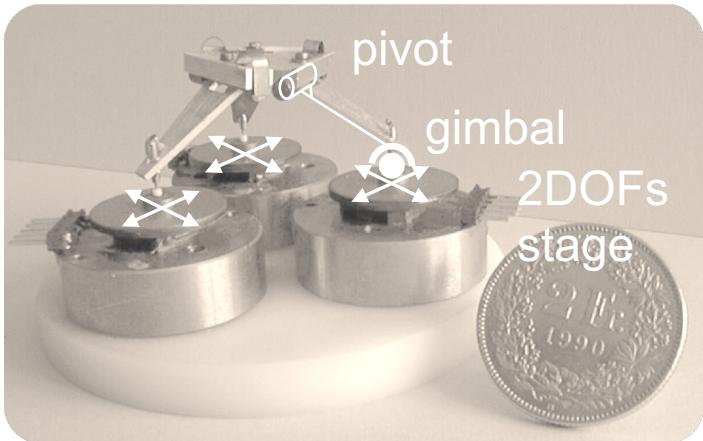


HEXAPOD (6 DOFs)

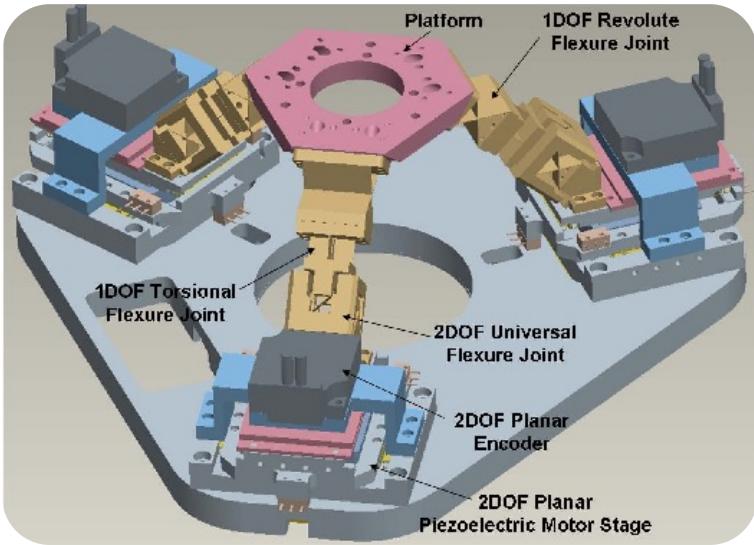


Flexure building blocs (bricks, joints, mechanisms)

TRIBIAS 6 DOFs



[Compact Light Weight Mechanisms for High Precision Manipulators, Jean-Marc Breguet, Yves Bellouard, Simon Henein, Cédric Aymon, Stefano Bottinelli, Raymond Clavel]



[D. Kang, D. Gweon, Analysis of large range rotational flexure in precision 6-DOF tripod robot, 2012 12th International Conference on Control, Automation and Systems]

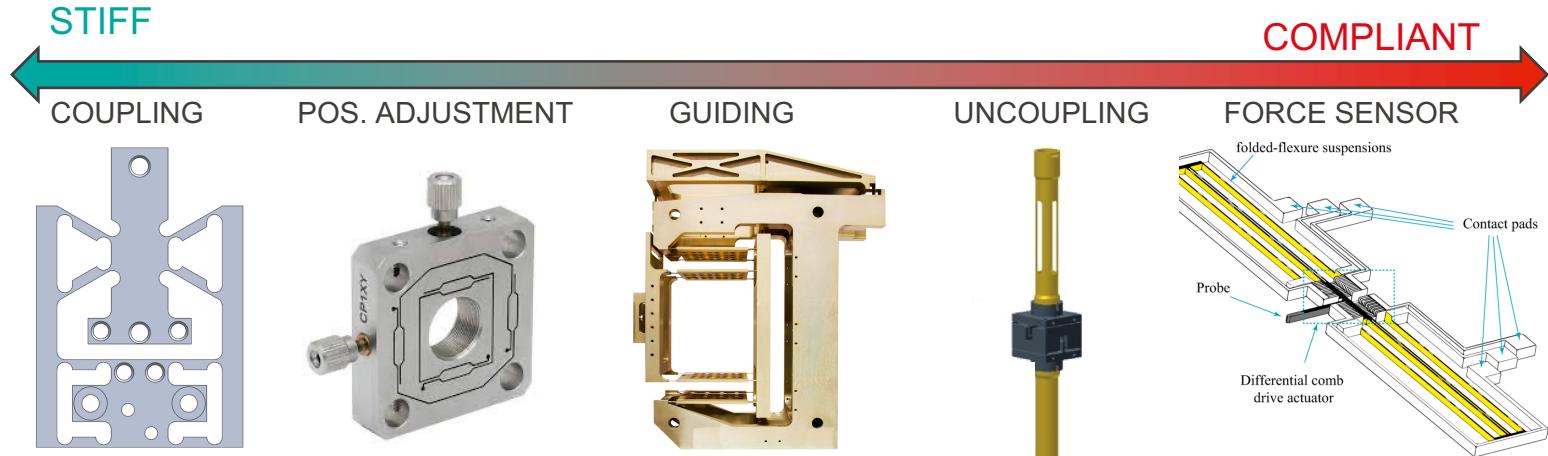
Flexure building blocs (bricks, joints, mechanisms)

		1 DoF	2 DoF	3 DoF	4 DoF	5 DoF	6 DoF
Translation	1	Linear stage Pair of membranes Sarrus Orthogryres Pseudorthogryres	2	Planar joint * XY stage bellow Serial XY stage Linear Isospring	3 Delta Serial XYZ stage	3	
Rotation	0		0		1	2	Bellow
Translation	0	Hinge * Torsion bar Pivot	1	Double hinge * Blade and rod	2 Spatial parallelo. * XYRz Stage	2 Double uni-joint *	
Rotation	1		1		2	3 Rod L-shaped blade	
Translation			0	Universal joint *** Circular hinge *** Spherical Isospring	1 Blade Membrane Tripod Tip-tilt piston	1	
Rotation			2		3		
Translation				0	Gimbal		
Rotation				3			

Functions of flexure mechanisms

- **Guiding** → high stiffness ratio
- **Amplifier / reducer** → precision, loads
- **Spring (preload)** → energy storage
- **Bistable / multistable** → tunable stiffness / force behavior
- **Transmission (coupling)** → customizable coupling DOFs
- **Uncoupling** → isostatic kinematics
- **Motion converter** (e.g. rotation/translation) → various kinematics
- **Motion inverter** → tunable motion
- **Path generator** → tunable motion

Flexures “style” versus stiffness



Stroke	Almost no stroke	Small strokes	Medium to large stroke	Small strokes	Medium to large stroke
Actuation	DOFs not actuated	Manual or motorized, no sensors	Motorized DOFs, with position sensors	No actuation	Act as a transducer
Function	Add compliance in some DOFs	Allows small static displacement	Large stiffness in blocked DOF, small stiffness in DOFs	Free some DOFs	Force to motion transducer
Features	Thick flexures and high stiffness	No slaving of internal DOFs	Optimized flexures dimensions	Low stiffness and low motion range	Very low stiffness, high sensitivity
Reference	[ESA project EXPOSITION]	[www.thorlabs.com]	[Baumann, H., Eichenberger, A., Jeckelmann, B., Cosandier, F., Clavel, R., Reber, D., Tommasini, D., <i>Design of the new METAS watt balance experiment Mark II</i> , in <i>Metrologia</i> , 2013]	[https://www.researchgate.net/publication/331905737_Modeling_and_experimental_characterization_of_an_active_MEMS-based_force_sensor]	

Week 5 exercises and homework

- Exercise
 - on MOODLE : EXO_5_Stiffness_calculation.pdf
 - with excel file: EXO_5.xlsx
- Homework
 - No homework