Exercise 9 – Part I:

Design and force balancing of a flexure-based parallel stage

Context:

A flexure-based parallel stage is subjected to fluctuating linear accelerations along the x-axis due to motion of its base. To be insensitive to this specific perturbation, we wish to force balance the mechanism.

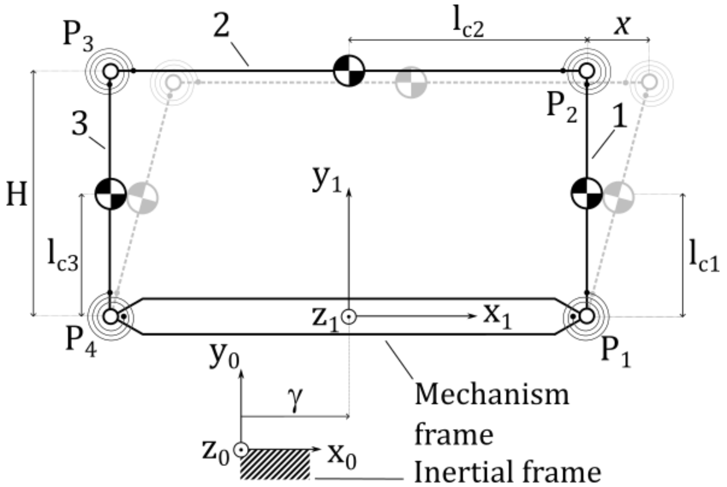

Fig. 1: Flexure-based parallel stage Pseudo-Rigid-Body-Model before its force balancing.

Figure 1 shows the Pseudo-Rigid-Body-Model (PRBM) of the studied flexure-based parallel stage before its force balancing. The mechanism is shown in its rest position (black lines) and in its deformed position (transparent black lines). The mechanism has one degree-of-freedom and is composed of three moving rigid bodies: solids **1**, **2** and **3** articulated in P_1, P_2, P_3 and P_4.

Each solid $i = 1 \dots 3$, has:

- a mass m_i,
- a center of mass (COM) c_i (represented by the symbol ◕) located by a distance l_{ci},
- an inertia tensor $\bar{\bar{J}}_i$ given at c_i.

Each pivot $P_{i=1\dots4}$ has an angular stiffness k_i that represents the intrinsic stiffness of the future flexure-based implementation.

First, the parallel stage mechanism is actuated by a voice coil whose permanent magnet is fixed to the mechanism's base and whose coil is fixed to solid **2**. The voice coil applies a force F at c_2 along the x-axis. The parameter x is used to locate the displacement of solid **2** relative to its base.

Last, the linear displacement that corresponds to the motion of the mechanism's base is represented by the parameter γ.

Part I: Force balancing of the flexure-based parallel stage

In order to force balance the mechanism, we propose to redistribute its mass by lowering the COM of solids 1 and 3 thanks to two identical supplementary masses m_{bal} having an inertia tensor $\bar{\bar{J}}_{\mathrm{bal}}$ at their COM. These two masses are located by the distance l_{bal}.

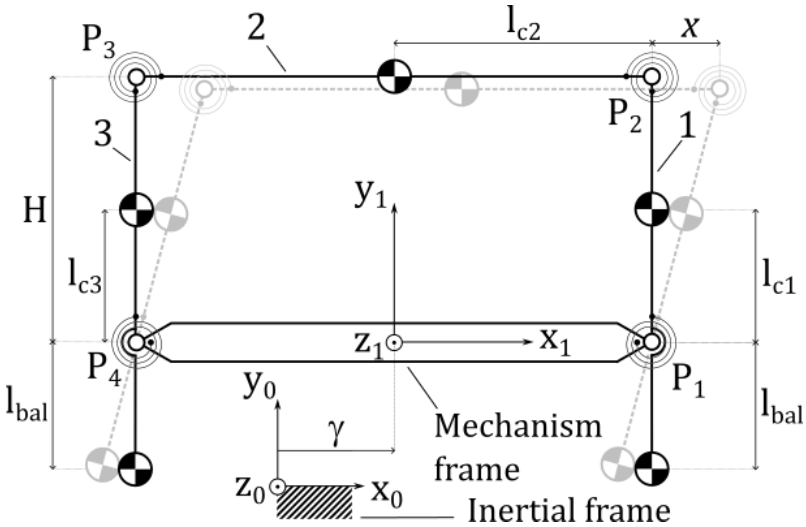

Fig. 2: Flexure-based parallel stage Pseudo-Rigid-Body-Model after its force balancing.

Numerical Values:

- $H = 20\text{e-}3$ [m]
- m_1=1.7138708e-02 [kg]
- m_2=6.8122300e-02 [kg]
- m_3=m_1
- m_{bal}=3.5565044e-02 [kg]
- l_{c1}=1.0411598e-2 [m]
- l_{c2} : not needed
- l_{c3}=l_{c1}
- l_{bal} : to be found
- $\bar{\bar{J}}_1 = \begin{bmatrix} 5.3559891\text{e-}6 & 0 & 0 \\ 0 & 1.9478338\text{e-}6 & 0 \\ 0 & 0 & 5.3569824\text{e-}6 \end{bmatrix} [\mathrm{kg.m^2}]$
- $\bar{\bar{J}}_2$: not needed
- $\bar{\bar{J}}_3 = \bar{\bar{J}}_1$
- $\bar{\bar{J}}_{\mathrm{bal}} = \begin{bmatrix} 2.0034975\text{e-}6 & 0 & 0 \\ 0 & 2.5379149\text{e-}6 & 0 \\ 0 & 0 & 1.1271682\text{e-}6 \end{bmatrix} [\mathrm{kg.m^2}]$

1. Using the Euler-Lagrange formalism, calculate the equation of motion of the parallel stage as a function of F, x and γ, assuming small deformations around the rest position. The equation of motion is of the form:

$$F = m_{\mathrm{eq}}\ddot{x} + k_{\mathrm{eq}}x + \tau_{\mathrm{coupling}}\ddot{\gamma}\ .$$

   Reminder: The Euler-Lagrange equation that needs to be solved is

$$F = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x},$$

   where $L = K - V$ is the Lagrangian; K and V are the kinetic and potential energies of the system, respectively.
2. Identify the dynamic coupling term τ_{coupling} that couples the mechanism motion to its base acceleration $\ddot{\gamma}$.
3. Find the length l_{bal} that cancels τ_{coupling}.
4. Compute the linear equivalent masses $m_{\mathrm{eq,init}}$ and $m_{\mathrm{eq,final}}$ of the parallel stage before and after its force balancing.
5. Considering $F = 0$, write the analytical transfer functions $T_{\mathrm{init}}(\omega)$ and $T_{\mathrm{final}}(\omega)$ of the parallel stage before and after it is force balanced:

$$T(\omega) = \frac{X(\omega)}{(j\omega)^2\Gamma(\omega)}\ .$$