Exercise 8:
Direct and Inverse Geometric Model of a flexure-based lamba robot

Context:

We wish to establish the direct (DGM) and inverse geometric models (IGM) of the parallel kinematic structure of a lambda planar robot shown in Figure 1. The motor positions are located at points A and B and the tool position is located at point D. Point C represents the joint between the long and short connecting rods. The DGM and IGM will be used to:

a) The crosschecking of the two models.
b) The determination of the linear strokes of the motors at points A and B to reach all the positions desired by the tool in the working surface.
c) The calculation of the maximum angular deflections of each of the joints at A, B and C in the extreme positions.
d) Based on the joint deflections and linear strokes of the motors, flexible joints can then be sized.
e) Following this, the lateral stiffnesses K_x and K_y at tool level will be calculated using the 'energy' method.

Points (a) to (e) will be calculated analytically and numerically using the Matlab script *EXO_lambda.m* to be filled in, and point (e) will be validated using finite elements based on the pre-established Comsol model *EXO_lambda.mph*.

The numerical values are listed:

- Length **L = 200 mm**
- Angle $\mathbf{Alpha_0}$ **= 60°**
- Workspace **Ws = 40 mm** (+/- 20 mm)
- Motor 1 initial position $\mathbf{q_{10}}$ **= 100 mm**
- Motor increments size **step = 5 mm**
- Flexures material young modulus **E = 200 GPa**
- Flexures admissible stress **sigma_adm = 900 MPa**
- Flexures width **b = 10 mm**
- Flexures length of the linear stages **l_stage = 80 mm**

- Flexures length of the separate crossed beam pivots **l_pivot = 14 mm**

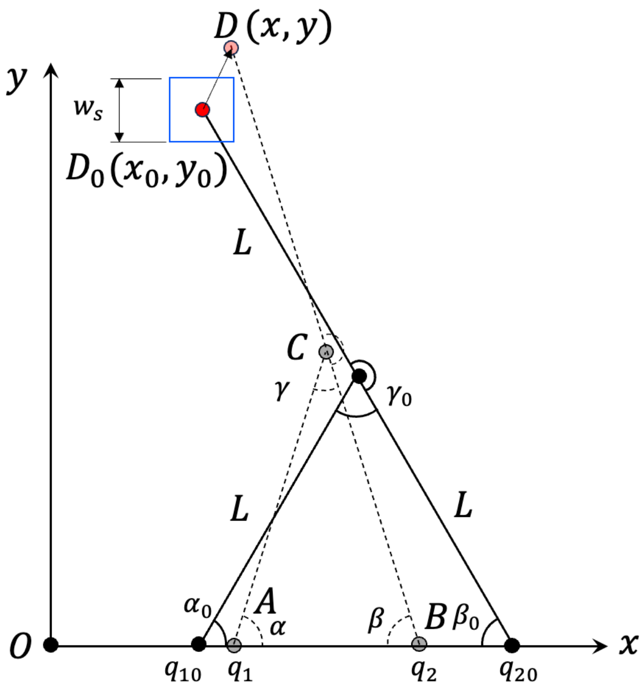

Figure 1: Schema of the parallel kinematic structure of a lambda planar robot

Remarks:

- All units displayed in the script are SI units, i.e. [m], [rad] and [Pa]. Quantities are only multiplied or divided when displayed.
- Both flexible linear stages are dimensioned with the same permissible motion range, corresponding to the maximum motion range required between the left and right motors. The same applies to the pivots. All are sized to withstand the maximum motion range of all pivots.
- Motor strokes are considered symmetrical around the nominal points q_{10} and q_{20}. I.e. the total stroke is twice the "stroke" value given in the script.
- In the Comsol simulation, only the flexible elements are deformed. The two rods connecting points *A* to *C* and *B* to *D* are attached to rigid connectors and are there solely for visual representation and are not included in the actual calculations.

Script filling procedure:

0. Open the Matlab script *EXO8_lambda.m*. Go through the script briefly, trying to understand what the different blocks correspond to.

1. Complete the *lambda_DGM* function by replacing the ***1*** stars with formulas describing the tool position as a function of the motor positions.

2. Complete the *lambda_IGM* function by replacing the ***2*** stars with formulas describing motor positions as a function of tool position.

3. Complete the angle calculation function *calculate_angles* by replacing the ***3*** stars with unit vectors or the position vectors of joints *A, B or C.*

4. Define and vectorially calculate the positions of the joint points (*A, B and C*) and tool (*D*) as a function of q_1, q_2, x and y and the positions already known by replacing the ***4*** stars. Copy the result twice (lines 55 to 58 and lines 118 to 121).

5. Check that the script works by running it.

6. Check the reciprocity of the direct and inverse geometric models by observing the equality of the q_{10} and q_{20} values with q_{10_check} and q_{20_check}.

7. Check the angular calculation by observing that the sum of the three calculated angles is always 180° (line 69).

8. Determine the motion range required on each motor to reach all positions in the robot's work area. This is done by dichotomy, while keeping the proposed resolution of 5 mm per step. Replace the values next to the ***8*** stars.

9. Calculate the thicknesses of the flexible blades of the linear stages and flexible pivots, based on the admissible stress, the blade dimensions and the Young's modulus of the material. Replace stars ***9*** and uncomment corresponding lines.

10. Calculate the stiffnesses of linear stages K_{stage} and flexible pivots K_{pivot} as a function of blade dimensions and Young's modulus of the material. Replace stars ***10*** and uncomment corresponding lines.

11. Calculate the K_x and K_y stiffnesses of the structure at tool level by replacing the ***11*** stars. If necessary, use the energy method, i.e. equating the energy of the structure with the summed individual energies of each of the joints. Report the values in Table 1.

12. Open the COMSOL file *EXO8_lambda.mph*. In *Global Definitions -> Parameters*, enter the numerical values of the blades thickness (h_stage and h_pivot) that you have calculated at point 9. Round up to the tens of microns.

13. Calculate the lateral stiffness K_x by imposing a 1 mm displacement at the output (*Rigid Connector "Output"*) in the *x*-direction and 0 mm in the *y*-direction. Run the simulation and evaluate the stiffness value through the *Line Integration 1*, in *Results -> Derived Values*. Report the value in Table 1.

14. Calculate the lateral stiffness K_y by imposing a 1 mm displacement at the output (*Rigid Connector "Output"*) in the *y* direction and 0 mm in the *x* direction. Run the study *Stiffness calculation* and evaluate the stiffness value through the *Line Integration 2*, in *Results -> Derived Values*. Report the value in Table 1.

Table 1: Analytical and FEM Stiffness evaluation of the flexure lambda robot

Axis	$K_{Analytic}$ [N/m]	K_{FEM} [N/m]	Delta [%] = $(K_{FEM}-K_{Anlt})/K_{FEM}$
x			
y			

15. Uncheck the *Prescribed Displacements* at the output and check the *Imposed Displacements* within the *Input1* and *Input2* nodes. Run the *Position sweep* study. Click on the Animation node in Results-> Export, click the play button on the Graphic window. Enjoy watching the mechanism move across the entire work surface!