Exercise 7: Fatigue test on flexure beam samples with vibration shaker

A vibration shaker is used to carry out fatigue tests on samples of stainless steel and titanium flexible blades. The shaker vibrates at the blade's natural frequencies. A 15-gram mass is attached to the end of each blade. The blade's own mass is assumed to be negligible. A typical vibration shaker as well as a schema of the test bench are depicted on Figure 1.

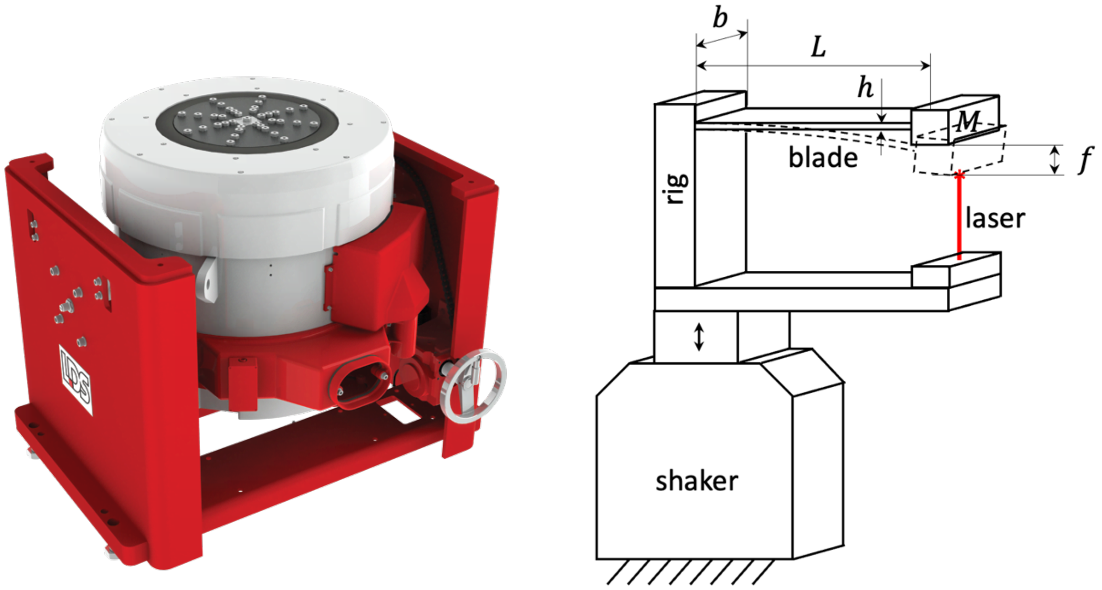

Figure 1: Left: Electromagnetic vibrating shaker. Right: Experimental setup schema of a blade mounted on top of a vibrating shaker.

The theoretical fatigue curves are approximated by the following equations for stainless **steel**:

$N_{cycles} < 20M$:

$$\sigma_{adm_{SS}} = \sigma_{adm_{0SS}} - \frac{\sigma_{adm_{0SS}} - \sigma_{adm_{20M}}}{\log_{10}(20 \cdot 10^6)} \cdot \log_{10}(N_{cycles})$$

$N_{cycles} > 20M$:

$$\sigma_{adm_{SS}} = \sigma_{adm_{20M}}$$

With $\sigma_{adm_{0SS}} = 1200\ MPa$ and $\sigma_{adm_{20M}} = 500\ MPa$. And the following equation for **titanium**:

$$\sigma_{adm_{Ti}} = \frac{\sigma_{adm_{0Ti}}}{0.5 \cdot \log_{10}(N_{cycles}) + 1}$$

with $\sigma_{adm_{0Ti}} = 900\ MPa$. These theoretical curves are plotted in the semi-log graph of Figure 2.

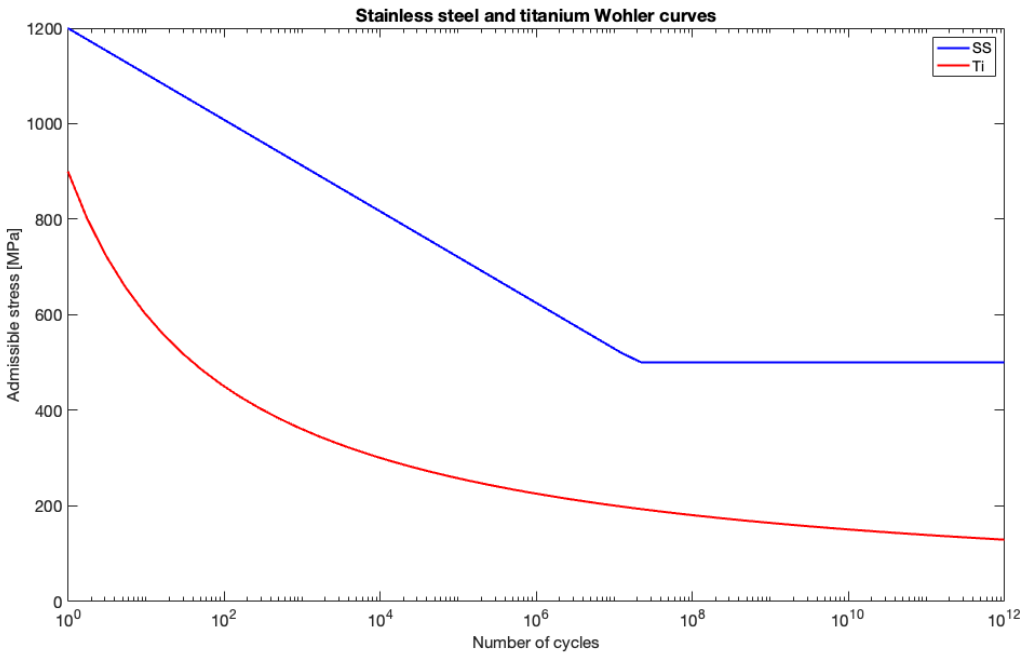

Figure 2 : S-N graph with Wohler curves for stainless steel and titanium.

A laser measures the vertical deflection of the blades during measurement and feedback is given to the shaker to impose its vertical stroke to 10 mm. The blades have the following dimensions and properties:

- Length: $L = 40$ mm
- Width: $b = 10$ mm
- Thicknesses (SS): $h_1 = 300$ um, $h_2 = 350$ um and $h_3 = 400$ um
- Thicknesses (Ti): $h_1 = 200$ um, $h_2 = 300$ um and $h_3 = 400$ um
- Young modulus: $E_{SS} = 200$ GPa and $E_{Ti} = 114$ GPa

The following assumptions are made:

- The length of the blade is considered from the root to the center of the 15-gram mass, i.e. the center of mass of the mass M is situated at the blade's moving extremity.
- The mass of the blade itself is neglected, as well as the rotational inertia of the mass M.
- The angular rotation of the mass is neglected when measuring the vertical stroke of the blade.
- The time to change one blade with another on the shaker is neglected.
- The blades deformation corresponds to its first bending mode (cantilever mode).

The question is:

When the three blade samples are tested successively, which material is expected to have the longest total fatigue test duration?

Plot the corresponding **working points** for each blade on the S-N graph. (fill your values within the MATLAB file **EXO_7.m**) The calculation procedure (**fill in voids in Table 1 in the excel file EXO_7.xlsx**) is as follows:

- **Calculate analytically** the maximal von Mises **stress** in each blade for a vertical stroke of 10 mm. The maximal stress will be located at the root of the blade.
- Calculate the linear **stiffness** of the blade at their free end.
- Calculate the **eigenfrequency** of each blade.
- Calculate the **number of cycles** for each blade, given the von Mises stress and the theoretical Wohler curves equations.
- Calculate the **duration** of the test for each blade in seconds and convert this result in days.
- Compare with the **FEM stress values** found previously (EXO 3).

Table 1:

Quantity	**Symbole**	**Value mat1**	**Value mat2**	**Unit**
Material	Mat	Ac	Ti	/
Young modulus	E	2.00E+11	1.14E+11	Pa
Yield stress static	sigma_D_0	1.20E+09	9.00E+08	Pa
Yield stress 20M cycles	sigma_D_20Mcyc	5.00E+08	/	Pa
Blade length	L	4.00E-02	4.00E-02	m
Blade width	b	1.00E-02	1.00E-02	m
Blade thickness 1	h1	3.00E-04	2.00E-04	m
Blade thickness 2	h2	3.50E-04	3.00E-04	m
Blade thickness 3	h3	4.00E-04	4.00E-04	m
Blade vertical deflection	f	0.01	0.01	m
Von Mises stress in blade 1	sigma1			Pa
Von Mises stress in blade 2	sigma2			Pa
Von Mises stress in blade 3	sigma3			Pa
Stiffness of blade 1	K1			N/m
Stiffness of blade 2	K2			N/m
Stiffness of blade 3	K3			N/m
Mass	M	0.015	0.015	kg
Eigenfrequency of blade 1	freq1			Hz
Eigenfrequency of blade 2	freq2			Hz
Eigenfrequency of blade 3	freq3			Hz
Number of cycles of blade 1	Ncycles1			cycles
Number of cycles of blade 2	Ncycles2			cycles
Number of cycles of blade 3	Ncycles3			cycles

Duration of test on blade 1	D1			s
Duration of test on blade 2	D2			s
Duration of test on blade 3	D3			s
Total duration of tests in hours	Dtot hours			h
Total duration of tests in days	Dtot days			d