

Exercise 12: RehearsalEXO A: Lagrangian formalism applied to a flexible linear stage

Through **Lagrange methodology**, determine the **motion equation** of the system shown on Figure 1. Then, based on the motion equation, identify the system **eigenfrequency** and determine the **sag due to gravity** (vertical static position) by expressing the dynamic condition corresponding to the static case. The **Young modulus** is 200 GPa, the **length L** is 50 mm, the **width b** is 10 mm and the **thickness h** is 300 um. The **mass m** of the stage is 1 kg.

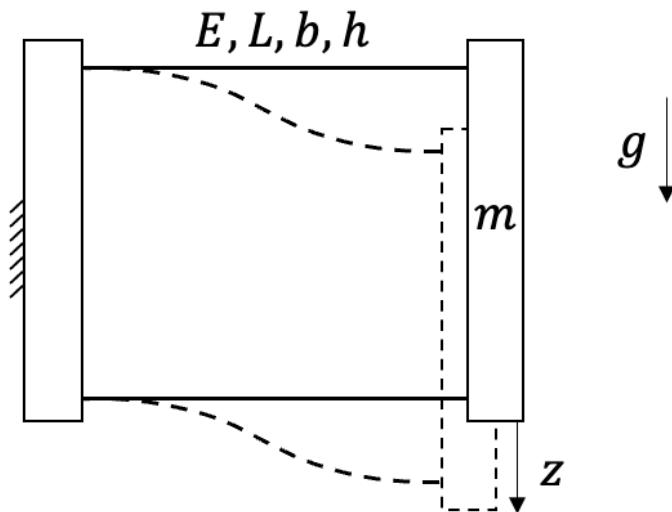


Figure 1 : Linear stage with gravity acting on the mobile shuttle.

EXO B: Mercurian optomechanical payload rotative stage

In the frame of a solar system exploration mission, a **rotative flexure stage** is designed to be used in an optomechanical system on Mercury to study its atmosphere. The **gravity** of Mercury equals 3.7 m/s^2 . You must size the width **b** and the thickness **h** of the blades of an overconstrained flexure pivot (Figure 2) to respect the following constraints: the **sag** due to Mercurian gravity must be equal to **645 nm** and the **rotative eigenfrequency** must be equal to **10 Hz**. The optical payload has a **mass** of **0.8 kg** and an **inertia** $J = 1.72\text{e-}4 \text{ kg}\cdot\text{m}^2$. The length of the pivot's blades **length** is imposed (**80 mm**) and the material is **Titanium** (Young modulus **E = 116 GPa**). Also, the inner roots of the blades are considered colinear with the pivot rotation axis (**p = 0**) and no radius is considered at the roots of the blades.

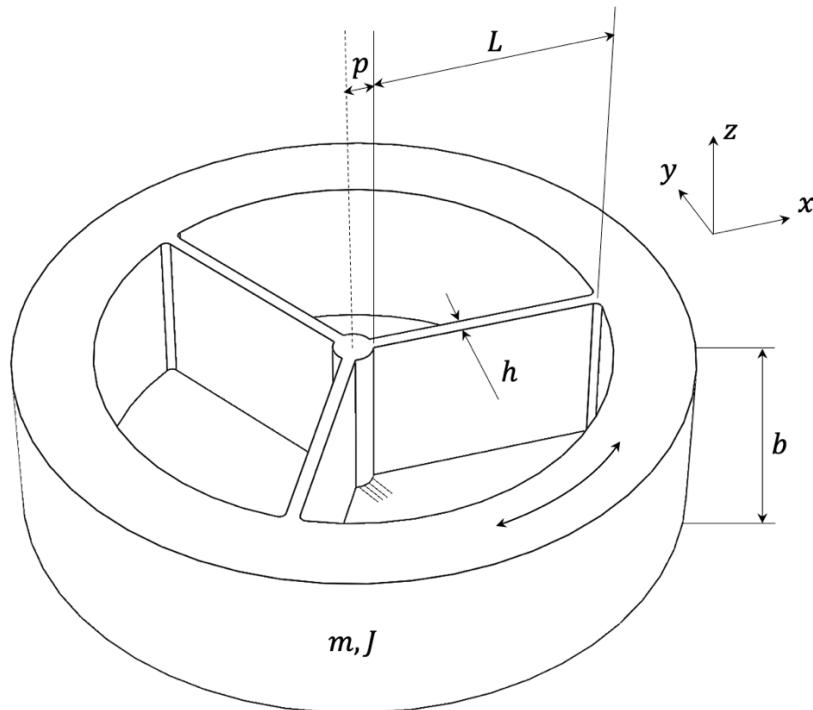


Figure 2 : Flexible rotation stage for orientation of an optical payload aboard a rover on Mercury.

EXO C: Crank-linear stage linkage geometric model

In a machine, a crank-linear stage is implemented using flexures. The system geometry is modeled as illustrated in Figure 3. The **input** of the mechanism is θ angle. It's motion ranges from -15° to $+15^\circ$. The **length** L of both connecting rods is **50 mm**. Calculate the **maximum deflections** of angles α and β as well as the linear motion range of the linear stage (z translation).

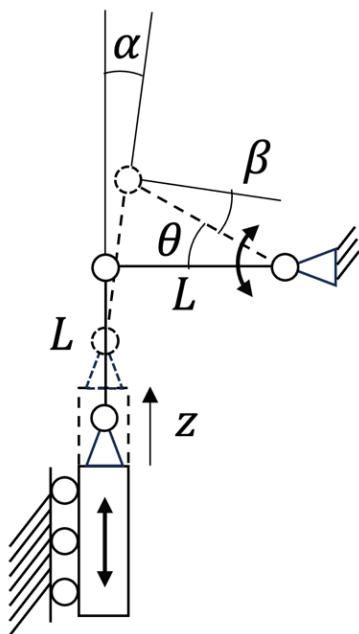


Figure 3: Geometric model of a crank-linear stage linkage.

The joints of the structure are then implemented in flexible elements. The pivots have separated crossed blades, whose **length** is **15 mm**. Calculate their thickness ***h_alpha***, ***h_beta*** and ***h_theta*** to reach the fatigue stress $\sigma_D = 100 \text{ MPa}$ with maximal deflection for each pivot. The material is Aluminum Avional. The Young modulus is $E = 73 \text{ GPa}$.