EPFL

Solution for exercise 9 – Part III:
Design and force balancing of a flexure-based parallel stage

Context:

A flexure-base parallel stage is subjected to fluctuating linear accelerations along the x-axis due to motion of its base. To be insensitive to this specific perturbation, we wish to force balance the mechanism.

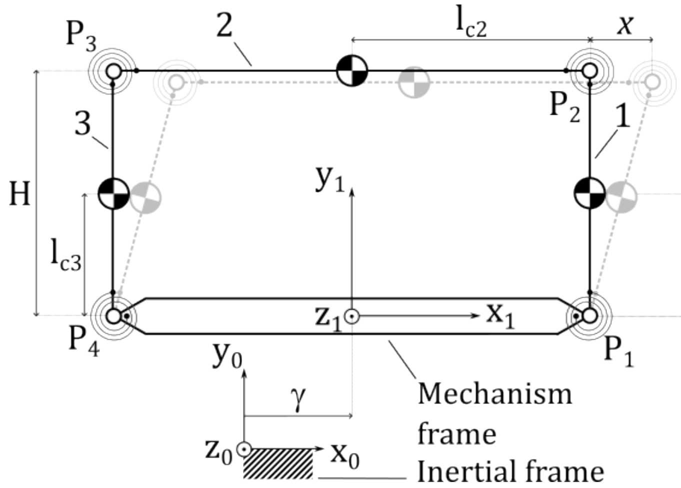

Fig. 1: Flexure-based parallel stage Pseudo-Rigid-Body-Model before its force balancing.

Figure 1 shows the Pseudo-Rigid-Body-Model (PRBM) of the studied flexure-based parallel stage before its force balancing. The mechanism has one degree-of-freedom and is composed of three moving rigid bodies: solids **1**, **2** and **3** articulated in P_1, P_2, P_3 and P_4.

Each solid $i = 1 \dots 3$, has:

- a mass m_i,
- a center of mass (COM) c_i (represented by the symbol ◕) located by a distance l_{ci},
- an inertia tensor $\bar{\bar{\boldsymbol{J}}}_i$ written at c_i.

Each pivot $\mathrm{P}_{i=1\dots4}$ has an angular stiffness k_i that represents the intrinsic stiffness of the future flexure-based implementation.

First, the parallel stage mechanism is actuated by a voice coil whose permanent magnet is fixed to the mechanism's base and whose coil is fixed to solid **2**. The voice coil applies a force **F** at c_2 along the x-axis. The parameter x is used to locate the displacement of solid **2** relative to its base.

Last, the linear displacement that corresponds to the motion of the mechanism's base is represented by the parameter γ.

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Part III: Finite Element verification using COMSOL

Open the file **Exo_Equilibrage.mph** in COMSOL Multiphysics 6.2

GLOBAL DEFINITIONS

1. In **Global Definitions** > Click on **Geometric Parameters**.
2. In the **Settings** window, locate the **Parameters** section.
3. In the stage, assign to the **L**, **my_h** and **b** parameters (currently set to 100) the numerical values you previously found:

Settings
Parameters

Label: Geometric Parameters

▾ Parameters

Name	Expression	Value	Description
H	20 [mm]	0.02 m	Vertical pivot to pivot dist...
W	100 [mm]	0.1 m	Horizontal pivot to pivot...
p	2 [mm]	0.002 m	Distance between pivot a...
L	100 [mm]	0.1 m	Blade length
my_h	100 [um]	1E-4 m	Blade thickness
b	100 [mm]	0.1 m	Blade depth

4. In **Global Definitions >** Click on **Inertial Parameters**.
5. In the **Settings** window, locate the **Parameters** section.
6. In the stage, assign to the **l_bal** parameter (currently set to 100) the numerical value you previously found:

Settings
Parameters

Label: Inertial Parameters

▾ Parameters

Name	Expression	Value	Description
m_1	1.7138708e-02 [kg]	0.017139 kg	Mass of solid 1
m_2	6.8122300e-02 [kg]	0.068122 kg	Mass of solid 2
m_3	m_1	0.017139 kg	Mass of solid 3
m_bal	3.5565044e-02 [kg]	0.035565 kg	Mass of balancing mass
J_1xx	5.3559891e-6 [kg*m^2]	5.356E-6 kg·m²	xx component inertia tensor solid 1
J_1yy	1.9478338e-6 [kg*m^2]	1.9478E-6 kg·m²	yy component inertia tensor solid 1
J_1zz	5.3569824e-6 [kg*m^2]	5.357E-6 kg·m²	zz component inertia tensor solid 1
J_3xx	J_1xx	5.356E-6 kg·m²	xx component inertia tensor solid 1
J_3yy	J_1yy	1.9478E-6 kg·m²	yy component inertia tensor solid 1
J_3zz	J_1zz	5.357E-6 kg·m²	zz component inertia tensor solid 1
J_balxx	2.0034975e-6 [kg*m^2]	2.0035E-6 kg·m²	xx component inertia tensor balancing mass
J_balyy	2.5379149e-6 [kg*m^2]	2.5379E-6 kg·m²	yy component inertia tensor balancing mass
J_balzz	1.1271682e-6 [kg*m^2]	1.1272E-6 kg·m²	zz component inertia tensor balancing mass
l_c1	1.0411598e-2 [m]	0.010412 m	Location of the COM of solid 1
l_c3	l_c1	0.010412 m	Location of the COM of solid 3
l_bal	100 [mm]	0.1 m	Location of the COM of the balancing mass

Remark: l_bal is positively defined. Put a minus sign in front of the value.

GEOMETRY

In **Component 1 (comp 1)** > Click on **Geometry** 1 > Click the **Build All** action button.

MATERIAL

In **Component 1 (comp 1)** > Click on **Materials** > Verify that the Ti-6Al-4V material is applied for all flexures.

SOLID MECHANICS

In **Component 1 (comp 1)** > **Solid Mechanics (solid)**

Rigid Connector 1: Mechanism's base

1. Observe that **Rigid Connector 1** connects the eight fixed parts of the flexures.
   Remark: **Rigid Connector 1** could have been replaced by a **Fixed Constraint.** Here, we use a rigid connector as we will later use the **Harmonic perturbation** feature (currently disabled).

Rigid Connector 2: Solid 1

1. Observe that **Rigid Connector 2** connects the four parts of the flexures that belong to Solid 1.
   a. Note that Mass and Moment of Inertia 1 corresponds to the contribution of the mass and inertia of Solid 1 without its balancing mass.
   b. Note that Mass and Moment of Inertia 2 corresponds to the contribution of the mass and inertia of the balancing mass attached to Solid 1 (currently disabled).

Rigid Connector 3: Solid 2

1. Observe that **Rigid Connector 3** connects the four parts of the flexures that belong to Solid 2.
   a. Note that Mass and Moment of Inertia 1 corresponds to the contribution of the mass of Solid 2.

Rigid Connector 4: Solid 3

1. Observe that **Rigid Connector 4** connects the four parts of the flexures that belong to Solid 3.
   a. Note that Mass and Moment of Inertia 1 corresponds to the contribution of the mass and inertia of Solid 3 without its balancing mass.
   b. Note that Mass and Moment of Inertia 2 corresponds to the contribution of the mass and inertia of the balancing mass attached to Solid 3.

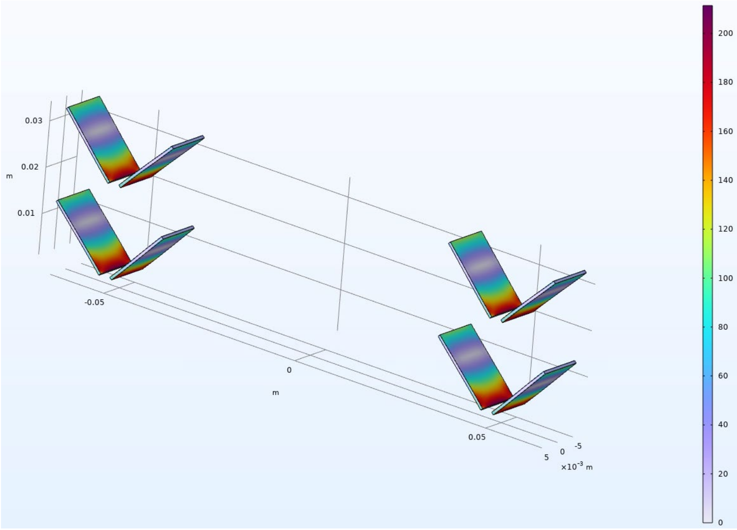
MESH

Verify that the flexures are correctly meshed with the same mapped mesh.

STUDY 1: Stationary

Use this first study to verify the stress in the flexures when the maximal stroke (±410 μm) of the mechanism is reached. Using the results, measure the linear equivalent stiffness (k_{eq}) of the stage.

Maximum stress at 410 μm of 214 [MPa] → $S \approx 2.3$



Linear equivalent stiffness $k_{eq} \approx 1.64e5\ [\mathrm{N.m^{-1}}]$ → 5.6% stiffer than the prediction. This deviation can be explained by not considering the Poisson coefficient in the analytical model. If the Poisson coefficient is set to 0 in COMSOL, then the relative error drops to 0.8%.

STUDY 2: Eigenfrequency

Use this second study to verify the 1st eigenfrequency of the parallel stage without its balancing masses.

Remark 1: do not forget to disable **Mass and Moment of Inertia 2** from **solid_1** and **solid_2.**

Remark 2: do not forget to remove the prescribed displacement of **solid_2**.

SOLUTION:

- First eigenfrequency at 196.32 [Hz]→with the new k_{eq}, the analytical eigenfrequency is 199.6 [Hz]. This 1.64% relative error is mainly due to the fact that the mass and inertia of the blades are neglected in the analytical model. The relative error drops to 0.1% if the density of the blades is set to 0.1 $[\mathrm{kg.m^{-3}}]$ in COMSOL.
- Second eigenfrequency at 787 [Hz]

STUDY 3: Eigenfrequency

Use this study to verify the 1st eigenfrequency of the parallel stage with its balancing masses.

Remark: do not forget to enable **Mass and Moment of Inertia 2** from **solid_1** and **solid_2.**

SOLUTION:

- First eigenfrequency at 138.37 [Hz]→with the new k_{eq}, the analytical eigenfrequency is 139.3 [Hz]. This 0.7% relative error is mainly due to the fact that the mass and inertia of the blade are neglected in the analytical model. The relative error drops to 0.2% if the density of the blades is set to 0.1 $[\mathrm{kg.m^{-3}}]$ in COMSOL.

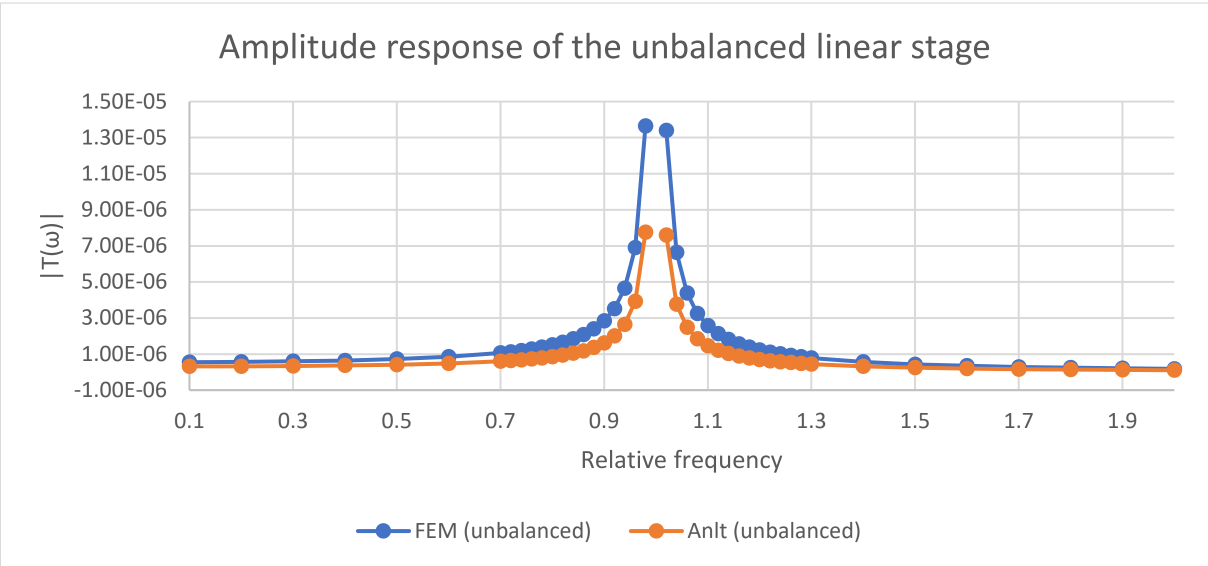
- Second eigenfrequency at 419 [Hz]

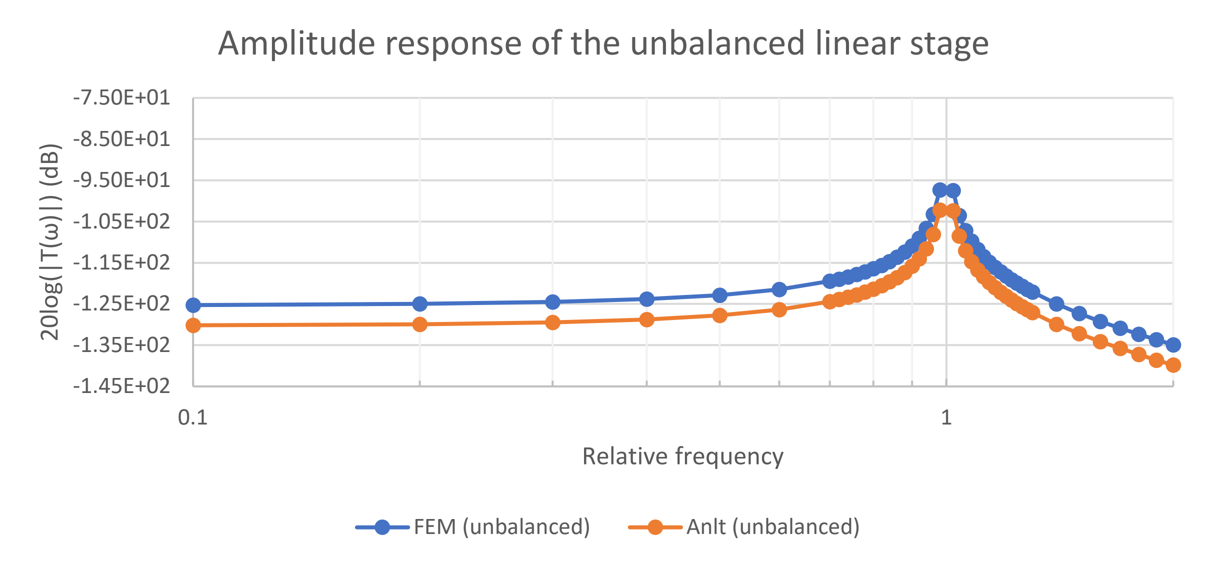
STUDY 4: Frequency Domain

In this study, we verify the sensitivity to linear accelerations around the main frequency of the parallel stage without its balancing masses.

1. In Mechanism's base (**Rigid Connector** 1), write "dx" in the **Prescribed in x direction** field.
2. Right click on Mechanism's base and add a **Harmonic Perturbation**.
3. In **Harmonic Perturbation** 1, write "dx" in the **Displacement constraint, x-component** field.
4. In **Study 4** > **Settings** windows > **Study Settings** section > Define a list of frequencies for which you want the effect of harmonic perturbation to be analyzed. Remark: avoid analyzing the stage at its eigenfrequency as this will create a simulation error.
5. Using the results of Study 4, plot a graph that shows the absolute displacement value of Solid_2 along x in function of the frequencies the mechanism's base was excited. Make sure you normalize the abscissa of the graph relative to the eigenfrequency of the mechanism.

SOLUTION:





STUDY 5: Frequency domain

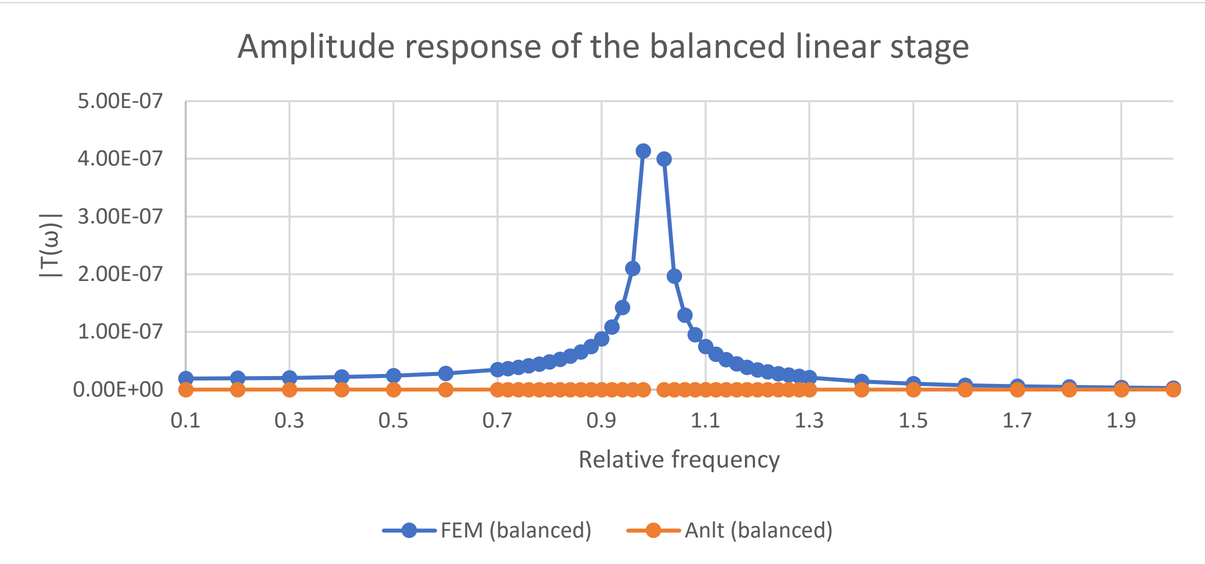
In this study, we verify the sensitivity to linear accelerations around the main frequency of the parallel stage with its balancing masses.

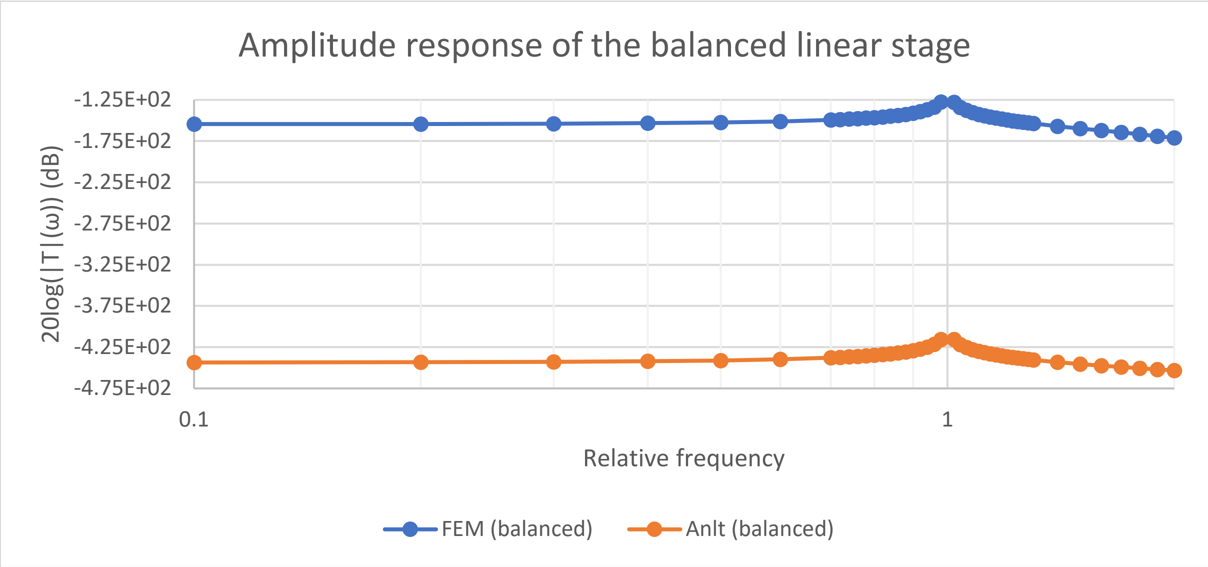
Remark 1: do not forget to enable **Mass and Moment of Inertia 2** from **solid_1** and **solid_2.**

Remark 2: Redo the same steps as in **Study 4** but study the response of the mechanism around the eigenfrequency you measured in **Study 3.**

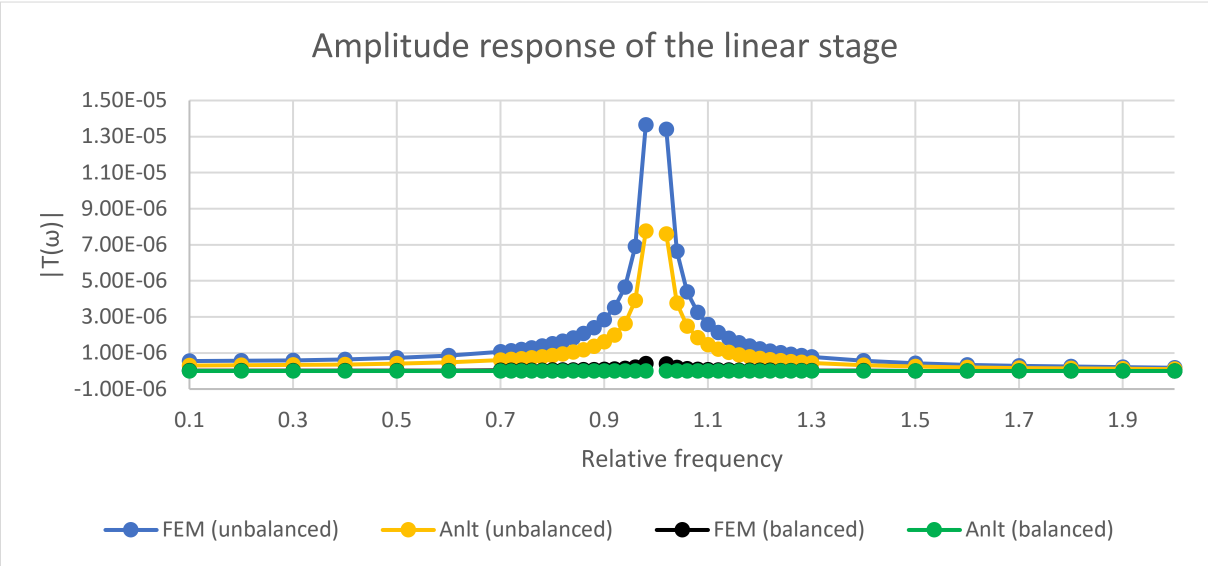
Compare the normalized graph from study 4 and study 5. How the balancing masses did affect the mechanism response? How much does the sensitivity to linear acceleration of the system decrease around its natural frequency? Using the transfer function from **Part 1, Question 5**, try to predict these two behaviors.

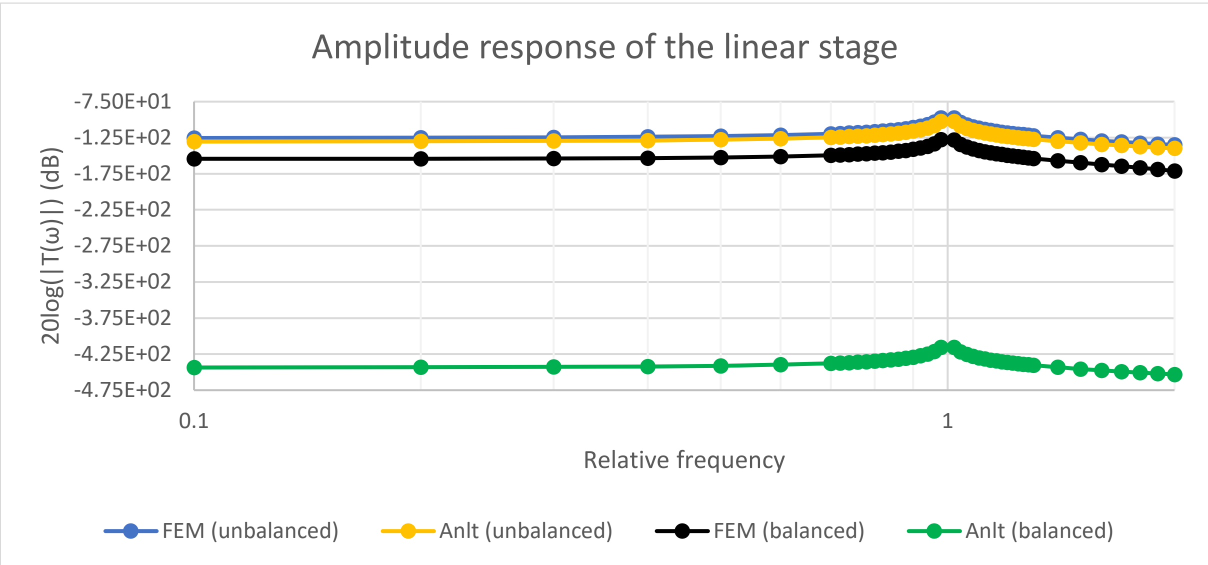
SOLUTION:




Amplitude response of the balanced linear stage
20log(|T|(ω)) (dB)
-1.25E+02
-1.75E+02
-2.25E+02
-2.75E+02
-3.25E+02
-3.75E+02
-4.25E+02
-4.75E+02
0.1
1
Relative frequency
FEM (balanced)
Anlt (balanced)

Comparison:


Amplitude response of the linear stage
|T(ω)|
1.50E-05
1.30E-05
1.10E-05
9.00E-06
7.00E-06
5.00E-06
3.00E-06
1.00E-06
-1.00E-06
0.1
0.3
0.5
0.7
0.9
1.1
1.3
1.5
1.7
1.9
Relative frequency
FEM (unbalanced)
Anlt (unbalanced)
FEM (balanced)
Anlt (balanced)


Amplitude response of the linear stage
20log(|T(ω)|) (dB)
-7.50E+01
-1.25E+02
-1.75E+02
-2.25E+02
-2.75E+02
-3.25E+02
-3.75E+02
-4.25E+02
-4.75E+02
0.1
1
Relative frequency
FEM (unbalanced)
Anlt (unbalanced)
FEM (balanced)
Anlt (balanced)

One can see that the amplitude response of the system after it is force balanced is approximatively 30 times lower than its initial amplitude response. The residual balancing error should mainly come from the non-consideration of the flexures mass and inertia.

STUDY 6: Stationary

Use this study to check the safety factor of the flexures for the plastification when the quasi-static acceleration G_rms you calculated in **Part 1, Question 1** is applied.

Remark 1: do not forget to rigidly fix **Solid_2** relative to the mechanism frame.

Remark 2: apply the acceleration load at CoM of Solid_1.

SOLUTION:

The maximum stress for a static load of 389 [N] is 114.8 [MPa]. The new estimated safety factor is $S = 4.4$ for plastification.

STUDY 7: Linear Buckling

Use this study to check the safety factor of the flexures for the buckling when the quasi-static acceleration G_rms you calculated in **Part 1, Question 1** is applied.

1. Replace the value of the load applied on Solid_1 by a load of 1 [N] in the

Remark 1: do not forget to rigidly fix **Solid_2** relative to the mechanism frame.

Remark 2: apply the acceleration load at CoM of Solid_1.

SOLUTION:

Buckling load is equal to 2848 [N]. The new estimated safety factor is $S = 7.3$ for buckling.