Solution for exercise 9 – Part II:
Design and force balancing of a flexure-based parallel stage

Context:

A flexure-base parallel stage is subjected to fluctuating linear accelerations along the x-axis due to motion of its base. To be insensitive to this specific perturbation, we wish to force balance the mechanism.

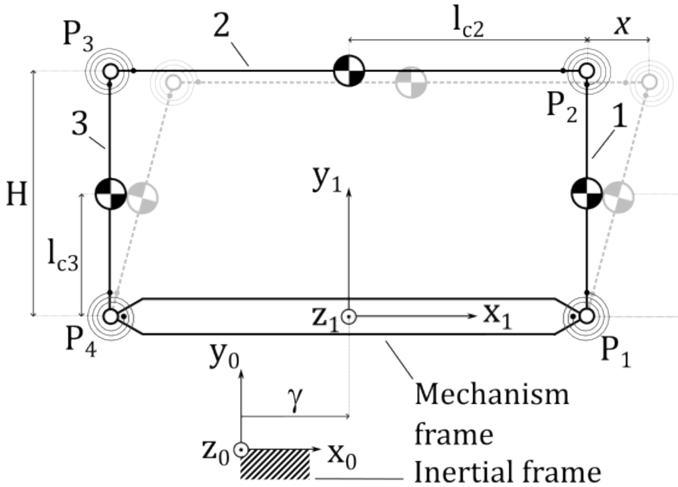


Fig. 1: Flexure-based parallel stage Pseudo-Rigid-Body-Model before its force balancing.

Figure 1 shows the Pseudo-Rigid-Body-Model (PRBM) of the studied flexure-based parallel stage before its force balancing. The mechanism has one degree-of-freedom and is composed of three moving rigid bodies: solids **1**, **2** and **3** articulated in P_1, P_2, P_3 and P_4.

Each solid $i = 1 \ldots 3$, has:

- a mass m_i,
- a center of mass (COM) c_i (represented by the symbol ◕) located by a distance l_{ci},
- an inertia tensor $\bar{\bar{J}}_i$ written at c_i.

Each pivot $P_{i=1\ldots4}$ has an angular stiffness k_i that represents the intrinsic stiffness of the future flexure-based implementation.

First, the parallel stage mechanism is actuated by a voice coil whose permanent magnet is fixed to the mechanism's base and whose coil is fixed to solid **2**. The voice coil applies a force **F** at c_2 along the x-axis. The parameter x is used to locate the displacement of solid **2** relative to its base.

Last, the linear displacement that corresponds to the motion of the mechanism's base is represented by the parameter γ.

Part II: Design of the flexure-based pivots

The flexure implementation of the PRBM shown in Figure 2 is illustrated in Figure 3. Pivot joints P_1 to P_4 along with their torsion springs of stiffness k_1 to k_4 are replaced by *remote center of compliance* (RCC) flexure pivots (red lines on Figure 3).

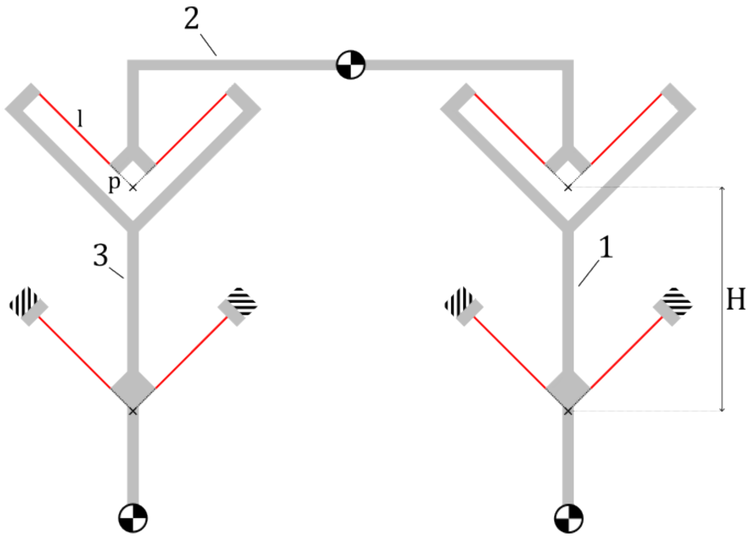


Fig. 2: Flexure-base implementation of the force balanced parallel stage.

The flexure-based parallel stage will be carried on board a rocket. During take-off, the mechanism will be subjected to vibrations along the x-axis. To protect it, solid **2** is rigidly locked (no translations and no rotations) relative to the base of the mechanism so that solids 1 and 3 are kinematically immobilized. Thus, during take-off, the acceleration forces that apply on solids 1 and 3 go through pivots P_1 and P_2, and P_3 and P_4, respectively.

Technical specifications:

Solid **2** of the flexure-based parallel stage has an admissible stroke of ± 410 µm. RCC flexure-based pivot should withstand the take-off vibrations specified from the ASD given by Figure 4.

The flexures are made out of Ti-6Al-4V.

Alliage	E [GPa]	R_m [MPa]	$R_{0.2}$ [MPa]	$\sigma_D(10^7)$ [MPa]
Titane 6Al-4V	114	900	830	500

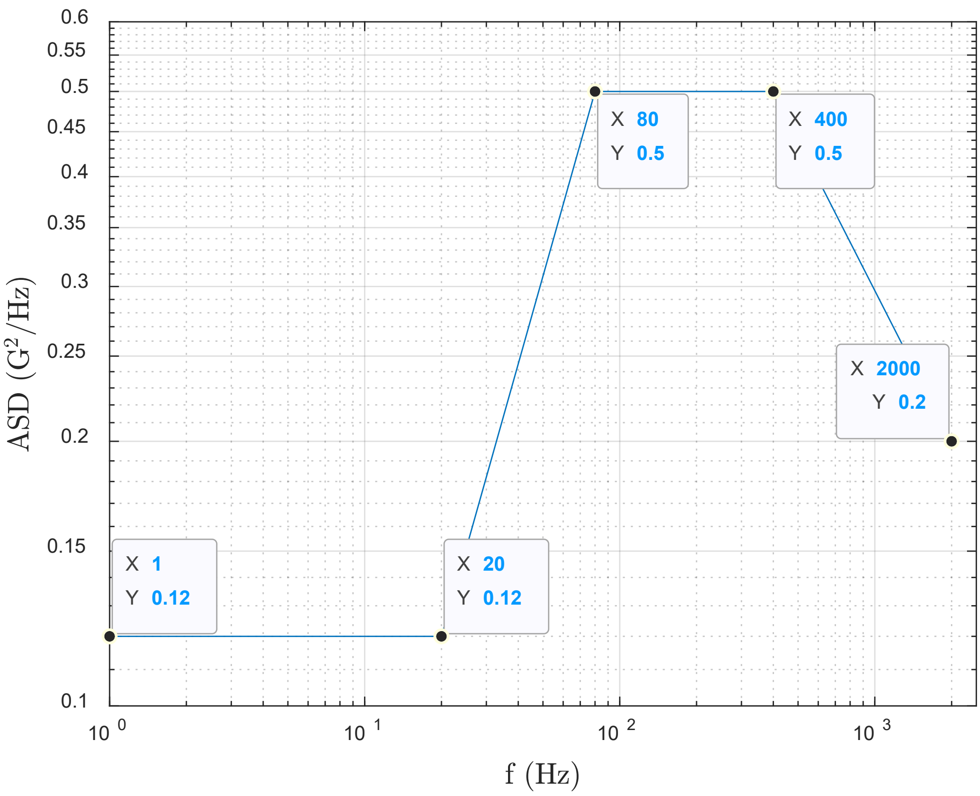


Fig. 3: ASD that the flexure-based parallel stage must withstand.

1. Considering $p = 2$ mm, find the parameters:
   - l: length of the blade
   - h: thickness of the blade
   - b: depth of the blade

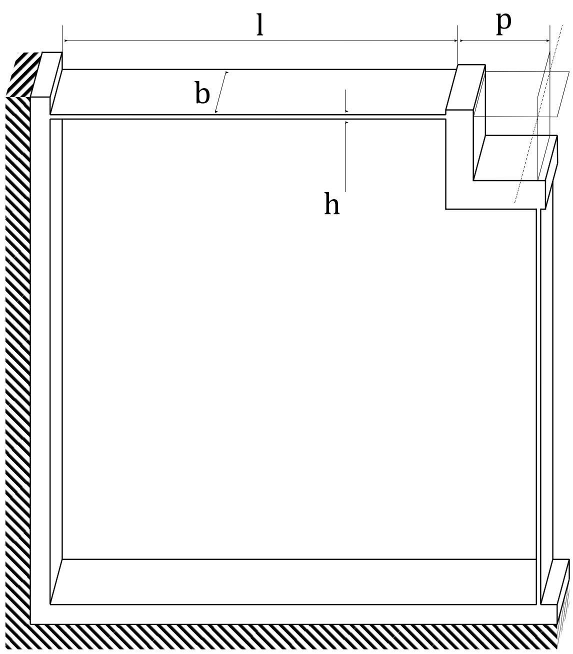


Fig. 4: Illustration of an RCC flexure pivot with its main parameters.

So that the flexures:

- withstand the stroke of solid **2** for a number of cycles $n > 10^7$ with a safety factor S≥2.5.
- do not buckle for the worst-case scenario specified by the ASD with a safety factor S≥2.5.
- do not plastify for the worst-case scenario specified by the ASD with a safety factor S≥2.5.

Reminder: to identify the worst-case scenario specified by the ASD, use the Mile's equation for all frequencies using a quality factor of $Q = 100$.

SOLUTION:

To withstand the ± 410 μm stroke, we can use:

$$\theta_{stroke} = \frac{x_{stroke}}{H} \approx 1.17°$$

We know from Simon's thesis that

$$\theta_{stroke} = \frac{1}{S} \frac{\sigma_D l^2}{Eh(2l + 3p)} \quad (1)$$

The formula that gives the buckling load of the RCC flexure-based pivot is

$$F_c = \frac{\sqrt{2}}{S} \frac{\pi^2 Ebh^3}{12\,(0.5l)^2} \quad (2)$$

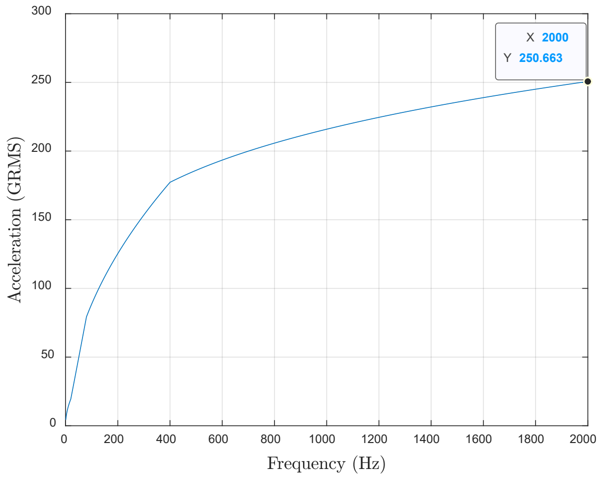
The formula that gives the plastification load of the RCC flexure-based pivot is

$$F_p = \frac{\sqrt{2}}{S} \sigma_D hb \quad (3)$$

To evaluate the worst-case scenario, we use the Mile's formula:

$$G_{rms} = \sqrt{\frac{\pi}{2} fQASD}$$

Applying the Mile's Formula on the ASD graph gives the following graph:



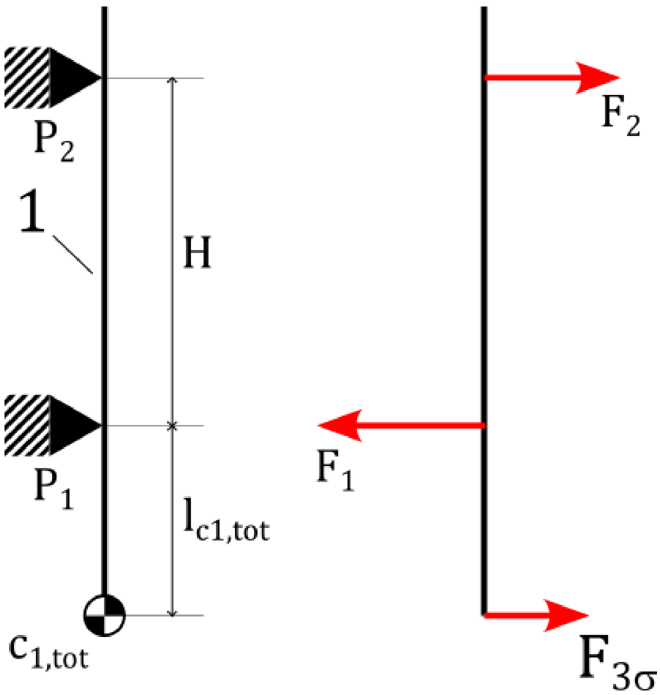
The worst-case scenario is to have a quasi-static acceleration

$$G_{rms} \approx 251 \text{ g } [\text{m/s}^2]$$

The 3σ force that apply at COM of Solid **1** (or Solid **2**) is

$$F_{3\sigma} = 3G_{rms} * g * m_{1,\text{tot}} \approx 389 \text{ [N]}$$

Finally, due to the mass distribution of Solid **1**, the force that goes through P1 is higher than the one that goes through P2:



We find

$$F_1 = F_{3\sigma}\left(1 + \frac{l_{c1,\text{tot}}}{H}\right) > F_2 = F_{3\sigma}\frac{l_{c1,\text{tot}}}{H}$$

Which gives us

$$F_1 \approx 641 \text{[N]}$$

Using Eqs. (1), (2) and (3), we find

$$h = \frac{9Sp\theta_{stroke}}{\pi^2 - 6S\pi\sqrt{\frac{E}{3\sigma_D}}\theta_{stroke}} \approx 637\ [\mu\text{m}]$$

$$l = \pi\sqrt{\frac{E}{3\sigma_D}}h \approx 17.5\ [\text{mm}]$$

$$b \geq \frac{F_1 S}{h\sigma_D\sqrt{2}} \approx 3.6\ [\text{mm}]$$

Development:
Using Eqs. (2) and (3), we have:

$$F_c = F_p$$

$$\rightarrow \frac{\sqrt{2}}{S}\frac{\pi^2 Ebh^3}{12\,(0.5l)^2} = \frac{\sqrt{2}}{S}\sigma_D hb$$

$$\rightarrow \frac{\pi^2 Eh^2}{12\,(0.5l)^2} = \sigma_D$$

$$\rightarrow l = \pi\sqrt{\frac{E}{3\sigma_D}}h$$

Injecting our last result into Eq. (1), we have:

$$\theta_{stroke} = \frac{1}{S}\frac{\sigma_D\pi^2 h^2\frac{E}{3\sigma_D}}{Eh\left(2\pi h\sqrt{\frac{E}{3\sigma_D}} + 3p\right)}$$

$$= \frac{1}{S}\frac{\pi^2 h}{3\left(2\pi\sqrt{\frac{E}{3\sigma_D}}h + 3p\right)}$$

$$\rightarrow h = \frac{9Sp\theta_{stroke}}{\pi^2 - 6S\pi\sqrt{\frac{E}{3\sigma_D}}\theta_{stroke}}$$

Returning Eq. (3), we have:

$$b \geq \frac{F_1 S}{h\sigma_D\sqrt{2}}$$

From the manufacturing constraints, we consider $b = 10\ [\text{mm}]$.
With these new values, we recompute the safety factors for:

- the stroke: ~2.5
- plastification load: ~7
- buckling load: ~7

2. Compute the linear equivalent stiffness k_{eq} of the parallel stage.

   SOLUTION:

   The stiffness k_1 of one RCC flexure pivot is given by

$$k_1 = \frac{8E\frac{bh^3}{12}(l^2 + 3pl + 3p^2)}{l^3} \approx 15.51\ [\mathrm{Nm.rad^{-1}}]$$

   We know that

$$k_{\mathrm{eq}} = \frac{4k_1}{H^2} \approx 1.55e5\ [\mathrm{N.m^{-1}}]$$

3. Compute the frequency f_{init} and f_{final} of the parallel stage before and after its force balancing.

   SOLUTION:

$$f_{init} = \frac{1}{2\pi}\sqrt{\frac{k_{\mathrm{eq}}}{m_{\mathrm{eq,init}}}} \approx 194.2\ [\mathrm{Hz}]$$

$$f_{\mathrm{final}} = \frac{1}{2\pi}\sqrt{\frac{k_{\mathrm{eq}}}{m_{\mathrm{eq,final}}}} \approx 135.6\ [\mathrm{Hz}]$$