MICRO-330 CAPTEURS
Formulaire d'examen 2025

Constants

$h \cong 6.6 \times 10^{-34}\mathrm{Js}$ $e \cong -1.6 \times 10^{-19}\mathrm{C}$ $c \cong 3.0 \times 10^{8}\mathrm{m/s}$ $k_B \cong 1.4 \times 10^{-23}\mathrm{m^2kgs^{-2}K^{-1}}$ $\Phi_0 \cong 2.1 \times 10^{-15}\mathrm{Tm^2}$

$\epsilon_0 \cong 8.8 \times 10^{-12}\mathrm{F/m}$ $\mu_0 \cong 1.3 \times 10^{-6}\mathrm{H/m}$ $m_e \cong 9.1 \times 10^{-31}\mathrm{kg}$

1. Mesure

$$\Delta x_{\min} = \frac{y_{n,rms}}{S_0} \qquad S_0 = \frac{dy}{dx} \qquad D = 20\log_{10}(x_{FS}/\Delta x_{\min})$$

$$y_{n,rms} = \sqrt{\frac{1}{t_2-t_1}\int_{t_1}^{t_2}(y(t)-y_{avg})^2dt} \qquad y_{n,rms} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i-y_{avg})^2} \qquad y_{n,rms} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(y_i-y_{avg})^2}$$

$$y_{avg} = \frac{1}{t_2-t_1}\int_{t_1}^{t_2}y(t)dt \qquad y_{avg} = \frac{1}{N}\sum_{i=1}^{N}y_i$$

$$X(s) = L(x(t)) = \int_{0^-}^{\infty} e^{-st}x(t)dt$$
$$s = \sigma + j\omega, \quad (\sigma,\omega) \in \Re e$$
$$L\left(\frac{dx(t)}{dt}\right) = sX(s) - x(0^-)$$
$$L\left(\frac{d^2x(t)}{dt^2}\right) = s^2X(s) - sx(0^-) - \frac{dx(0^-)}{dt}$$
$$L\left(\int_0^t x(\tau)d\tau\right) = \frac{1}{s}X(s)$$

$$\Delta A_c = \left(\frac{\partial f}{\partial B}\Delta B_c\right) + \left(\frac{\partial f}{\partial C}\Delta C_c\right) + \left(\frac{\partial f}{\partial D}\Delta D_c\right) + \ldots$$
$$\Delta A = \left|\frac{\partial f}{\partial B}\Delta B\right| + \left|\frac{\partial f}{\partial C}\Delta C\right| + \left|\frac{\partial f}{\partial D}\Delta D\right| + \ldots$$
$$\sigma_A = \sqrt{\left(\frac{\partial f}{\partial B}\right)^2\sigma_B^2 + \left(\frac{\partial f}{\partial C}\right)^2\sigma_C^2 + \left(\frac{\partial f}{\partial D}\right)^2\sigma_D^2 + \ldots}$$

$$S_n(f) = \lim_{T\to\infty}\frac{1}{T}\left|\int_0^T v_n(t)e^{-i2\pi ft}dt\right|^2$$
$$v_{n,rms} = \sqrt{\int_0^\infty S_n(f)df}$$

$$y(t) = S_0x(t) \qquad G(j\omega) = S_0$$
$$\tau\dot{y}(t) + y(t) = S_0x(t) \qquad G(j\omega) = \frac{S_0}{1+j\omega\tau}$$
$$\frac{1}{\omega_0^2}\ddot{y}(t) + \frac{2\xi}{\omega_0}\dot{y}(t) + y(t) = S_0x(t) \qquad G(j\omega) = \frac{S_0}{1-(\omega/\omega_0)^2+j2\xi(\omega/\omega_0)}$$

$$y(t) = \frac{S_0x_0}{\sqrt{1+\omega^2\tau^2}}\cos(\omega t+\phi)$$
$$y(t) = S_0x_0\left(1-e^{-t/\tau}\right)$$
$$y(t) = S_0\alpha\left(t-\tau(1-e^{-t/\tau})\right)$$

$$y(t) = \frac{S_0x_0}{\sqrt{\left[1-(\omega/\omega_0)^2\right]^2+4\xi^2(\omega/\omega_0)^2}}\cos(\omega t+\phi)$$
$$y(t) = S_0x_0\left(1-\frac{1}{\sqrt{1-\xi^2}}e^{-\xi\omega_0 t}\sin\left(\omega_0\sqrt{1-\xi^2}t+\varphi\right)\right)$$

$$S_n(f) = \frac{4Rhf}{e^{\frac{hf}{k_BT}}-1} \qquad S_n(f) = 4k_BTR \qquad v_{n,rms} = \sqrt{4k_BTR}\sqrt{\Delta f}$$
$$S_n(f) = \frac{4hf/R}{e^{\frac{hf}{k_BT}}-1} \qquad S_n(f) = 4k_BT/R \qquad i_{n,rms} = \sqrt{4k_BT}\sqrt{\Delta f}/\sqrt{R}$$

$$S_n(f) = 2ei_0$$
$$i_{n,rms} = \sqrt{2ei_0}\sqrt{\Delta f}$$

$$\overline{v_{n1}v_{n2}} = \frac{1}{T}\int_0^T v_{n1}(t)v_{n2}(t)dt$$

$$S_n(f) = (i_nR_2)^2 + \left(e_n\frac{R+R_1+R_2}{R+R_1}\right)^2 + \left(4k_BTR\left(\frac{R_2}{R_1+R}\right)^2\right) + \left(4k_BTR_1\left(\frac{R_2}{R_1+R}\right)^2\right) + (4k_BTR_2)$$
$$S_n(f) = (i_nR_2)^2 + \left(i_nR\frac{R_1+R_2}{R_1}\right)^2 + \left(e_n\frac{R_1+R_2}{R_1}\right)^2 + \left(4k_BTR_1\left(\frac{R_2}{R_1}\right)^2\right) + (4k_BTR_2) + \left(4k_BTR\left(\frac{R_1+R_2}{R_1}\right)^2\right)$$

$$\delta = \sqrt{1/\pi f\mu\sigma} = \sqrt{\rho/\pi f\mu}$$

$$S_n(f) = 4k_BT\left(\Re e\left[Z_{RC}\right]\right) = 4k_BT\frac{R}{1+\omega^2R^2C^2}$$
$$v_{n,rms} = \sqrt{\frac{k_BT}{C}}$$

2. Capacitifs

$$C = \varepsilon_0\int_V\varepsilon_r(\mathbf{x})\left|\mathbf{E}_u(\mathbf{x})\right|^2d\mathbf{x}^3 \qquad C \cong \frac{\varepsilon_0\varepsilon_rA}{d} \qquad W_{el} = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} \qquad F_{el} = \frac{\partial W_{el}}{\partial d} \qquad \frac{\partial V}{\partial d} \cong \frac{\sigma_sd_1}{\varepsilon_0}\frac{1}{d}$$

3. Inductifs

$$L = \frac{1}{\mu_0}\int\limits_V \frac{1}{\mu_r(\mathbf{x})}\left|\mathbf{B}_u(\mathbf{x})\right|^2 d\mathbf{x}^3$$

$$L \cong \mu_0\mu_r\frac{N^2}{l}\pi R^2$$

$$V_0 = \left[R_b + \underbrace{\frac{\omega^2 {L_{12}}^2}{{R_c}^2+\omega^2{L_c}^2}R_c}_{\Delta R(x)} + j\omega\left(L_b - \underbrace{\frac{\omega^2 {L_{12}}^2}{{R_c}^2+\omega^2{L_c}^2}L_c}_{\Delta L(x)}\right)\right] i_b$$

$$\delta = 1/\sqrt{\pi f\mu\sigma} = \sqrt{\rho/\pi f\mu}$$

$$\Re = \sum \frac{l_i}{\mu_r\mu_0 A} = \int \frac{dl}{\mu_r\mu_0 A} \qquad L = \frac{N^2}{\Re}$$

$$S_\sigma = \frac{1}{\sigma}\frac{\Delta\mu}{\mu} \qquad L \cong \frac{N^2\mu A}{l}$$

4. Magnétiques

$$R(\theta) = R_{//} - \Delta R_{MR}\sin^2\theta \qquad R(\theta) = R_\perp + \Delta R_{MR}\cos^2\theta$$

$$R(\theta) = R_{\phi=180^\circ} - \Delta R_{GMR}(1+\cos\phi)/2$$

$$\mathbf{F} = e(\mathbf{E} + \mathbf{v}\wedge\mathbf{B})$$
$$\mathbf{F} = m\dot{\mathbf{v}} = m\Delta\mathbf{v}/\Delta t \cong m\mathbf{v}/\tau$$
$$V_H = \int\limits_{-w/2}^{w/2} E_y dy = v_x B_z w = \frac{\mu\rho}{h} I B_z = \frac{1}{neh} I B_z$$
$$\mu = \frac{v_x}{E_x} = \frac{e\tau}{m};\ \rho = \frac{m}{en\tau} = \frac{1}{\mu n};\ J = env_x;\ I = Jwh$$
$$V_H = \frac{1}{enh} IB = S_I IB = SB$$

$$R_B = R_0\left(1 + m(\mu B)^2\right)$$

$$B = \mu_0(H+M) \cong \mu_0\left(H + \mu_{r\,\max}H - \alpha H^3\right) \cong \mu_0\mu_{r\,\max}H - \alpha H^3 \text{ (for } \mu_{r\,\max} >> 1)$$
$$V_{ind} = \underbrace{\mu_{\max}H_{exc}\omega\cos\omega t}_{f} - \underbrace{3\alpha H_{ext}^2 H_{exc}\omega\cos\omega t}_{f} - \underbrace{6\alpha H_{ext}H_{exc}^2\omega\sin\omega t\cos\omega t}_{2f} - \underbrace{3\alpha H_{exc}^3\omega\cos\omega t\sin^2\omega t}_{3f}$$

$$\Phi_B = n\Phi_0 \quad n\in\mathbb{N}, \qquad I_c \cong 2i_c\left|\cos\left(\pi\frac{\Phi_{B_{ext}}}{\Phi_0}\right)\right|$$

5. Optiques

$$E_{ph} = h\nu \qquad \lambda = \frac{c*}{\nu} = \frac{hc*}{E_{ph}}$$

$$\Delta n = n_{on} - n_{off} = g\tau_n$$
$$g = \frac{1}{lwd}\frac{\eta(1-r)\Phi}{h\nu}$$
$$R = \frac{1}{\sigma}\frac{l}{wd} = \frac{1}{e\mu n}\frac{l}{wd}$$
$$\Delta I = e\Delta n\frac{wdl}{\tau_{tr}} = eg\frac{\tau_n}{\tau_{tr}}wdl$$
$$S = \frac{\Delta I}{\Phi} = e\frac{\tau_n}{\tau_{tr}}\frac{\eta(1-r)}{h\nu}$$

$$I = I_d - I_{ph}$$
$$I_{ph} = e\eta(1-r)\frac{\lambda\Phi(x)}{hc} = e\eta(1-r)\frac{\Phi(x)}{h\nu}$$
$$\Phi(x) = \Phi_0\exp(-\alpha x)$$
$$I_d = I_0\left(\exp\left(\frac{eV_d}{k_B T}\right) - 1\right)$$
$$S = \frac{I_{ph}}{\Phi_0} = \frac{e\eta(1-r)\frac{\lambda\Phi_0\exp(-\alpha x)}{hc}}{\Phi_0} = e\eta(1-r)\frac{\lambda\exp(-\alpha x)}{hc}$$

$$f_D = f_r - f_e = -\frac{f_e}{c}\mathbf{v}\cdot(\hat{\mathbf{r}} - \hat{\mathbf{e}}) \qquad f_D \cong -f_e\frac{2v}{c}\cos\theta$$

$$\beta = C_{ver}Bd$$

$$\Delta\Phi \cong \frac{8\pi^2R^2}{c\lambda}\Omega$$

$$I_{signal} = I_L M$$
$$I_{noise} = \sqrt{2eI_{signal}MFB} = \sqrt{2eI_L M^2 FB} = M\sqrt{2eI_L FB}$$
$$F \cong \alpha M + (1-\alpha)(2-(1/M))$$

Pour $M \gg 1$ et $\alpha \ll 1 : F \cong (\alpha M + 2)$

Pour $M \cong 1$ et $\alpha \ll 1 : F \cong 1$

$$S(\lambda, T) \cong \frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{-\frac{hc}{\lambda k_B T}} - 1}$$

Pour $h\nu \gg k_B T : S(\lambda,T) \cong \frac{2\pi c^2 h}{\lambda^5}e^{-\frac{hc}{\lambda k_B T}}$

$$\int_0^\infty S(\lambda,T)d\lambda = \sigma T^4 \qquad \sigma \cong 5.7\times 10^{-8}\frac{\mathrm{W}}{\mathrm{m^2K^4}}$$
$$\lambda_{\max}T = 2.9\times 10^{-3}\ \mathrm{m\ K}$$
$$V_m(T) = \int\limits_{\lambda_0-\Delta\lambda}^{\lambda_0+\Delta\lambda}\varepsilon(\lambda,T)S(\lambda,T)S_c(\lambda)d\lambda$$

Pour $h\nu \gg kT : V_m(T) = \varepsilon(\lambda_1,T)C_1\lambda_1^{-5}e^{-C_2/\lambda_1 T}S_c(\lambda_1)2\Delta\lambda$

$$C_1 = 2\pi c^2 h \qquad C_2 = hc/k$$

6. Thermiques

$$V_{bruit-thermique,rms} = \sqrt{4k_B T R \Delta f}$$

$$R(T) = R_0[1 + \alpha_R(T - T_0) + \beta_R(T - T_0)^2 + ...]$$

$$\Phi(t) = C_{th}\frac{dT}{dt} + (T - T_0)G_{th} \qquad \tau_{thermique} = \frac{C_{th}}{G_{th}} = \frac{mc_v}{G_{th}}$$

PTAT : $V_1 - V_N = \frac{k_B T}{e}\ln(N)$ Thermocouple : $\Delta V = S_{AB}(T_2 - T_1)$

$$V_m = S_0\left[\varepsilon_{obj}T^4_{obj} - T^4_{capteur} + (1 - \varepsilon_{obj})\,T^4_{ambiant}\right]$$

7. Piézorésistifs

$$\frac{\Delta R}{R} = K\frac{\Delta l}{l} = K\varepsilon \qquad \frac{\Delta R}{R} = [\alpha_R + K(\alpha_e - \alpha_j)]\,\Delta T$$

$$V_m = \frac{V_0}{4}\frac{\Delta R_4}{R}\left(1 - \frac{1}{2}\frac{\Delta R_4}{R} + \ldots\right) \qquad V_m = \frac{V_0}{4}K\frac{\Delta l}{l}\left(1 - \frac{1}{2}K\frac{\Delta l}{l} + \ldots\right)$$

$$\frac{\Delta\rho_i}{\rho} = \pi_{ij}\sigma_j \qquad \pi_{transverse} = -\frac{1}{2}\pi_{44} \qquad \pi_{longitudinal} = \frac{1}{2}\pi_{44}$$

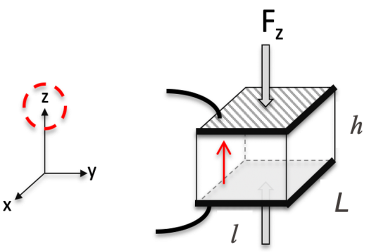
Effet longitudinal: $\frac{\Delta R}{R} = \frac{1}{2}\pi_{44}\sigma_{[110]}$ Effet transverse: $\frac{\Delta R}{R} = -\frac{1}{2}\pi_{44}\sigma_{[1\underline{1}0]}$

8. Piézoélectriques

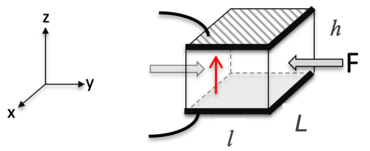
d_{ik} avec i : direction du champ électrique k : direction de la force (ou couple)

$$\begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} = \begin{pmatrix} d_{11} & -d_{11} & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & -d_{14} & -2d_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}(\sigma_{1..6}) \quad \text{(pour quartz)}$$

$$\begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}(\sigma_{1..6}) \quad \text{(pour PZT)}$$



$$V_z = \frac{Q_z}{C} = \frac{Q_z h}{\varepsilon\varepsilon_0 Ll} = \frac{d_{33}}{\varepsilon\varepsilon_0}\frac{h}{A}F_z$$



$$V_z = \frac{Q_z}{C} = d_{31}F\frac{1}{L}\frac{1}{\varepsilon\varepsilon_0}$$

$$d_{ultrason} = v_{ultrason}\frac{1}{2}TOF \qquad Q_{débit} = \pi R^2\frac{v_{ultrason}}{\cos(\alpha)}\frac{\Delta f_{doppler,max}}{f_0}$$

9. Résonants

$$Q = \frac{k}{\lambda\omega_0} = \frac{A_{max}}{A_0} \qquad \frac{\Delta f}{\Delta m} = -\frac{2f_0^2}{A\sqrt{\rho_q G}}$$

$$F_{coriolis,débitmètre} = 2\dot{M}\Omega L \qquad F_{y,gyro} = -2m_2\,|\Omega|\,v_x \qquad y_{max} = \frac{2\Omega Q_y}{\omega_0}x$$

10. Chimiques

$$\boxed{\frac{\Delta R_{reaction}}{R_0} = \alpha_R \Delta T_{reaction} = \frac{\alpha_R \Delta H_{reac} \gamma}{G_{therm}} C_0}$$

$$\boxed{E = E^0 + \frac{RT}{nF} \ln\left(\frac{C_{ox}}{C_{red}}\right) \qquad \text{à } 25°\text{C} : E = E^0 + \frac{0.0256}{n} 2.3 \log_{10}\left(\frac{C_{ox}}{C_{red}}\right)}$$

$$\boxed{E = E_{ref} + 0.059\left(pH_{ref} - pH\right)}$$