Last Name:

First Name:

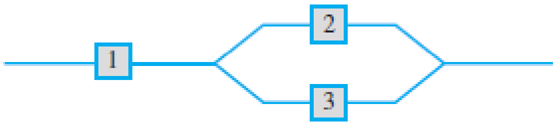
SCIPER:

Practice Problems: Exercise 4 – Microengineering 110

Spring 2025

Prof. Vivek Subramanian

1. Three components are connected to form a system as shown in the accompanying diagram. Because the components in the 2–3 subsystem are connected in parallel, that subsystem will function if at least one of the two individual components functions. For the entire system to function, component 1 must function and so must the 2–3 subsystem.



The experiment consists of determining the condition of each component [S (success) for a functioning component and F (failure) for a nonfunctioning component].

a. Which outcomes are contained in the event A that exactly two out of the three components function?

$A = \{SSF, SFS, FSS\}$.

b. Which outcomes are contained in the event B that at least two of the components function?

$B = \{SSS, SSF, SFS, FSS\}$.

c. Which outcomes are contained in the event C that the system functions?

For event C to occur, the system must have component 1 working (S in the first position), then at least one of the other two components must work (at least one S in the second and third positions): C = {SSS, SSF, SFS}.

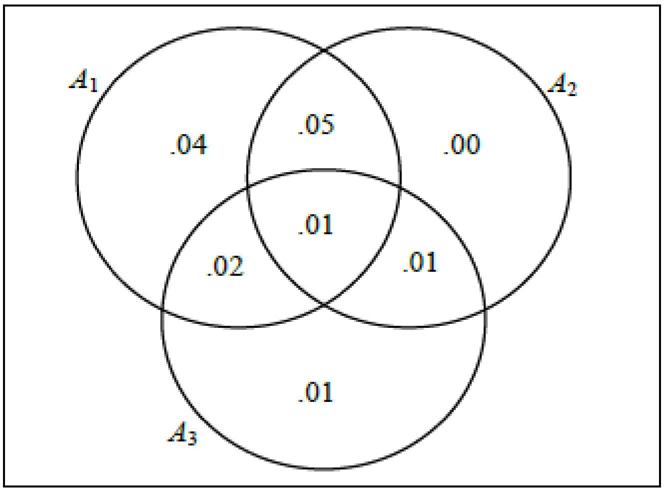
d. List outcomes in C', A ∪ C, A ∩ C, B ∪ C, and B ∩ C.

$C' = \{SFF, FSS, FSF, FFS, FFF\}$.
$A \cup C = \{SSS, SSF, SFS, FSS\}$.
$A \cap C = \{SSF, SFS\}$.
$B \cup C = \{SSS, SSF, SFS, FSS\}$. Notice that B contains C, so $B \cup C = B$.
$B \cap C = \{SSS\ SSF, SFS\}$. Since B contains C, $B \cap C = C$.

2. A certain system can experience three different types of defects. Let A_i (i = 1,2,3) denote the event that the system has a defect of type i. Suppose that

$$P(A_1) = .12 \quad P(A_2) = .07 \quad P(A_3) = .05$$
$$P(A_1 \cup A_2) = .13 \quad P(A_1 \cup A_3) = .14$$
$$P(A_2 \cup A_3) = .10 \quad P(A_1 \cap A_2 \cap A_3) = .01$$

To proceed, it is recommended that you populate the Venn diagram below.



a. What is the probability that the system does not have a type 1 defect?

$P(A_1') = 1 - P(A_1) = 1 - .12 = .88.$

b. What is the probability that the system has both type 1 and type 2 defects?

The addition rule says $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Solving for the intersection ("and") probability, you get $P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = .12 + .07 - .13 = .06.$

c. What is the probability that the system has both type 1 and type 2 defects but not a type 3 defect?

A Venn diagram shows that $P(A \cap B') = P(A) - P(A \cap B)$. Applying that here with $A = A_1 \cap A_2$ and $B = A_3$, you get $P([A_1 \cap A_2] \cap A_3') = P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3) =$ $.06 - .01 = .05.$

d. What is the probability that the system has at most two of these defects?

The event "at most two defects" is the complement of "all three defects," so the answer is just $1 - P(A_1 \cap A_2 \cap A_3) = 1 - .01 = .99.$

3. The composer Beethoven wrote 9 symphonies, 5 piano concertos (music for piano and orchestra), and 32 piano sonatas (music for solo piano).

   a. How many ways are there to play first a Beethoven symphony and then a Beethoven piano concerto?

      (9)(5) = 45.

   b. The manager of a radio station decides that on each successive evening (7 days per week), a Beethoven symphony will be played followed by a Beethoven piano concerto followed by a Beethoven piano sonata. For how many years could this policy be continued before exactly the same program would have to be repeated?

      By the same reasoning, there are (9)(5)(32) = 1440 such sequences, so such a policy could be carried out for 1440 successive nights, or almost 4 years, without repeating exactly the same program.

4. Computer keyboard failures can be attributed to electrical defects or mechanical defects. A repair facility currently has 25 failed keyboards, 6 of which have electrical defects and 19 of which have mechanical defects.

   a. How many ways are there to randomly select 5 of these keyboards for a thorough inspection (without regard to order)?

      Since order doesn't matter, the number of ways to randomly select 5 keyboards from the 25 available is $\binom{25}{5} = 53{,}130$.

   b. In how many ways can a sample of 5 keyboards be selected so that exactly two have an electrical defect?

      Sample in two stages. First, there are 6 keyboards with an electrical defect, so the number of ways to select exactly 2 of them is $\binom{6}{2}$. Next, the remaining 5 – 2 = 3 keyboards in the sample must have mechanical defects; as there are 19 such keyboards, the number of ways to randomly select 3 is $\binom{19}{3}$. So, the number of ways to achieve both of these in the sample of 5 is the product of these two counting numbers: $\binom{6}{2}\binom{19}{3} = (15)(969) = 14{,}535$.

   c. If a sample of 5 keyboards is randomly selected, what is the probability that at least 4 of these will have a mechanical defect?

      Following the analogy from **b**, the number of samples with exactly 4 mechanical defects is $\binom{19}{4}\binom{6}{1}$, and the number with exactly 5 mechanical defects is $\binom{19}{5}\binom{6}{0}$. So, the number of samples with <u>at least</u> 4 mechanical defects is $\binom{19}{4}\binom{6}{1} + \binom{19}{5}\binom{6}{0}$, and the probability of this event is

$$\frac{\binom{19}{4}\binom{6}{1} + \binom{19}{5}\binom{6}{0}}{\binom{25}{5}} = \frac{34{,}884}{53{,}130} = .657.$$ (The denominator comes from **a**.)