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Practice Problems: Exercise 3 – Microengineering 110

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1. Exposure to microbial products, especially endotoxin, may have an impact on vulnerability to allergic diseases. The following data on concentration (EU/mg) in settled dust for one sample of urban homes and another of farm homes was obtained:

   U: 6.0 5.0 11.0 33.0 4.0 5.0 80.0 18.0 35.0 17.0 23.0
   F: 4.0 14.0 11.0 9.0 9.0 8.0 4.0 20.0 5.0 8.9 21.0 9.2 3.0 2.0 0.3

   a. Determine the sample mean for each sample. How do they compare?

      For urban homes, mean = 21.55 EU/mg; for farm homes, mean = 8.56 EU/mg.

      The average endotoxin concentration in urban homes is more than double the average endotoxin concentration in farm homes.

   b. Determine the sample median for each sample. How do they compare? Why is the median for the urban sample so different from the mean for that sample?

      For urban homes, median = 17.00 EU/mg; for farm homes, median = 8.90 EU/mg.

      The median endotoxin concentration in urban homes is nearly double the median endotoxin concentration in farm homes. The mean and median endotoxin concentration for urban homes are so different because the few large values, especially the extreme value of 80.0, raise the mean but not the median.

   c. Calculate the trimmed mean for each sample by deleting the smallest and largest observation. What are the corresponding trimming percentages? How do the values of these trimmed means compare to the corresponding means and medians?

      For urban homes, deleting the smallest (x = 4.0) and largest (x = 80.0) values gives a trimmed mean of 153/9 = 17 EU/mg. The corresponding trimming percentage is 100(1/11) ≈ 9.1%. The trimmed mean is less than the mean of the entire sample, since the sample was positively skewed. Coincidentally, the median and trimmed mean are equal.

      For farm homes, deleting the smallest (x = 0.3) and largest (x = 21.0) values gives a trimmed mean of xtr = 107.1/13 = 8.24 EU/mg. The corresponding trimming percentage is 100(1/15) ≈ 6.7%. The trimmed mean is below, though not far from, the mean and median of the entire sample.

2. Poly(3-hydroxybutyrate) (PHB), a semicrystalline polymer that is fully biodegradable and biocompatible, is obtained from renewable resources. From a sustainability perspective, PHB offers many attractive properties though it is more expensive to produce than standard plastics. The accompanying data on melting point (°C) for each of 12 specimens of the polymer using a differential scanning calorimeter was obtained:

180.5 181.7 180.9 181.6 182.6 181.6 181.3 182.1 182.1 180.3 181.7 180.5

Compute the following:

a. The sample range

   The maximum and minimum values are 182.6 and 180.3, respectively, so the range is 182.6 – 180.3 = 2.3°C.

b. The sample variance s^2

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	x_i^2
180.5	−0.90833	0.82507	32580.3
181.7	0.29167	0.08507	33014.9
180.9	−0.50833	0.25840	32724.8
181.6	0.19167	0.03674	32978.6
182.6	1.19167	1.42007	33342.8
181.6	0.19167	0.03674	32978.6
181.3	−0.10833	0.01174	32869.7
182.1	0.69167	0.47840	33160.4
182.1	0.69167	0.47840	33160.4
180.3	−1.10833	1.22840	32508.1
181.7	0.29167	0.08507	33014.9
180.5	−0.90833	0.82507	32580.3
sums: 2176.9	0	5.769167	394913.6
$\bar{x} = 181.41$			

$$s^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2 / (n-1) = 5.769167/(12-1) = 0.52447.$$

c. The sample standard deviation

$$s = \sqrt{0.52447} = 0.724.$$

3. Four universities—1, 2, 3, and 4—are participating in a holiday basketball tournament. In the first round, 1 will play 2 and 3 will play 4. Then the two winners will play for the championship, and the two losers will also play. The outcome is listed as the rank order of the teams at the end of the tournament. One possible outcome can be denoted by 1324 (1 beats 2 and 3 beats 4 in first-round games, and then 1 beats 3 and 2 beats 4).

    a. List all outcomes in S.

$S = \{1324, 1342, 1423, 1432, 2314, 2341, 2413, 2431, 3124, 3142, 4123, 4132, 3214, 3241, 4213, 4231\}$.

    b. Let A denote the event that 1 wins the tournament. List outcomes in A.

Event A contains the outcomes where 1 is first in the list:
$A = \{1324, 1342, 1423, 1432\}$.

    c. Let B denote the event that 2 gets into the championship game. List outcomes in B.

Event B contains the outcomes where 2 is first or second:
$B = \{2314, 2341, 2413, 2431, 3214, 3241, 4213, 4231\}$.