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**Practice Final Exam – Microengineering 110**

**Spring 2025**

**Prof. Vivek Subramanian**

- You **MUST** enter your answers **BOTH** on this paper version and on the moodle. The paper version will be used as a backup in case your moodle version is corrupted, etc.
- If there is a submission problem on moodle and your paper version is incomplete, you will not be given any allowance. **IT IS YOUR RESPONSIBILITY TO ENSURE THAT YOU DUPLICATE ALL ANSWERS ON THE MOODLE AND PAPER**
- No communications are allowed with others. Therefore, **YOU MUST EXIT ALL COMMUNICATION PROGRAMS** (Chat windows, etc.). Should a proctor notice that you have an open communication program, it will be considered to be a violation of academic integrity standards.
- **YOU MUST USE DECIMAL POINTS WHEN SPECIFYING NON-INTEGER VALUES. YOU WILL NOT RECEIVE POINTS IF YOU USE COMMAS AS THE SEPARATOR FOR SUCH VALUES.**
- Use of ChatGPT and similar AI-based programs is not allowed
- The moodle will allow you to enter information for exactly 2 hours, at which point it will lock you out.
- You may find the following website useful, since it has many online calculators that are relevant to this course:

<https://www.socscistatistics.com/tests/>

1. A company is planning to purchase a robot to perform a particular manufacturing task. Three different vendors submit robots for consideration. Vendor A will sell the robot for 1 Million CHF. Vendor B will sell the robot for 1.1 Million CHF. Vendor C will sell the robot for 1.2 Million CHF.

The company generally plans to buy the robot that completes the task most quickly. On the other hand, if there is no statistical difference in the speed of the robots, the company will buy the cheapest robot.

The company plans to run repeated trials with each robot, and determine the time it takes for each robot to complete the task. This information will then be used to decide which robot to purchase.

What type of test should the company use for this evaluation, and why?

- Regression, since the time taken is scale data and we are therefore comparing scale vs scale data
- ANOVA, since the companies are nominal data, and the time taken is scale data
- T-test, since the companies are nominal data, and the time taken is scale data

2. The table below shows the results of the test performed, with each entry showing the time taken (in seconds) to perform the task during a particular trial.

Vendor A	Vendor B	Vendor C
8	8	10
10	9	9
9	9	10
11	8	11
10	10	9

We will use this data to perform an ANOVA analysis. Based on the analysis, which vendor should the company select?

- Vendor A
- Vendor B
- Vendor C

3. In the above analysis, how many degrees of freedom are available for the R sub-table?

4. Suppose Vendor D also submits a robot consideration. The robot costs 1.2 Million CHF, and achieves time trial results of 7, 6, 8, 8, and 8 seconds. Under this new scenario, which robot should the company purchase?

- Vendor A
- Vendor B
- Vendor C
- Vendor D

5. An economist in England performed analysis of household income vs household expenditures. She obtained the following table:

Income (in 1000 £)	Expenditure (in 1000 £)
1	7
5	6
9	9
13	8
17	10

Perform linear regression of this data to obtain a best-fit line. What is the slope of the line?

6. What is the Pearson correlation coefficient for this line?

7. For a family to survive, the household income must at least equal to the household expenditure. Using the best fit line, determine the minimum household income for a family to survive, in thousands of £.

8. The economist uses the data to predict the expected household consumption for a family earning 6000 £ and for a family earning 25000 £. Would you trust these estimates?

- No, I would not trust either estimate since the R value is poor
- Yes, I would trust both estimates since the R value is good
- I would only trust the value at 6000 £ since we shouldn't extrapolate too far beyond the bounds of the data
- I would only trust the value at 25000 £ since the 6000 £ data is close to an influential outlier.

9. A statistician performs an ANOVA analysis. He provides us with the following table. Unfortunately, some data in the table is missing. The missing data is indicated by a letter a-g in the table below.

	Sum of Squares	Degrees of Freedom	Mean Square	F
Between Treatments	2.124	a	0.708	0.75
Within Treatments	b	20	c	
Total	d	e		

What is the value of a?

10. What is the value of b?

11. What is the value of c?

12. What is the value of d?

13. What is the value of e?

14. In the silicon chips used in automotive electronics, the wiring is commonly done using Aluminium with 0.85% silicon. The silicon content of each of 25 randomly selected wiring samples was determined, and the following results were obtained:

Silicon content:

Mean: 0.888%

Standard deviation: 0.1807%

T score: 1.05

P value: 0.3

What type of test was used to obtain these results?

- 1 sample t test
- 2 sample t test
- Paired t test
- z test

15. In the above test, what is the null hypothesis?

- $\mu = 0.85$
- $\mu < 0.85$
- $\mu = 0.888, \sigma = 0.1807$

16. Is this analysis one-tailed or two tailed?

- One-tailed
- Two-tailed

17. Does the sample set fall within the expected specifications for the wiring material?

- Yes
- No

18. As you may know, the traditional method of pouring champagne is to hold the glass vertically, under the belief that this preserves tiny gas bubbles that improve flavor and aroma. This is different from beer, where the glass is usually tilted to reduce the frothing.



To determine the best way to pour, measurements were performed while pouring champagne at different temperatures, and the following data was obtained:

Temperature (°C)	Pour type	n	CO2 Loss (g/L)	SD
18	Vertical	4	4	0.5
18	Tilted	4	3.7	0.3
12	Vertical	4	3.3	0.2
12	Tilted	4	2	0.3

At 18°C, is there a difference between a vertical pour or a tilted pour?

- Yes
- No

19. Repeat this analysis at 12°C. What is the p-value?

20. So, to retain bubbles with a chilled champagne, should you pour vertically or tilted?

- Vertically
- Tilted

21. During breastfeeding, women often temporarily lose bone mass, since the body uses calcium to produce milk. To test this hypothesis, data was collected from 10 women while they were breastfeeding, and again shortly after their children had grown up enough to stop breastfeeding. The data is provided below.

Female	Bone Mineral Content (g)	
	Lactating	Post-lactation
1	1928	2126
2	2549	2885
3	2825	2895
4	1924	1942
5	1628	1750
6	2175	2184
7	2114	2164
8	2621	2626
9	1843	2006
10	2541	2627

Perform both paired and 2-sample t-tests on this data to determine if there is in-fact mineral loss during lactation.

What is the t-value for the paired t-test?

22. What is the t-value for the 2-sample t-test?

23. Does lactation cause statistically significant bone mineral loss?

- Yes
- No

24. A sample of students was categorized according to gender and eye color, as listed in the table below:

Gender	Eye Color				TOTAL
	Blue	Brown	Green	Hazel	
Male	370	352	198	187	
Female	359	290	110	160	
TOTAL					

Suppose that one of these students is randomly selected. Let  $F$  denote the event that the selected individual is a female, and  $BL$ ,  $BR$ ,  $G$ , and  $H$  represent the events that he or she has blue, brown, green, and hazel eyes, respectively.

Calculate  $P(F)$

25. Calculate  $P(F \cap G)$

26. Are events  $F$  and  $G$  independent?

- Yes
- No

27. If the selected individual has green eyes, what is the probability that he or she is a female?

28. If the selected individual is female, what is the probability that she has green eyes?

29. A restaurant serves three fixed-price dinners costing CHF12, CHF15, and CHF20. For a randomly selected brother and sister dining at this restaurant, let  $X$  = the cost of the brother's dinner and  $Y$  = the cost of the sister's dinner. The joint pmf of  $X$  and  $Y$  is given in the following table:

p(x,y)		y		
		12	15	20
x	12	0.05	0.05	0.1
	15	0.05	0.1	0.35
	20	0	0.2	0.1

What is the probability that the brother's and sister's dinner cost at most CHF15 each?

30. Are X and Y independent?

- Yes
- No

31. What is the expected total cost of the dinner for the two people?

32. Suppose that when a pair of diners opens fortune cookies at the conclusion of the meal, they find the message “You will receive as a refund the difference between the cost of the more expensive and the less expensive meal that you have chosen.” How much would the restaurant expect to refund?

33. Let  $\tau > 0$  and let  $X_1, X_2, \dots, X_n$  be random samples from a distribution with a pdf given by:

$$f(x; \tau) = \frac{\tau^5}{8} x^{14} e^{-\tau x^3}, \quad x > 0.$$

Which of the following is the maximum likelihood estimator of  $\tau$ ?

- $\hat{\tau} = \frac{n}{\sum_{i=1}^n X_i^3}$
- $\hat{\tau} = \frac{5n}{\sum_{i=1}^n X_i^3}$
- $\hat{\tau} = \frac{n}{5 \cdot \sum_{i=1}^n X_i^3}$
- $\hat{\tau} = \frac{\frac{5}{8}n}{\sum_{i=1}^n X_i^3}$

34. Let  $X_1, X_2, \dots, X_n$  be a random sample from a population with pdf:

$$f(x|\theta) = \frac{1}{\theta} x^{(1-\theta)/\theta}$$

Where  $0 < x < 1, 0 < \theta < \infty$

Obtain a method of moments estimator for  $\theta$ , and use this to calculate an estimate when  $x_1 = 0.1, x_2 = 0.22, x_3 = 0.54, x_4 = 0.36$ . What is the value of the estimator?