Electrotechique-II
MICRO-101: week 9

I.-C. Benea-Chelmus

- What you have seen last time:
  - Time-response of L (ch. 6.2.3.), RL circuits (ch. 6.3.2)

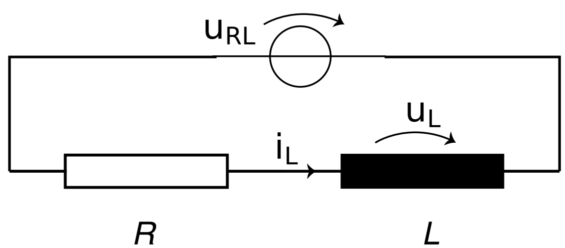
- Today:
  - Wrap up RL circuits (ch. 6.3.2)
  - Transient behavior of AC-powered RL circuits (ch. 6.4)
  - Solving of past exam (during exercise class)

- Next weeks
  - Lab work

1-slide recap of last week

RL circuit (Ch. 6.3.2)

- Rule 1: In real systems (with non-zero resistance) – the current flowing through an inductor can not change suddenly (no current jump)



- Boundary conditions:

$$t < 0$$
$$i_L = 0$$
$$u_L = 0$$

$$t \geq 0$$
$$i_L = \frac{u_0}{R}(1 - e^{-\frac{tR}{L}})$$
$$u_L = L\frac{di_L}{dt} = u_0 e^{-\frac{tR}{L}}$$

- Rule 2: at $t = \infty$, the inductor behaves like a short circuit

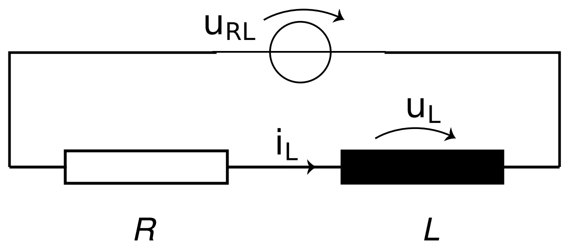
Today

I.-C. Benea-Chelmus

RL circuit (Ch. 6.3.2)

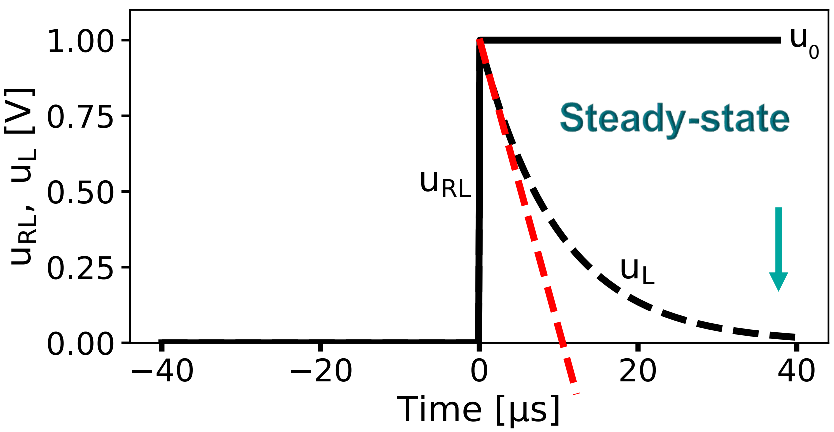
- Rule 1: In real systems (with non-zero resistance) – the current flowing through an inductor can not change suddenly (no current jump)

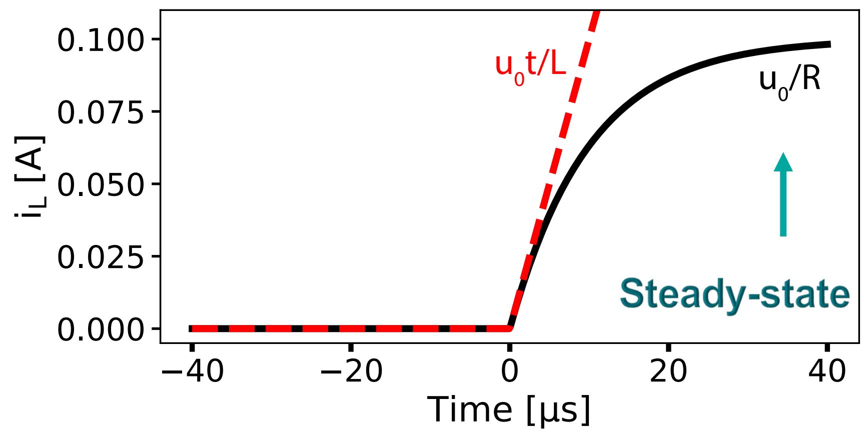
$$t \geq 0$$



$$i_L = \frac{u_0}{R}(1 - e^{-\frac{tR}{L}})$$

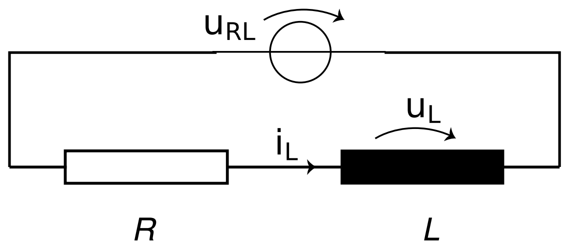
$$u_L = L\frac{di_L}{dt} = u_0 e^{-\frac{tR}{L}}$$





RL circuit (Ch. 6.3.2)

- Rule 2: In real systems (with non-zero resistance) – the current flowing through an inductor can not change suddenly (no current jump)



$$t \geq 0$$

$$i_L = \frac{u_0}{R}(1 - e^{-\frac{tR}{L}})$$

$$u_L = L\frac{di_L}{dt} = u_0 e^{-\frac{tR}{L}}$$

We find that the current ramps up with a characteristic time constant $\tau_{RL} = \frac{L}{R}$. The RL time constant can be reduced for example by decreasing L.

At t = 0, the voltage spikes to a value of $u_L\,(t = 0) = u_0$ after which it decreases again with the same slope as the current. We note that for L → 0 we find back the case of the resistor connected to a voltage source which is switched on.

Summary: RC versus RL circuit

I.-C. Benea-Chelmus

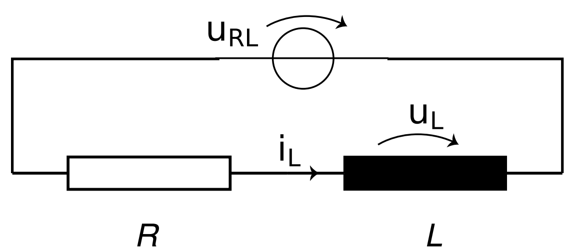
- RC
  - No voltage jump
  - at $t = \infty$, the capacitor behaves like an open circuit
- You can use these rules to compute steady-state values

- RL
  - No current jump
  - at $t = \infty$, the inductor behaves like a short circuit
- You can use these rules to compute steady-state values

AC-powered RL circuit (ch. 6.4.)

- Applied voltage source is alternating at frequency ω with peak value $\hat{U}$ and phase offset α:

$$u_{RL} = \hat{U}\cos(\omega t + \alpha)h(t)$$



(solving at the blackboard)

- Boundary conditions

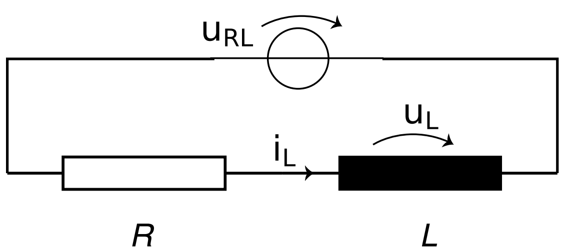
$$t < 0$$
$$i_L = 0$$
$$u_L = 0$$

AC-powered RL circuit (ch. 6.4.)

- Applied voltage source is alternating at frequency ω with peak value $\hat{U}$ and phase offset α:

$$u_{RL} = \hat{U}\cos(\omega t + \alpha)h(t) \longrightarrow t \geq 0:$$

$$\hat{U}\cos(\omega t + \alpha) = i_L R + L\frac{di_L}{dt}$$



Find solution to the homogeneous diff. eq. i_h and a particular solution i_p

- Boundary conditions

$$t < 0$$

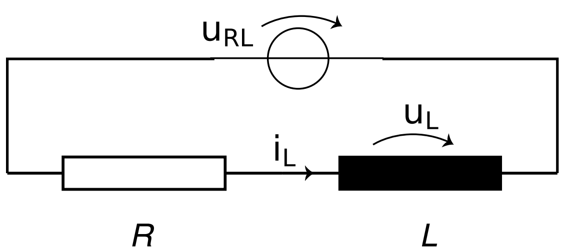
$$i_L = 0$$

$$u_L = 0$$

AC-powered RL circuit (ch. 6.4.)

- Applied voltage source is alternating at frequency ω with peak value $\hat{U}$ and phase offset α:

$$u_{RL} = \hat{U}\cos(\omega t + \alpha)h(t) \longrightarrow t \geq 0:$$



$$\hat{U}\cos(\omega t + \alpha) = i_L R + L\frac{di_L}{dt}$$

Find solution to the homogeneous diff. eq. i_h and a particular solution i_p

$$i_h = i_0 e^{-\frac{tR}{L}}$$

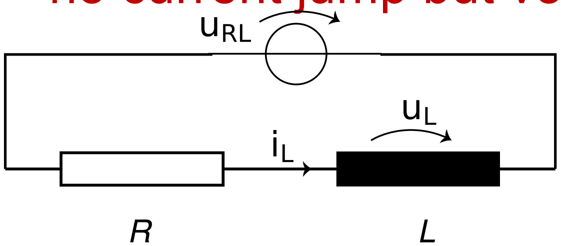
$$i_p = \frac{\hat{U}}{Z}\cos\left(\omega t + \alpha - \phi_Z\right) = \frac{\hat{U}}{\sqrt{R^2 + \omega^2 L^2}}\cos(\omega t + \alpha - \arctan\frac{\omega L}{R})$$

$$i_L = i_p + i_h = \frac{\hat{U}}{Z}\cos\left(\omega t + \alpha - \phi_Z\right) + i_0 e^{-\frac{tR}{L}}$$

$$i_0 = -\frac{\hat{U}}{Z}\cos\left(\alpha - \phi_Z\right)$$

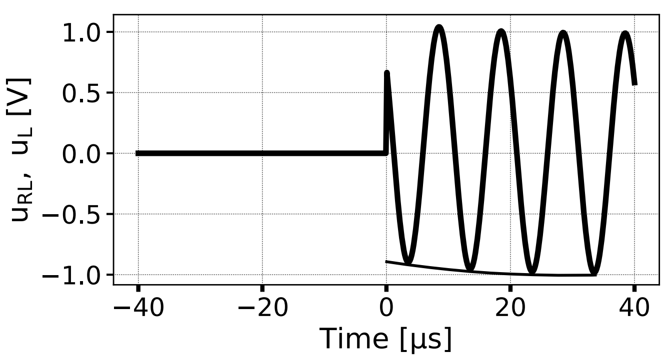
AC-powered RL circuit (ch. 6.4.)

- no current jump but voltage jump!

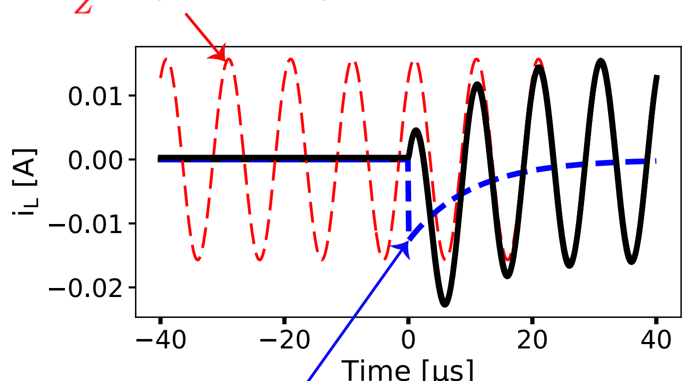


$$i_L = \frac{\hat{U}}{Z}\left(\cos\left(\omega t + \alpha - \phi_Z\right) - \cos\left(\alpha - \phi_Z\right)e^{-\frac{tR}{L}}\right)$$

$$u_L = u_{RL} - i_L R = \hat{U}\cos(\omega t + \alpha) - \frac{\hat{U}R}{Z}\left(\cos\left(\omega t + \alpha - \phi_Z\right) - \cos\left(\alpha - \phi_Z\right)e^{-\frac{tR}{L}}\right)$$



$\frac{\hat{U}}{Z}\cos\left(\omega t + \alpha - \phi_Z\right)$



$-\frac{\hat{U}}{Z}\cos\left(\alpha - \phi_Z\right)e^{-\frac{tR}{L}}$

Written exam

- Counts 80% of final grade (TP counts 20%)
- You can bring along any **printed** material (script, books, notes)
- **No electronics** allowed (no cell phone, no smart watch, no tablets etc). Only **exception**: calculator without internet functionality.
- Exam duration: 3 hours
- Seat map sent out 1 week before exam
- Students who took the TP in spring 2024 have the choice between:
  - re-register for the TP and retake TP exam (counts 20% of the grade)
  - or keep last year's TP grade.