Some Exercises for Chapter 3 of Advanced Control Systems

Problem 3.1: Consider a PI controller in a unity feedback system as:

$$K(s) = K_P + \frac{K_I}{s}$$

1. Find a discrete representation of the PI controller $K(q^{-1})$. You can use the transformation $s = (1 - q^{-1})/h$ where h is the sampling period.
2. Find an equivalent RST controller for the PI controller.

Problem 3.2: Consider a control system with a feedback and feedforward controller as shown below:

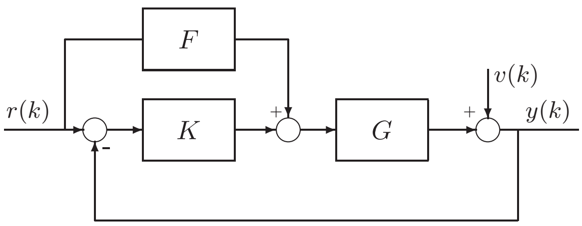

Fig. 1: Two-degree of freedom controller

where $K(q^{-1}) = \frac{N_k(q^{-1})}{D_k(q^{-1})}$ and $F(q^{-1}) = \frac{N_f(q^{-1})}{D_f(q^{-1})}$. Find an equivalent RST controller.

Problem 3.3: Consider the following plant model:

$$G(z) = \frac{0.5(z - 0.8)}{(z - 0.7)(z - 0.5)}$$

Take $P_d(q^{-1}) = 1 - 0.7q^{-1}$ and the same dynamics for tracking and regulation.

1. Compute an RST controller.
2. Redesign the RST controller such that it includes an integrator.
3. Redesign the RST controller such that it includes an integrator using Q-parametrization (see Example 3.8).

Problem 3.4: Consider the following plant model:

$$G(q^{-1}) = \frac{0.5q^{-1} - 0.4q^{-2}}{1 - 1.2q^{-1} + 0.35q^{-2}}$$

For the regulation dynamic choose the dominant closed loop pole to have a damping of $\zeta = 0.8$ and a natural frequency of $\omega_n = 1$ rad/s. Note that $h = 1$ s.

1. Design an RST controller for Model Reference Control problem.
2. Add a fixed term $H_R(q^{-1}) = 1 + q^{-1}$ in the controller.
3. Repeat the second item using Q-parametrization.

Problem 3.5: Consider the following plant model:

$$G(q^{-1}) = \frac{B(q^{-1})}{A(q^{-1})} = \frac{0.5q^{-1} - 0.4q^{-2}}{1 - 1.2q^{-1} + 0.35q^{-2}}$$

Take $P_d(q^{-1}) = 1 - 1.2q^{-1} + 0.35q^{-2}$ (choosing closed-loop poles equal to the poles of the plant model is called internal model control) and the same dynamics for tracking and regulation. Design an RST controller that includes an integrator. Note that R should include the polynomial $A(q^{-1})$ in order that the Bezout equation has a solution.

Problem 3.6: Consider the following plant model

$$G(z) = \frac{2}{z^2 - 1.6z + 0.7}$$

with $h = 0.2s$.

(a) Design an RST controller such that the regulation dynamic has a natural frequency of $\omega_n = 2$ and a damping factor of $\zeta = 0.7$.

(b) Add the fixed terms $H_R(q^{-1}) = 1 + q^{-1}$ and $H_S(q^{-1}) = 1 - q^{-1}$ in the controller using Q parameterization.

In both items $T(q^{-1})$ should be computed to have the same dynamics for tracking and regulation.

Problem 3.7: Show that for minimizing the criterion with constant forgetting factor:

$$J(k) = \sum_{i=1}^{k} \lambda_1^{(k-i)} [y(i) - \hat{\theta}^T(k)\phi(i-1)]^2$$

the following PAA can be used.

$$F^{-1}(k+1) = \lambda_1 F^{-1}(k) + \phi(k)\phi^T(k)$$
$$\hat{\theta}(k+1) = \hat{\theta}(k) + F(k+1)\phi(k)\epsilon^\circ(k+1)$$

Problem 3.8: In a PAA the parameters are updated using:

$$\hat{\theta}(k+1) = \hat{\theta}(k) + F(k+1)\phi(k)\epsilon^\circ(k+1)$$

Show that it can be reformulated as:

$$\hat{\theta}(k+1) = \hat{\theta}(k) + F(k)\phi(k)\epsilon(k+1)$$

where

$$\epsilon(k+1) = y(k+1) - \hat{\theta}^T(k+1)\phi(k) = \frac{\epsilon^\circ(k+1)}{1 + \phi^T(k)F(k)\phi(k)}$$

$$F(k+1) = F(k) - \frac{F(k)\phi(k)\phi^T(k)F(k)}{1 + \phi^T(k)F(k)\phi(k)}$$

Problem 3.9: Consider the unknown discrete-time model of a plant as:

$$y(k+1) = -ay(k) + bu(k)$$

Compute the parameters of an RST indirect adaptive controller based on pole placement (same dynamics for tracking and regulation) after one sampling period (using $\hat{\theta}(1)$ estimated by recursive LS algorithm with decreasing gain).

The following data are measured : $u(0) = 2, y(0) = 1, y(1) = 5.5$. Assume $\hat{\theta}(0) = [0\,,\,0]^T$, $F(0) = 2I$, $P_d(q^{-1}) = 1 - 0.8q^{-1}$.