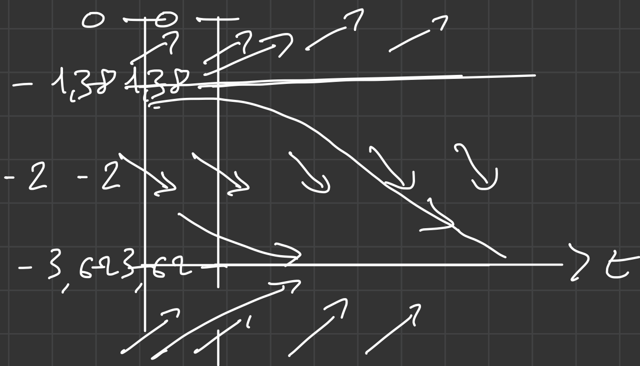
Commande Non Linéaire

Série 1

indications.

Série 1 I.1

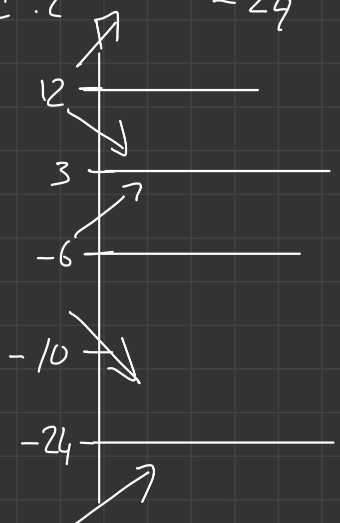
I.1 $\bar{x} = \frac{1}{2}\left(-5 \pm \sqrt{5}\right)$ $\rightarrow -1,382$; $\rightarrow -3,618$



$x^2 + 5x + 5$

$4 - 10 + 5 = -1 < 0$

I.2 $-24 \quad -6 \quad 3, \quad 12$



$(x-3)(x+6)(x-12)(x+24)$

$- \quad - \quad - \quad +$

$-$

$$\begin{cases} \dot{x}_1 = 2x_1 + 3x_2 + 6x_1^2 + 6x_2^3 & = f_1 \\ \dot{x}_2 = x_1 - x_2 + 7x_2^2 & = f_2 \end{cases}$$

$\bar{x}_1 = ?$

$\bar{x}_2 = ?$ $\longrightarrow$ 2 équilibres (réels)

$\bar{x}_1 = 0$ $\qquad$ $\bar{x}_1 = -0{,}55$

$\bar{x}_2 = 0$ $\qquad$ $\bar{x}_2 = -0{,}218$

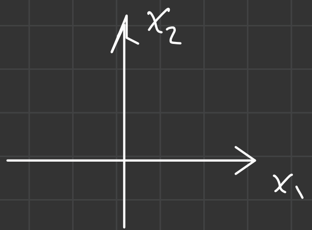
$$\left.\begin{aligned} 0 &= 2x_1 + 3x_2 + 6x_1^2 + 6x_2^3 \\ 0 &= x_1 - x_2 + 7x_2^2 \end{aligned}\right)$$

$x_1 = x_2 - 7x_2^2$

$\Longrightarrow$ d° 4 $\rightarrow$ 4 équilibres, seuls 2 sont réels.

en x_2

$$A = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} \\[2ex] \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} \end{bmatrix} \Bigg|_{\begin{cases} x_1 = \bar{x}_1 \\ x_2 = \bar{x}_2 \end{cases}}$$



A_1

$\bar{x}_1 = -0{,}55$

$\bar{x}_2 = -0{,}218$

A_2

$$A = \begin{bmatrix} 12x_1 + 2 & 18x_2^2 + 3 \\ 1 & 14x_2 - 1 \end{bmatrix}$$

$\bar{x}_1 = 0$

$\bar{x}_2 = 0$

$$A_1 = \begin{bmatrix} -4{,}6 & 3{,}86 \\ 1 & -4{,}06 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$$

$$|\lambda I - A_1| = 0$$

$$|\lambda I - A_2| = 0$$

?

?

1
2
3
4
5
6

+ vecteurs propres

$$A_1 V_1 = \lambda_{1,1} V_1$$
$$A_1 V_2 = \lambda_{1,2} V_2$$

$$A_2 V_1 = \lambda_{2,1} V_1$$
$$A_2 V_2 = \lambda_{2,2} V_2$$

noeud stable

$$\lambda_{11} = -6{,}31$$
$$\lambda_{12} = -2{,}34$$

point selle

$$\lambda_{21} = +2{,}79$$
$$\lambda_{22} = -1{,}79$$

Toutes ses quantités sont associées à une interprétation "graphique" dans le plan de phase

Vecteurs propres

$$V_{1,1} = \begin{pmatrix} 1 \\ -0{,}44 \end{pmatrix} \qquad V_{1,2} = \begin{pmatrix} 1 \\ 0{,}585 \end{pmatrix}$$

$$\boxed{\begin{aligned} \bar{x}_1 &= -0{,}55 \\ \bar{x}_2 &= -0{,}218 \end{aligned}}$$

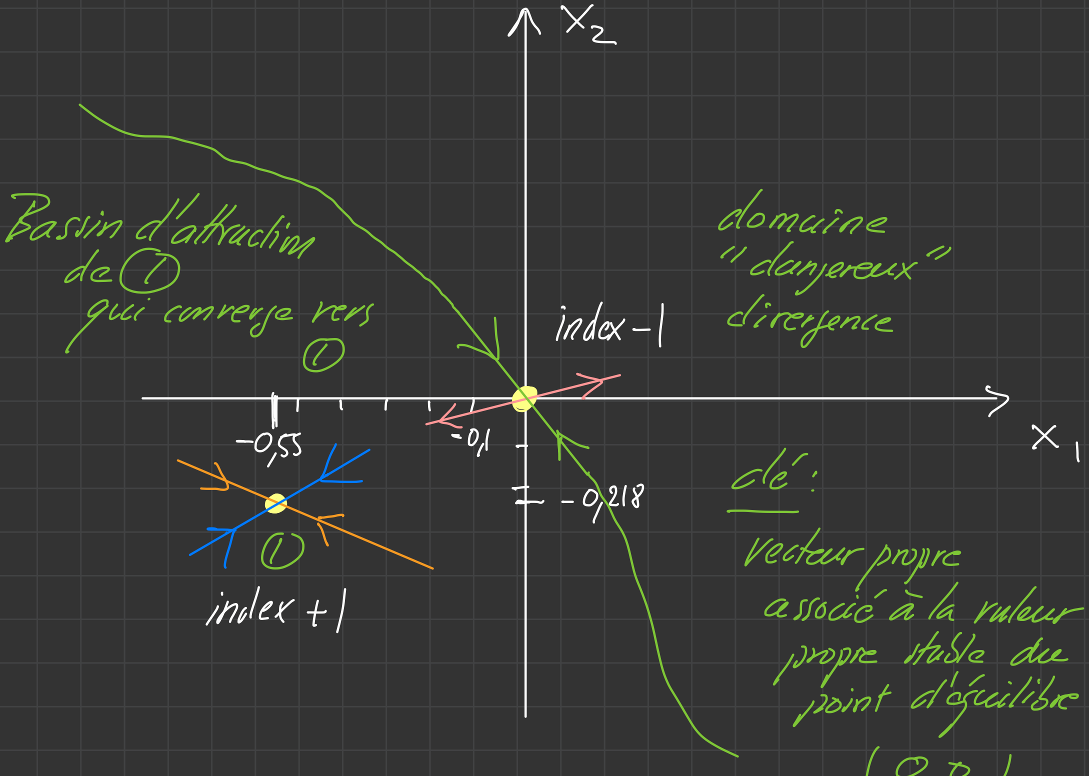
$$\begin{vmatrix} \lambda-2 & -3 \\ -1 & \lambda+1 \end{vmatrix}$$

$$\lambda^2 - \lambda - 5 = 0$$

$$\frac{1 \pm \sqrt{1+20}}{2} = \begin{cases} +2{,}79 \\ -1{,}79 \end{cases}$$

$$V_{2,1} = \begin{pmatrix} 1 \\ 0{,}26 \end{pmatrix} \qquad V_{2,2} = \begin{pmatrix} 1 \\ -1{,}26 \end{pmatrix}$$

$$\bar{x}_1 = 0, \quad \bar{x}_2 = 0$$



<u>Expérience :</u> numérique.

on inverse le temps

$$t = -\bar{t}$$
$$dt = -d\bar{t}$$

$$\dot{x} = f(x)$$

$$\frac{dx}{dt} = f(x) \qquad -\frac{dx}{d\bar{t}} = f(x)$$

$$\frac{dx}{dt} = -f(x)$$

"pour remonter le temps"

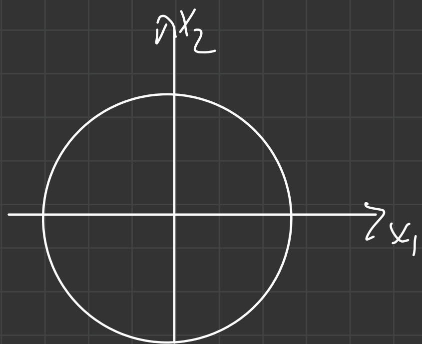
signe det A_i = index

$$\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = -5$$

signe $= -1$

index -1

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 \end{cases}$$



$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$PAP^{-1} = \bar{A}$$

$$\longrightarrow \quad \bar{A} \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix}$$

$$\lambda^2 + 1 = 0$$

$$\lambda = \pm i \longrightarrow \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix}$$

$x_1 \in \mathbb{C}$

$x_2 \in \mathbb{C}$

$$\begin{cases} \dot{x}_1 = +i x_1 \\ \dot{x}_2 = -i x_2 \end{cases}$$

$$\chi_1(\underline{x}_0, t) = x_{01} e^{it}$$

$$\frac{d}{dt}\chi_1 = +i x_0 e^{it} = +i \chi_1$$

$$\chi_2 = x_{02} e^{-it}$$

$x_{01} = a + ib$

$x_{02} = c + id$

$$\frac{\chi_1 + \chi_2}{2}$$

$$\frac{\chi_1 - \chi_2}{2i}$$

$$\begin{cases} (a+ib) e^{+it} = \chi_1 \\ (c+id) e^{-it} = \chi_2 \end{cases}$$

$\dot{x} = Ax$

$\mathbb{R}$

Schur

$A =$