10 CHAPTER

Quality Control

CHAPTER OUTLINE

LEARNING OBJECTIVES

After completing this chapter, you should be able to:

LO10.1 Explain the need for quality control.

LO10.2 Discuss the basic issues of inspection.

LO10.3 List and briefly explain the elements of the control process.

LO10.4 Explain how control charts are used to monitor a process and the concepts that underlie their use.

LO10.5 Use and interpret control charts.

LO10.6 Perform run tests to check for nonrandomness in process output.

LO10.7 Assess process capability.

This chapter covers quality control. The purpose of quality control is to assure that processes are performing in an acceptable manner. Companies accomplish this by monitoring process output using statistical techniques. **Quality control** is a process that measures output relative to a standard and takes corrective action when output does not meet standards. If the results are acceptable, no further action is required; unacceptable results call for corrective action.

Quality control A process that evaluates output relative to a standard and takes corrective action when output doesn't meet standards.

Every process generates output that exhibits random variability. That is natural and cannot be corrected. However, if there are nonrandom variations in process output, that can be corrected. Quality control tools are used to decide when corrective action is needed.

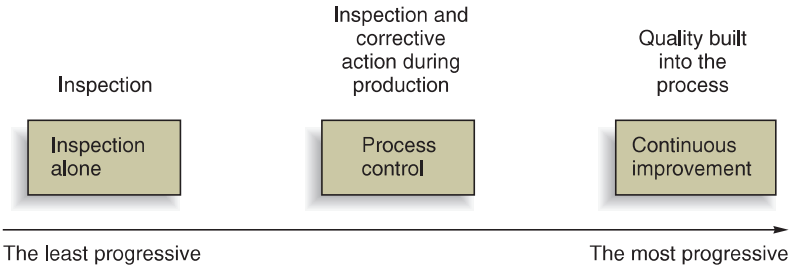
10.1 INTRODUCTION

LO10.1 Explain the need for quality control.

Quality assurance that relies primarily on inspection of lots (batches) of previously produced items is referred to as *acceptance sampling*. It is described in the chapter supplement. Quality control efforts that occur during production are referred to as *statistical process control,* and these we examine in the following sections.

The best companies emphasize *designing quality into the process,* thereby greatly reducing the need for inspection or control efforts. As you might expect, different business organizations are in different stages of this evolutionary process: Some rely heavily on inspection. However, inspection alone is generally not sufficient to achieve a reasonable level of quality. Many occupy a middle ground that involves some inspection and a great deal of process control. Figure 10.1 illustrates these phases of quality assurance.

FIGURE 10.1
Approaches to quality assurance



10.2 INSPECTION

Inspection Appraisal of goods or services.

Inspection is an appraisal activity that compares goods or services to a standard. Inspection is a vital but often unappreciated aspect of quality control. Although for well-designed processes little inspection is necessary, inspection cannot be completely eliminated. And with increased outsourcing of products and services, inspection has taken on a new level of significance. In lean organizations, inspection is less of an issue than it is for other organizations because lean organizations place extra emphasis on quality in the design of both products and processes. Moreover, in lean operations, workers have responsibility for quality (quality at the source). However, many organizations do not operate in a lean mode, so inspection is important for them. This is particularly true of service operations, where quality continues to be a challenge for management.

Inspection can occur at three points: before production, during production, and after production. The logic of checking conformance before production is to make sure that inputs are acceptable. The logic of checking conformance during production is to make sure that the conversion of inputs into outputs is proceeding in an acceptable manner. The logic of checking conformance of output is to make a final verification of conformance before passing goods on to customers.

Inspection before and after production often involves *acceptance sampling* procedures; monitoring during the production process is referred to as *process control.* Figure 10.2 gives an overview of where these two procedures are applied in the production proces.

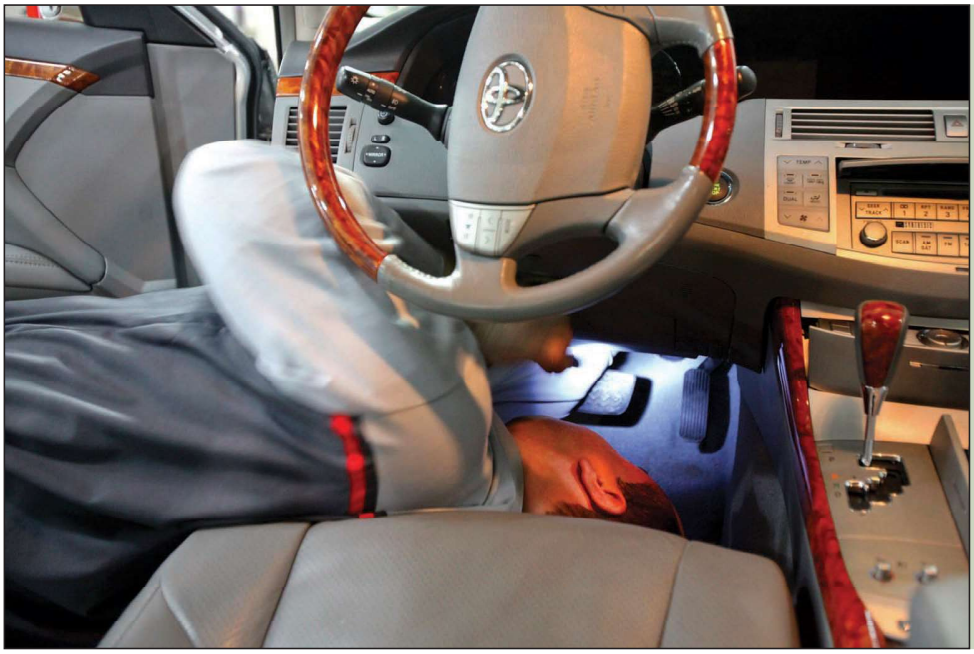
To determine whether a process is functioning as intended or to verify that a batch or lot of raw materials or final products does not contain more than a specified percentage of defective goods, it is necessary to physically examine at least some of the items in question. The purpose of inspection is to provide information on the degree to which items conform to a standard. The basic issues are

LO10.2 Discuss the basic issues of inspection.

1. How much to inspect and how often.
2. At what points in the process inspection should occur.
3. Whether to inspect in a centralized or on-site location.
4. Whether to inspect attributes (i.e., *count* the number of times something occurs) or variables (i.e., *measure* the value of a characteristic).

Consider, for example, inspection at an intermediate step in the manufacture of personal computers. Because inspection costs are often significant, questions naturally arise on whether one needs to inspect every computer or whether a small sample of computers will suffice. Moreover, although inspections could be made at numerous points in the production process, it is not generally cost-effective to make inspections at every point. Hence, the question comes up of which points should be designated for inspections. Once these points have been identified, a manager must decide whether to remove the computers from the line and take them to

FIGURE 10.2
Acceptance sampling and process control



A Toyota technician prepares to remove the accelerator assembly in a recalled Toyota Avalon.

a lab, where specialized equipment might be available to perform certain tests, or to test them where they are being made. We will examine these points in the following sections.

How Much to Inspect and How Often

The amount of inspection can range from no inspection whatsoever to inspection of each item numerous times. Low-cost, high-volume items such as paper clips, roofing nails, and wooden pencils often require little inspection because (1) the cost associated with passing defective items is quite low and (2) the processes that produce these items are usually highly reliable, so defects are rare. Conversely, high-cost, low-volume items that have large costs associated with passing defective products often require more intensive inspections. Thus, critical components of a manned-flight space vehicle are closely scrutinized because of the risk to human safety and the high cost of mission failure. In high-volume systems, *automated* inspection is one option that may be employed.

The majority of quality control applications lie somewhere between the two extremes. Most require some inspection, but it is neither possible nor economically feasible to critically examine every part of a product or every aspect of a service for control purposes. The cost of inspection, resulting interruptions of a process or delays caused by inspection, and the manner of testing typically outweigh the benefits of 100 percent inspection. Note that for manual inspection, even 100 percent inspection does not guarantee that all defects will be found and removed. Inspection is a process, and hence, subject to variation. Boredom and fatigue are factors that cause inspection mistakes. Moreover, when destructive testing is involved (items are destroyed by testing), that must be taken into account. However, the cost of letting undetected defects slip through is sufficiently high that inspection cannot be completely ignored. The amount of inspection needed is governed by the costs of inspection and the expected costs of passing defective items. As illustrated in Figure 10.3, if inspection activities increase, inspection costs increase, but the costs of undetected defects decrease. The traditional goal was to minimize the sum of these two costs. In other words, it may not pay to attempt to catch every defect, particularly if the cost of inspection exceeds the penalties associated with letting some defects get through. Current thinking is that every reduction in defective output reduces costs, although not primarily by inspection.

As a rule, operations with a high proportion of human involvement necessitate more inspection effort than mechanical operations, which tend to be more reliable.

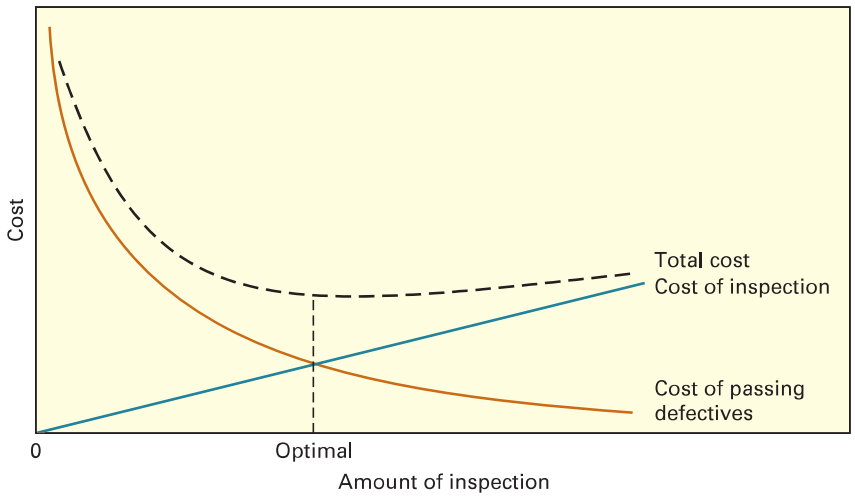


FIGURE 10.3
Traditional view: The amount of inspection is optimal when the sum of the costs of inspection and passing defectives is minimized

The frequency of inspection depends largely on the rate at which a process may go out of control or on the number of lots being inspected. A stable process will require only infrequent checks, whereas an unstable one or one that has recently given trouble will require more frequent checks. Likewise, many small lots will require more samples than a few large lots because it is important to obtain sample data from each lot. For high-volume, repetitive operations, computerized automatic inspections at critical points in a process are cost effective.

Where to Inspect in the Process

Many operations have numerous possible inspection points. Because each inspection adds to the cost of the product or service, it is important to restrict inspection efforts to the points where they can do the most good. In manufacturing, some of the typical inspection points are

1. **Raw materials and purchased parts.** There is little sense in paying for goods that do not meet quality standards and in expending time and effort on material that is bad to begin with. Supplier certification programs can reduce or eliminate the need for inspection.
2. **Finished products.** Customer satisfaction and the firm's image are at stake here, and repairing or replacing products in the field is usually much more costly than doing it at the factory. Likewise, the seller is usually responsible for shipping costs on returns, and payments for goods or service may be held up pending delivery of satisfactory goods or remedial service. Well-designed processes, products and services, quality at the source, and process monitoring can reduce or eliminate the need for inspection.
3. **Before a costly operation.** The point is to not waste costly labor or machine time on items that are already defective.
4. **Before an irreversible process.** In many cases, items can be reworked up to a certain point; beyond that point they cannot. For example, pottery can be reworked prior to firing. After that, defective pottery must be discarded or sold as seconds at a lower price.
5. **Before a covering process.** Painting, plating, and assemblies often mask defects.

Inspection can be used as part of an effort to improve process yield. One measure of process yield is the ratio of output of good product to the total output. Inspection at key points can help guide process improvement efforts to reduce the scrap rate and improve the overall process yield, and reduce or eliminate the need for inspection.

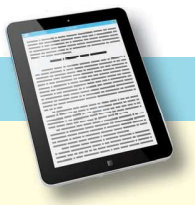
In the service sector, inspection points are incoming purchased materials and supplies, personnel, service interfaces (e.g., service counter), and outgoing completed work (e.g., repaired appliances). Table 10.1 illustrates a number of examples.

TABLE 10.1
Examples of inspection points in service organizations

Type of Business	Inspection Points	Characteristics
Fast food	Cashier	Accuracy
	Counter area	Appearance, productivity
	Eating area	Cleanliness, no loitering
	Building and grounds	Appearance, safety hazards
	Kitchen	Cleanliness, purity of food, food storage, health regulations
	Parking lot	Safety, good lighting
Hotel/motel	Accounting/billing	Accuracy, timeliness
	Building and grounds	Appearance and safety
	Main desk	Appearance, waiting times, accuracy of bills
	Maid service	Completeness, productivity
	Personnel	Appearance, manners, productivity
	Reservations/occupancy	Over/underbooking, percent occupancy
	Restaurants	Kitchen, menus, meals, bills
	Room service	Waiting time, quality of food
	Supplies	Ordering, receiving, inventories
Supermarket	Cashiers	Accuracy, courtesy, productivity
	Deliveries	Quality, quantity
	Produce	Freshness, ample stock
	Aisles and stockrooms	Uncluttered layout
	Inventory control	Stock-outs
	Shelf stock	Ample supply, rotation of perishables
	Shelf displays	Appearance
	Checkouts	Waiting time
	Shopping carts	Good working condition, ample supply, theft/vandalism
	Parking lot	Safety, good lighting
	Personnel	Appearance, productivity
Doctor's office	Waiting room	Appearance, comfortable
	Examination room	Clean, temperature controlled
	Doctor	Neat, friendly, concerned, skillful, knowledgeable
	Doctor's assistant	Neat, friendly, concerned, skillful
	Patient records	Accurate, up-to-date
	Billing	Accurate
	Other	Waiting time minimal, adequate time with doctor

In the Chips at Jays

READING



Neil Steinberg

A potato chip is a delicate thing. Fragile. A pound of pressure will crush it. So when you're moving 250 tons of chips through your plant, as they do every day at Jays Foods, you need to have a system.

"You don't buy potato crumbs, you buy potato chips," said Tom Howe, CEO and co-owner of the Chicago company, at 99th and Cottage Grove. Jays makes 125 different types and brands of chips and several hundred varieties of popcorn, puffs, twists, pretzels and assorted bagged munchies.

Jays combats the tendency of potato chips to crush into flinders with a variety of conveyor belts, radial filling chutes and gently vibrating slides, where masses of chips, a yard deep, are gradually massaged forward, the outer layer of chips shearing away like the face of a glacier.

The raw material is far easier to handle. An entire semi-trailer of sturdy North Dakota "chipping" potatoes can be emptied in a matter of minutes, by backing the trailer onto a hydraulic lift, tilting it 45 degrees and letting the potatoes—grown for their thin skins and low moisture—tumble out.

About a dozen semi-trailers' worth of potatoes arrive every day. The potatoes are immediately separated into big and small sizes for a purpose both reasonable and extraordinary: Big potatoes make big chips that go into large bags; small potatoes make small chips for lunch-size bags.

(*continued*)

(*concluded*)

"Nobody wants to open a small bag and find three big potato chips in it," Howe said.

Computers keep track of everything, shunting potatoes to 15,000-pound holding bins. Each bin feeds into a pipe containing a turning screw—a version of the ancient Archimedes screw used to pump water—that moves the potatoes from the bin to conveyor belts, to where they are washed and skinned—the skin scrubbed off by metal bristle brushes.

No machine can detect if a potato is rotten inside. So a pair of human inspectors reach into the passing brown parade and give the potatoes a quick squeeze. Occasionally, they snatch one and slice it open, usually revealing black areas of rot, a skill they attribute to experience.

"I *know,*" said Alicia Jimenez, asked to explain what about a potato tips her off to slice it open and find rot.

The naked potatoes are sent into high-speed chippers—spinning brass rings, each with eight blades inside, straight blades for straight chips, ripple blades for ripple chips.

The blades cut the potatoes, but the potatoes take their revenge. Every three hours the blades are dulled and the line must be stopped so the old rings can be replaced by new rings with sharpened blades.

The sheer quantity of slicing spews big foamy banks of starch from either side of the chipper, which calls to mind a washing machine gone berserk.

Potato chips account for about 55 percent of Jays' business.

The raw chips spend three minutes cooking in hot corn oil, which is constantly circulated and filtered. Then they are salted, and flavorings—barbecue, for instance, or sour cream and onion—are added.

After the chips are fried, there is another quality check, in which workers pluck burned and deformed chips out of the masses passing by. The chips are conveyed on a link grid, wide enough to let broken chips fall through.

The chips also are laser-inspected, rushing, in a single layer, over a complex device called an Opti-Sort Scanner. Chips with dark spots or holes are detected by a laser, which instructs one of 82 small tubes to fire a puff of air that knocks the substandard chip off the line, into a discard bin.

The discards—about 3 percent of production—are gathered up and used: Starch is drawn out and sold to cornstarch makers; the rest goes to hog feed. Just as the stockyards were said to use every part of the pig but the squeal, at Jays every part of the potato is used but the rich, earthy smell.

Jays even tried to sell burnt chips to the public once, about 20 years ago. "Consumers kept telling us they liked the brown chips," said Len Japp Jr., recalling the "Brownies" variety. "It went over like a lead balloon." Japp and his father, now 93 and honorary chairman of the board, sold the company to Borden in 1986. "They almost ruined it," Howe said, citing a slump in product quality and neglect of the Jays distribution system. "They lost the connection with the consumer."

By 1994, Jays was on the rocks and the Japps, allied with Howe, bought the company back. "Not too many people have a second chance in life," said Japp, whose children are in the company.

Getting the chips in the bags is another challenge: You can't just fill up bags and seal them; the chips would be smashed. Rather, a conveyor pours chips—gently—onto the central hub of a large, wheel-like device, where the chips scatter into 15 buckets that are, basically, scales. A computer monitors the weight of each bucket and opens up the exact combination that, in this case, will fill a 14-ounce bag. The bags are packed into boxes that read: "HANDLE LIKE EGGS."

While not exactly perishable, potato chips do have a shelf life of about eight weeks, only one day of which is spent at the plant.

"Potatoes that are in this morning will be in our branches tomorrow morning, ready to hit the streets," Howe said. Jays is still a regional brand, sold in Illinois, Indiana, Michigan, Wisconsin and Missouri. But business has grown 50 percent in the past two years.

"We connect to people's lifestyle," Howe said. "People treat themselves with Jays. We're in the fun food business."

Questions

1. What characteristics of potato chips concern Jays in terms of quality?
2. Do you feel that Jays is overdoing it with its concern for quality? Explain.

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Centralized versus On-Site Inspection

Some situations require that inspections be performed *on site.* For example, inspecting the hull of a ship for cracks requires inspectors to visit the ship. At other times, specialized tests can best be performed in a lab (e.g., performing medical tests, analyzing food samples, testing metals for hardness, running viscosity tests on lubricants).

The central issue in the decision concerning on-site or lab inspections is whether the advantages of specialized lab tests are worth the time and interruption needed to obtain the results. Reasons favoring on-site inspection include quicker decisions and avoidance of introduction

A Mattel technician in China does a pulling test with a Dora the Explorer doll in the name of product safety. Mattel has 10 labs in six countries and has set up strict requirements for vendors because of safety recalls.

of extraneous factors (e.g., damage or other alteration of samples during transportation to the lab). On the other hand, specialized equipment and a more favorable test environment (less noise and confusion, lack of vibrations, absence of dust, and no workers "helping" with inspections) offer strong arguments for using a lab.

Some companies rely on self-inspections by operators if errors can be traced back to specific operators. This places responsibility for errors at their source (*quality at the source*).

10.3 STATISTICAL PROCESS CONTROL

Quality control is concerned with the **quality of conformance** of a process: Does the output of a process conform to the intent of design? Variations in characteristics of process output provide the rationale for process control. **Statistical process control (SPC)** is used to evaluate process output to decide if a process is "in control" or if corrective action is needed.

Quality of conformance A product or service conforms to specifications.

Statistical process control (SPC) Statistical evaluation of the output of a process.

Process Variability

All processes generate output that exhibits some degree of variability. The issue is whether the output variations are within an acceptable range. The issue is addressed by answering two basic questions about the process variations:

1. Are the variations random? If nonrandom variations are present, the process is considered to be unstable. Corrective action will need to be taken to improve the process by eliminating the causes of nonrandomness to achieve a stable process.
2. Given a stable process, is the inherent variability of process output within a range that conforms to performance criteria? This involves assessment of a process's capability to meet standards. If a process is not capable, this situation will need to be addressed.

The natural or inherent process variations in process output are referred to as *chance* or **random variations.** Such variations are due to the combined influences of countless minor factors, each one so unimportant that even if it could be eliminated, the impact on process variations would be negligible. In Deming's terms, this is referred to as *common variability.* The amount of inherent variability differs from process to process. For instance, older machines

Random variation Natural variation in the output of a process, created by countless minor factors.

generally exhibit a higher degree of natural variability than newer machines, partly because of worn parts and partly because new machines may incorporate design improvements that lessen the variability in their output.

Assignable variation In process output, a variation whose cause can be identified. A nonrandom variation.

A second kind of variability in process output is called **assignable variation,** or *nonrandom variation.* In Deming's terms, this is referred to as *special variation.* Unlike natural variation, the main sources of assignable variation can usually be identified (assigned to a specific cause) and eliminated. Tool wear, equipment that needs adjustment, defective materials, human factors (carelessness, fatigue, noise and other distractions, failure to follow correct procedures, and so on) and problems with measuring devices are typical sources of assignable variation.

Sampling and Sampling Distributions

In statistical process control, periodic samples of process output are taken and sample statistics, such as sample means or the number of occurrences of a certain type of outcome, are determined. The sample statistics can be used to judge randomness of process variations. The sample statistics exhibit variation, just as processes do. The variability of sample statistics can be described by its **sampling distribution,** a theoretical distribution that describes the *random* variability of sample statistics. The most frequently used distribution is the normal distribution, for a variety of revasons.

Sampling distribution A theoretical distribution of sample statistics.

Figure 10.4A illustrates a sampling distribution and a process distribution (i.e., the distribution of process variations). Note three important things in Figure 10.4: (1) both distributions have the same mean; (2) the variability of the sampling distribution is less than the variability of the process; and (3) the sampling distribution is normal. This is true even if the process distribution is not normal.

Central limit theorem The distribution of sample averages tends to be normal regardless of the shape of the process distribution.

In the case of sample means, the **central limit theorem** states that as the sample size increases, the distribution of sample averages approaches a normal distribution regardless of the shape of the sampled population. This tends to be the case even for fairly small sample sizes. For other sample statistics, the normal distribution serves as a reasonable approximation to the shape of the actual sampling distribution.

Figure 10.4B illustrates what happens to the shape of the sampling distribution relative to the sample size. The larger the sample size, the narrower the sampling distribution. This means that the likelihood that a sample statistic is close to the true value in the population is higher for large samples than for small samples.

A sampling distribution serves as the theoretical basis for distinguishing between random and nonrandom values of a sampling statistic. Very simply, limits are selected within which

FIGURE 10.4A The sampling distribution of means is normal, and it has less variability than the process distribution, which might not be normal

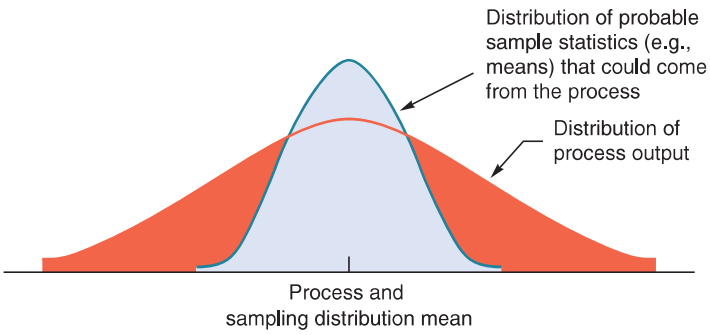
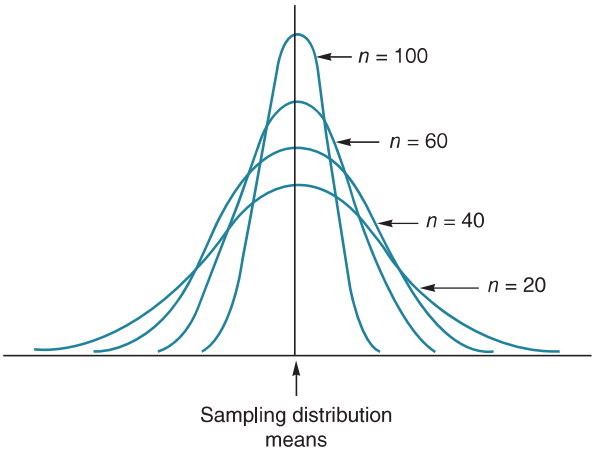


FIGURE 10.4B The larger the sample size, the narrower the sampling distribution



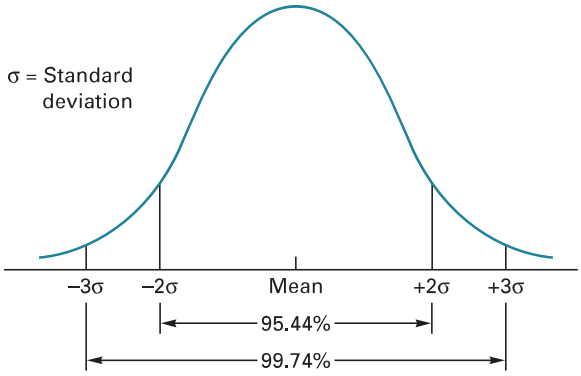


FIGURE 10.5
Percentage of values within given ranges in a normal distribution

most values of a sample statistic should fall if its variations are random. The limits are stated in terms of number of standard deviations from the distribution mean. Typical limits are ± 2 standard deviations or ± 3 standard deviations. Figure 10.5 illustrates these possible limits and the probability that a sample statistic would fall within those limits if only random variations are present. Conversely, if the value of a sample statistic falls outside those limits, there is only a small probability (1 − 99.74 = .0026 for ± 3 limits, and 1 − 95.44 = .0456 for ± 2 limits) that the value reflects randomness. Instead, such a value would suggest nonrandomness.

The Control Process

LO10.3 List and briefly explain the elements of the control process.

Sampling and corrective action are only a part of the control process. Effective control requires the following steps:

Define. The first step is to define in sufficient detail what is to be controlled. It is not enough, for example, to simply refer to a painted surface. The paint can have a number of important characteristics such as its thickness, hardness, and resistance to fading or chipping. Different characteristics may require different approaches for control purposes.

Food and beverage companies use Omron Electronics' fiber optic sensors to monitor processes and to perform quality inspections such as checking beverage content and caps.

Measure. Only those characteristics that can be counted or measured are candidates for control. Thus, it is important to consider how measurement will be accomplished.

Compare. There must be a standard of comparison that can be used to evaluate the measurements. This will relate to the level of quality being sought.

Evaluate. Management must establish a definition of *out of control.* Even a process that is functioning as it should will not yield output that conforms exactly to a standard, simply because of the natural (i.e., random) variations inherent in all processes, manual or mechanical—a certain amount of variation is inevitable. The main task of quality control is to distinguish random from *nonrandom* variability, because nonrandom variability means that a process is out of control.

Correct. When a process is judged to be out of control, corrective action must be taken. This involves uncovering the cause of nonrandom variability (e.g., worn equipment, incorrect methods, failure to follow specified procedures) and correcting it.

Monitor results. To ensure that corrective action is effective, the output of a process must be monitored for a sufficient period of time to verify that the problem has been eliminated.

In sum, control is achieved by checking a portion of the goods or services, comparing the results to a predetermined standard, evaluating departures from the standard, taking corrective action when necessary, and following up to ensure that problems have been corrected.

Control Charts: The Voice of the Process

Control chart A time-ordered plot of sample statistics, used to distinguish between random and nonrandom variability.

LO10.4 Explain how control charts are used to monitor a process and the concepts that underlie their use.

An important tool in statistical process control is the control chart, which was developed by Walter Shewhart. A **control chart** is a *time-ordered* plot of sample statistics. It is used to distinguish between random variability and nonrandom variability. It has upper and lower limits, called *control limits,* that define the range of acceptable (i.e., random) variation for the sample statistic. A control chart is illustrated in Figure 10.6. The purpose of a control chart is to monitor process output to see if it is random. A necessary (but not sufficient) condition for a process to be deemed "in control," or stable, is for all the data points to fall between the upper and lower control limits. Conversely, a data point that falls outside of either limit would be taken as evidence that the process output may be nonrandom and, therefore, not "in control." If that happens, the process would be halted to find and correct the cause of the nonrandom variation. The essence of statistical process control is to assure that the output of a process is random so that *future output* will be random.

The basis for the control chart is the sampling distribution, which essentially describes random variability. There is, however, one minor difficulty relating to the use of a normal sampling distribution. The theoretical distribution extends in either direction to *infinity.* Therefore, *any* value is theoretically possible, even one that is a considerable distance from the mean of the distribution. However, as a practical matter, we know that, say, 99.7 percent of the values will be within ± 3 standard deviations of the mean of the distribution. Therefore, we could decide to set the limit, so to speak, at values that represent ± 3 standard deviations from the

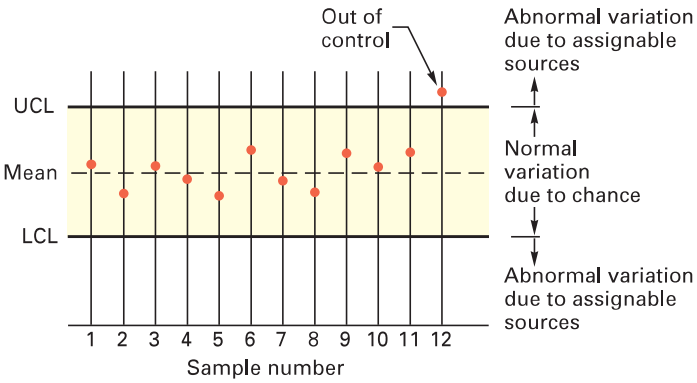


FIGURE 10.6
Example of a control chart

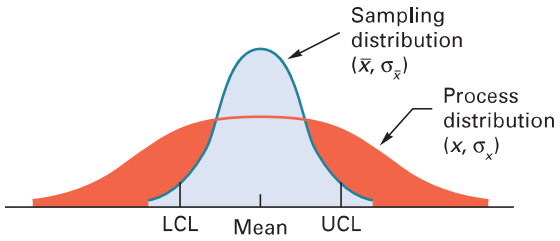


FIGURE 10.7
Control limits are based on the sampling distribution

mean, and conclude that any value that was farther away than these limits was a nonrandom variation.

In effect, these limits are **control limits:** the dividing lines between what will be designated as random deviations from the mean of the distribution and what will be designated as nonrandom deviations from the mean of the distribution. Figure 10.7 illustrates how control limits are based on the sampling distribution.

Control limits The dividing lines between random and nonrandom deviations from the mean of the distribution.

Control charts have two limits that separate random variation and nonrandom variation. The larger value is the *upper control limit* (UCL), and the smaller value is the *lower control limit* (LCL). A sample statistic that falls between these two limits suggests (but does not prove) randomness, while a value outside or on either limit suggests (but does not prove) nonrandomness.

It is important to recognize that because any limits will leave some area in the *tails* of the distribution, there is a small probability that a value will fall outside the limits *even though only random variations are present.* For example, if ± 2 sigma (standard deviation) limits are used, they would include 95.5 percent of the values. Consequently, the complement of that number (100 percent = 95.5 percent = 4.5 percent) would not be included. That percentage (or *probability*) is sometimes referred to as the probability of a **Type I error,** where the "error" is concluding that nonrandomness is present when only randomness is present. It is also referred to as an *alpha* risk, where alpha (α) is the sum of the probabilities in the two tails. Figure 10.8 illustrates this concept.

Type I error Concluding a process is not in control when it actually is.

Using wider limits (e.g., ± 3 sigma limits) reduces the probability of a Type I error because it decreases the area in the tails. However, wider limits make it more difficult to detect nonrandom variations *if* they are present. For example, the mean of the process might shift (an assignable cause of variation) enough to be detected by two-sigma limits, but not enough to be readily apparent using three-sigma limits. That could lead to a second kind of error, known as a **Type II error,** which is concluding that a process is in control when it is really out of control (i.e., concluding nonrandom variations are not present, when they are). In theory, the costs of making each error should be balanced by their probabilities. However, in practice, two-sigma limits and three-sigma limits are commonly used without specifically referring to the probability of a Type II error.

Type II error Concluding a process is in control when it is not.

Table 10.2 illustrates how Type I and Type II errors occur.

Each sample is represented by a single value (e.g., the sample mean) on a control chart. Moreover, each value is compared to the extremes of the sampling distribution (the control

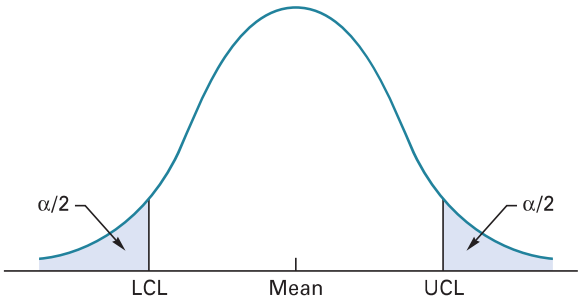
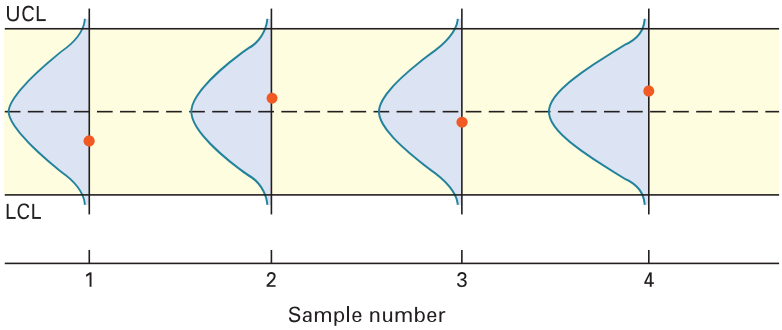


FIGURE 10.8
The probability of a Type I error

TABLE 10.2
Type I and Type II errors

If a process is actually:		*And the conclusion is that it is:* In Control	Out of Control
	In control	No error	**Type I error** (producer's risk)
	Out of control	**Type II error** (consumer's risk)	No error

FIGURE 10.9
Each observation is compared to the selected limits of the sampling distribution



limits) to judge if it is within the acceptable (random) range. Figure 10.9 illustrates this concept.

There are four commonly used control charts. Two are used for **variables,** and two are used for **attributes.** Attribute data are *counted* (e.g., the number of defective parts in a sample, the number of calls per day); variables data are *measured,* usually on a continuous scale (e.g., amount of time needed to complete a task, length or width of a part).

Variables Generate data that are *measured.*

Attributes Generate data that are *counted.*

The two control charts for variables data are described in the next section, and the two control charts for attribute data are described in the section following that.

Control Charts for Variables

Mean and range charts are used to monitor variables. Control charts for means monitor the *central tendency* of a process, and range charts monitor the *dispersion* of a process.

Mean control chart Control chart used to monitor the central tendency of a process.

Mean Charts. A **mean control chart,** sometimes referred to as an $\bar{x}$ ("x–bar") chart, is based on a normal distribution. It can be constructed in one of two ways. The choice depends on what information is available. Although the value of the standard deviation of a process, σ, is often unknown, if a reasonable estimate is available, one can compute control limits using these formulas:

$$\text{Upper control limit (UCL):} = \bar{\bar{x}} + z\sigma_{\bar{x}} \qquad (10\text{–}1)$$

$$\text{Lower control limit (LCL):} = \bar{\bar{x}} - z\sigma_{\bar{x}}$$

where

$\sigma_{\bar{x}} = \sigma/\sqrt{n}$

$\sigma_{\bar{x}}$ = Standard deviation of distribution of sample means

σ = Estimate of the process standard deviation

n = Sample size

z = The number of standard deviations that control limits are based on

$\bar{\bar{x}}$ = Average of sample means

The following example illustrates the use of these formulas.

EXAMPLE 1

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A quality inspector took five samples, each with four observations ($n = 4$), of the length of time for glue to dry. The analyst computed the mean of each sample and then computed the grand mean. All values are in minutes. Use this information to obtain three-sigma (i.e., $z = 3$) control limits for means of future times. It is known from previous experience that the standard deviation of the process is .02 minute.

		SAMPLE				
		1	2	3	4	5
Observation	1	12.11	12.15	12.09	12.12	12.09
	2	12.10	12.12	12.09	12.10	12.14
	3	12.11	12.10	12.11	12.08	12.13
	4	12.08	12.11	12.15	12.10	12.12
	$\bar{x}$	12.10	12.12	12.11	12.10	12.12

SOLUTION

$$\bar{\bar{x}} = \frac{12.10 + 12.12 + 12.11 + 12.10 + 12.12}{5} = 12.11$$

Using Formula 10–1, with $z = 3$, $n = 4$ observations per sample, and $\sigma = .02$, we find

$$\text{UCL: } 12.11 + 3\left(\frac{.02}{\sqrt{4}}\right) = 12.14$$

$$\text{LCL: } 12.11 - 3\left(\frac{.02}{\sqrt{4}}\right) = 12.08$$

Note: If one applied these control limits to the means, one would judge the process to be *in control* because all of the sample means have values that fall within the control limits. The fact that some of the *individual* measurements fall outside of the control limits (e.g., the first observation in Sample 2 and the last observation in Sample 3) is irrelevant. You can see why by referring to Figure 10.7: *Individual* values are represented by the process distribution, a large portion of which lies outside of the control limits for *means*.

This and similar problems can also be solved using the Excel templates that are available on the book's Web site. The solution for Example 1 using Excel is shown here.

Mean Control Chart (σ known)

<Back Basic Clear

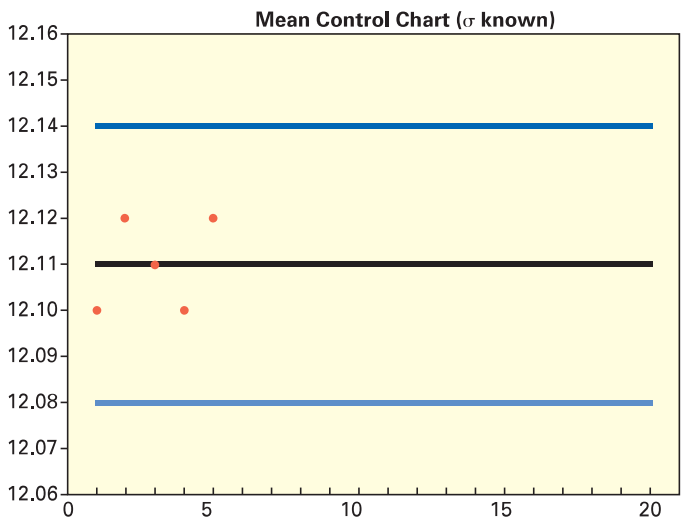
Average of sample means		12.11
Process standard deviation	σ =	0.02
Sample size	n =	4
	z =	3
	Δz =	0.1

UCL =	12.14
LCL =	12.08

Calculations:

	Mean
Average	12.11

Sample	Mean
1	12.1
2	12.12
3	12.11
4	12.1
5	12.12
6	



Note: To display more data on the above graph, right click on the x-axis, select Format Axis, and set Maximum to higher value. The size of the graph may also be increased.

LO10.5 Use and interpret control charts.

If an observation on a control chart is on or outside of either control limit, the process is stopped to investigate the cause of that value, such as operator error, machine out of adjustment, or similar assignable cause of variation. If no source of error is found, the value could simply be due to chance, and the process will be restarted. However, the output should then be monitored to see if additional values occur that are beyond the control limits, in which case a more thorough investigation would be needed to uncover the source of the problem so that it could be corrected.

If the standard deviation of the process is unknown, another approach is to use the sample *range* as a measure of process variability. The appropriate formulas for control limits are

$$\begin{aligned} \text{UCL} &= \bar{\bar{x}} + A_2\bar{R} \\ \text{LCL} &= \bar{\bar{x}} - A_2\bar{R} \end{aligned} \qquad (10\text{–}2)$$

where

A_2 = A factor from Table 10.3
$\bar{R}$ = Average of sample ranges

EXAMPLE 2

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Refer to the data given in Example 1. In order to use Formula 10–2, we need to compute the grand mean for the data and the average sample range. In Example 1, the grand mean is 12.11. The range for each sample is the difference between the largest and smallest sample values. For the first sample, the largest value is 12.11 and the smallest value is 12.08. The range is the difference between these two values, which is 12.11 − 12.08 = .03. For Sample 2, the range is 12.15 − 12.10 = 0.05. The other ranges can be computed in similar fashion. The average range is:

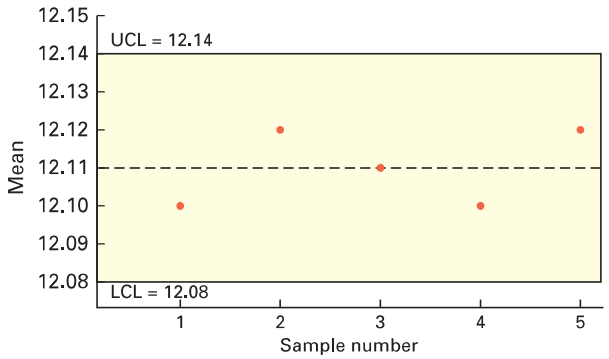
$$\bar{R} = (.03 + .05 + .06 + .04 + .05)/5 = .046.$$

SOLUTION

$\bar{\bar{x}} = 12.11$, $\bar{R} = .046$, and $A_2 = .73$ for $n = 4$ (from Table 10.3). Using Formula 10–2, we can compute the upper and lower limits for a mean control chart:

$$\text{UCL} = 12.11 + .73(.046) = 12.14 \text{ minutes}$$
$$\text{LCL} = 12.11 - .73(.046) = 12.08 \text{ minutes}$$

Except for rounding, these results are the same as those computed in Example 1. Usually that will be the case, but not always.



Range control chart Control chart used to monitor process dispersion.

Range Charts. **Range control charts** (R-charts) are used to monitor process dispersion; they are sensitive to changes in process dispersion. Although the underlying sampling distribution is not normal, the concepts for the use of range charts are much the same as those for

the use of mean charts. Control limits for range charts are found using the average sample range in conjunction with these formulas:

$$\text{UCL} = D_4\bar{\text{R}}$$
$$\text{LCL} = D_3\bar{\text{R}} \qquad (10\text{–}3)$$

where values of D_3 and D_4 are obtained from Table 10.3.[1]

EXAMPLE 3

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Using the average range found in Example 2 and Formula 10–3, we can compute the control limits for a range chart.

SOLUTION

From Table 10.3, for $n = 4$, $D_4 = 2.28$ and $D_3 = 0$. Thus,

$$\text{UCL} = 2.28(.046) = .105 \text{ minutes}$$
$$\text{LCL} = 0(.046) = 0 \text{ minutes}$$

Note that the five sample ranges shown in Example 2 are within these control limits.

TABLE 10.3
Factors for three-sigma control limits for $\bar{X}$ and R charts

Number of Observations in Sample, n	Factor for $\bar{X}$ Chart, A_2	FACTORS FOR R CHARTS	
		Lower Control Limit, D_3	Upper Control Limit, D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72
13	0.25	0.31	1.69
14	0.24	0.33	1.67
15	0.22	0.35	1.65
16	0.21	0.36	1.64
17	0.20	0.38	1.62
18	0.19	0.39	1.61
19	0.19	0.40	1.60
20	0.18	0.41	1.59

Source: Adapted from Eugene Grant and Richard Leavenworth, *Statistical Quality Control,* 5th ed. Copyright © 1980 McGraw-Hill Companies, Inc. Used with permission.

[1]If the process standard deviation is known, control limits for a range chart can be calculated using values from Table 10.3:

$$\text{LCL} = \frac{3D_3\sigma}{A_2\sqrt{n}}, \quad \text{UCL} = \frac{3D_4\sigma}{A_2\sqrt{n}}$$

Food inspectors inspect cooked hamburger meat for food safety at McDonald's.

Using Mean and Range Charts. Mean control charts and range control charts provide different perspectives on a process. As we have seen, mean charts are sensitive to shifts in the process mean, whereas range charts are sensitive to changes in process dispersion. Because of this difference in perspective, both types of charts might be used to monitor the same process. The logic of using both is readily apparent in Figure 10.10. In Figure 10.10A, the mean chart picks up the shift in the process mean, but because the dispersion is not changing, the range

FIGURE 10.10
Mean and range charts used together complement each other

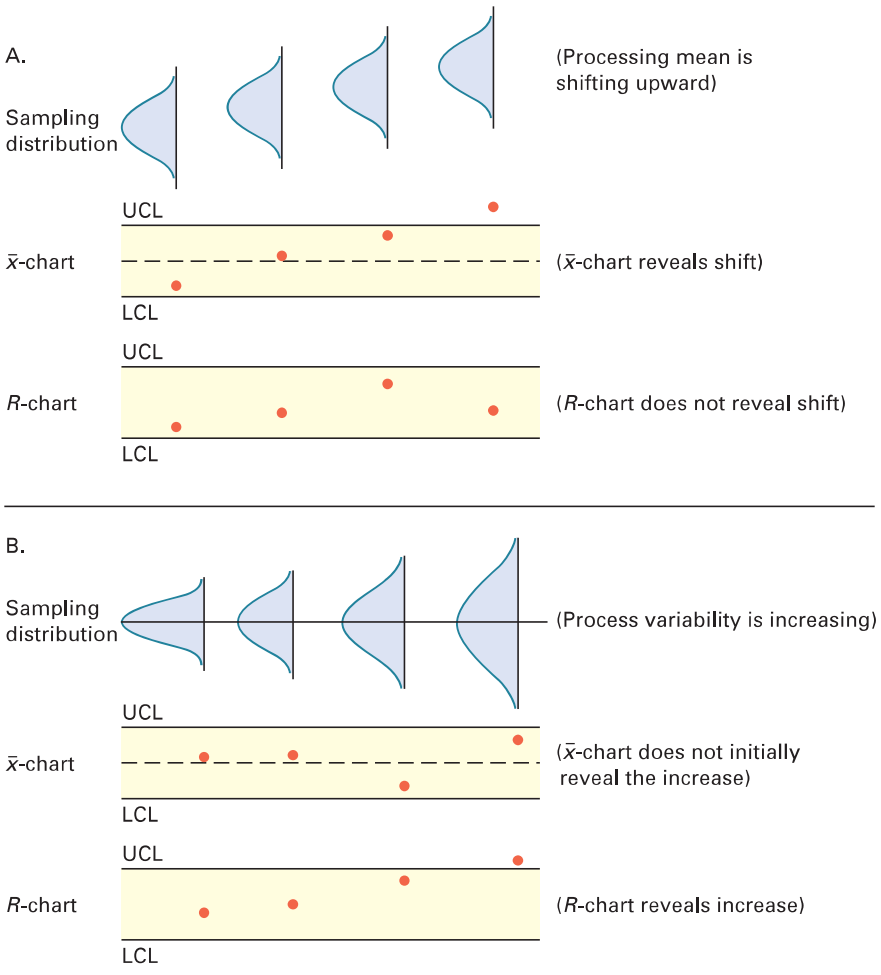

chart fails to indicate a problem. Conversely, in Figure 10.10B, a change in process dispersion is less apt to be detected by the mean chart than by the range chart. Thus, use of both charts provides more complete information than either chart alone. Even so, a single chart may suffice in some cases. For example, a process may be more susceptible to changes in the process mean than to changes in dispersion, so it might be unnecessary to monitor dispersion. Because of the time and cost of constructing control charts, gathering the necessary data, and evaluating the results, only those aspects of a process that tend to cause problems should be monitored.

Once control charts have been set up, they can serve as a basis for deciding when to interrupt a process and search for assignable causes of variation. To determine initial control limits, one can use the following procedure:

1. Obtain 20 to 25 samples. Compute the appropriate sample statistic(s) for each sample (e.g., mean).
2. Establish preliminary control limits using the formulas.
3. Determine if any points fall outside the control limits.
4. Plot the data on the control chart and check for patterns.
5. If no out-of-control signals are found, assume that the process is in control. If any out-of-control signals are found, investigate and correct causes of variation. Then resume the process and collect another set of observations upon which control limits can be based.

Control Charts for Attributes

Control charts for attributes are used when the process characteristic is *counted* rather than measured. For example, the number of defective items in a sample is counted, whereas the length of each item is measured. There are two types of attribute control charts, one for the fraction of defective items in a sample (a p-chart) and one for the number of defects per unit (a c-chart). A *p-chart* is appropriate when the data consist of two categories of items. For instance, if glass bottles are inspected for chipping and cracking, both the good bottles and the defective ones can be counted. However, one can count the number of accidents that occur during a given period of time but *not* the number of accidents that did not occur. Similarly, one can count the number of scratches on a polished surface, the number of bacteria present in a water sample, and the number of crimes committed during the month of August, but one cannot count the number of nonoccurrences. In such cases, a *c-chart* is appropriate. See Table 10.4.

p-chart Control chart for attributes, used to monitor the proportion of defective items in a process.

p-Chart. A ***p*-chart** is used to monitor the proportion of defective items generated by a process. The theoretical basis for a p-chart is the binomial distribution, although for large sample sizes, the normal distribution provides a good approximation to it. Conceptually, a p-chart is constructed and used in much the same way as a mean chart.

The centerline on a p-chart is the average fraction defective in the population, p. The standard deviation of the sampling distribution when p is known is

$$\sigma_p = \sqrt{\frac{p(1-p)}{n}}$$

Control limits are computed using the formulas

$$\begin{aligned} \text{UCL}_p &= p + z\sigma_p \\ \text{LCL}_p &= p - z\sigma_p \end{aligned} \qquad (10\text{–}4)$$

If p is unknown, which is generally the case, it can be estimated from samples. That estimate, $\bar{p}$, replaces p in the preceding formulas, and $\hat{\sigma}_p$ replaces σ_p, as illustrated in Example 4.

Note: Because the formula is an approximation, it sometimes happens that the computed LCL is negative. In those instances, zero is used as the lower limit.

TABLE 10.4
p-chart or *c*-chart?

The following tips should help you select the type of control chart, a *p*-chart or a *c*-chart, that is appropriate for a particular application:

Use a *p*-chart:

1. When observations can be placed into one of *two* categories. Examples include items (observations) that can be classified as
   a. Good or bad.
   b. Pass or fail.
   c. Operate or don't operate.
2. When the data consist of multiple samples of n observations each (e.g., 15 samples of $n = 20$ observations each).

Use a *c*-chart:

When only the number of occurrences per unit of measure can be counted; nonoccurrences cannot be counted. Examples of occurrences and units of measure include
a. Scratches, chips, dents, or errors per item.
b. Cracks or faults per unit of distance (e.g., meters, miles).
c. Breaks or tears, per unit of area (e.g., square yard, square meter).
d. Bacteria or pollutants per unit of volume (e.g., gallon, cubic foot, cubic yard).
e. Calls, complaints, failures, equipment breakdowns, or crimes per unit of time (e.g., hour, day, month, year).

EXAMPLE 4

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An inspector counted the number of defective monthly billing statements of a telephone company in each of 20 samples. Using the following information, construct a control chart that will describe 99.74 percent of the chance variation in the process when the process is in control. Each sample contained 100 statements.

Sample	Number of Defectives	Sample	Number of Defectives	Sample	Number of Defectives	Sample	Number of Defectives
1	7	6	11	11	8	16	10
2	10	7	10	12	12	17	8
3	12	8	18	13	9	18	12
4	4	9	13	14	10	19	10
5	9	10	10	15	16	20	21
							220

SOLUTION

To find z, divide .9974 by 2 to obtain .4987, and using that value, refer to Appendix B, Table A to find $z = 3.00$.

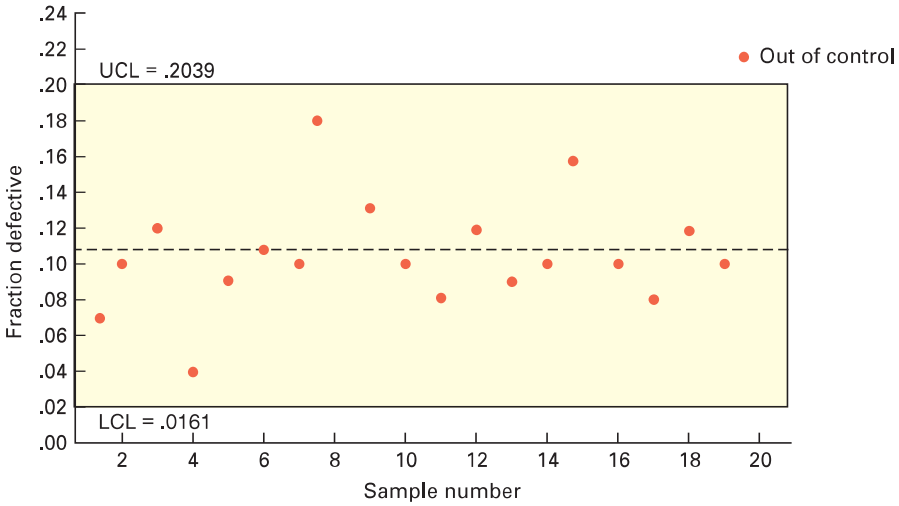
$$\bar{p} = \frac{\text{Total number of defectives}}{\text{Total number of observations}} = \frac{220}{20(100)} = .11$$

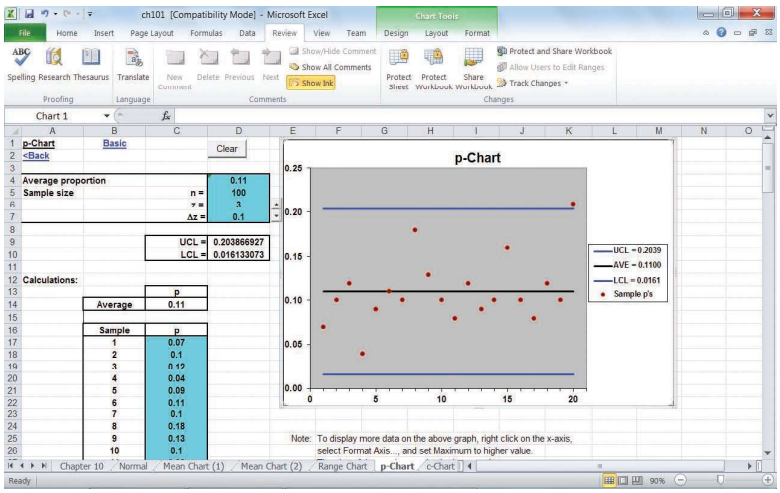
$$\hat{\sigma}_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{.11(1-.11)}{100}} = .0313$$

Control limits are

$$\text{UCL}_p = \bar{p} + z(\hat{\sigma}_p) = .11 + 3.00(.0313) = .2039$$
$$\text{LCL}_p = \bar{p} - z(\hat{\sigma}_p) = .11 - 3.00(.0313) = .0161$$

Plotting the control limits and the sample fraction defective, you can see that the last value is above the upper control limit. The process would be stopped at that point to find and correct the possible cause. Then new data would be collected to establish new control limits. If no cause is found, this could be due to chance. The new limits would remain, but future output would be monitored to assure the process remains in control.





c-Chart. When the goal is to control the number of *occurrences* (e.g., defects) *per unit,* a **c-chart** is used. Units might be automobiles, hotel rooms, typed pages, or rolls of carpet. The underlying sampling distribution is the Poisson distribution. Use of the Poisson distribution assumes that defects occur over some *continuous* region and that the probability of more than one defect at any particular point is negligible. The mean number of defects per unit is c and the standard deviation is $\sqrt{c}$. For practical reasons, the normal approximation to the Poisson is used. The control limits are

c-chart Control chart for attributes, used to monitor the number of defects per unit.

$$\begin{aligned} \text{UCL}_c &= c + z\sqrt{c} \\ \text{LCL}_c &= c - z\sqrt{c} \end{aligned} \qquad (10\text{–}5)$$

If the value of c is unknown, as is generally the case, the sample estimate, $\bar{c}$, is used in place of c, using $\bar{c}$ = Number of defects ÷ Number of samples.

EXAMPLE 5

Rolls of coiled wire are monitored using a c-chart. Eighteen rolls have been examined, and the number of defects per roll has been recorded in the following table. Is the process in control? Plot the values on a control chart using three standard deviation control limits.

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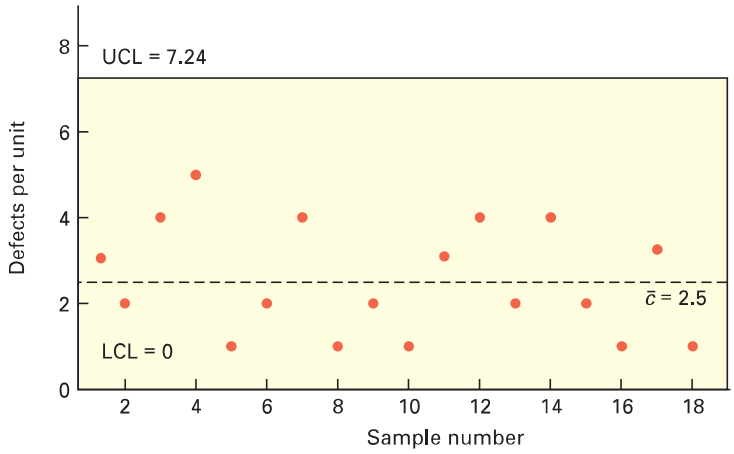
Sample	Number of Defects	Sample	Number of Defects	Sample	Number of Defects
1	3	7	4	13	2
2	2	8	1	14	4
3	4	9	2	15	2
4	5	10	1	16	1
5	1	11	3	17	3
6	2	12	4	18	1
					45

SOLUTION

$$\bar{c} = 45/18 = 2.5 = \text{Average number of defects per coil}$$

$$\text{UCL}_c = \bar{c} + 3\sqrt{\bar{c}} = 2.5 + 3\sqrt{2.5} = 7.24$$

$$\text{LCL}_c = \bar{c} - 3\sqrt{\bar{c}} = 2.5 - 3\sqrt{2.5} = -2.24 \rightarrow 0$$



When the computed lower control limit is negative, the effective lower limit is zero. In such cases, if a control chart point is zero, it should not be deemed to be out of control. The calculation sometimes produces a negative lower limit due to the use of the normal distribution to approximate the Poisson distribution: The normal is symmetrical, whereas the Poisson is not symmetrical when c is close to zero.

Note that if an observation falls below the lower control limit on a p-chart or a c-chart, the cause should be investigated, just as it would be for a mean or range chart, even though such a point would imply that the process is exhibiting better than expected quality. It may turn out to be the result of an undesirable overuse of resources. On the other hand, it may lead to a discovery that can improve the quality of the process.

Managerial Considerations Concerning Control Charts

Using control charts adds to the cost and time needed to obtain output. Ideally a process is so good that the desired level of quality could be achieved without the use of any control charts. The best organizations strive to reach this level, but many are not yet there, so they employ control charts at various points in their processes. In those organizations, managers must make a number of important decisions about the use of control charts:

1. At what points in the process to use control charts.
2. What size samples to take.
3. What type of control chart to use (i.e., variables or attribute).
4. How often should samples be taken.

The decision about where to use control charts should focus on those aspects of the process that (1) have a tendency to go out of control and (2) are critical to the successful operation of the product or service (i.e., variables that affect product or service characteristics).

Sample size is important for two reasons. One is that cost and time are functions of sample size; the greater the sample size, the greater the cost to inspect those items (and the greater the lost product if destructive testing is involved) and the longer the process must be held up while waiting for the results of sampling. The second reason is that smaller samples are more likely to reveal a change in the process than larger samples because a change is more likely to take place *within* the large sample, but *between* small samples. Consequently, a sample statistic such as the sample mean in the large sample could combine both "before-change" and "after-change" observations, whereas in two smaller samples, the first could contain "before" observations and the second "after" observations, making detection of the change more likely.

In some instances, a manager can choose between using a control chart for variables (a mean chart) and a control chart for attributes (a *p*-chart). If the manager is monitoring the diameter of a drive shaft, either the diameter could be measured and a mean chart used for control, or the shafts could be inspected using a *go, no-go gauge*—which simply indicates whether a particular shaft is within specification without giving its exact dimensions—and a *p*-chart could be used. Measuring is more costly and time-consuming per unit than the yes-no inspection using a go, no-go gauge, but because measuring supplies more information than merely counting items as good or bad, one needs a much smaller sample size for a mean chart than a *p*-chart. Hence, a manager must weigh the time and cost of sampling against the information provided.

Sampling frequency can be a function of the stability of a process and the cost to sample.

Run Tests

Control charts test for points that are too extreme to be considered random (e.g., points that are outside of the control limits). However, even if all points are within the control limits, the data may still not reflect a random process. In fact, any sort of pattern in the data would suggest a nonrandom process. Figure 10.11 illustrates some patterns that might be present.

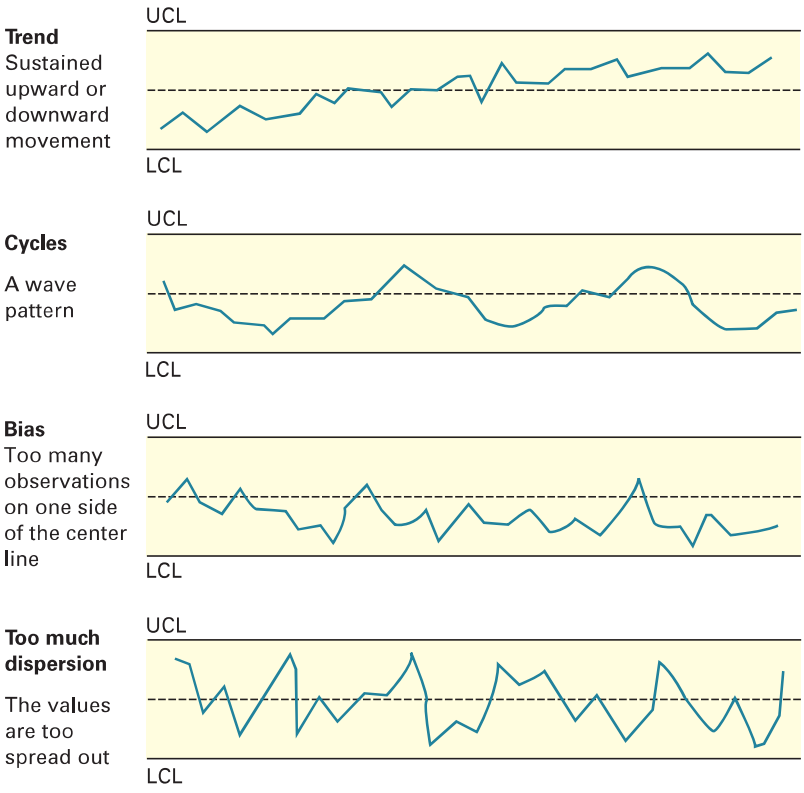


FIGURE 10.11
Some examples of nonrandom patterns in control chart plots

Run test A test for patterns in a sequence.

LO10.6 Perform run tests to check for nonrandomness in process output.

Analysts often supplement control charts with a **run test,** which checks for patterns in a sequence of observations. This enables an analyst to do a better job of detecting abnormalities in a process and provides insights into correcting a process that is out of control. A variety of run tests are available; this section describes two that are widely used.

When a process is stable or in statistical control, the output it generates will exhibit random variability over a period of time. The presence of patterns, such as trends, cycles, or bias in the output indicates that assignable, or nonrandom, causes of variation exist. Hence, a process that produces output with such patterns is not in a state of statistical control. This is true even though all points on a control chart may be within the control limits. For this reason, it is usually prudent to subject control chart data to run tests to determine whether patterns can be detected.

Run Sequence of observations with a certain characteristic.

A **run** is defined as a sequence of observations with a certain characteristic, followed by one or more observations with a different characteristic. The characteristic can be anything that is observable. For example, in the series A A A B, there are two runs: a run of three As followed by a run of one B. Underlining each run helps in counting them. In the series A A B B B A, the underlining indicates three runs.

Two useful run tests involve examination of the number of runs *up and down* and runs above and below the *median.*[2] In order to count these runs, the data are transformed into a series of Us and Ds (for *up* and *down*) and into a series of As and Bs (for *above* and *below* the median). Consider the following sequence, which has a median of 36.5. The first two values are below the median, the next two are above it, the next to last is below, and the last is above. Thus, there are four runs:

25	29	42	40	35	38
B	B	A	A	B	A

In terms of up and down, there are three runs in the same data. The second value is up from the first value, the third is up from the second, the fourth is down from the third, and so on:

25	29	42	40	35	38
—	U	U	D	D	U

(The first value does not receive either a U or a D because nothing precedes it.)

If a plot is available, the runs can be easily counted directly from the plot, as illustrated in Figures 10.12 and 10.13.

FIGURE 10.12
Counting above/below median runs

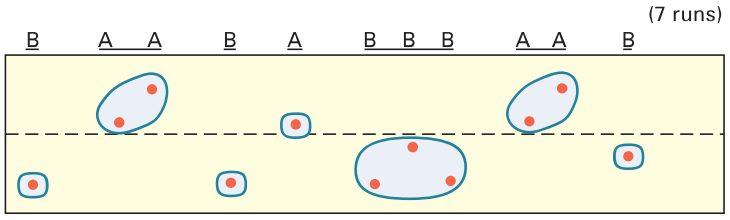
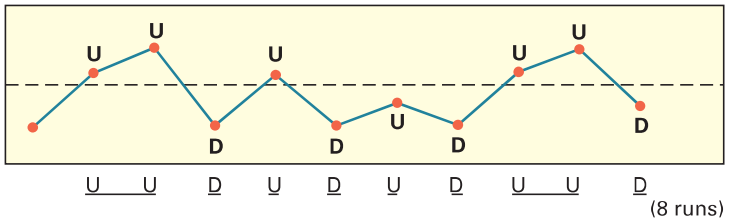


FIGURE 10.13
Counting up/down runs



[2]The median and mean are approximately equal for control charts. The use of the median depends on its ease of determination; use the mean instead of the median if it is given.

To determine whether any patterns are present in control chart data, one must transform the data into both As and Bs and Us and Ds, and then count the number of runs in each case. These numbers must then be compared with the number of runs that would be expected in a completely random series. For both the median and the up/down run tests, the expected number of runs is a function of the number of observations in the series. The formulas are

$$E(r)_{\text{med}} = \frac{N}{2} + 1 \tag{10–6a}$$

$$E(r)_{u/d} = \frac{2N - 1}{3} \tag{10–7a}$$

where N is the number of observations or data points, and $E(r)$ is the expected number of runs.

The actual number of runs in any given set of observations will vary from the expected number, due to chance and any patterns that might be present. Chance variability is measured by the standard deviation of runs. The formulas are

$$\sigma_{\text{med}} = \sqrt{\frac{N - 1}{4}} \tag{10–6b}$$

$$\sigma_{u/d} = \sqrt{\frac{16N - 29}{90}} \tag{10–7b}$$

Distinguishing chance variability from patterns requires use of the sampling distributions for median runs and up/down runs. Both distributions are approximately normal. Thus, for example, 95.5 percent of the time a random process will produce an observed number of runs within two standard deviations of the expected number. If the observed number of runs falls in that range, there are probably no nonrandom patterns; for observed numbers of runs beyond such limits, we begin to suspect that patterns are present. Too few or too many runs can be an indication of nonrandomness.

In practice, it is often easiest to compute the number of standard deviations, z, by which an observed number of runs differs from the expected number. This z value would then be compared to the value ± 2 (z for 95.5 percent) or some other desired value (e.g., ± 1.96 for 95 percent, ± 2.33 for 98 percent). A test z that exceeds the desired limits indicates patterns are present. (See Figure 10.14.) The computation of z takes the form

$$z_{\text{test}} = \frac{\text{Observed number of runs} - \text{Expected number of runs}}{\text{Standard deviation of number of runs}}$$

For the median and up/down tests, one can find z using these formulas:

$$\text{Median:} \qquad z = \frac{r - [(N/2) + 1]}{\sqrt{(N - 1)/4}} \tag{10–8}$$

$$\text{Up and down:} \qquad z = \frac{r - [(2N - 1)/3]}{\sqrt{(16N - 29)/90}} \tag{10–9}$$

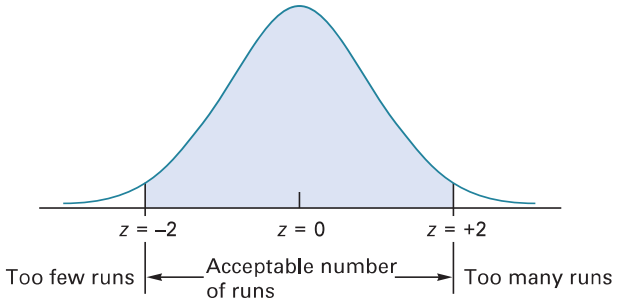


FIGURE 10.14
A sampling distribution for runs is used to distinguish chance variation from patterns

where

N = Total number of observations

r = Observed number of runs of either As and Bs or Us and Ds, depending on which test is involved.

It is desirable to apply both run tests to any given set of observations because each test is different in terms of the types of patterns it can detect. Sometimes both tests will pick up a certain pattern, but sometimes only one will detect nonrandomness. If either does, the implication is that some sort of nonrandomness is present in the data.

EXAMPLE 6

eXcel

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Twenty sample means have been taken from a process. The means are shown in the following table. Use median and up/down run tests with $z = 2$ to determine if assignable causes of variation are present. Assume the median is 11.0.

SOLUTION

The means are marked according to above/below the median and up/down. The solid lines represent the runs.

Sample	A/B	Mean	U/D	Sample	A/B	Mean	U/D
1	B	10.0	—	11	B	10.7	D
2	B	10.4	U	12	A	11.3	U
3	B	10.2	D	13	B	10.8	D
4	A	11.5	U	14	A	11.8	U
5	B	10.8	D	15	A	11.2	D
6	A	11.6	U	16	A	11.6	U
7	A	11.1	D	17	A	11.2	D
8	A	11.2	U	18	B	10.6	D
9	B	10.6	D	19	B	10.7	U
10	B	10.9	U	20	A	11.9	U
		A/B: 10 runs			U/D: 17 runs		

The expected number of runs for each test is

$$E(r)_{\text{med}} = \frac{N}{2} + 1 = \frac{20}{2} + 1 = 11$$

$$E(r)_{u/d} = \frac{2N - 1}{3} = \frac{2(20) - 1}{3} = 13$$

The standard deviations are

$$\sigma_{\text{med}} = \sqrt{\frac{N - 1}{4}} = \sqrt{\frac{20 - 1}{4}} = 2.18$$

$$\sigma_{u/d} = \sqrt{\frac{16N - 29}{90}} = \sqrt{\frac{16(20) - 29}{90}} = 1.80$$

The z_{test} values are

$$z_{\text{med}} = \frac{10 - 11}{2.18} = -.46$$

$$z_{u/d} = \frac{17 - 13}{1.80} = +2.22$$

Although the median test does not reveal any pattern, because its z_{test} value is within the range $\pm$ 2, the up/down test does; its value exceeds +2. Consequently, nonrandom variations are probably present in the data and, hence, the process is not in control.

If ties occur in either test (e.g., a value equals the median or two values in a row are the same), assign A/B or U/D in such a manner that that z_{test} is as large as possible. If z_{test} still does not exceed $\pm$ 2 ($\pm$ 1.96, etc.), you can be reasonably confident that a conclusion of randomness is justified.

Using Control Charts and Run Tests Together

Although for instructional purposes most of the examples, solved problems, and problems focus on either control charts or run tests, ideally both control charts and run tests should be used to analyze process output, along with a plot of the data. The procedure involves the following three steps:

1. Compute control limits for the process output.
   a. Determine which type of control chart is appropriate (see Figure 10.18).
   b. Compute control limits using the appropriate formulas. If no probability is given, use a value of $z = 2.00$ to compute the control limits.
   c. If any sample statistics fall outside of the control limits, the process is not in control. If all values are within the control limits, proceed to Step 2.
2. Conduct median and up/down run tests. Use $z = \pm 2.00$ for comparing the test scores. If either or both test scores are not within $z = \pm 2.00$, the output is probably not random. If both test scores are within $z = \pm 2.00$, proceed to Step 3.
3. *Note:* If you are at this point, there is no indication so far that the process output is nonrandom. Plot the sample data and visually check for patterns (e.g., cycling). If you see a pattern, the output is probably not random. Otherwise, conclude the output is random and that the process is in control.

What Happens When a Process Exhibits Possible Nonrandom Variation?

Nonrandom variation is indicated when a point is observed that is outside the control limits, or a run test produces a large z-value (e.g., greater than $\pm$ 1.96). Managers should have response plans in place to investigate the cause. It may be a false alarm (i.e., a Type I error), or it may be a real indication of the presence of an assignable cause of variation. If it appears to be a false alarm, resume the process but monitor it for a while to confirm this. If an assignable cause can be found, it needs to be addressed. If it is a good result (e.g., an observation below the lower control limit of a p-chart, a c-chart, or a range chart would indicate unusually good quality), it may be possible to change the process to achieve similar results on an ongoing basis. The more typical case is that there is a problem that needs to be corrected. Operators can be trained to handle simple problems, while teams may be needed to handle more complex problems. Problem solving often requires the use of various tools, described in Chapter 9, to find the root cause of the problem. Once the cause has been found, changes can be made to reduce the chance of recurrence.

10.4 PROCESS CAPABILITY

Once the stability of a process has been established (i.e., no nonrandom variations are present), it is necessary to determine if the process is capable of producing output that is within an acceptable range. The variability of a process becomes the focal point of the analysis.

Three commonly used terms refer to the variability of process output. Each term relates to a slightly different aspect of that variability, so it is important to differentiate these terms.

Specifications or *tolerances* are established by engineering design or customer requirements. They indicate a range of values in which individual units of output must fall in order to be acceptable.

Specifications A range of acceptable values established by engineering design or customer requirements.

To maximize production of a machine run in a paper mill, the machine's alignment must be correct. If not, performance and quality will be affected, which could result in machine downtime and expensive repairs. The on-board processor calculates the position of the paper in relationship to the machine datum. Two points are then measured on the roller. With the simple press of a button the operator is provided with any deviations on a display panel.

Control limits are statistical limits that reflect the extent to which *sample statistics* such as means and ranges can vary due to randomness alone.

Process variability Natural or inherent variability in a process.

Process variability reflects the natural or inherent (i.e., random) variability in a process. It is measured in terms of the process standard deviation.

Control limits and process variability are directly related: Control limits are based on sampling variability, and sampling variability is a function of process variability. On the other hand, there is *no* direct link between specifications and either control limits or process variability. They are specified in terms of the output of a product or service, not in terms of the *process* by which the output is generated. Hence, in a given instance, the output of a process may or may not conform to specifications, even though the process may be statistically in control. That is why it is also necessary to take into account the *capability* of a process. The term **process capability** refers to the inherent variability of process output *relative to* the variation allowed by the design specifications. The following section describes capability analysis.

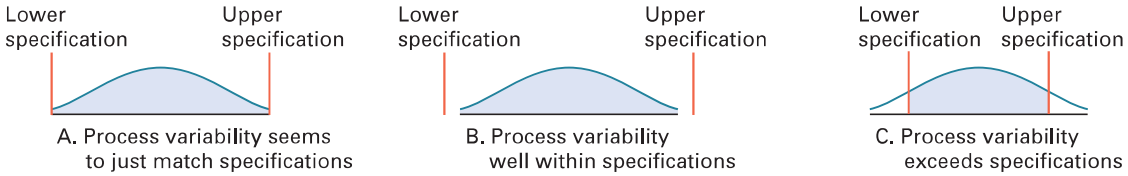
Process capability The inherent variability of process output relative to the variation allowed by the design specification.

Capability Analysis

Capability analysis is performed on a process that is in control (i.e., the process exhibits only random variation) for the purpose of determining if the range of variation is within design specifications that would make the output acceptable for its intended use. If it is within the specifications, the process is said to be "capable." If it is not, the manager must decide how to correct the situation.

Consider the three cases illustrated in Figure 10.15. In the first case, process capability and output specifications are well matched, so that nearly all of the process output can be expected to meet the specifications. In the second case, the process variability is much less than what is called for, so that virtually 100 percent of the output should be well within tolerance. In the third case, however, the specifications are tighter than what the process is capable of, so that even when the process is functioning as it should, a sizable percentage of the output will fail to meet the specifications. In other words, the process could be in control and still generate unacceptable output. Thus, we cannot automatically assume that a process that is in control will provide desired output. Instead, we must specifically check whether a process is *capable* of meeting specifications and not simply set up a control chart to monitor it. A process should be both in control and within specifications *before* production begins—in essence, "Set the toaster correctly at the start. Don't burn the toast and then scrape it!"

FIGURE 10.15 Process capability and specifications may or may not match



In instances such as case C in Figure 10.15, a manager might consider a range of possible solutions: (1) redesign the process so that it can achieve the desired output, (2) use an alternative process that can achieve the desired output, (3) retain the current process but attempt to eliminate unacceptable output u-sing 100 percent inspection, and (4) examine the specifications to see whether they are necessary or could be relaxed without adversely affecting customer satisfaction.

Obviously, process variability is the key factor in process capability. It is measured in terms of the process standard deviation. To determine whether the process is capable, compare ± 3 standard deviations (i.e., 6 standard deviations) of the process to the specifications for the process. For example, suppose the ideal length of time to perform a service is 10 minutes, and an acceptable range of variation around this time is ± 1 minute. If the process has a standard deviation of .5 minute, it would not be capable because ± 3 standard deviations would be ± 1.5 minutes, exceeding the specification of ± 1 minute.

EXAMPLE 7

eXcel

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A manager has the option of using any one of three machines for a job. The processes and their standard deviations are listed below. Determine which machines are capable if the specifications are 10.00 mm and 10.80 mm.

Process	Standard Deviation (mm)
A	.13
B	.08
C	.16

SOLUTION

Determine the extent of process variability (the process width) of each process (i.e., six standard deviations) and compare that value to the specification *difference* of .80 mm.

Process	Standard Deviation (mm)	Process Width
A	.13	.78
B	.08	.48
C	.16	.96

C_p

To assess the capability of a machine or process, a **capability index** can be computed using the following formula:

capability index Used to assess the ability of a process to meet specifications.

LO10.7 Assess process capability.

$$\text{Process capability index, } C_p = \frac{\text{Specification width}}{\text{Process width}} = \frac{\text{Upper specification} - \text{Lower specification}}{6\sigma \text{ of the process}} \quad (10\text{–}10)$$

For a process to be deemed to be capable, it must have a capability index of at least 1.00. However, an index of 1.00 would mean that the process is just barely capable. The current trend is to aim for an index of at least 1.33. An index of 1.33 allows some leeway. Consider driving a car into a garage that has a door opening that is 1 inch wider than the car versus driving into a garage where the door opening is 20 inches wider than the car, and you'll understand

why this book and many companies use 1.33 as the standard for judging process capability instead of 1.00. So use 1.33 as the standard to achieve in judging process capability.

An index of 1.00 implies about 2700 parts per million (ppm) can be expected to not be within the specifications, while an index of 1.33 implies only about 30 ppm won't be within specs. Moreover, the greater the capability index, the greater the probability that the output of a process will fall within design specifications.

EXAMPLE 8

Compute the process capability index for each process in Example 7.

SOLUTION

The specification width in Example 7 is .80 mm. Hence, to determine the capability index for each process, divide .80 by the process width (i.e., six standard deviations) of each machine. The results are shown in the following table:

Process	Standard Deviation (mm)	Process Capability	C_P
A	.13	.78	.80/.78 = 1.03
B	.08	.48	.80/.48 = 1.67
C	.16	.96	.80/.96 = 0.83

We can see that only process B is capable because its index is not less than 1.33. (See Figure 10.15 for a visual portrayal of these results.)

For processes that are not capable, several options might be considered, such as performing 100 percent inspection to weed out unacceptable items, improving the process to reduce variability, switching to a capable process, outsourcing, etc.

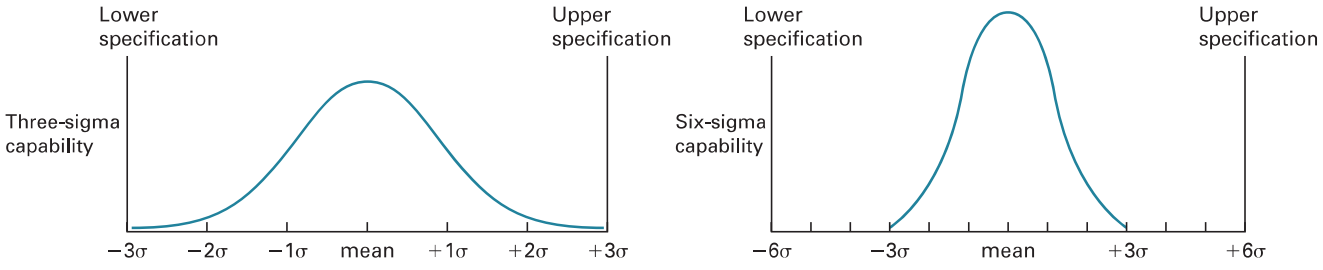
The Motorola Corporation is well known for its use of the term *six sigma,* which refers to its goal of achieving a process variability so small that the design specifications represent six standard deviations above *and* below the process mean. That means a process capability index equal to 2.00, resulting in an extremely small probability of getting any output not within the design specifications. This is illustrated in Figure 10.16.

To get an idea of how a capability index of 2.00 compares to an index of, say, 1.00 in terms of defective items, consider that if the U.S. Postal Service had a capability index of 1.00 for delivery errors of first-class mail, this would translate into about 10,000 misdelivered pieces per day; if the capability index was 2.00, that number would drop to about 1,000 pieces a day.

Care must be taken when interpreting the C_p index, because its computation does not involve the process mean. Unless the target value (i.e., process mean) is *centered* between the upper and lower specifications, the C_p index can be misleading. For example, suppose the specifications are 10 and 11, and the standard deviation of the process is equal to .10. The C_p would seem to be very favorable:

$$\frac{11 - 10}{6(.10)} = 1.67$$

FIGURE 10.16 Three-sigma versus six-sigma capability



However, suppose that the process mean is 12, with a standard deviation of .10; ± 3 standard deviations would be 11.70 to 12.30, so it is very unlikely that *any* of the output would be within the specifications of 10 to 11!

There are situations in which the target value is not centered between the specifications, either intentionally or unavoidably. In such instances, a more appropriate measure of process capability is the C_{pk} index, because it does take the process mean into account.

C_{pk}

If a process is not centered, a slightly different measure is used to compute its capability. This index is represented by the symbol C_{pk}. It is computed by finding the difference between each of the specification limits and the mean, identifying the smaller difference, and dividing that difference by three standard deviations of the process. Thus, C_{pk} is equal to the *smaller* of

$$\frac{\text{Upper specification} - \text{Process mean}}{3\sigma} \tag{10–11}$$

and

$$\frac{\text{Process mean} - \text{Lower specification}}{3\sigma}$$

EXAMPLE 9

A process has a mean of 9.20 grams and a standard deviation of .30 gram. The lower specification limit is 7.50 grams and the upper specification limit is 10.50 grams. Compute C_{pk}.

SOLUTION

1. Compute the index for the lower specification:

$$\frac{\text{Process mean} - \text{Lower specification}}{3\sigma} = \frac{9.20 - 7.50}{3(.30)} = \frac{1.70}{.90} = 1.89$$

2. Compute the index for the upper specification:

$$\frac{\text{Upper specification} - \text{Process mean}}{3\sigma} = \frac{10.50 - 9.20}{3(.30)} = \frac{1.30}{.90} = 1.44$$

The *smaller* of the two indexes is 1.44, so this is the C_{pk}. Because the C_{pk} is more than 1.33, the process is capable.

You might be wondering why a process wouldn't be centered as a matter of course. One reason is that only a range of acceptable values, not a target value, may be specified. A more compelling reason is that the cost of nonconformance is greater for one specification limit than it is for nonconformance for the other specification limit. In that case, it would make sense to have the target value be closer to the spec that has the lower cost of nonconformance. This would result in a noncentered process.

Improving Process Capability

Improving process capability requires changing the process target value and/or reducing the process variability that is inherent in a process. This might involve simplifying, standardizing, making the process mistake-proof, upgrading equipment, or automating. See Table 10.5 for examples.

Improved process capability means less need for inspection, lower warranty costs, fewer complaints about service, and higher productivity. For process control purposes, it means narrower control limits.

TABLE 10.5
Process capability improvement

Method	Examples
Simplify	Eliminate steps, reduce the number of parts, use modular design
Standardize	Use standard parts, standard procedures
Make mistake-proof	Design parts that can only be assembled the correct way; have simple checks to verify a procedure has been performed correctly
Upgrade equipment	Replace worn-out equipment; take advantage of technological improvements
Automate	Substitute automated processing for manual processing

Taguchi Loss Function

Gnocchi Taguchi, a Japanese quality expert, holds a nontraditional view of what constitutes poor quality, and hence the cost of poor quality. The traditional view is that as long as output is within specifications, there is no cost. Taguchi believes that any deviation from the target value represents poor quality, and that the farther away from target a deviation is, the greater the cost. Figure 10.17 illustrates the two views. The implication for Taguchi is that reducing the variation inherent in a process (i.e., increasing its capability ratio) will result in lowering the cost of poor quality, and consequently, the loss to society.

Limitations of Capability Indexes

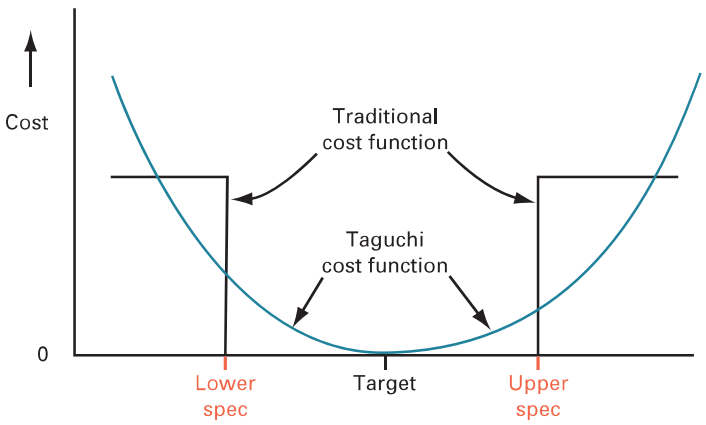
There are several risks of using a capability index:

1. The process may not be stable, in which case a capability index is meaningless.
2. The process output may not be normally distributed, in which case inferences about the fraction of output that isn't acceptable will be incorrect.
3. The process is not centered but the C_p index is used, giving a misleading result.

10.5 OPERATIONS STRATEGY

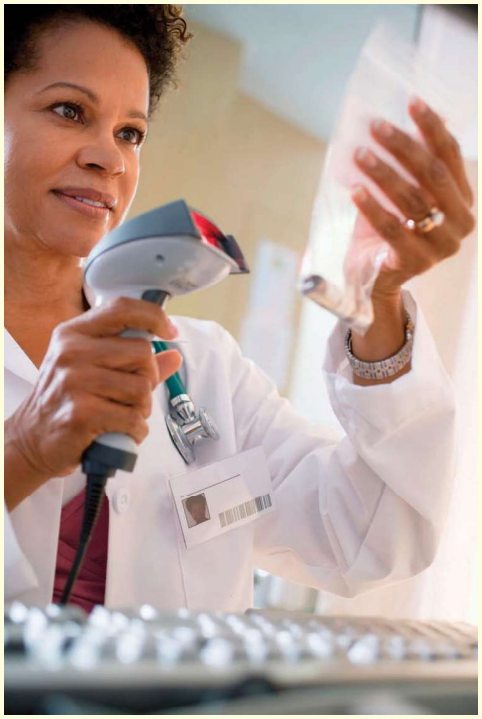
Quality is a major consideration for virtually all customers, so achieving and maintaining quality standards is of strategic importance to all business organizations. Quality assurance and product and service design are two vital links in the process. Organizations should continually seek to increase the capability of the processes they use, so that they can move from a position of using inspection or extensive use of control charts to achieve desired levels of quality to one where quality is built into products and processes, so that little or no effort is needed to assure quality. Processes that exhibit evidence of nonrandomness, or processes that are deemed to not be capable, should be viewed as opportunities for continuous process improvement.

FIGURE 10.17
Taguchi and traditional views of the cost of poor quality



Bar Codes Might Cut Drug Errors in Hospitals

READING



It's estimated that more than 7,000 hospital patients die each year because of drug errors, and many others suffer ill effects from being given the wrong drug or the wrong dosage. Some hospitals are using bar codes attached to patients' wristbands that allow hospital personnel who administer drugs to patients to electronically check to make sure the drug and dosage are appropriate. Before administering a drug, the doctor or nurse scans the bar code attached to the patient to see what drug is needed and when, and then the drug's bar code is scanned to verify that the medication is correct.

But bar codes are not foolproof, as a recent study of hospitals showed. Nurses may develop a workaround that involves using photo copies of a group of patients' bar codes which are then used to obtain drugs for the entire group. The nurse would then have a tray that may contain drugs of different dosages intended for different patients. At that point, the bar code protection has been circumvented.

Questions

1. Why are bar codes being used in hospitals?
2. What action would you suggest to avoid the problem of workarounds?

Source: Based on "Bar Codes Might Cut Drug Errors," *Rochester Democrat and Chronicle,* March 14, 2003, p. 9A; and "Bar Codes Are Not Foolproof in Hospitals, says Study," *Rochester Democrat and Chronicle,* July 3, 2008, p. 3A.

SUMMARY

This chapter describes inspection and statistical process control. Inspection means examining the output of a process to determine whether it is acceptable. Key issues in inspection include where to inspect in the process, how often to inspect, and whether to inspect on-site or in a laboratory.

Statistical process control focuses on detecting departures from randomness in a process. Two basic tools of process control are control charts and run tests. Figure 10.18 gives an overview of quality control. The general theory of control charts is discussed, and four types of control charts—two for variables and two for attributes—and two types of run tests are described in the chapter. The chapter ends with a discussion of process capability. Process capability studies are used to determine if the output of a process will satisfy specifications. They can provide valuable information for managers in terms of reducing costs and avoiding problems created by generating output that is not within specifications. Table 10.6 provides a summary of formulas.

KEY POINTS

1. All processes exhibit random variation. Quality control's purpose is to identify a process that also exhibits nonrandom (correctable) variation on the basis of sample statistics (e.g., sample means) obtained from the process.
2. Control charts and run tests can be used to detect nonrandom variation in sample statistics. It is also advisable to plot the data to visually check for patterns.
3. If a process does not exhibit nonrandom variation, its capability to produce output that meets specifications can be assessed.

KEY TERMS

assignable variation, 418
attributes, 422
capability index, 437
c-chart, 429
central limit theorem, 418
control chart, 420
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