Numerical Methods in Biomechanics

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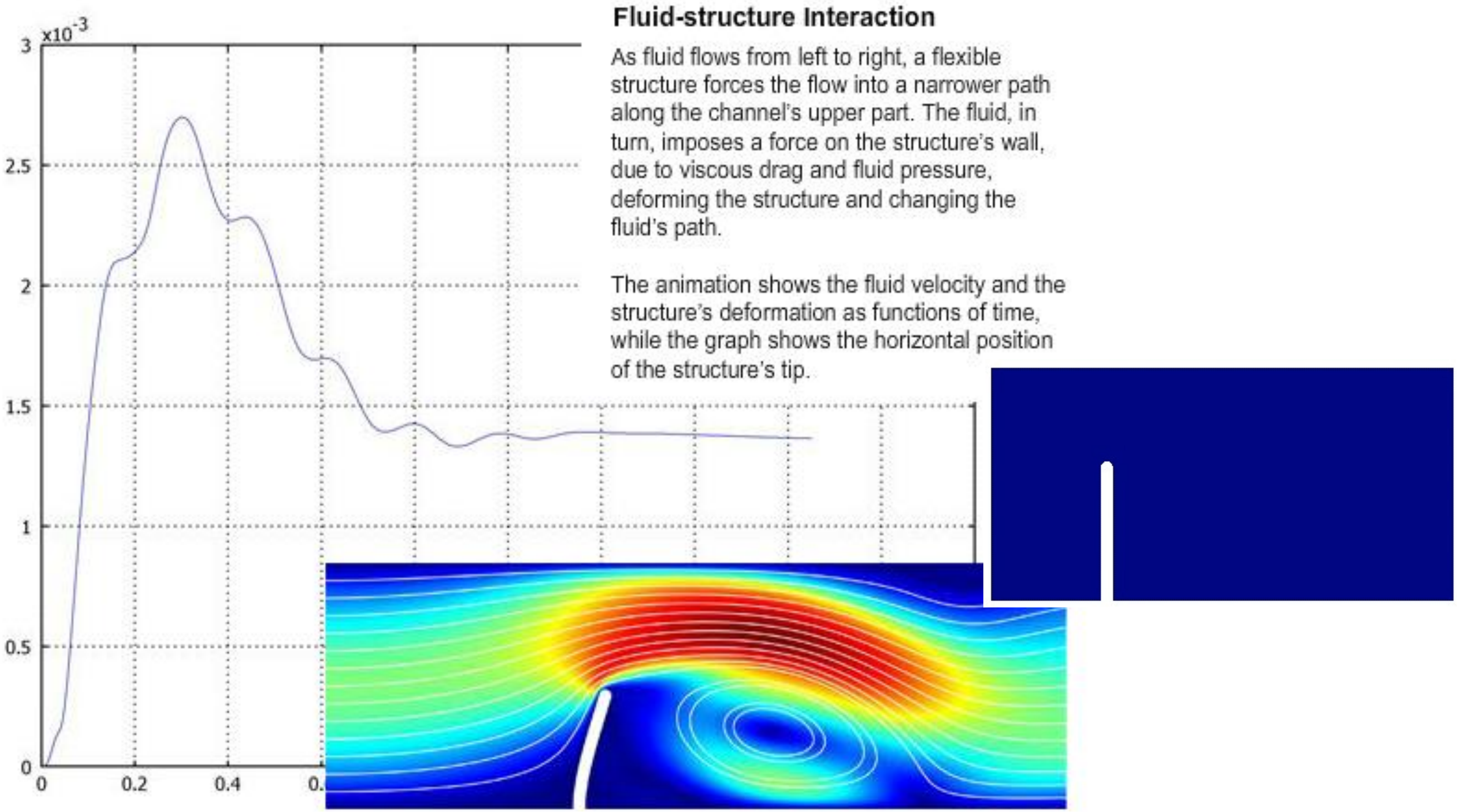
Content - schedule

W01: Organization, introduction and examples
W02: External lecturers
W03: Partial Differential Equations
W04: Solid mechanics in numerical biomechanics
W05: Fluid mechanics in numerical biomechanics
W06: Midterm project presentations
W07: The Finite Element Method (FEM), and its extensions
W08: Midterm evaluation
W09: Multiphysics, coupling, Comsol
W10: Multiphysics example 1
W11: Multiphysics example 2
W12: Multiphysics example 3
W13: Final project presentation

Fluid-structure example



www.comsol.com/model/fluid-structure-interaction-361

Physics

- Solid mechanics
- Fluids mechanics (Navier-Stokes)
- Heat: conduction, convection, radiation
- Transport: advection, diffusion, reaction
- Electrodynamics (Maxwell)
- Adaptation law of living tissues

Physics coupling

In some situations (experiments), physical (and chemical) phenomena are strongly interdependent and Physics (PDE) must be coupled to get a reasonable solution

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multiphysics modeling

Coupling in biomechanics

- Cardiovascular (fluid-solid)
- Orthopedics (solid-solid contacts)
- Tissue engineering (poroelasticity)
- Cell mechanics (advection-diffusion-reaction)
- Medical devices (EM, heat, piezo)
- Adaptive process (bone, muscle, arteries)

Where appears the coupling?

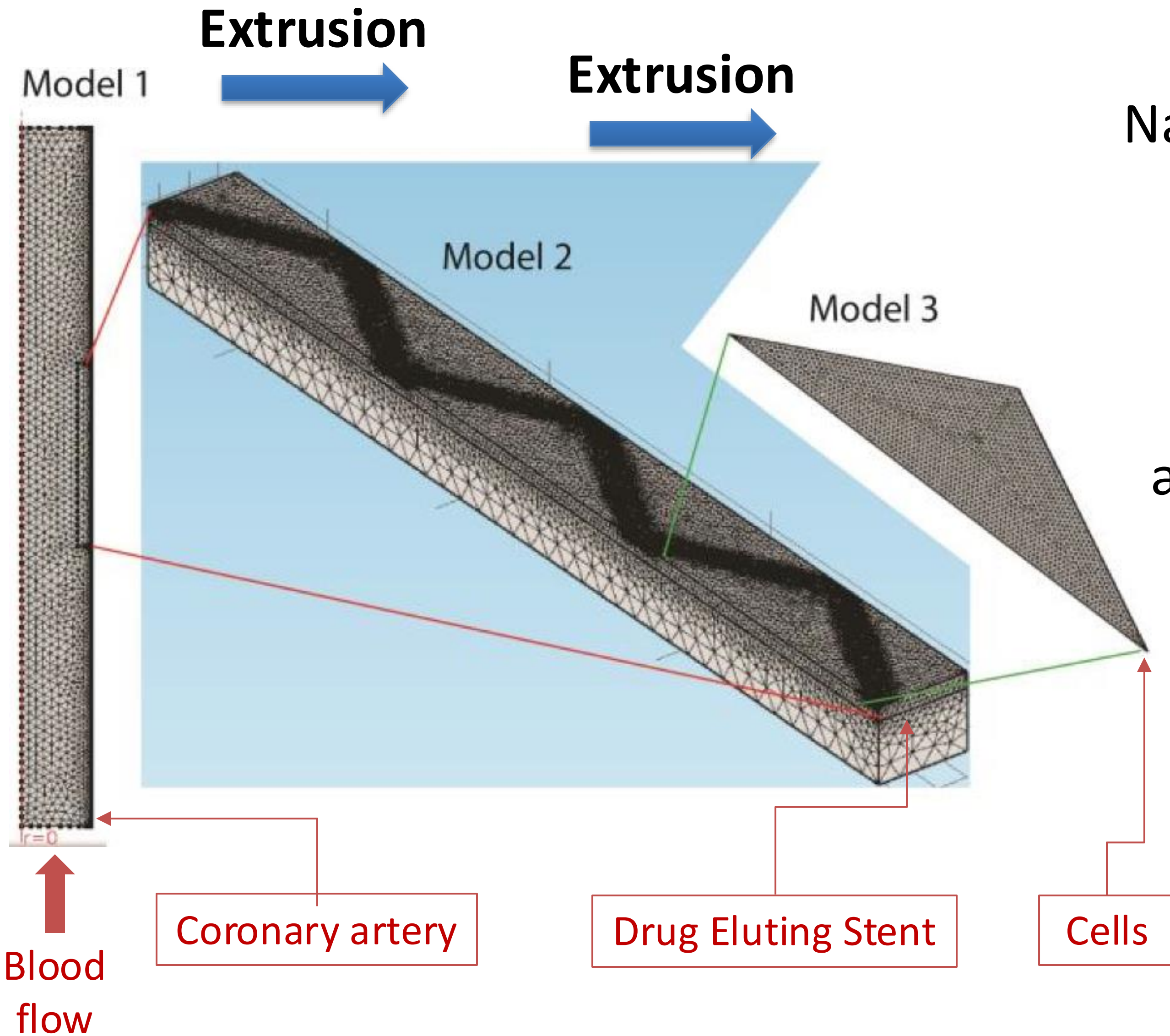
Coupling between variables of different physics in:

- PDE (convection, poroelasticity)
- Constitutive laws (non-isothermal flow)
- Boundary conditions (fluid-solid interaction)
- Contacts between solids
- Other (medical devices control, MEMS)

Types of coupling

- Coupling between physics (field)
- Coupling within materials (constitutive laws)
- Coupling at interfaces
- Coupling operators (modeling, not physical)
  (modeling simplification between source and destination sub-models)
  - Extrusion (2D → 3D)
  - Projection (3D → 2D)
  - Integration (2D/3D → 1D)

Extrusion operator



Model 1

Navier-Stokes (2D axisym.)

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fluid flow

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Model 2

advection-diffusion (3D)

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drug concentration

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Model 3

ODE (1D)

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cell reaction

Drug Eluting Stent Modelling. Elena Cambria, Gaëlle Minger, Aline Zbinden. Project course Numerical Methods in Biomechanics, 2013

Protein release in flow chamber

- Flow chamber 2D (500x250 µm)
- Cell layer (50 µm, 50µm from inlet)
- Diffusion: D = 100 $\mu m^2/s$
- Protein release = 10^{-6} $mol/(m^2 s)$
- Laminar flow (u_{avg} = 1 mm/s)

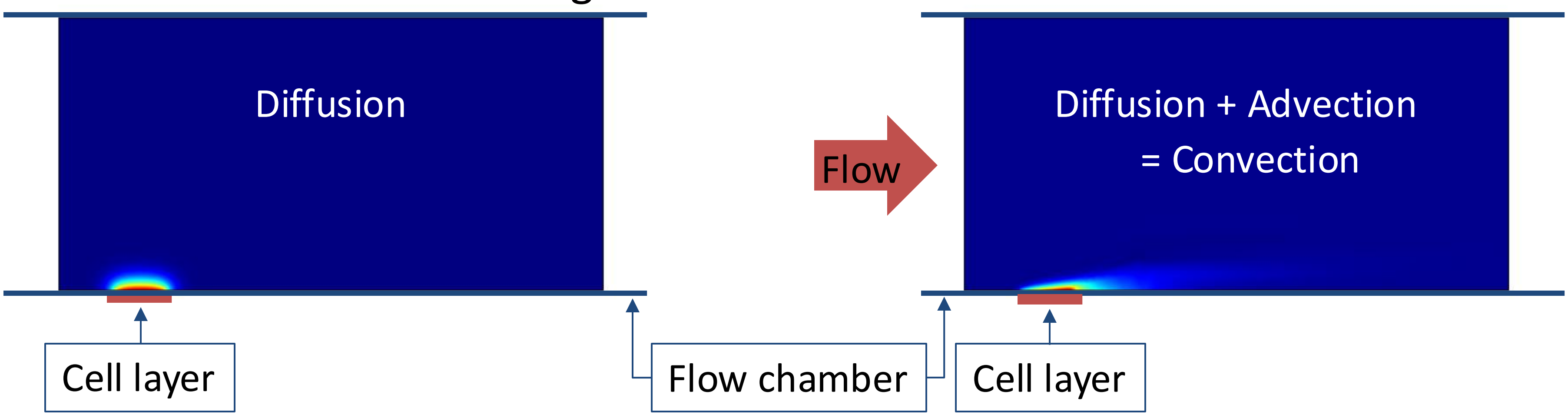
Protein concentration

$$\frac{\partial c}{\partial t} + \nabla \cdot (-D \nabla c) = 0$$

$$-\boldsymbol{n} \cdot \boldsymbol{N} = N_{0,c}$$

Advection

$$\frac{\partial c}{\partial t} + \nabla \cdot (-D \nabla c) + u \cdot \nabla c = 0$$



Coupling modeling

- 1-way coupling
  - One physics influence the other one, but it is not reciprocal
- 2-way coupling
  - Both physics influence each other

- Weak coupling: slight coupling effect
- Strong coupling: important coupling effect

Coupling modeling

- 1-way coupling
  - Thermal expansion: T → ε
  - Viscosity: friction → T
- 2-way coupling
  - Joule heating: j ↔ T (through conductivity)
- 1 or 2-way coupling
  - Fluid-structure interaction (FSI): v → ε, ε → v, v ↔ ε

Numerical techniques in coupling

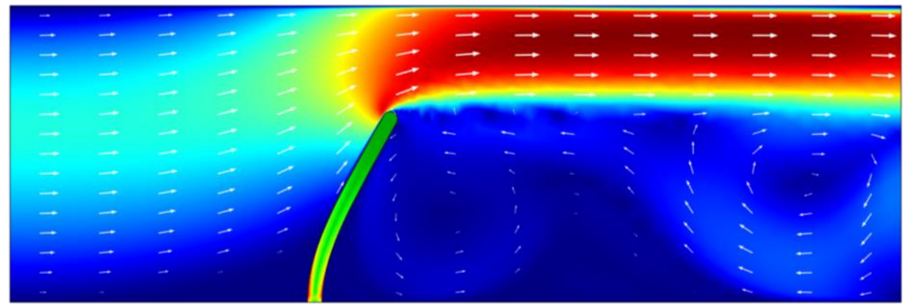
Multi-physics solvers

- One unique solver for several physics
  - No additional approximation
  - Same mesh, same time increment
  - Physics can’t be optimized separately (Fluid-Solid)
  - Thermo-mechanics (one more degree of freedom: T)
- Several mono-physics solvers talking to each others
  - Fully coupled solver
  - Segregated solver

Fully coupled vs. segregated solver

- Fully coupled solver
  - Solves “exactly” (fixed accuracy) **simultaneously**
  - Iterative methods
- Segregated solver
  - Solves “approximately” (substeps solving) **sequentially**
  - Efficient for weak coupling
  - Use less memory
  - Slow or no convergence if strong nonlinearity

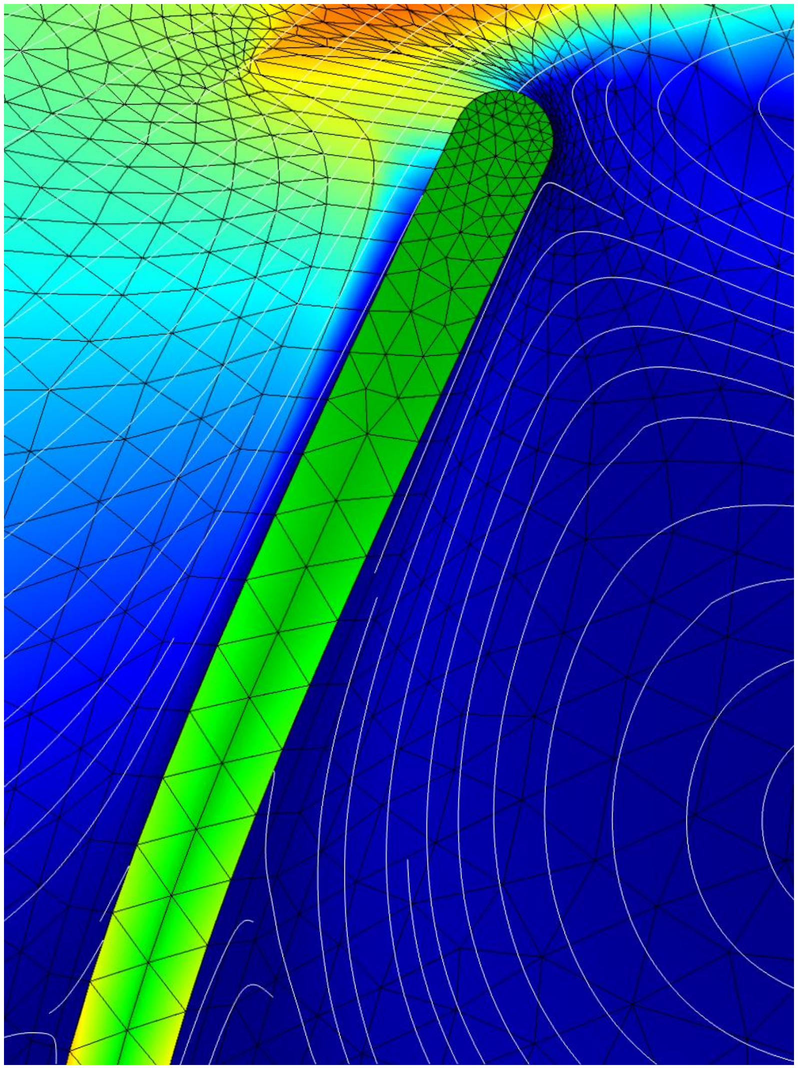
Fluid-Structure Interaction



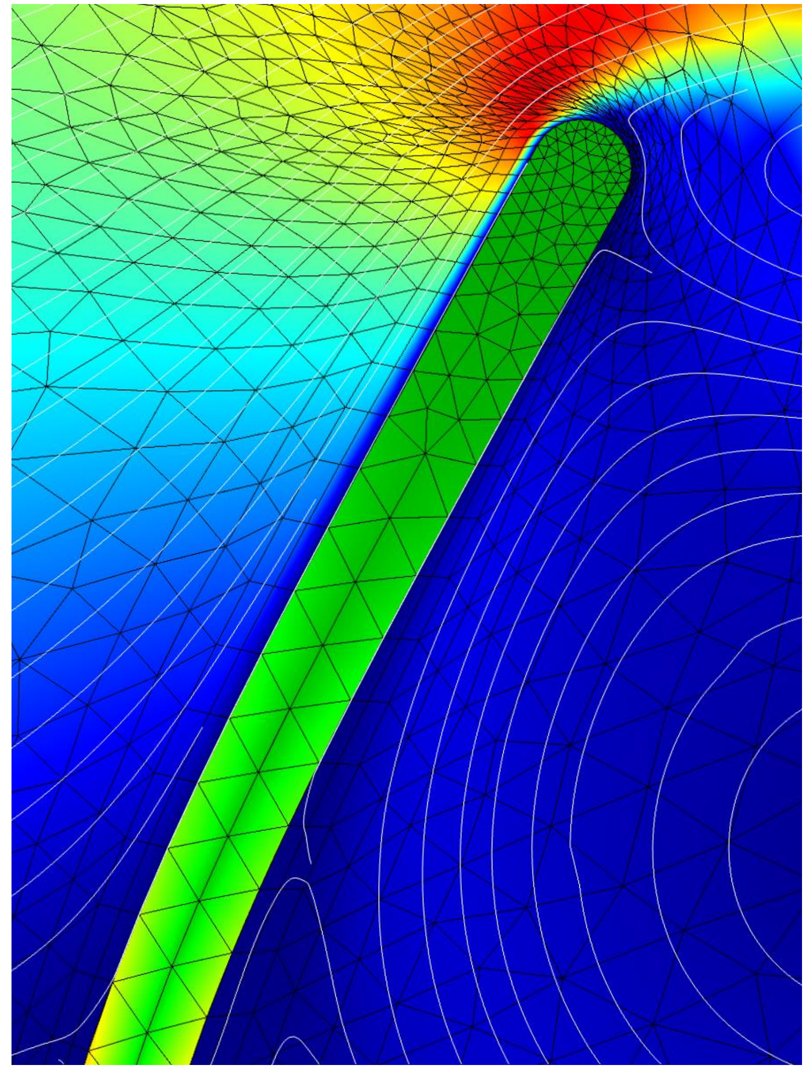
- Geometry:
  - 2D
  - canal 30 x 10 mm
  - obstacle 0.5 x 7 mm (0.25 mm fillet), 10 mm from inlet
- Material:
  - Water: ρ = 1e3 [kg/m^3], μ = 1e-3 [Pa s]
  - Medical silicone: E = 10 MPa, ν = 0.4, ρ = 1e3 kg/m^3
- Flow: Inlet 500 mm/s (average laminar), outlet P = 0

Mesh deformation type

Winslow (t=6.7s)

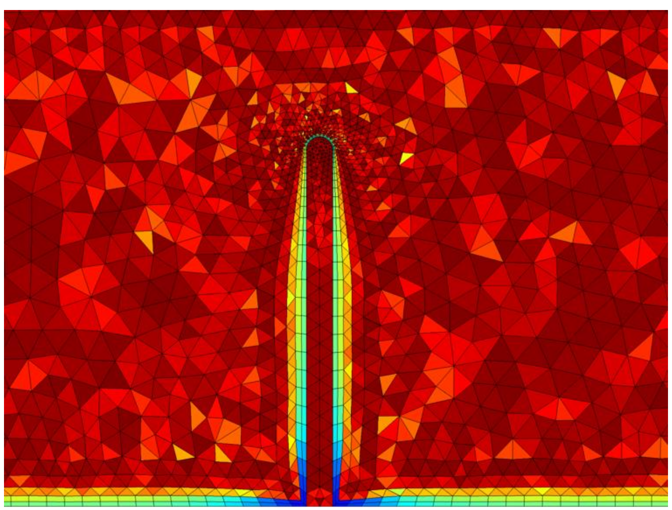


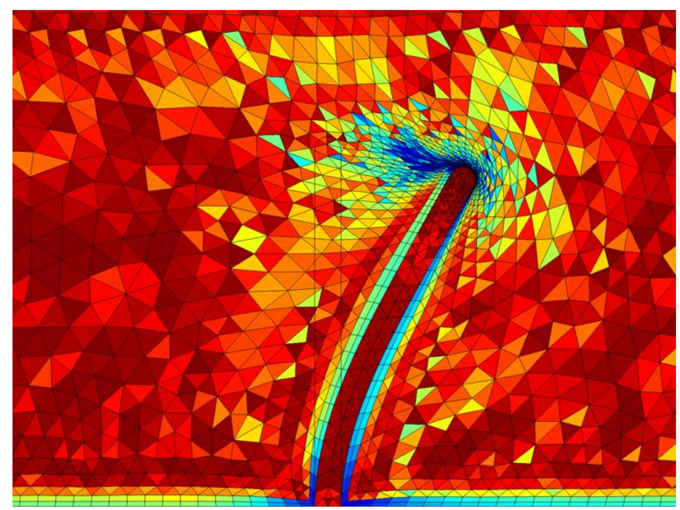
Hyperelastic (t=10 s)



Fluid-Structure Interaction

- Arbitrary Lagrangian-Eulerian (ALE) method
  - Solid → Lagrangian description (material frame)
  - Fluid → Eulerian description (spatial frame)
- Mesh degradation: stop, mesh deformed, map last results on new mesh, continue





Optimization of Dielectrophoretic Cell Trapping in a Microfluidic Channel

Claire AMADIO[1]; Alexandre KEHREN[1]; Hadrien MERIAH[2]

1. Bioengineering; 2. Mechanical.

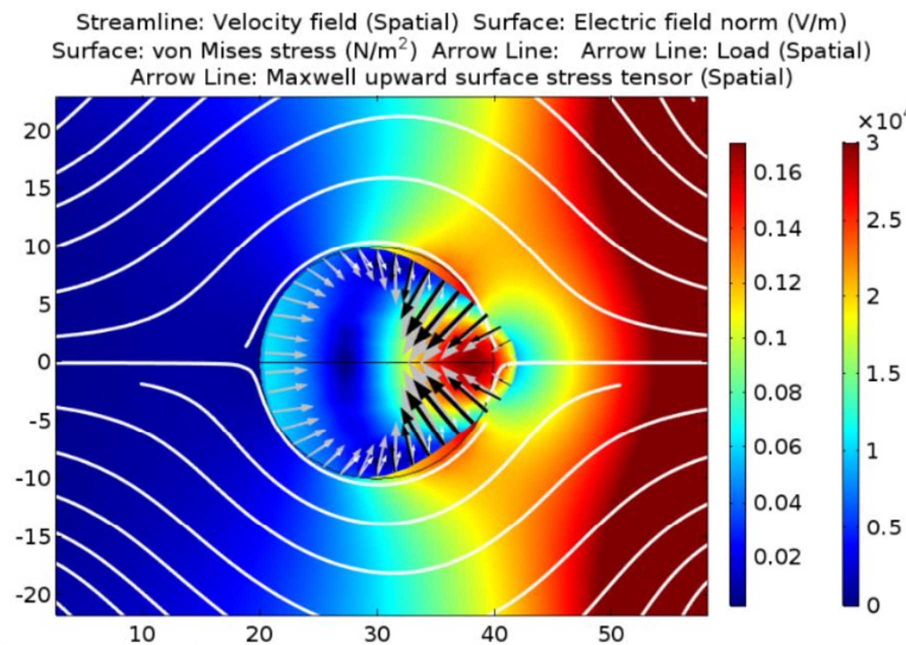


Figure 3.10: Visualization of the cell deformation under a DC electrical field. The arrows represent the different surface loads : fluid (white), DEP from the electrical field (black) and total (grey). The white lines represent flow field, and the rainbow coloring the electrical field.