Numerical Methods in Biomechanics

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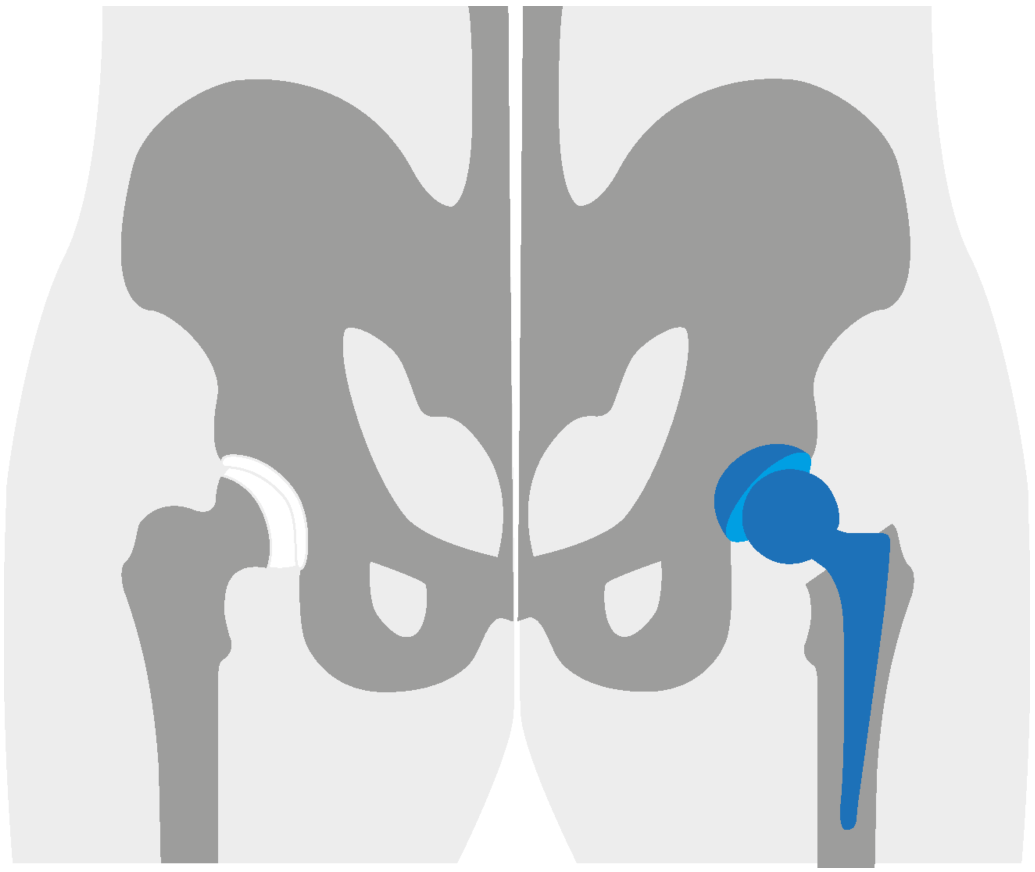
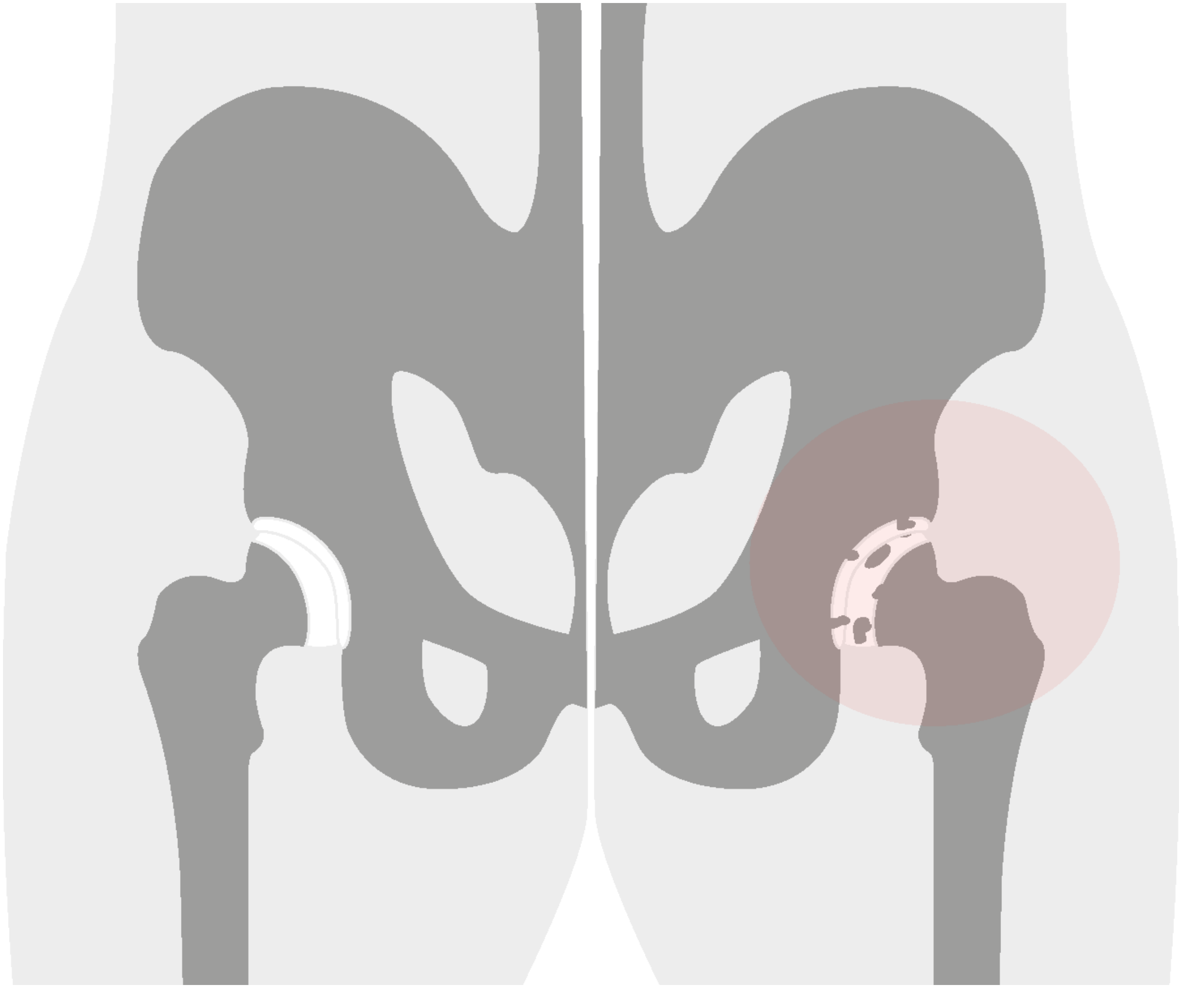


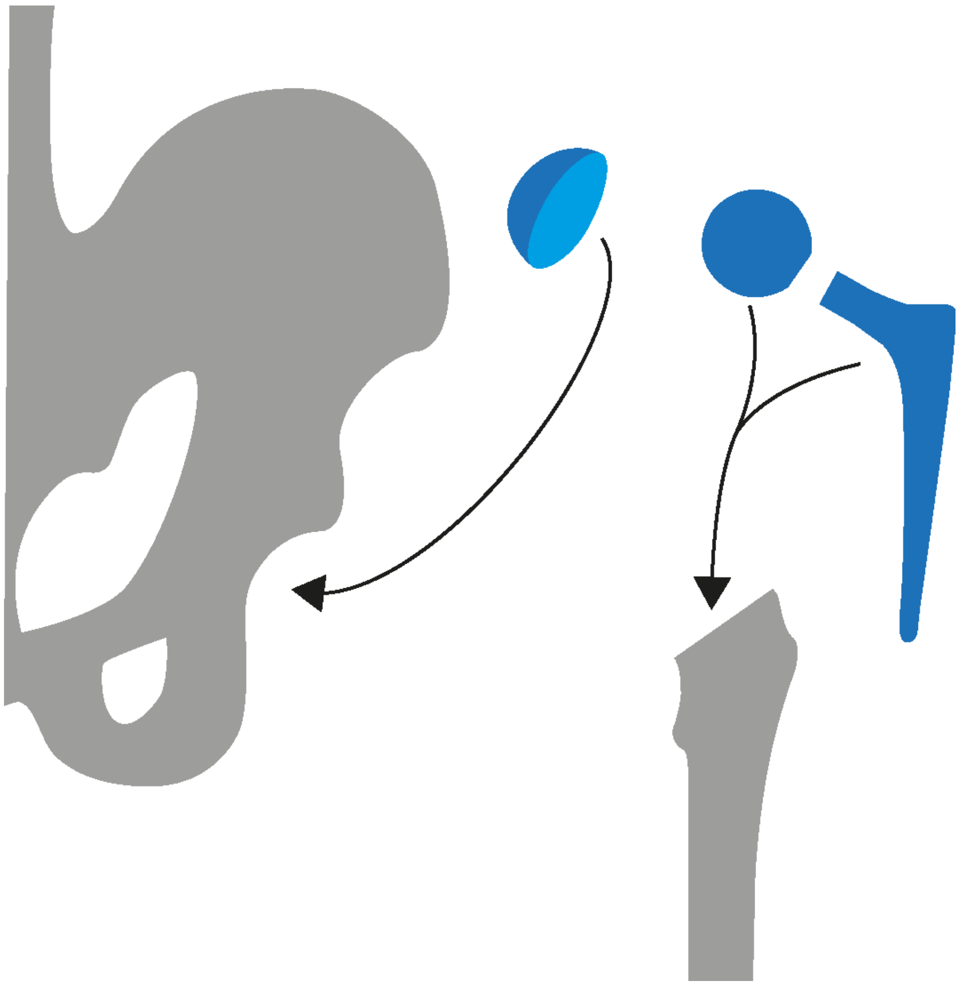
Numerical modeling to investigate aseptic loosening of hip implants

Valérie Malfroy Camine

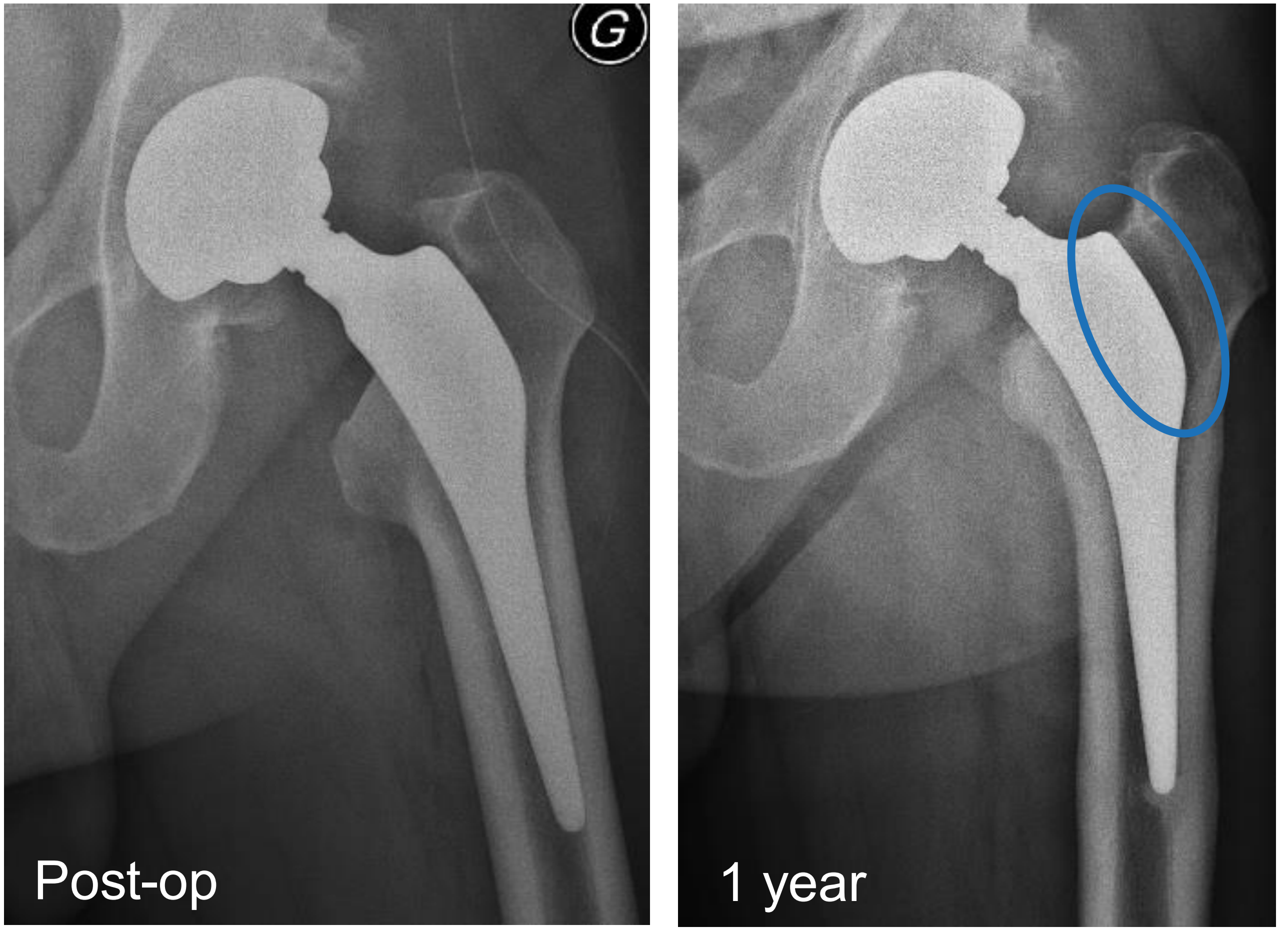
Osteoarthritis



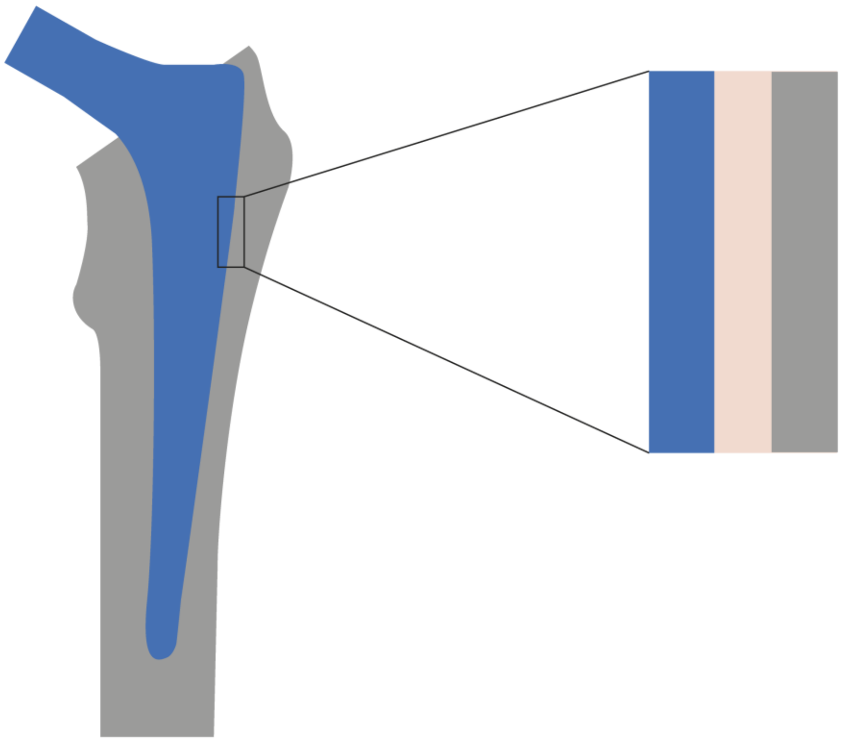
Total Hip Replacement



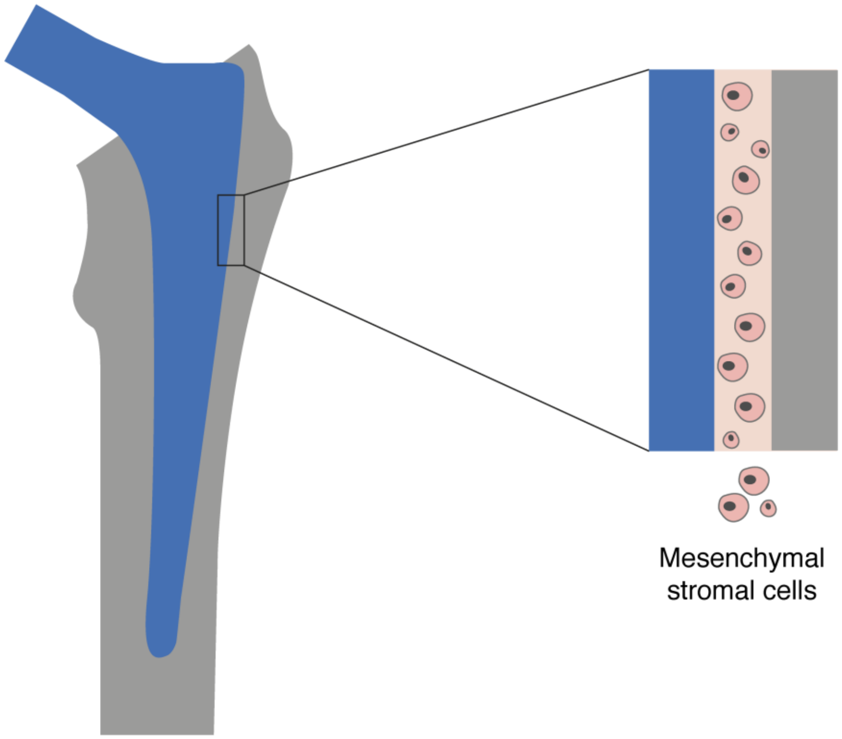
Aseptic Loosening



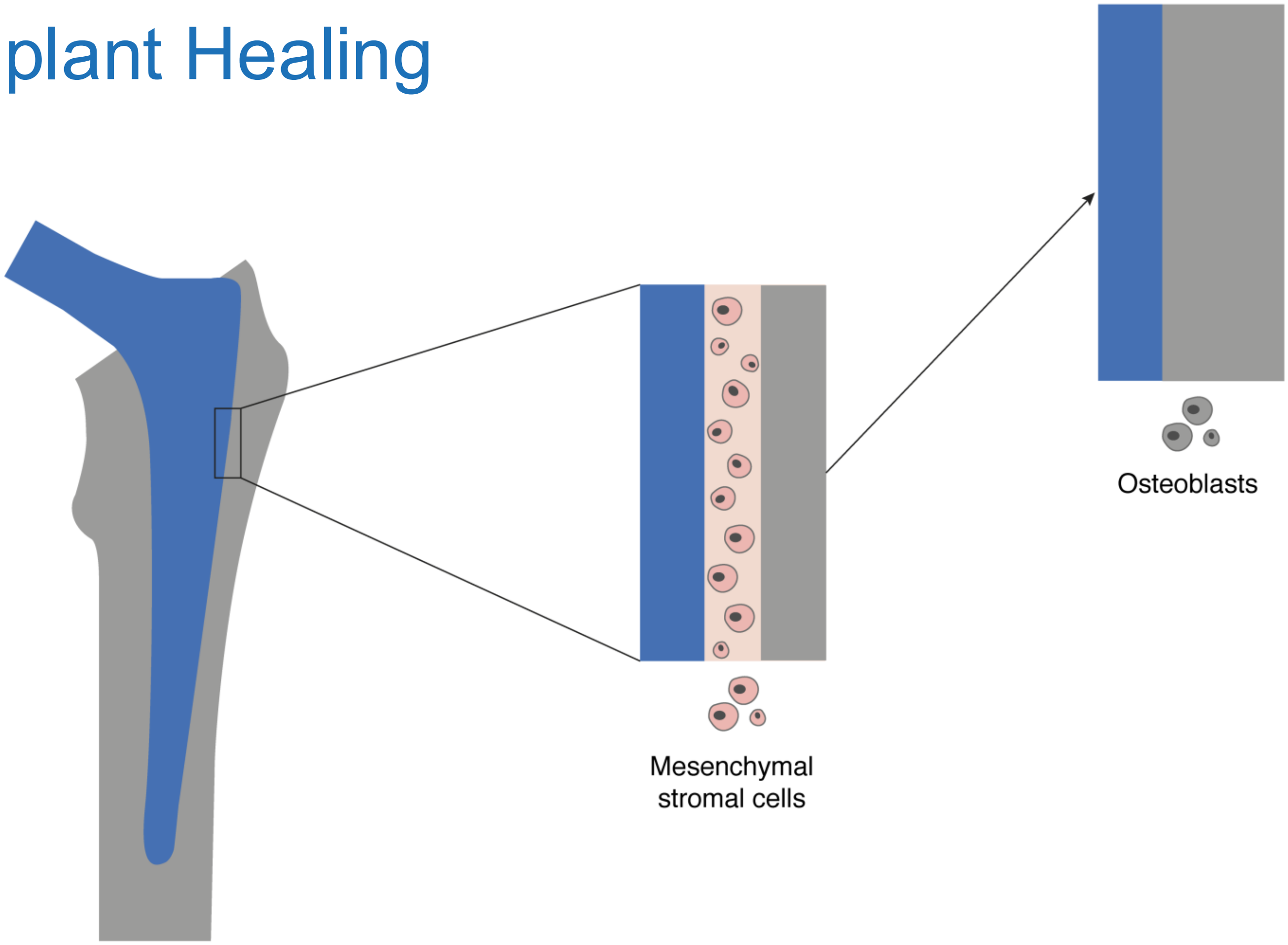
Peri-implant Healing



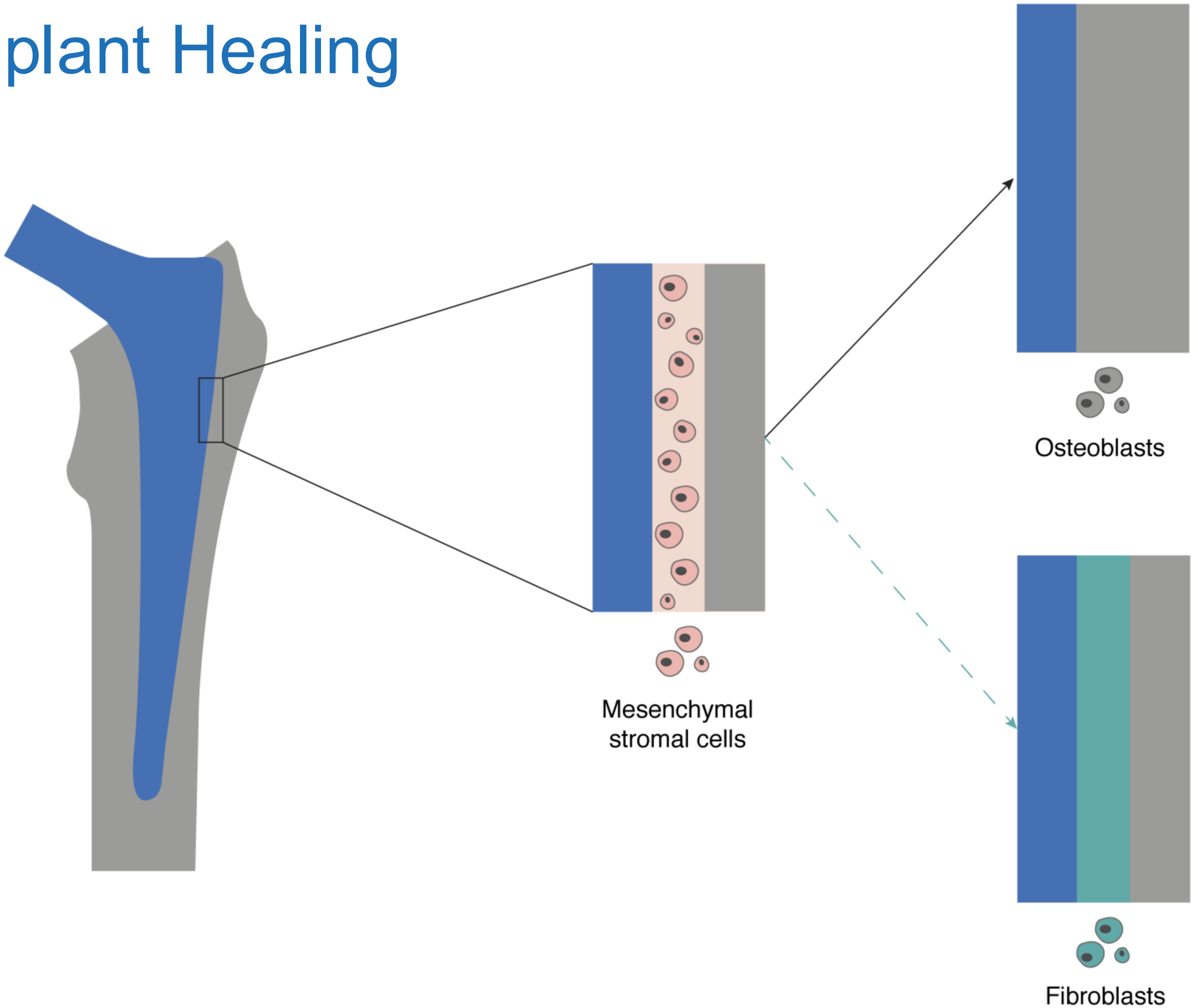
Peri-implant Healing



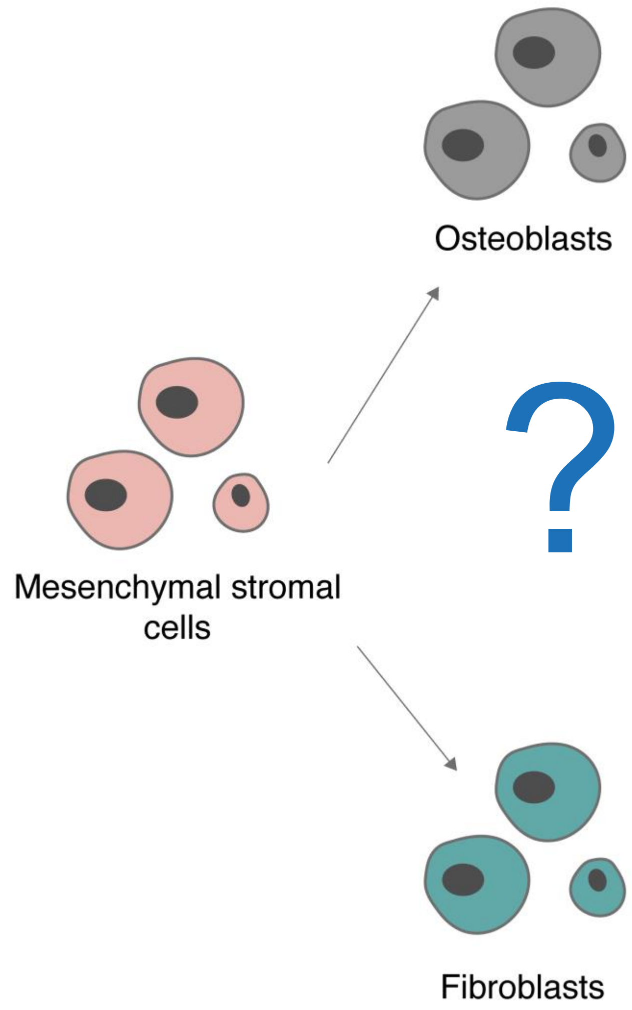
Peri-implant Healing



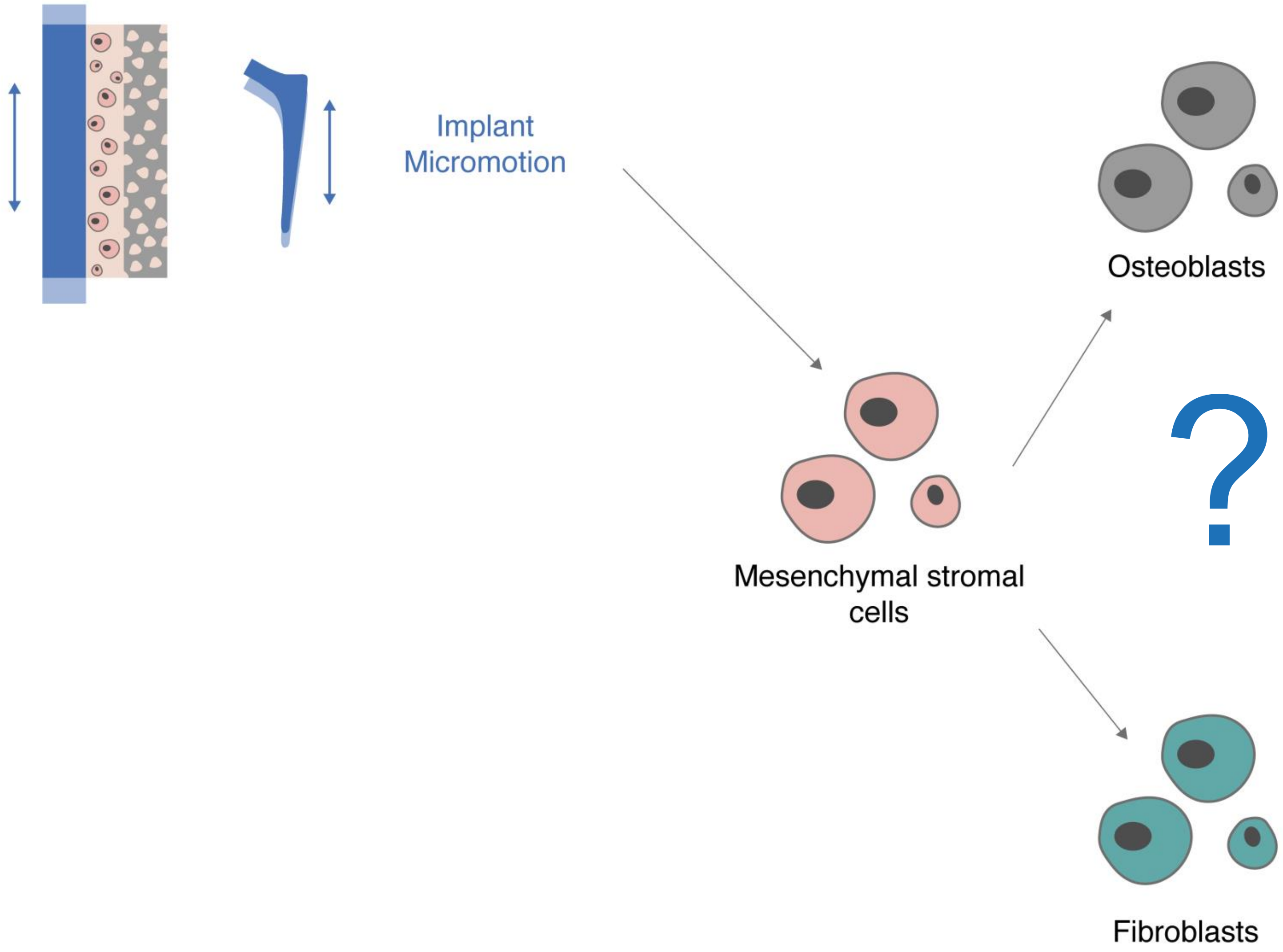
Peri-implant Healing



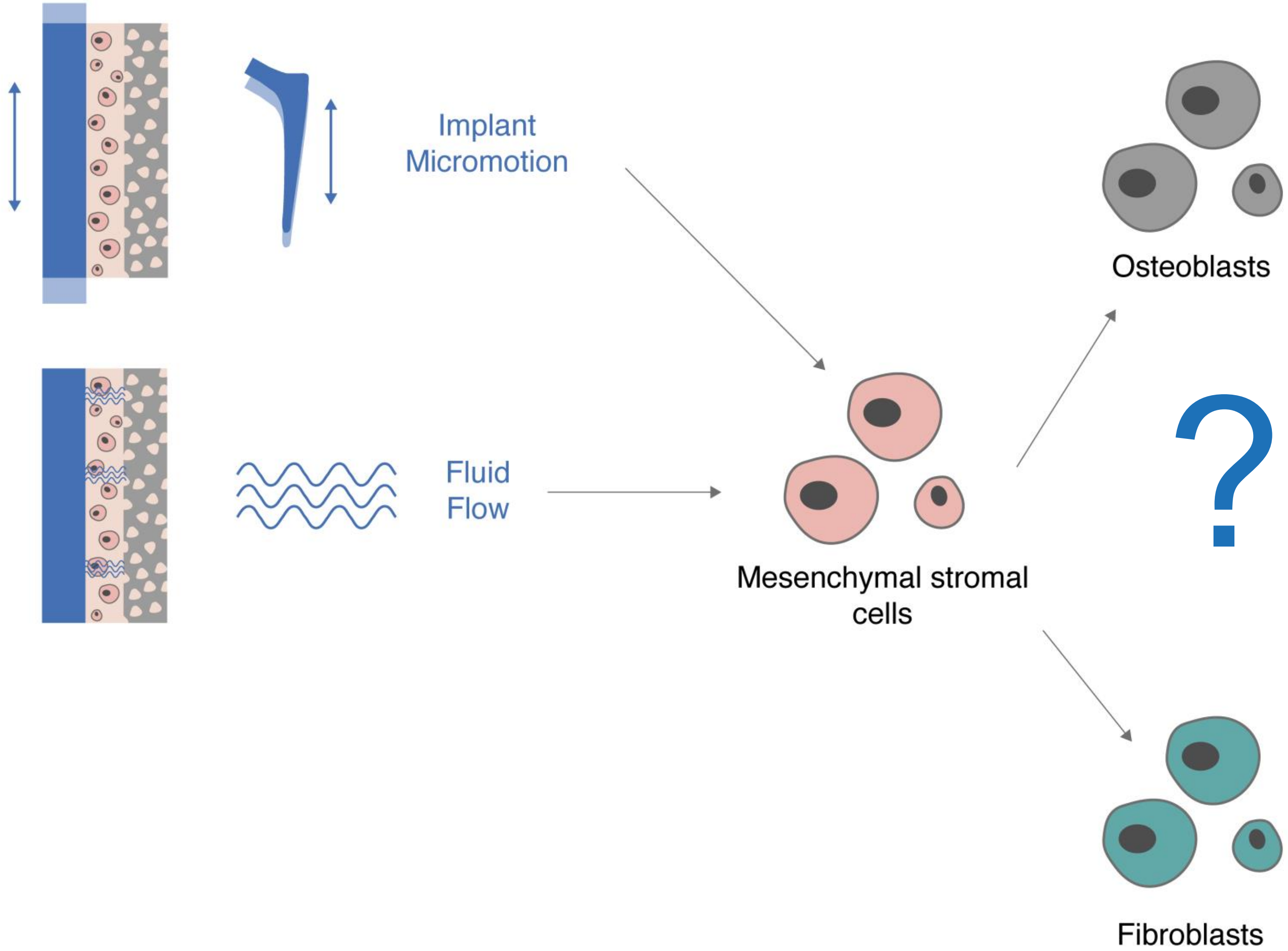
MSCs Differentiation



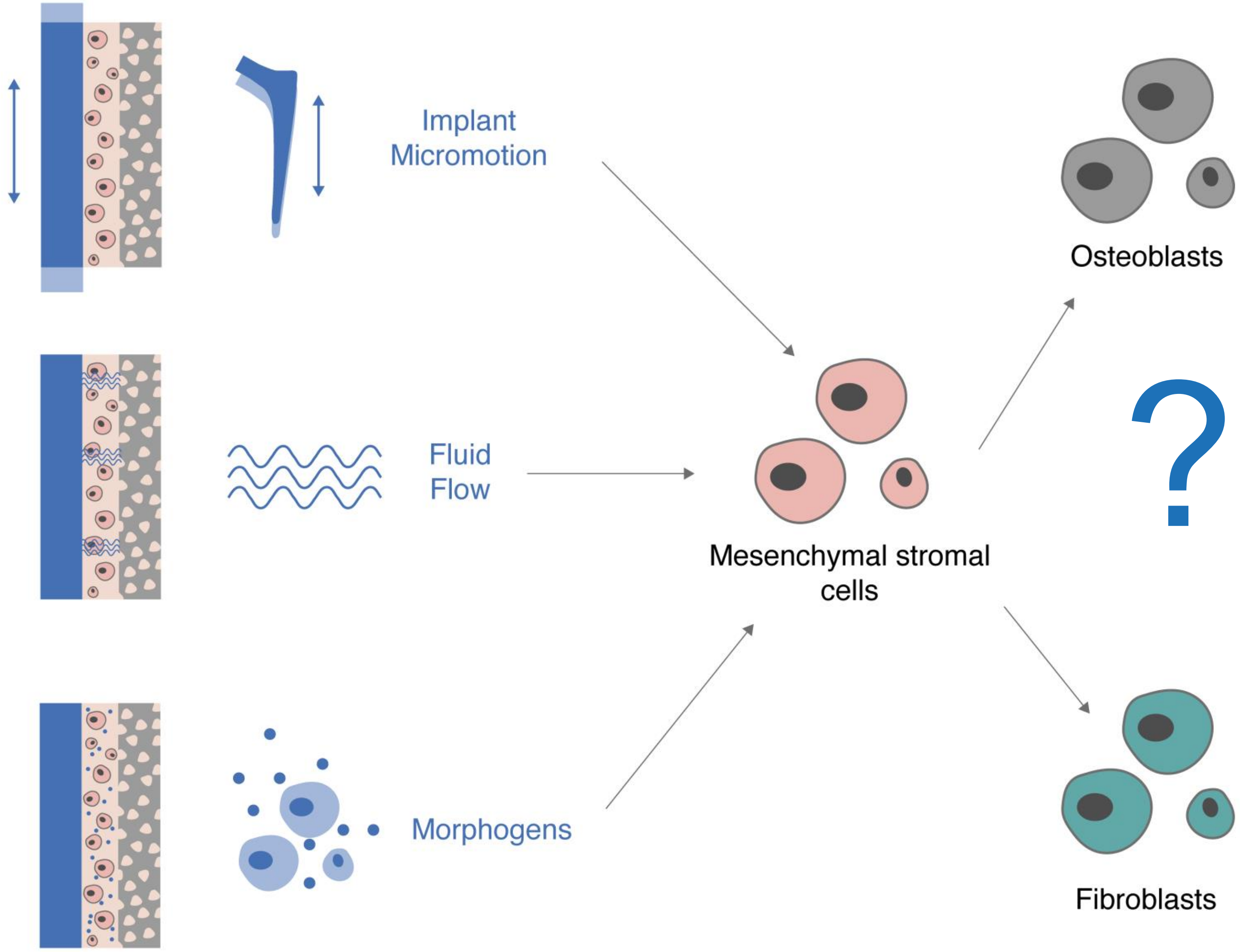
MSCs Differentiation



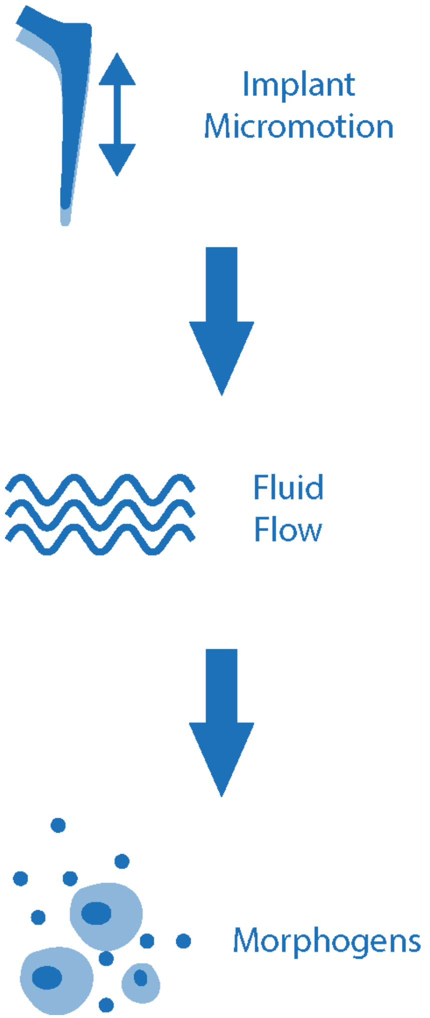
MSCs Differentiation



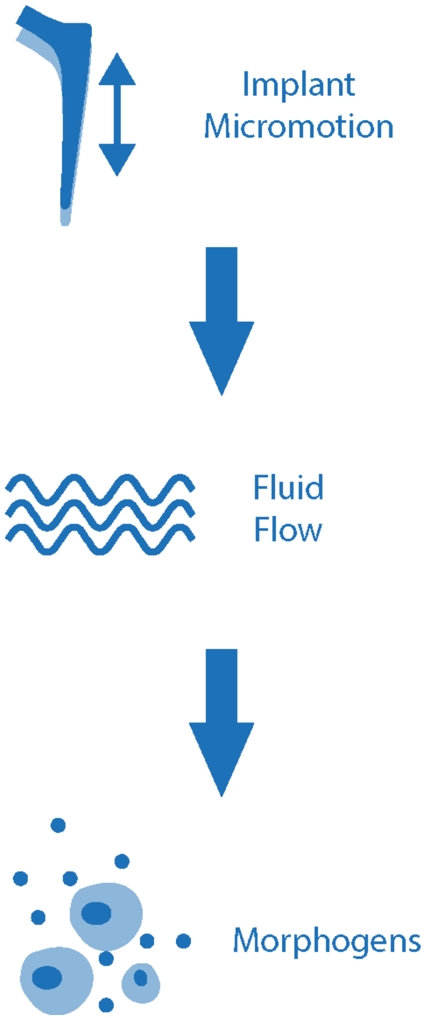
MSCs Differentiation



Hypothesis



Modeling Strategy



Quantify implant micromotion

Estimate micromotion-induced fluid flow in peri-implant tissue

Study the effects of fluid on morphogens distribution and MSCs differentiation

Quantify
implant micromotion

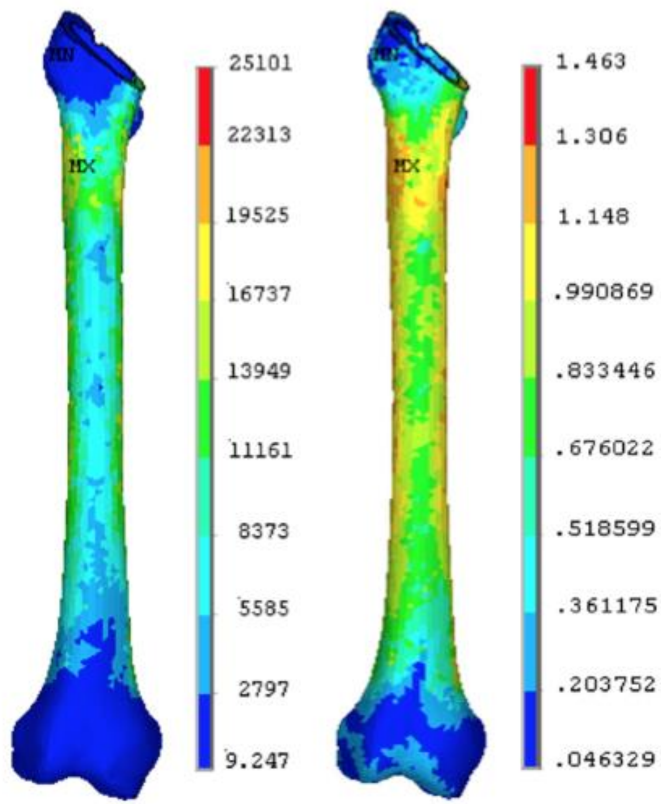
Geometry

Abdul-Kadir 2008

Bone: From patients CT-scans

Implant: From CAD file

Material



Elastic Modulus

Mineral Density

Bone: Linear elastic with an elastic modulus dependent on mineral density obtained from the CT scan

Implant: Linear elastic (titanium alloy)

Reggiani 2007

Governing Equations

Solid Mechanics

Navier's equation for solid:

$$-\nabla \sigma = F$$

Hooke's law constitutive equation for linear elastic material:

$$\sigma = C : \varepsilon$$

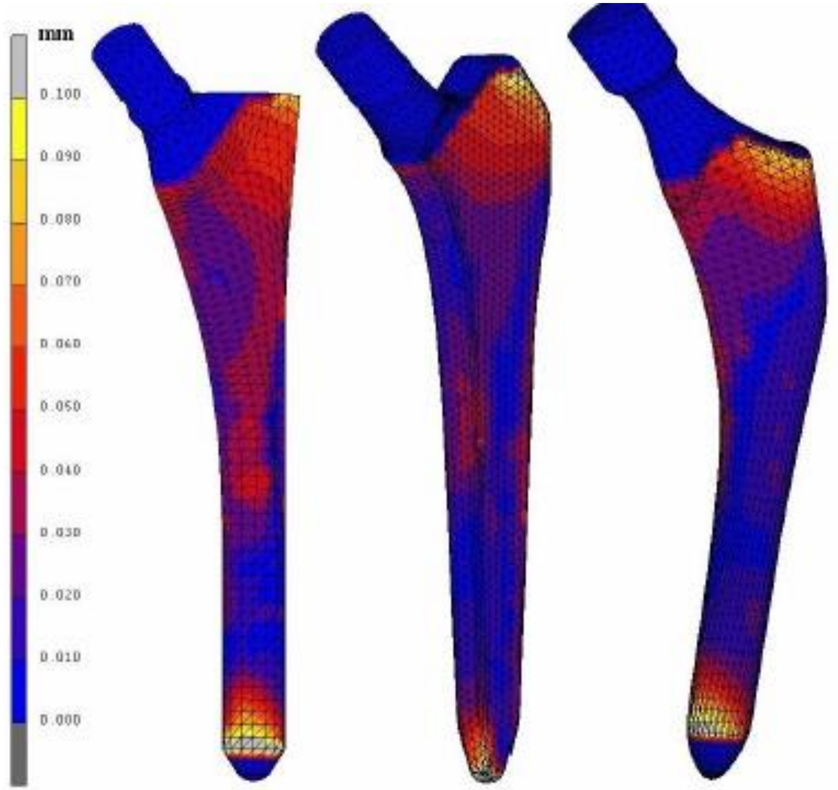
Boundary Conditions

Reggiani 2007

Loads and constraints:

- Experimental measurements in instrumented prostheses
- ISO standards for implant testing

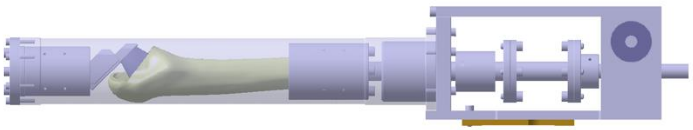
Results



Micromotion extends locally from a few µm to 100 µm

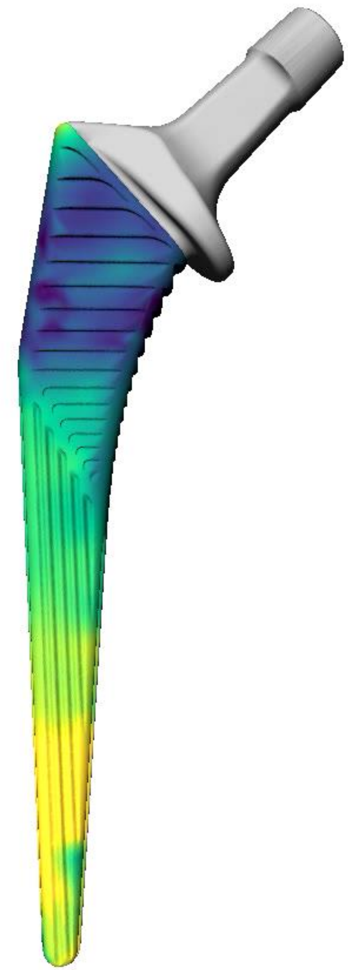
Abdul-Kadir 2008

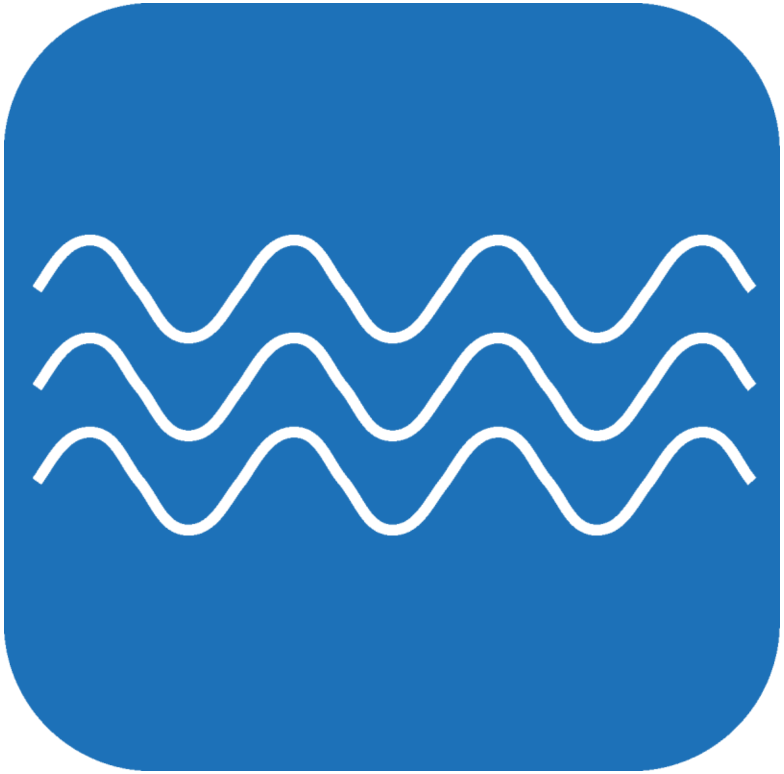
Experimental Validation



Experimental loading of implanted cadaveric femur

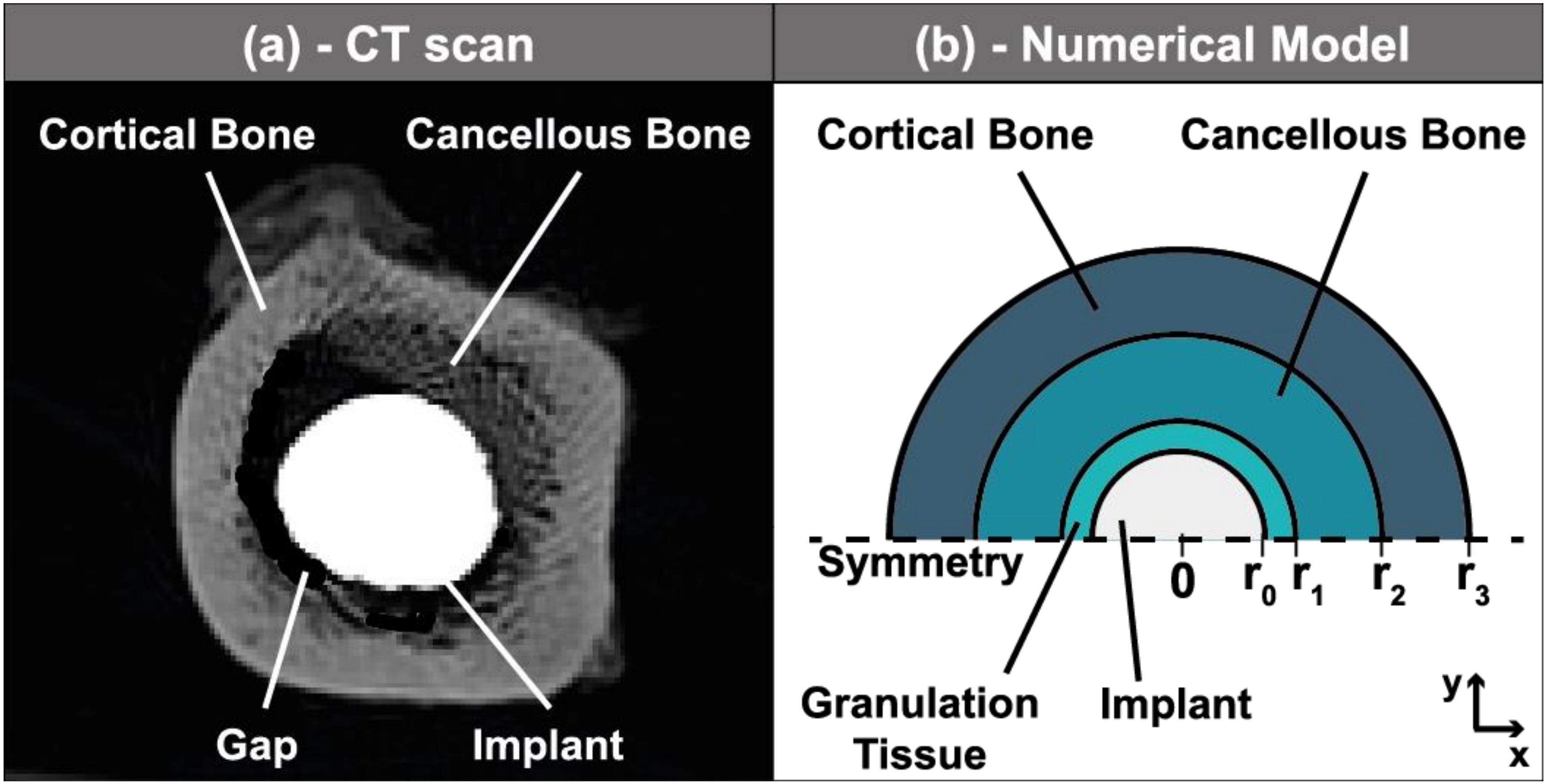
Analyze the displacements of radiopaque markers in a μ-CT scan





Quantify micromotion-induced fluid flow with a poroelastic FE model

Geometry



http://dx.doi.org/10.1080/10255842.2017.1296954

Governing Equations

Biot's poroelasticity

Darcy's (fluid) velocity

Navier's equation for solid:

$$-\nabla\sigma = 0$$

Darcy's law combined with continuity equation:

$$S\frac{\partial p_f}{\partial t} + \nabla\cdot\left[-\frac{\kappa}{\mu}\nabla p_f\right] = -\alpha_B\frac{\partial \varepsilon_{vol}}{\partial t}$$

Coupled through Biot's constitutive relations:

$$\sigma = \mathbf{C}(E,\nu)\varepsilon(\mathbf{u}) - \alpha_B p_f$$

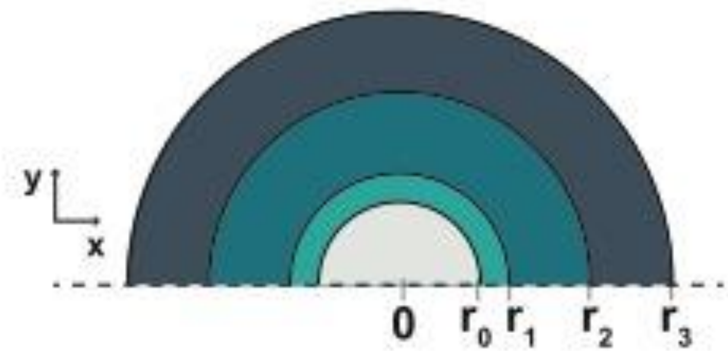
$$p_f = \frac{1}{S}(\zeta - \alpha_B\varepsilon_{vol})$$

σ : stress tensor
ε : strain tensor
ε_{vol} : volumetric strain
C : elastic tensor
E : Young modulus
ν : Poisson ratio

p_f : fluid (pore) pressure
k : permeability
μ : viscosity
α_B : Biot-Willis coefficient
S : storage coefficient
S = S(α, porosity, fluid bulk modulus, solid bulk modulus)

ζ : fluid volume per unit volume

Initial and Boundary Conditions



$$\mathbf{u_0} =$$

Initial conditions:

$$p_f = 0, \mathbf{u} = 0$$

Boundary conditions:

Solid

$$\begin{cases} \mathbf{u} = 0 & , \forall \mathbf{r} = \mathbf{r_3} \\ \mathbf{u} = \mathbf{u_0} \cdot \frac{1}{2} sin(2\pi f t - \frac{\pi}{2}) & , \forall \mathbf{r} = \mathbf{r_0} \end{cases}$$

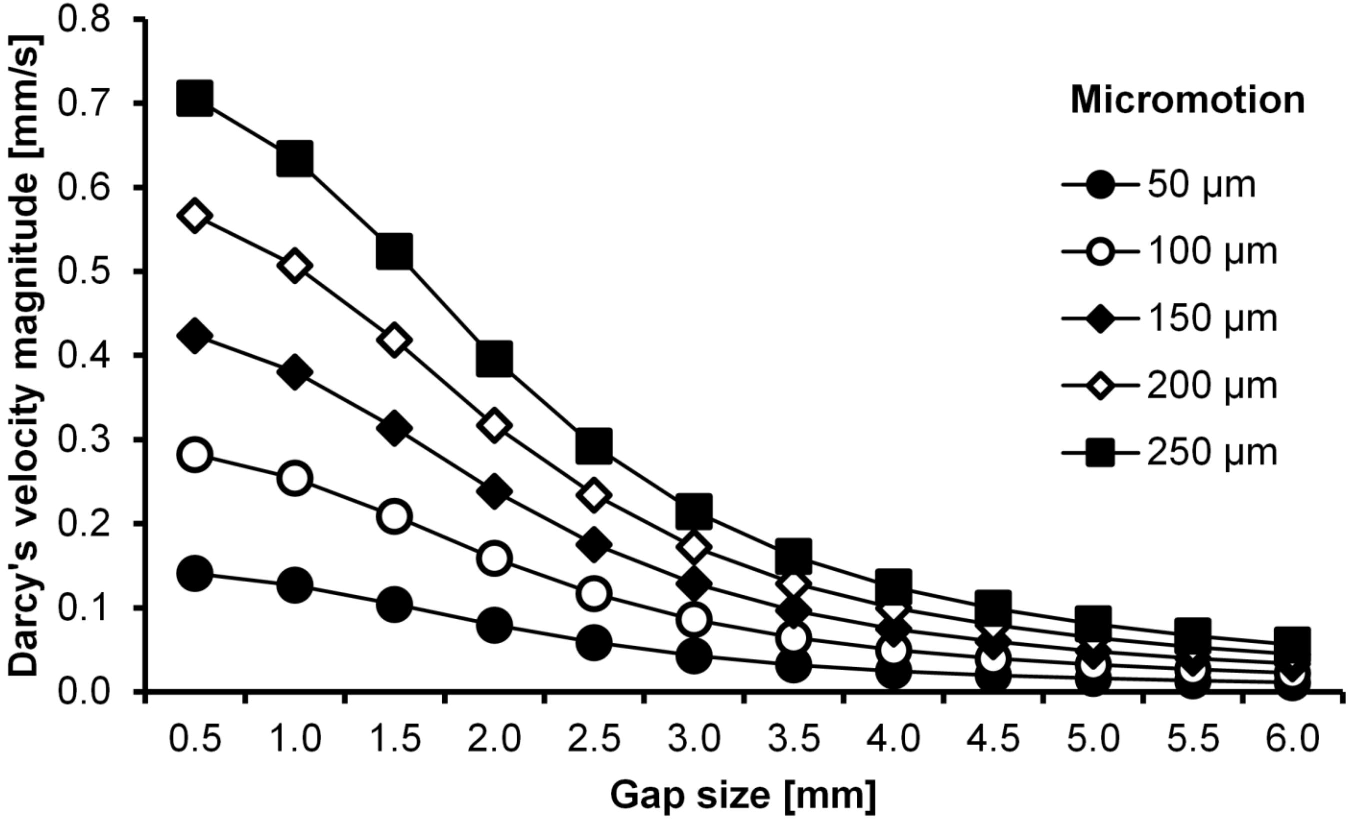
Fluid

$$\begin{cases} \mathbf{n} \cdot \nabla p_f = 0 & , \forall \mathbf{r} = \mathbf{r_0}, \mathbf{r_3} \\ p_f = 0 & , \forall \mathbf{r} = \mathbf{r_1}, \mathbf{r_2} \end{cases}$$

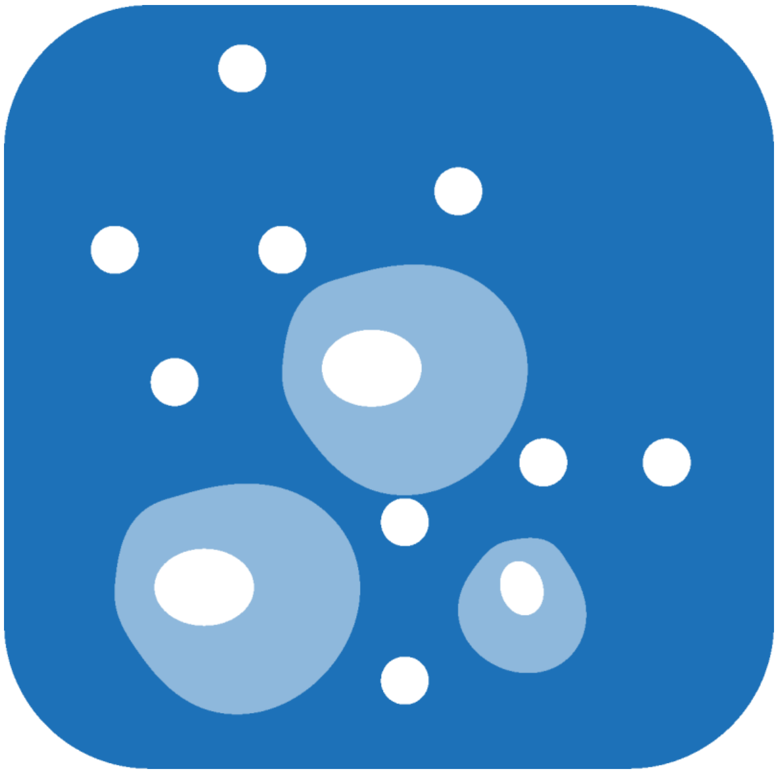
Material

Property	Material		
	Granulation Tissue	Cancellous Bone	Cortical Bone
Young's Modulus (E)	1 MPa	6 GPa	15.75 GPa
Poisson's Ratio (ν)	0.167	0.325	0.325
Porosity (ϵ_p)	0.8	0.8	0.04
Permea-bility (κ)	$1e^{-17}$ m^2	$3.7e^{-16}$ m^2	$1e^{-20}$ m^2

Results

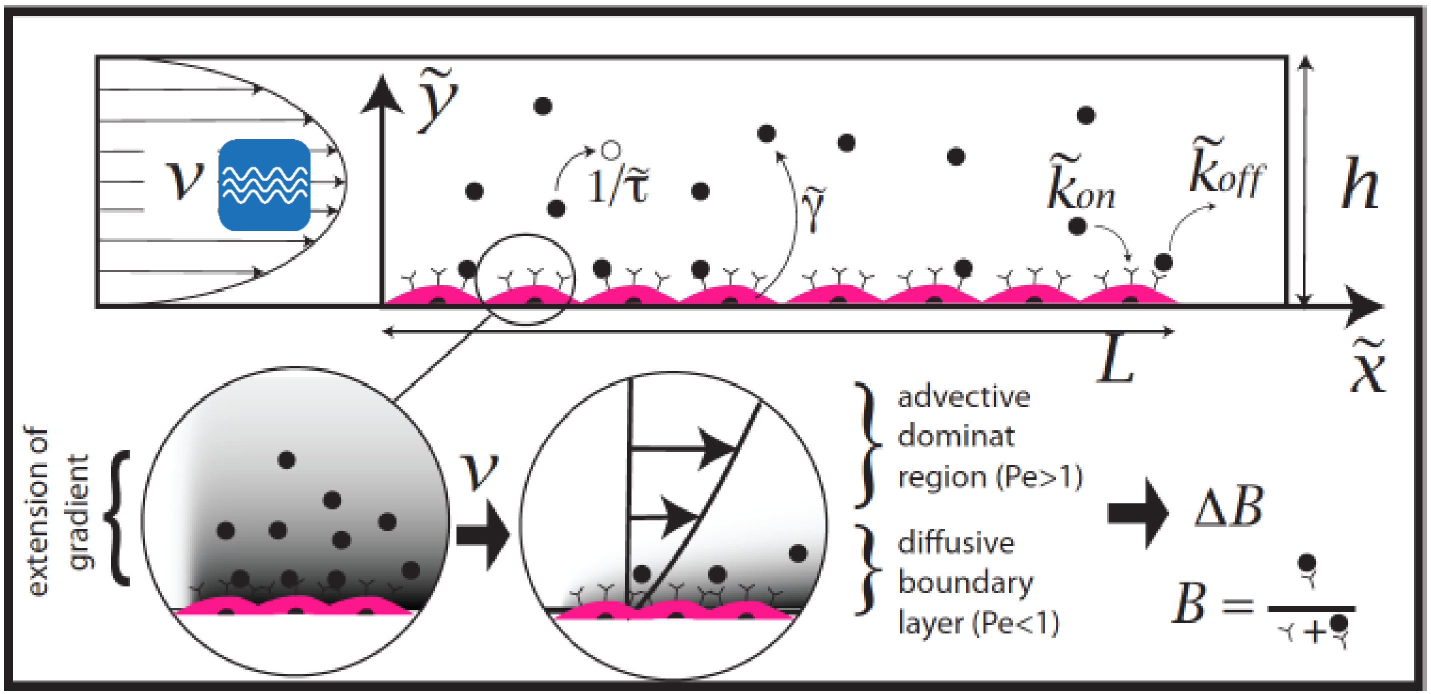


Micromotion-induced fluid flow in granulation tissue extends 1 µm/s to 700 µm/s



Study the effects of fluid on morphogens distribution and MSCs differentiation

Model



$$B = \frac{\text{surface density of } \textit{bound} \text{ receptors}}{\text{surface density of receptors}}$$

Governing Equations

Mass transport

Diffusion (D) – Advection (A) – Reaction (R) in bulk:

$$\frac{\partial}{\partial t}C = \underbrace{\nabla \cdot [D(\nabla C)]}_{D} - \underbrace{\nabla \cdot (C\,u)}_{A} - \underbrace{\frac{1}{\tau}C}_{R}$$

Binding (B) – Unbinding (UB) reaction on the wall :

$$\frac{\partial B}{\partial t} = \underbrace{k_{on}(R_t - B)C\big|_{y=0}}_{B} - \underbrace{k_{off}B}_{UB}$$

Secretion(S) of morphogens:

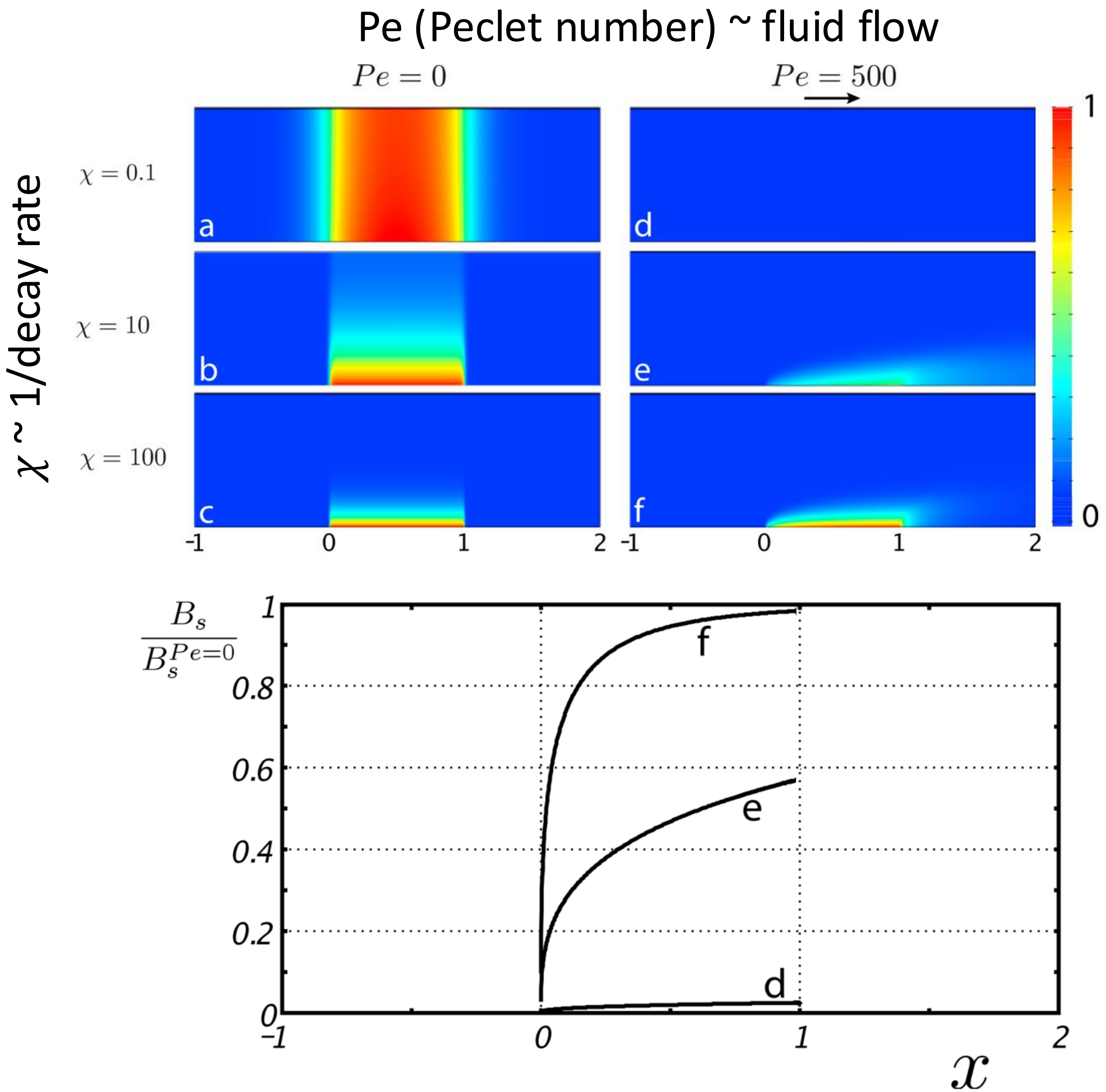
$$\underbrace{\gamma}_{S} - \frac{\partial B}{\partial t} = -D\frac{\partial}{\partial y}C\Big|_{y=0}$$

C - concentration

R_t – total number of receptors

B – number of bound receptors

Results



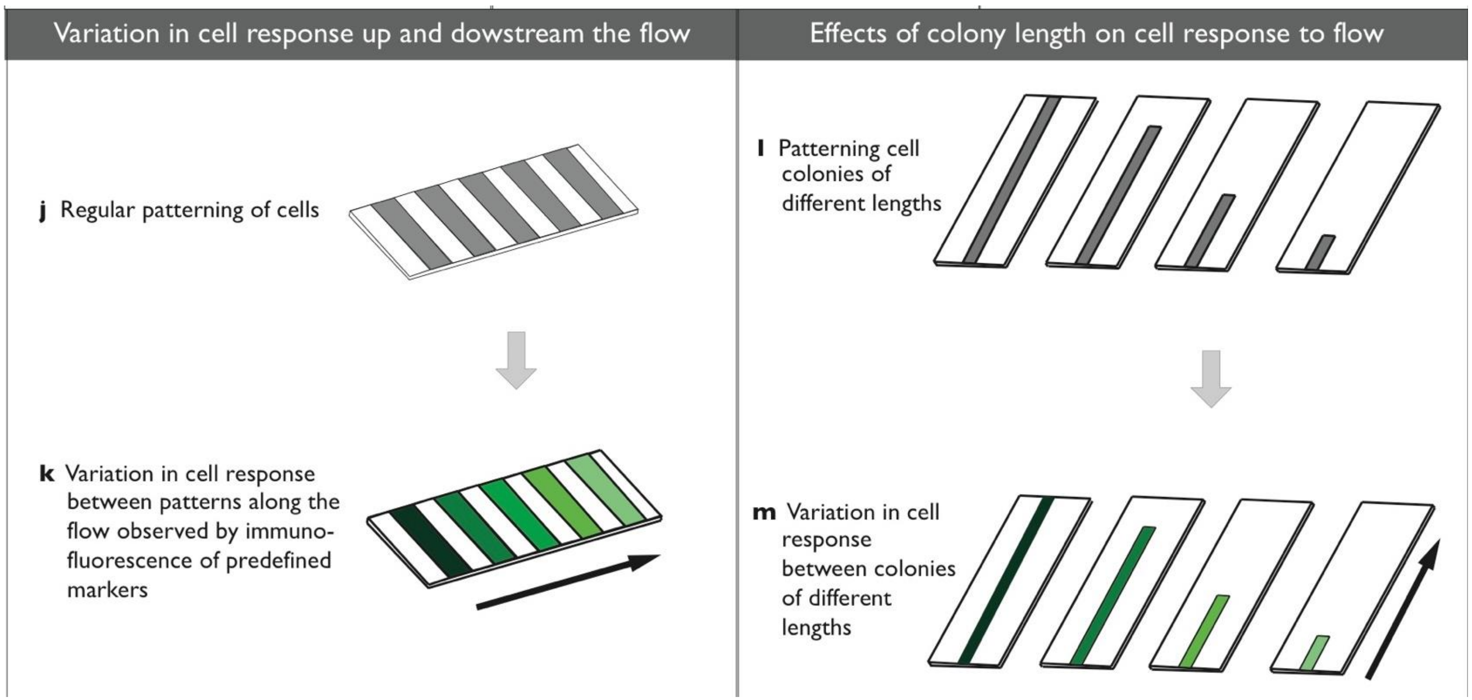
Gortchacow, Biophysical Journal, 2013

Fluid flow is strong enough to:

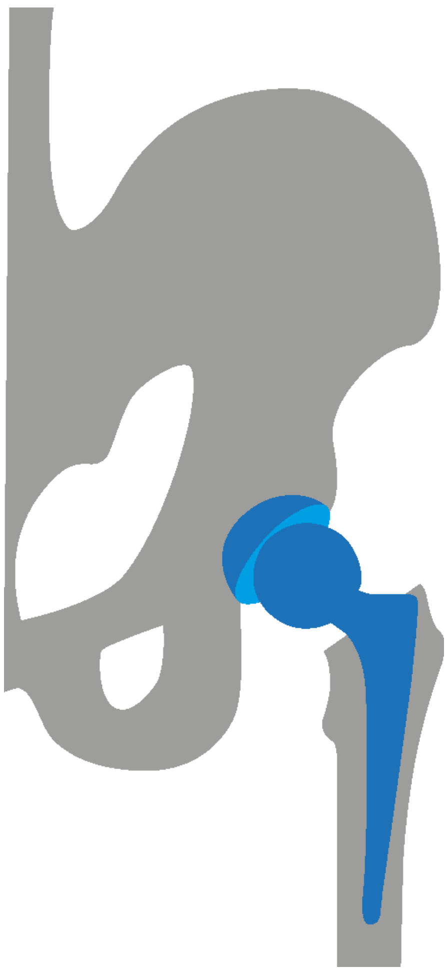
- Disturb the concentration profile of morphogens
- Change the number of bound receptors (i.e. have an effect on cell differentiation)

Experimental Validation

Microfluidics experiments

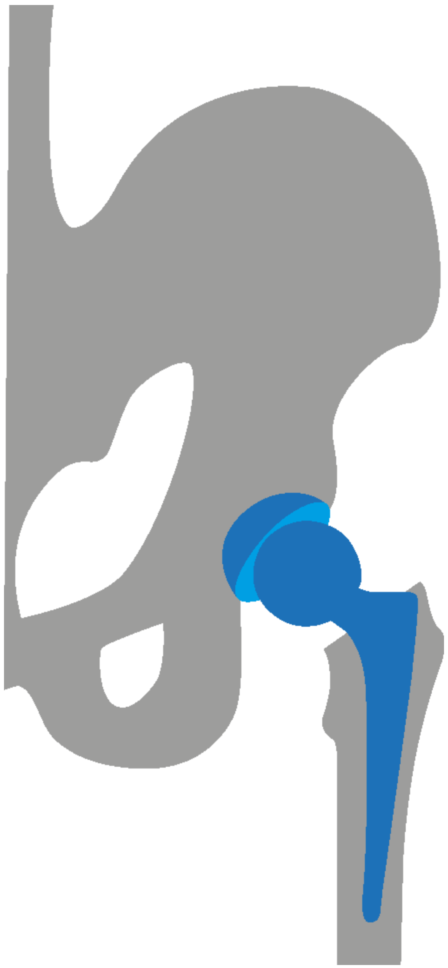


Conclusion



A better understanding of conditions promoting aseptic loosening of hip implants can lead to better implant designs or surgical techniques and benefit patients

Conclusion



Numerical modeling helps to investigate complex multi-scale hypotheses.

However, experimental validation is essential to assess the predictive capabilities of models.