Multi physics 2: fluid-solid interaction

This exercise is a continuation of the previous artificial bladder exercise. Now, we would like to predict the effect of an external compressive force on the pressure of the fluid inside the bladder, to evaluate the release of the urea by the artificial sphincter.

Starting from the model of exercise 7, we add the fluid, and its interaction with the solid spherical silicon bladder. Instead of modeling completely the inner volume with finite elements, we can more simply and efficiently use a relationship between volume and pressure, and couple it to the solid deformation of the silicon sphere. We assume an isothermal compression of the inner fluid

$$p = p_0 + k\,(V_0/V - 1)$$

where V_0 and V are the initial and actual inner volumes, and $k = -V\, dV/dp$ is bulk modulus. We assume that the liquid is water (k = 2.2 GPa), that the initial fluid pressure p_o = 1 atm, and that the external pressure is p_e = 1 atm.

Using *Global Definition/Parameters*, set

> *V0_intBladder* = V_0 = *4/3*pi*R_sph^3 (R_sph is the internal radius of the sphere)*
> *p0_intBladder* = p_0
> *bulkModulus* = k
> *p_extBladder* = p_e

Using *Model/Definitions/Components Couplings/Integration*, define the operator *Vintegration* with the inner boundary (curve) of the sphere as a source.

Using *Model/Definitions/Variables*, define on the entire model

> *V_intBladder = Vintegration(-pi*r^2*solid.nr)*

to get the actual inner volume *V*. The expression *(-pi*r*r*solid.nr)* comes from the Gauss integration formula (divergence theorem in the plane). The terms *pi*r^2* represents the contribution of Gauss integration to transform a surface integral into a line integral (v=∬2*pi*r=∮pi*r^2), and solid.nr represents the radial component of the unit vector normal to the integration curve. This term being negative along the curve, we need to add the minus sign. We only need to integrate along the inner boundary of the curve, since the product inside the other parts of the closing loop are zero.

Using *Model/Definitions/Variables*, define on the entire model

> *p_intBladder* = *p0_intBladder* + *bulkModulus* *(*V0_intBladder*/*V_intBladder* -1)

Set *p_intBladder* and *p_extBladder* as boundary condition of the internal and external surface of the sphere respectively.

Evaluate the relationship between the applied force *F* and $\triangle p = p - p_e$ in mmHg.

Which force is required to increase the internal pressure up to 200 mmHg?