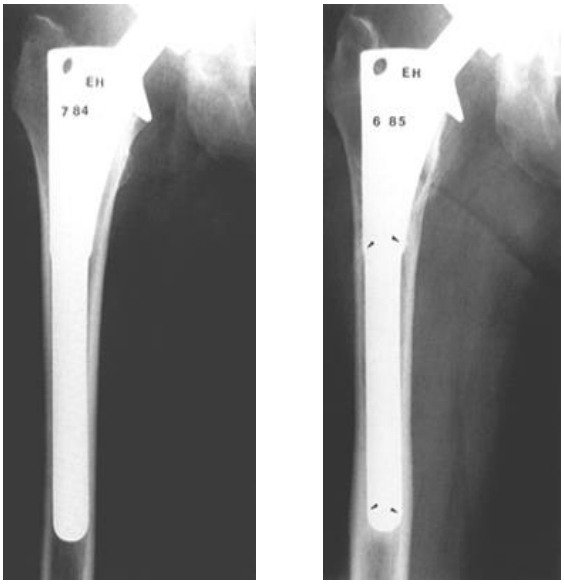
Simple physics: solid mechanics

Hip joint prostheses usually require a long metallic stem to be inserted into the femur. One year after the surgery, we can observe bone resorption in the proximal part and bone densification at the tip of the stem. This phenomenon is caused by a change of the mechanical environment (e.g. strain energy density) within bone, and is named “stress shielding”.



Post surgery After 1 year

J. D. Bobyn and C. A. Engh, Non-Cemented THA, Raven Press (1988)

Build a simplified axisymmetric (2D) femur represented by a hollow cylinder, with an external diameter of 2.8 cm, a thickness of 7 mm, and a length of 48 cm. Assume an homogeneous and elastic material with a Young’s modulus of 18 GPa, a Poisson’s ratio of 0.3, and a density of 1800 kg/m^3. Apply an axial load of 2 kN on the upper (proximal) part, corresponding to peak load during walking (2.5 bodyweight). Prescribe the displacement of the lower (distal) part to be zero in the axial direction.

Verify that only the axial (zz) component of the stress tensor (solid.sz) is non-null. Check that the stress is homogeneous (constant within bone) and consistent (stress = force/surface). Output the axial component of the strain tensor in µstrain (= 10^{-6}). Check the value is consistent (strain = stress/Young’s modulus). Compare to the ultimate (failure) strain (in compression), which is about 1500 µstrain. Compare to in-vivo measurements (see review paper Yang2011). Output the strain energy density.

Insert a titanium cylindrical stem having a length of 10 cm, with a Young’s modulus of 110 GPa, a Poisson’s ratio of 0.3, and a density of 4500 kg/m^3. Output the strain energy density. Where are zones of lower and higher energy compared to the normal case?