Protein concentration in a flow chamber

The objective of this exercise is to predict the stationary concentration of paracrine proteins within a flow chamber. The model includes a layer of cells, which release, bind and unbind proteins at the cell surface. The proteins are transported within the fluid by diffusion and advection. In addition, the proteins degrade with a decay rate of 1/tau. The flow chamber is modeled in 2D. Its dimensions are 1200 μm and 250 μm. The fluid flows along the longer dimension. The layer of cell is 400 μm, in the middle of the longer edge.

Build the geometry of the system. Use the three following "physics" to model the system. Set the initial and boundary conditions of each "physics", and the coupling between them. You may define constants in *Global Definitions → Parameters*. You should define the protein binding process at the surface (*react_surf*) in *Model → Definitions → Variable*.

Fluid Flow → Single-Phase Flow → Laminar Flow (spf)
Domain is flow chamber
Fluid is water
Initial velocity and pressure is 0.
Inlet flow rate is 3 ml/min (entrance thickness 2[cm])
Outlet pressure is 0 Pa

Chemical Species Transport → Surface Reactions (chsr)
Domain is cell layer
Dependent variable: surface concentration of bound proteins cs [mol/m^2]
Initial concentration: *cs* = 0 [mol/m^2]
Surface properties:
Density of sites: *Rt* = 1e-8 [mol/m^2] (total number of receptors on the surface)
Diffusion coefficient: *D* = 1e-8 [m^2/s]
Reaction:
Reaction rate for surface species: *react_surf = kon*(Rt-cs)*c-koff*cs,* with
kon = 30 [m^3/s/mol] (association rate),
koff = 6e-4 [1/s] (dissociation rate)
c defined below.

Chemical Species Transport → Transport of Diluted Species (chds)
Domain is flow chamber
Dependent variable: protein concentration *c*
Velocity field: velocity of laminar flow model
Coefficient of diffusion: *D* = 1e-8 [m^2/s]
Initial protein concentration: *c* = 0 [mol/m^3]
Inward flux at cell layer: *paracrine - react_surf, with*
paracrine = 8e-7[mol/m^2/s] (protein production by cells)
Reaction within fluid: *-c/tau*, with *tau* = 0.05 [s] (half -life of proteins)
Add Outflow condition to let morphogens leave the chamber.

Run a stationary study with the above flow rate, and one without flow (velocity = 0). Plot the morphogen concentration at the cell layer for these two situations.