Series 9 – Solutions

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Chapter 20: Wave Travel and Velocity

Exercise 20.1 – Solution

If Δt is the mean duration of a cardiac cycle and c_0 is the propagation speed of the pressure wave, the wavelength λ is given by:

$$\lambda = c_0 \cdot \Delta t \tag{1}$$

Using the values from the course notes we obtain:

$$\lambda = 4\ \frac{\text{m}}{\text{sec}} \cdot 0.86\ \text{sec} = 3.44\ \text{m} \tag{2}$$

If we compare the pressure wavelength to the length of the arterial system (typically ≈ 1.7 m), we observe that the pressure wave created during a cardiac cycle spans the whole arterial tree excluding the interference of a cardiac beat with the next one.

The factors that can modify the value of λ are the propagation speed and the heart rate.

Exercise 20.2 – Solution

1) At normal myogenic tone (green curve) and mean pressure p_1 = 100 mmHg (13.33 kPa) the inner diameter is:

$$D_{i,1} = 0.595\ \text{mm} \tag{1}$$

and the lumen area is:

$$A_1 = \frac{\pi D_{i,1}^2}{4} = \frac{\pi \cdot 0.595^2}{4}\ \text{mm}^2 \tag{2}$$

The tangent $\left(\frac{\Delta D_i}{\Delta p}\right)_1$ of the curve at this point is calculated using a central difference:

$$\left(\frac{\Delta D_i}{\Delta p}\right)_1 = \frac{0.6 - 0.58}{14 - 13} = 0.02\ \frac{\text{mm}}{\text{kPa}} \tag{3}$$

The area compliance $C_{A,1}$ at this point is calculated by:

$$C_{A,1} = \frac{\pi D_{i,1}}{2}\left(\frac{\Delta D_i}{\Delta p}\right)_1 = \frac{\pi \cdot 0.595}{2} 0.02 \ \frac{\text{mm}^2}{\text{kPa}} \tag{4}$$

And finally, taking the blood density ρ = 1050 kgr/m^3 the pulse wave velocity c_1 is calculated by the Newton-Young equation:

$$c_1 = \sqrt{\frac{A_1}{\rho C_{A,1}}} = \sqrt{\frac{\frac{\pi \cdot 0.595^2}{4} \ \text{mm}^2}{1050 \ \frac{\text{kgr}}{\text{m}^3} \ \frac{\pi \cdot 0.595}{2} 0.02 \ \frac{\text{mm}^2}{\text{kPa}} \ \frac{1}{1000} \ \frac{\text{kPa} \cdot \text{m} \cdot \text{sec}^2}{\text{kgr}}}} = 3.764 \ \frac{\text{m}}{\text{sec}} \tag{5}$$

2) After a strong dose of norepinephrine (still green curve) the mean pressure is raised to p_2 = 150 mmHg (20 kPa) and the inner diameter becomes:

$$D_{i,2} = 0.655 \text{ mm} \tag{6}$$

and the lumen area is:

$$A_2 = \frac{\pi D_{i,2}^2}{4} = \frac{\pi \cdot 0.655^2}{4} \ \text{mm}^2 \tag{7}$$

The tangent $\left(\frac{\Delta D_i}{\Delta p}\right)_2$ of the curve at this point is calculated using a central difference:

$$\left(\frac{\Delta D_i}{\Delta p}\right)_2 = \frac{0.665 - 0.645}{21 - 19} = 0.01 \ \frac{\text{mm}}{\text{kPa}} \tag{8}$$

The area compliance $C_{A,2}$ at this point is calculated by:

$$C_{A,2} = \frac{\pi D_{i,2}}{2}\left(\frac{\Delta D_i}{\Delta p}\right)_2 = \frac{\pi \cdot 0.655}{2} 0.01 \ \frac{\text{mm}^2}{\text{kPa}} \tag{9}$$

And finally, taking the blood density ρ = 1050 kgr/m^3 the new value of the pulse wave velocity c_2 is calculated by the Newton-Young equation:

$$c_2 = \sqrt{\frac{A_2}{\rho C_{A,2}}} = \sqrt{\frac{\frac{\pi \cdot 0.655^2}{4} \ \text{mm}^2}{1050 \ \frac{\text{kgr}}{\text{m}^3} \ \frac{\pi \cdot 0.655}{2} 0.01 \ \frac{\text{mm}^2}{\text{kPa}} \ \frac{1}{1000} \ \frac{\text{kPa} \cdot \text{m} \cdot \text{sec}^2}{\text{kgr}}}} = 5.585 \ \frac{\text{m}}{\text{sec}} \tag{10}$$

So we observe that an increase in pressure due to a strong dose of norepinephrine decreases the compliance of the arterial wall ($C_{A,2} < C_{A,1}$) and this in turn results in a higher pulse wave velocity ($c_2 > c_1$).

3) Following the increase in pressure to p_2 = 150 mmHg (20 kPa) the artery develops very strong myogenic tone and reaches maximal contraction (red curve). The inner diameter becomes:

$$D_{i,3} = 0.630 \text{ mm} \tag{11}$$

and the lumen area is:

$$A_3 = \frac{\pi D_{i,3}^2}{4} = \frac{\pi \cdot 0.630^2}{4} \text{ mm}^2 \tag{12}$$

The tangent $\left(\frac{\Delta D_i}{\Delta p}\right)_3$ of the curve at this point is calculated using a central difference:

$$\left(\frac{\Delta D_i}{\Delta p}\right)_3 = \frac{0.640 - 0.605}{21 - 19} = 0.0175 \ \frac{\text{mm}}{\text{kPa}} \tag{13}$$

The area compliance $C_{A,3}$ at this point is calculated by:

$$C_{A,3} = \frac{\pi D_{i,3}}{2}\left(\frac{\Delta D_i}{\Delta p}\right)_3 = \frac{\pi \cdot 0.630}{2} 0.0175 \ \frac{\text{mm}^2}{\text{kPa}} \tag{14}$$

And finally, taking the blood density ρ = 1050 kgr/m^3 the new value of the pulse wave velocity c_3 is calculated by the Newton-Young equation:

$$c_3 = \sqrt{\frac{A_3}{\rho C_{A,3}}} = \sqrt{\frac{\frac{\pi \cdot 0.630^2}{4} \text{ mm}^2}{1050 \ \frac{\text{kgr}}{\text{m}^3} \ \frac{\pi \cdot 0.630}{2} 0.0175 \ \frac{\text{mm}^2}{\text{kPa}} \ \frac{1}{1000} \ \frac{\text{kPa} \cdot \text{m} \cdot \text{sec}^2}{\text{kgr}}}} = 4.140 \ \frac{\text{m}}{\text{sec}} \tag{15}$$

So we observe that an increase in myogenic tone under constant pressure increases the compliance of the arterial wall ($C_{A,3} > C_{A,2}$) and this in turn results in a lower pulse wave velocity ($c_3 < c_2$).

4a) The incremental elastic modulus $E_{inc,1}$ under normal myogenic tone (green curve) and mean pressure p_1 = 100 mmHg (13.33 kPa) is calculated by the Moens-Korteweg equation:

$$E_{inc,1} = \frac{c_1^2 \rho}{\frac{h_1}{D_{i,1}}} = \frac{3.764^2 \ \frac{\text{m}^2}{\text{sec}^2} \ 1050 \ \frac{\text{kgr}}{\text{m}^3} \ \frac{1}{1000} \ \frac{\text{kPa} \cdot \text{m} \cdot \text{sec}^2}{\text{kgr}}}{\frac{1}{10}} = 148.761 \text{ kPa} \tag{16}$$

where h_1 is the thickness of the arterial wall under these conditions. The outer diameter is:

$$D_{o,1} = D_{i,1} + 2h_1 = D_{i,1} + 2D_{i,1}/10 = 0.595 + 2\cdot 0.595/10 = 0.714 \text{ mm} \tag{17}$$

4b) We employ the incompressibility condition (conservation of cross-sectional area) to calculate the outer diameter $D_{o,2}$ under normal myogenic tone (green curve) and mean pressure p_2 = 150 mmHg (20 kPa):

$$\frac{\pi}{4}\left(D_{o,2}^2 - D_{i,2}^2\right) = \frac{\pi}{4}\left(D_{o,1}^2 - D_{i,1}^2\right) \Rightarrow$$
$$\Rightarrow D_{o,2} = \sqrt{D_{i,2}^2 + D_{o,1}^2 - D_{i,1}^2} = \sqrt{0.655^2 + 0.714^2 - 0.595^2} = 0.765 \text{ mm} \tag{18}$$

The ratio of thickness to diameter then becomes:

$$\frac{h_2}{D_{i,2}} = \frac{D_{o,2} - D_{i,2}}{2D_{i,2}} = \frac{0.765 - 0.655}{2\cdot 0.655} = 0.084 \tag{19}$$

The new value of incremental elastic modulus $E_{inc,2}$ is calculated by the Moens-Korteweg equation:

$$E_{inc,2} = \frac{c_2^2 \rho}{\dfrac{h_2}{D_{i,2}}} = \frac{5.585^2 \dfrac{\text{m}^2}{\text{sec}^2}\, 1050 \dfrac{\text{kgr}}{\text{m}^3} \dfrac{1}{1000} \dfrac{\text{kPa}\cdot\text{m}\cdot\text{sec}^2}{\text{kgr}}}{0.084} = 389.903 \text{ kPa} \tag{20}$$

4c) We employ the incompressibility condition (conservation of cross-sectional area) to calculate the outer diameter $D_{o,3}$ under maximal contraction (red curve) and mean pressure p_2 = 150 mmHg (20 kPa):

$$\frac{\pi}{4}\left(D_{o,3}^2 - D_{i,3}^2\right) = \frac{\pi}{4}\left(D_{o,1}^2 - D_{i,1}^2\right) \Rightarrow$$
$$\Rightarrow D_{o,3} = \sqrt{D_{i,3}^2 + D_{o,1}^2 - D_{i,1}^2} = \sqrt{0.630^2 + 0.714^2 - 0.595^2} = 0.743 \text{ mm} \tag{21}$$

The ratio of thickness to diameter then becomes:

$$\frac{h_3}{D_{i,3}} = \frac{D_{o,3} - D_{i,3}}{2D_{i,3}} = \frac{0.743 - 0.630}{2\cdot 0.630} = 0.090 \tag{22}$$

The new value of incremental elastic modulus $E_{inc,3}$ is calculated by the Moens-Korteweg equation:

$$E_{inc,3} = \frac{c_3^2 \rho}{\frac{h_3}{D_{i,3}}} = \frac{4.140^2 \ \frac{\text{m}^2}{\text{sec}^2} \ 1050 \ \frac{\text{kgr}}{\text{m}^3} \ \frac{1}{1000} \ \frac{\text{kPa} \cdot \text{m} \cdot \text{sec}^2}{\text{kgr}}}{0.090} = 199.962 \ \text{kPa} \qquad (23)$$