Series 6 (25 March 2025)

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Chapter 10 : Elasticity

Exercise 10.1

Study the static behavior of a thin-walled tube made of elastic, homogeneous, incompressible and isotropic material. The tube has a constant length and is subjected to an internal pressure.

1. Supposing a hookean solid:
   a. Show that the pressure-radius relation, $p(r)$, is non-linear.
   b. Represent graphically the pressure radius relation, $p(r)$.
2. Propose a mathematical expression for the elastic modulus $E(r)$ that describes the non-hookean behavior of an arterial wall.

Exercise 10.2

Poiseuille's law is derived for a straight rigid tube. In an elastic tube, the local diameter is a function of the local pressure. Under flow, the continuous drop in pressure along the artery length will lead to a progressively smaller diameter. Determine the flow-pressure relation, $Q(p)$, in a blood vessel of constant length and with linearly elastic (Hookean) properties.
Hypotheses: steady flow; thin wall; homogeneous; constant elastic modulus, E.
Indications: Suppose a Poiseuille flow in which the radius depends on the axial position z, i.e. $r = r(z)$, then use the law of Laplace and the circumferential strain ($\varepsilon_{\vartheta\vartheta} = \sigma_{\vartheta\vartheta}/E$) to determine the relation $r(z,p)$.

Exercise 10.3

Determine the influence of vasomotion on the vascular resistance, R_v, comparing the fluidic resistance in a tube of fixed diameter ($d = D$) with that in a tube of diameter $d = D + \delta\sin\Theta$, where $\Theta = 2\pi t/T$ where T is the period of vasomotion.
Suppose a Poiseuille flow to graphically depict the ratio of resistances, κ, and the ratio of flows, q:

$$\kappa(\Theta) = \frac{R_v^{oscill}}{R_v^{rigid}}, \quad q(\Theta) = \frac{Q^{oscill}}{Q^{rigid}}$$

as a function of Θ and calculate the mean ratio, $\bar{\kappa}$, over the period of vasomotion; discuss the result as a function of the ratio $\alpha = \delta/D$.