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Chapter 8 : Oscillatory Flow Theory

Exercise 8.1

A fluid (μ = 0.004 Ns/m^2, ρ = 1000 kg/m^3) flows in a rigid tube having an inner radius of 2 mm. The flow consists of a steady component with a harmonically oscillating flow (frequency = 16 rad/s) superimposed. The pressure gradient (in N/m^3) is:

$$\frac{p_1 - p_2}{L} = 5 \cdot 10^3 + 10^3 \cdot \cos(\omega t)$$

a) Make use of the Womersley solution to determine the corresponding expression for the instantaneous flow.
b) What is the Reynolds number, based on the velocity of the steady flow component?

Exercise 8.2

In a blood vessel having an inner diameter of 0.5 mm the pressure drop per unit length, $\Delta p/l$, is measured and found to be a periodic function of time, f(t). The highest frequency of any significance in the Fourier series describing f(t) is 4 Hz. Assuming that blood (μ = 0.004 Ns/m^2, ρ = 1050 kg/m^3) can be treated as a continuum (without wall effects) determine an approximate, but accurate, relationship between the instantaneous flow, $Q(t)$, and the pressure gradient, f(t).

Exercise 8.3

Originally, Womersley developed his theory for deriving blood flow (Q) from pressure gradient measurements (-dP/dz). Pressure gradients were measured on a double sensor pressure catheter, the sensors being located on the distal end of the catheter and with a few cm apart, so that the difference in their signal divided by the distance between sensors would give the instantaneous pressure gradient. Please note that in the 1950's invasive pressure measurements were easy and flow measurements were impossible, as ultrasonic Doppler or MRI were not yet developed. Today, of course, the situation is reversed, as it is much easier to measure flow noninvasively, than pressure. The pressure gradient waveform in the human thoracic aorta is shown in the figure below and is also given in the file "Pressure Gradient CC.txt" in Moodle, where the first column is time, the second column is the pressure gradient (-dP/dz) and the third column is the flow (ml/s). The data are for one complete heart cycle. Use Mathematica® or MatLab® or do your own program (more tedious) to perform the following:

1) Do Fourier analysis of the pressure gradient pulse. Show the modulus of each harmonic as a function of frequency. How many harmonics are enough to include in your analysis? Hint: neglect higher harmonics with low modulus.

2) Use Womersley's theory to derive velocity as a function of radius and time. Produce an animated plot for visualizing velocity as a function of time. Plot the velocity profile for every 30° of the heart cycle.
3) Integrate the velocity over the cross-section to obtain the flow rate as a function of time, Q(t). Compare the flowrate waveform to that of the measured flow. What do you observe?

Parameters:

Common carotid radius:	5.5 mm
Blood density:	1060 kg/m^3
Blood viscosity:	0.004 N s /m^2