Series 3 (04 March 2025)

Prof: Nikos Stergiopulos
TA: Sokratis Anagnostopoulos

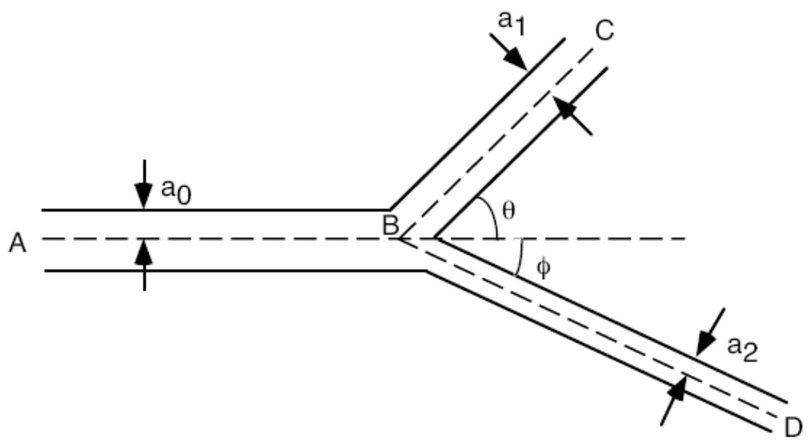
Chapter 2: Law of Poiseuille

Exercise 2.3

Murray (1926) proposed a «cost» function, the minimum of which describes an optimum geometry of a blood vessel. The latter is modeled as a rigid tube of radius a, the length L and flow Q of which are imposed. The cost function P, expressed in [W], is obtained considering the work provided to the blood as well as the metabolic energy absorbed by the wall:

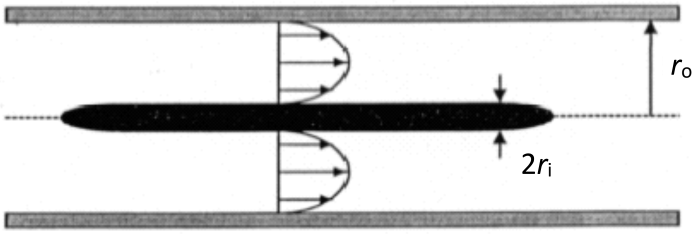
$$P = Q\Delta p + k\pi a^2 L$$, where p = pressure and k = constant

1) Determine the optimum relation between radius and flow when minimizing the cost function of a Poiseuille flow.
2) If the cost functions of different segments of the vascular tree are additive, determine the optimum geometry at point B, i.e., the angles θ and ϕ, of the bifurcation shown below, wherein the flows Q_0, Q_1 and Q_2 and the points A, C and D are fixed.
3) Supposing bifurcations with $a_1 = a_2$ (i.e., $\theta = \phi$), determine the number of successive bifurcations to pass from the aorta (a_0 = 1.3 cm) to the capillaries ($a = 5\cdot10^{-4}$ cm). Deduct the total number of capillaries and compare it with physiological data.



Exercise 2.4

Shear stress is considered to be an important biological stimulus for the endothelial cells. Under augmented shear stress the proliferation of the smooth muscle cells (which could potentially provoke a restenosis after dilation caused by angioplasty) is strongly reduced. An efficient way to locally increase the shear stress is to place in the artery a flow deflector which has a cylindrical shape of radius r_i. Hypothesizing a developed flow, calculate the velocity profile and the shear stress as a function of the flow Q, the viscosity μ and the radii r_i and r_o.



r_o
$2r_i$