Series 2 (25 February 2025)

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Chapter 2: Law of Poiseuille

Exercise 2.1

It has been suggested that a power law relationship

$$\tau = -b\left(\frac{\partial u}{\partial r}\right)^n$$

be used to characterize the relationship between the shear stress and velocity gradient for blood. The quantity *b* is a constant and the exponent *n* is an odd integer. Use this relationship and derive the corresponding velocity distribution for flow in a tube. Use all assumptions made in deriving Poiseuille's Law (except for shear stress relationship). Plot several velocity distributions (u/u_{max} vs r/R) for n = 1, 3, 5, etc. to show how the profile changes with *n*.

Exercise 2.2

The law of Poiseuille developed in class applies to Newtonian fluids. Blood, however, is a non- Newtonian fluid. Blood behaves like a solid when the shear rate $\gamma \rightarrow 0$. In this case, the relationship between shear stress and shear rate is given by the Casson equation:

$$\sqrt{\tau} = \sqrt{\tau_y} + \sqrt{\mu\gamma}$$

Where τ is the shear stress, τ_y is the yield stress.

Considering a rigid arterial wall determine:

a) the shear stress as a function of the pressure gradient for flow in a tube.
b) the velocity profile in the artery.
   Hint: Starting from Casson's equation and using the expression $\gamma = -\frac{\partial u}{\partial r}$, solve for the velocity profile for the fluid domain. Find the boundary between the fluid and "solid" domain, based on the yield stress.
c) the velocity near the axis of the vessel.
d) the flow in comparison with Poiseuille's relation.

Chapter 6: Resistance

Exercise 6.1

Supposing a Poiseuille blood flow along the whole arterial and venous circulation, determine the evolution of the blood pressure starting from the right heart ventricle, wherein blood pressure is approximately equal to 6 mmHg.

Given:

$$\frac{R_{arteries}}{R_{total}} = 0.19 \quad , \quad \frac{R_{arterioles}}{R_{total}} = 0.47 \quad , \quad \frac{R_{capillaries}}{R_{total}} = 0.27$$

$$R_{total} = R_{arteries} + R_{arterioles} + R_{capillaries} + R_{veins}$$